

THE AMERICAN MATHEMATICAL MONTHLY	MAA
Placing the Book and Posing Problems: Adventures with the Monthly's Editor	287
Book Reviews	
Is the Number of n -Integers Uniquely Determined by the Multiset of Its Divisors?	400
John A. Davis	
A Graphical Link Between Integer Partitions and Euler's Pentagonal Theorem	408
Michael J. Heule	
A Combinatorial Theorem for Elements in the Complex Plane	430
John J. and Peter J.	
The Mathematical Team Spirit: Welcome to the L.A. Math Olympiad	437
Thomas S. Lowther	
NOTES	
Newton's Theorem for Functions and Limits for Real Sequences	449
Carl H. and Peter S. Lowther	
A Group Characterization of Subgroups of the Free 2- and 3-Generated Groups	458
G. A. and Peter S. Lowther	
A Group Characterization of Subgroups of the Free 2- and 3-Generated Groups	459
Michael J. Heule	
A Group Characterization of Subgroups of the Free 2- and 3-Generated Groups	464
Michael J. Heule	
PROBLEMS AND SOLUTIONS	
PROBLEM 1000	
Problem and Solution: The Two and Underdog Story of a Problem in the History of the Monthly	477
James D. and	
PROBLEM 1001	
Problem and Solution: The Two and Underdog Story of a Problem in the History of the Monthly	480
James D. and	

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k th Power of a Partial Sum

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kth Power of a Partial Sum

Recently the following result appeared [1, Theorem 2.1].

Theorem 1. For any finite sequence of positive numbers $(a_j)_{j=1}^n$ whose partial sums are $(S_j)_{j=1}^n$ we have $\sum_{j=1}^n (a_j^2 + 2a_j S_{j-1}) = S_n^2$.

Here we prove the following extension of the previous result.

Theorem 2. For any finite sequence of positive numbers $(a_j)_{j=1}^n$ whose partial sums are $(S_j)_{j=1}^n$ and for all integers $k \geq 1$ we have

$$\sum_{j=1}^n \sum_{m=1}^k \binom{k}{m} S_{j-1}^{k-m} a_j^m = S_n^k.$$

Proof. The proof follows by telescoping. Since $S_j = S_{j-1} + a_j$, we have

$$S_j^k - S_{j-1}^k = (S_{j-1} + a_j)^k - S_{j-1}^k = \sum_{m=1}^k \binom{k}{m} S_{j-1}^{k-m} a_j^m.$$

Since $S_0 = 0$, we have $\sum_{j=1}^n \sum_{m=1}^k \binom{k}{m} S_{j-1}^{k-m} a_j^m = \sum_{j=1}^n (S_j^k - S_{j-1}^k) = S_n^k$. ■

Corollary. For $k = 3$ we have $\sum_{j=1}^n (a_j^3 + 3a_j^2 S_{j-1} + 3a_j S_{j-1}^2) = S_n^3$.

Example. Let $a_j = F_{2j-1}$. It is well known that $S_n = \sum_{j=1}^n F_{2j-1} = F_{2n}$. It follows that $a_j^3 + 3a_j^2 S_{j-1} + 3a_j S_{j-1}^2 = F_{2j-1}^3 + 3F_{2j-1} F_{2j-2} F_{2j}$, which implies

$$\sum_{j=1}^n (F_{2j-1}^3 + 3F_{2j-1} F_{2j-2} F_{2j}) = F_{2n}^3.$$

Many other identities may be found and proved using Theorem 2.

REFERENCE

- [1] Treeby, D. (2016). Further physical derivations of Fibonacci summations. *Fibonacci Quart.* 54(4): 327–334.

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