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Anisotropic Filtering with Nonlinear Structure Tensors

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ABSTRACT

We present an anisotropic filtering scheme which uses a nonlinear version of the local structure tensor to dynamically adapt the shape of the neighborhood used to perform the estimation. In this way, only the samples along the orthogonal direction to that of maximum signal variation are chosen to estimate the value at the current position, which helps to better preserve boundaries and structure information. This idea sets the basis of an anisotropic filtering framework which can be applied for different kinds of linear filters, such as Wiener or LMMSE, among others. In this paper, we describe the underlying idea using anisotropic gaussian filtering which allows us, at the same time, to study the influence of nonlinear structure tensors in filtering schemes, as we compare the performance to that obtained with classical definitions of the structure tensor.

Keywords: Anisotropic Filtering, Local Structure Tensor, Nonlinear Structure Tensor, Gaussian Smoothing, Adaptive Neighborhood

1. INTRODUCTION

The main goal of anisotropic filtering schemes is to reduce noise at the same time that data structure is neither delocalized nor blurred. This idea, which seems very simple at a first glance, requires a lot of care when trying to put it into practice. Literature shows different approaches based on linear filters,[1] PDEs[2] or more sophisticated methods.[3] Nevertheless, all of them are inspired on a common idea: to penalize smoothing along the direction of maximum signal variation while it is favored in the orthogonal one.

In this paper, we present a filtering algorithm based on the use of nonlinear structure tensor[4] to estimate the signal value at a given point using the samples in a neighborhood adapted to the image features, which avoids to mix information from different areas. For our purposes, the structure tensor is seen as a n-dimensional matrix, so in this paper, both terms (matrix and tensor) are used as synonymous. In addition, we propose the use of nonlinear structure tensors to perform that task, since these tensors take the advantage that delocalization is reduced and, hence, local structure is better preserved.

Section 2 presents the definition of nonlinear structure tensor and analyzes some of its properties. For our purposes, the structure tensor is seen as a n-dimensional matrix, so in this paper, both terms (matrix and tensor) are used as synonymous. Then, we present our filtering scheme in section 3. This algorithm can be easily applied to any kind of linear filter as Wiener, LMMSE or, as it is done in this paper, anisotropic gaussian filtering. Finally, we present in section 4 some encouraging results obtained with our approach in comparison to the use of the classical definition of the local structure tensor.

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2. LOCAL STRUCTURE TENSOR

The Local Structure Tensor (LST) was initially proposed by Bigün[5] and also by Förstner[6] and nowadays it has become a basic tool for image processing. This tensor (also called second order moment) is a matrix that codifies the structure information in the neighborhood of a point, that is, signal variation orientation and signal variation quantification in both directions, the one of maximum signal variation and in the orthogonal one.

Several methods have been proposed to estimate the local structure tensor. On one hand, Knutsson[7] proposed a method using a set of quadrature filters defined in the Fourier domain and oriented along predefined directions. On the other hand, Bigün[8] estimate it using the outer product of the gradient and convolving the result with a gaussian kernel, as it will be explained in section 2.1. Both approaches provide equivalent results for the estimated tensor, although each method has some advantages and disadvantages with respect to the other. In this paper, we use the method proposed by Bigün which is easily implemented and naturally leads to the definition of nonlinear structure tensors proposed by Brox[4] and presented in section 2.2.

2.1. Local Structure Tensor Estimation

Let $\Omega \subset \mathcal{R}^m$ denote the m-dimensional image domain and let $I(\mathbf{x}) : \Omega \rightarrow \mathcal{R}$ denote our grayscale image. The local structure tensor $\mathbf{T}$ at location $\mathbf{x_0}$, estimated as proposed by Bigün,[8] is calculated using the outer product of the image gradient convolved with a gaussian kernel $G_\sigma(\mathbf{x})$ with standard deviation σ, as indicates the following equation:

$$\mathbf{T}(\mathbf{x}) = G_\sigma(\mathbf{x}) * ((\nabla I(\mathbf{x}))(\nabla I(\mathbf{x}))^T) \tag{1}$$

The outer product of the image gradient provides an initial matrix with the same information of the gradient, since it is not full rank. However, it takes the advantage that it can be smoothed without cancellation effects, as it happens at both sides of a thin line, where the gradients take opposite signs.[9] In this way, gaussian smoothing not only reduces the noise level in the matrix field, but also introduces spatial coherence through the scale factor σ, which determines the size of the convolution kernel and, hence, the size of the neighborhood whose structure we want to quantify. Thus, after smoothing the no-full rank matrices, they become full rank and an analysis of their eigenvalues and eigenvectors provide the desired information about local structure, as presented in section 2.3.

Nevertheless, the scale factor σ has to be chosen carefully, since a blurring effect and structure delocalization appear. These effects are typical for isotropic linear filters and they become more important as σ is greater. A proposed approach to solve these effect was proposed by Brox[4] through nonlinear smoothing methods for matrix data.

2.2. Nonlinear Structure Tensors

Koenderink[10] showed that Gaussian smoothing can be modeled by the heat diffusion partial differential equation (PDE) shown Eq. 2, where the amount of smoothing of the image is determined by the diffusion time t which is related to the scale-space parameter σ through the following expression $t = \frac{\sigma^2}{2}$.

$$\partial_t u(\mathbf{x}) = div(\nabla u(\mathbf{x})), \quad u(\mathbf{x}, t = 0) = I(\mathbf{x}) \tag{2}$$

In this way, an equivalent formulation for the local structure tensor in Eq 1 can be achieved with a generalized heat equation for matrices.[11] Briefly, this generalization applies the heat equation 2 to the components t_{ij}, $i, j = 1 \ldots m$ of the outer product of the gradients $\mathbf{T}_0 = (\nabla I(\mathbf{x}))(\nabla I(\mathbf{x}))^T$:

$$\partial_t t_{ij}(\mathbf{x}) = div(\nabla t_{ij}(\mathbf{x})) \tag{3}$$

This formulation naturally leads to the definition of nonlinear structure tensors,[4] which are founded on a nonlinear heat diffusion equation.[2] The aim of nonlinear diffusion is to better preserve boundaries and to reduce structure delocalization, which are the main drawbacks of gaussian smoothing. For this purpose, a diffusivity function $g(|\nabla u(\mathbf{x})|)$ is introduced in Eq. 2 to correlate the amount of smoothing with the image gradient magnitude, as it shows Eq. 4. In this way, when the gradient is high (that means the presence of a boundary), the diffusivity must be low to avoid smoothing and vice versa.

$$\partial_t u(\mathbf{x}) = div(g(|\nabla u(\mathbf{x})|)\nabla u(\mathbf{x})), \quad u(\mathbf{x}, t=0) = I(\mathbf{x}) \tag{4}$$

Several diffusivity functions have been proposed in the literature[12] each of them with different properties. In our case, we will use the diffusivity function expressed in Eq. 5, also called *total variation flow*[13,14]:

$$g(|\nabla u(\mathbf{x})|) = \frac{1}{\sqrt{\epsilon^2 + |\nabla u(\mathbf{x})|^2}} \tag{5}$$

where the parameter ϵ is a small constant that avoids singularities and allows differentiability.

Hence, as in the definition of the linear structure tensor in Eq. 3, the nonlinear counterpart applies the nonlinear heat equation 4 to the components t_{ij} of the matrix $\mathbf{T}_0$:

$$\partial_t t_{ij}(\mathbf{x}) = div(g(\sum_{i,j=1}^{m} |\nabla t_{ij}(\mathbf{x})|)\nabla t_{ij}(\mathbf{x})) \tag{6}$$

It is remarkable the fact that the diffusivity function takes into account the gradient magnitude of the different components of the tensor, so a boundary in one of them will reduce the amount of smoothing in all.

2.3. Interpretation

The local structure tensor provides information about the complexity of the signal in a neighborhood.[15] Through an analysis of eigenvalues $\lambda_1 \geq \ldots \geq \lambda_m$ and eigenvectors $\mathbf{e_1}, \ldots, \mathbf{e_m}$ we can distinguish different cases in 3D $m = 3$ (an analogous classification can be done in 2D):

1. Planar case ($\lambda_1 \gg \lambda_2 \simeq \lambda_3 \simeq 0$): There is only one main direction of signal variation. This neighborhood is approximately a planar structure whose normal vector is given by $\mathbf{e_1}$.
2. Linear case ($\lambda_1 \simeq \lambda_2 \gg \lambda_3 \simeq 0$): In this case, there are two main directions of signal variation which yields a line-like neighborhood oriented along $\mathbf{e_3}$, as the edges of a cube.
3. Point Structure case ($\lambda_1 \simeq \lambda_2 \simeq \lambda_3 \gg 0$): There is no preferred orientation of signal variation, which represents a corner or a junction in 3D.
4. Homogeneous case ($\lambda_1 \simeq \lambda_2 \simeq \lambda_3 \simeq 0$): In this case, there is neither a preferred orientation of signal variation nor significant variation, which corresponds to homogeneous regions.

In this way, the structure tensor is an interesting tool for adaptive and anisotropic processing systems and algorithms. The next section presents a new algorithm to denoise images using the structure tensor to drive an anisotropic diffusion scheme.

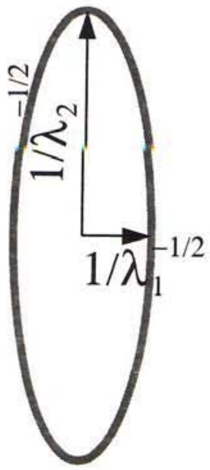

Figure 1. Ellipse represented by the quadratic form $\mathbf{xTx}^T = 1$.

3. ANISOTROPIC GAUSSIAN FILTERING

Anisotropic Gaussian filtering is a generalization of the isotropic case which reduces the problem of structure delocalization and blurring. The idea behind this generalization is to adapt the gaussian kernel variance σ in order to reduce smoothing along the direction of maximum signal variation while it is increased in the orthogonal direction. This can be achieved introducing a symmetric positive definite matrix $\mathbf{D}(\mathbf{x})$ as shows the following equation:

$$G(\mathbf{x}) = \frac{1}{4\pi\sqrt{det(\mathbf{D}(\mathbf{x}))}} \exp(-\frac{\mathbf{x}\mathbf{D}^{-1}(\mathbf{x})\mathbf{x}^T}{4}) \tag{7}$$

For a fixed matrix $\mathbf{D}(\mathbf{x}) = \mathbf{D}$, calculating the convolution $I_G(\mathbf{x}) = G(\mathbf{x}) * I(\mathbf{x})$ is equivalent to solve the linear anisotropic diffusion problem in Eq. 8 with $\mathbf{D}$ as diffusion tensor.[12] However, if the diffusion matrix is not constant, anisotropic Gaussian smoothing is no longer equivalent to an anisotropic diffusion problem, although a sophisticated relationship can be established.[16]

$$\partial_t u(\mathbf{x}) = div(\mathbf{D}\nabla u(\mathbf{x})), \quad u(\mathbf{x}, t = 0) = I(\mathbf{x}) \tag{8}$$

The main practical drawback of anisotropic diffusion schemes is the dependence on the diffusion time t, which is given by n iterations of a *time-step* ∂t, since there is no criterion to determine neither the best number of iterations nor the best *time-step*. On the other hand, anisotropic gaussian filtering schemes takes the advantage of using non-iterative approaches which include the use of adaptive kernels to perform the smoothing.

In this paper, we propose an anisotropic gaussian filtering scheme with adaptive neighborhoods using the information provided by the nonlinear structure tensor in Eq. 6. To develop our algorithm we will take a 2D example, but results are analogous in the 3D case. The main idea of the algorithm is to estimate the value at position $\mathbf{x}$ using a linear combination of the samples inside the ellipse $\mathbf{xT}(\mathbf{x})\mathbf{x}^T = 1$, where $\mathbf{T}(\mathbf{x})$ is the structure tensor.

Let us denote by $\lambda_1 \geq \lambda_2 > 0$ the eigenvalues of the structure tensor $\mathbf{T}(\mathbf{x})$ and by $\mathbf{e_1}, \mathbf{e_2}$ the associated eigenvectors. The quadratic form $\mathbf{xT}(\mathbf{x})\mathbf{x}^T = 1$ represented in Fig. 1 defines an ellipse whose main axis follows the direction $\mathbf{e_2}$ and its length is $\frac{1}{\sqrt{\lambda_2}}$. Respectively, the orthogonal axis is oriented in the direction given by $\mathbf{e_1}$ and its length is $\frac{1}{\sqrt{\lambda_1}}$.

In this way, for quite anisotropic structure tensors, the quadratic form $\mathbf{xT}(\mathbf{x})\mathbf{x}^T = 1$ provide elongated neighborhoods $\mathcal{N}(\mathbf{x})$ around point $\mathbf{x}$ oriented along the discontinuity and not across it, while quite isotropic tensors provide circular or homogeneous neighborhoods, as shown in Fig. 2.

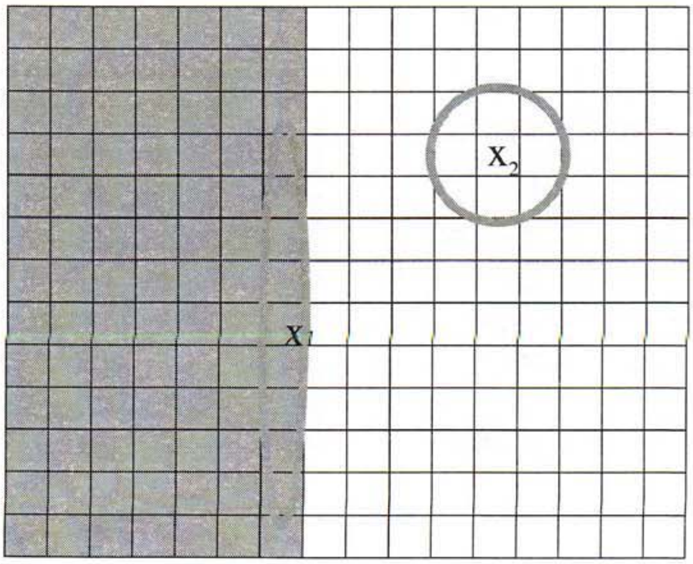

Figure 2. Different neighborhoods locally adapted to the structure of the image.

Once the neighborhood $\mathcal{N}(\mathbf{x_0})$ is calculated, we estimate the signal value at that point with a weighted mean, as shown in Eq. 9, whose weights are given by the anisotropic Gaussian kernel given in Eq. 7. To introduce the anisotropic behavior in that equation, we also use the nonlinear structure tensor, which is related to the diffusion tensor through the inverse $\mathbf{D}^{-1}(\mathbf{x}) = \mathbf{T}(\mathbf{x})$.

$$\hat{I}(\mathbf{x_0}) = \frac{\sum_{i=1}^{V(\mathcal{N}(\mathbf{x_0}))} G(\mathbf{x_0} - \mathbf{x_i}) I(\mathbf{x_0} - \mathbf{x_i})}{\sum_{i=1}^{V(\mathcal{N}(\mathbf{x_0}))} G(\mathbf{x_0} - \mathbf{x_i})} \tag{9}$$

It is remarkable that the idea behind this algorithm is quite general and, hence, it can be applied to build anisotropic versions of any kind of linear filters, such as Wiener or LMMSE, among others. In the next section we show the performance of the filtering scheme presented here and measure the improvement obtained compared to the utilization of linear structure tensors.

4. RESULTS

In this section we present some preliminary results in order to obtain an idea of the performance of the algorithm.

4.1. Experiment 1: Performance on Synthetic Images

For this experiment we use the synthetic image in Fig. 3.a. Then, we add white gaussian noise with zero mean and standard deviation $\sigma_N = 30$ to obtain the noisy version shown in 3.b. Finally we compare in Fig. 4 the result of our filtering scheme to the output of the adaptive Wiener filter proposed by Lim[17] and to the output of anisotropic gaussian smoothing driven by nonlinear structure tensors, but using $N \times N$ neighborhoods, with $N = 5$.

From Fig. 4 we can see that the blurring effect that appears in isotropic filtering schemes is negligible, although noise is not very well removed near boundaries using the Wiener filter (the figure in the middle). It is also remarkable that the image on the right shows a displaced boundary two pixels inside the cross due to the mixture of information achieved by the use of square neighborhoods. As it can be seen, neither the first, nor the second effect can be appreciated in the image on the left corresponding to the algorithm presented here. In addition, corners in that image are better preserved than in the right image.

Table 1 shows the MSE error between the original image and the outputs of the filtering algorithms involved in the experiment.

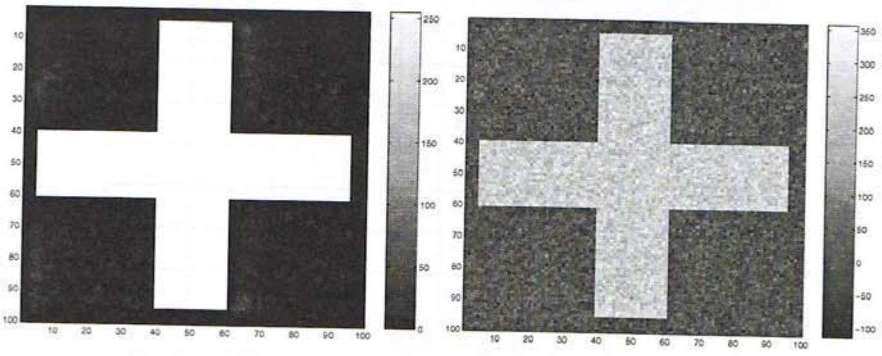

Figure 3. Left: Original Synthetic Image. Right: Noisy image $\sigma_N = 30$.

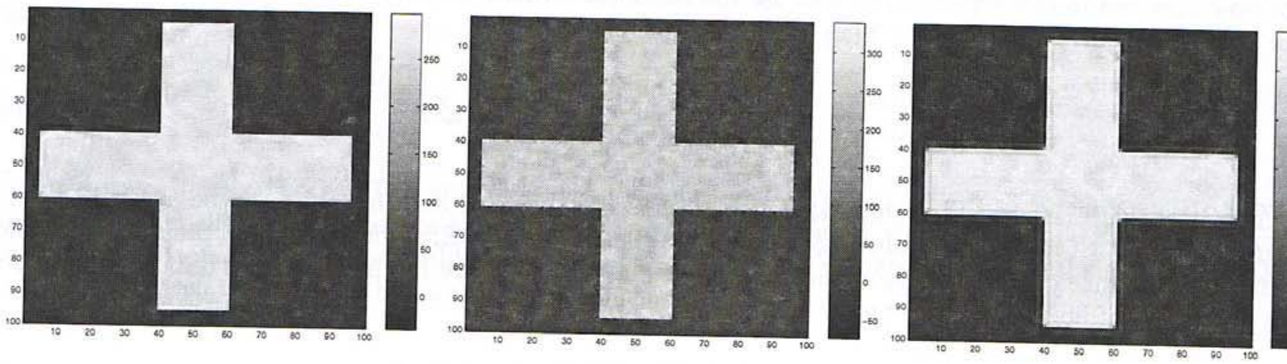

Figure 4. Left: Image restaured with the algorithm proposed in this paper. Middle: Image restaured with an adaptive Wiener Filter. Right: Image restaured with anisotropic gaussian filtering with a 5×5 neighborhood.

Restauration Method	Mean Squared Error
Anisotropic Gaussian (adaptive neighborhoods)	67.3603
Adaptive Wiener	154.3141
Anisotropic Gaussian (fixed 5×5 neighborhood)	106.8149

Table 1. Comparison of the Mean Square Error between the original image in Fig. 3.a and results in Fig. 4.

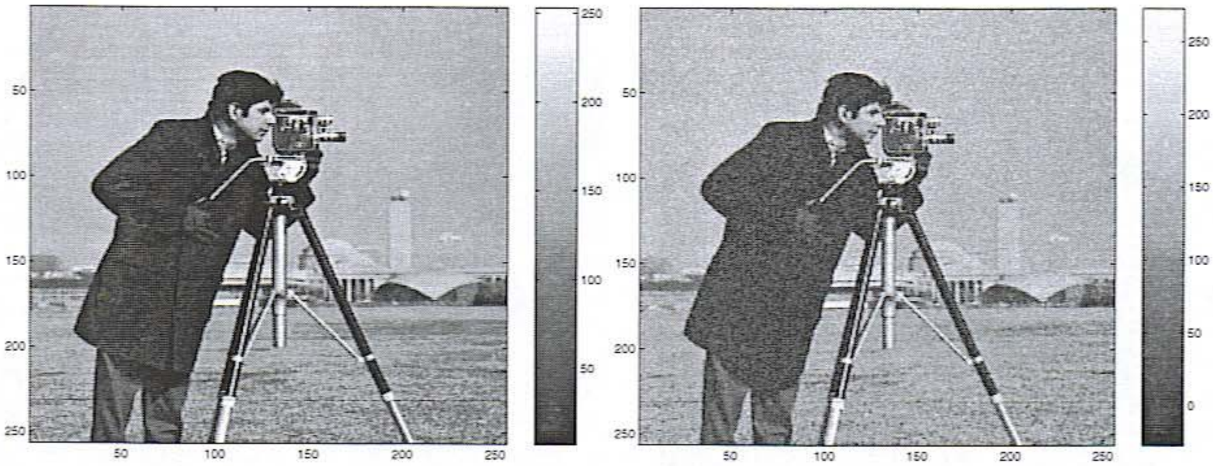

Figure 5. Left: Original Image. Right: Noisy image $\sigma_N = 10$.

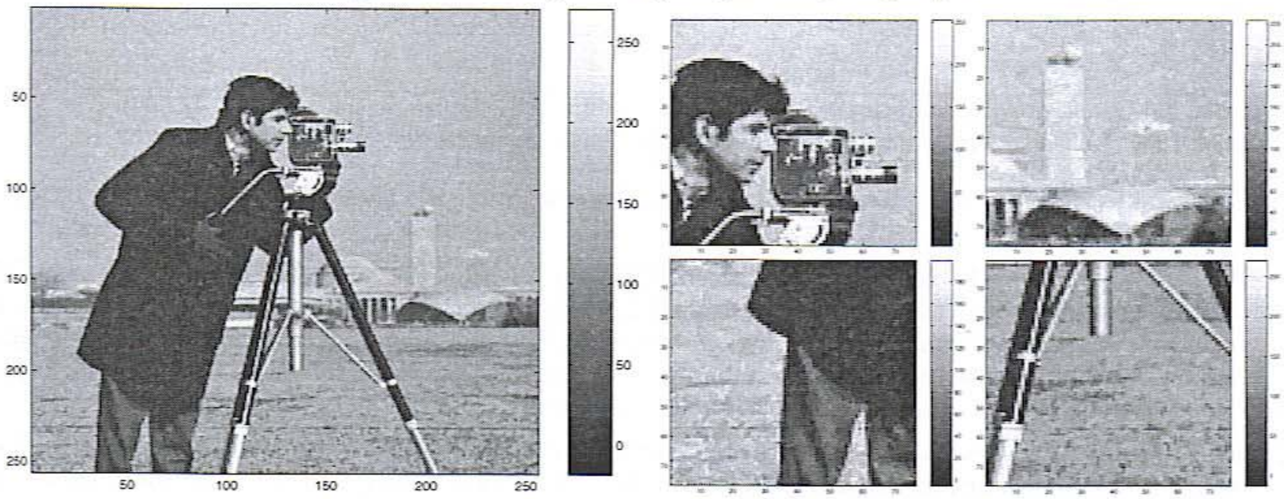

Figure 6. Left: Filtered image using the proposed algorithm and nonlinear structure tensors. Right: Several details of the image

4.2. Experiment 2: Influence of Nonlinear Structure Tensor

In this experiment we will use the image shown in Fig. 5.a. As in the precedent case, we add some white gaussian noise with zero mean and standard deviation $\sigma_N = 10$ and then we try to recover the original image with the filtering scheme proposed in this paper using, on one hand, the linear structure tensor and, on the other hand, the nonlinear counterpart. In this way, we will be able to determine whether the nonlinear structure tensor helps to better preserve structure in the image or not.

If we compare Fig. 6 and 7 it can be seen that structure and boundaries are better preserved in Fig. 6 which uses nonlinear structure tensor. Details on right side of Fig. 7 show that the linear structure tensor also introduces a bit of blurring in some structures and textures, although the boundaries are well preserved.

Table 2 shows the MSE error between the original image and the output of the proposed algorithm using the nonlinear structure tensor on one hand, and on the other the classical linear structure tensor.

5. CONCLUSIONS AND PRESENT WORK

In this paper, we present an anisotropic gaussian filtering scheme which uses nonlinear structure tensors to define the neighborhood to be used in the estimation and to drive an anisotropic gaussian function. On one hand, the

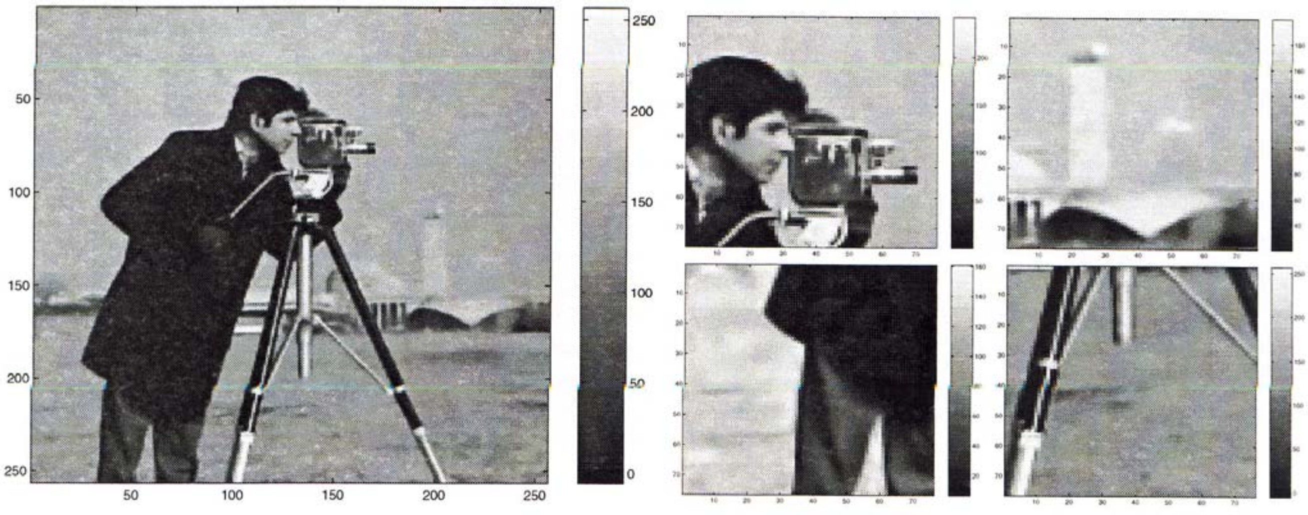

Figure 7. Left: Filtered image using the proposed algorithm and linear structure tensors. Right: Several details of the image

Restauration Method	Mean Squared Error
Anisotropic Gaussian with Adaptive Neighborhoods (nonlinear structure tensor)	42.0659
Anisotropic Gaussian with Adaptive Neighborhoods (linear structure tensor)	60.5272

Table 2. Comparison of the Mean Square Error between the original image and the output.

proposed method to define the neighborhoods lead us to a general framewok to build anisotropic versions of classical linear estimators, such as Wiener or constrained LMMSE, which still have to be studied. On the other hand, we introduce the use of nonlinear structure tensors in a well known filtering approach, as it is anisotropic gaussian filtering. A comparison of the results obtained with classical linear structure tensor and the nonlinear version is achieved and shows a superior performance of the latter.

Although presented results are encouraging, some work has still to be done for the final manuscript. In this sense, we are currently working on the definition of new experiments and on a quantitative approach to better understand the results, to relate this filter to anisotropic diffusion schemes and to provide an estimation of the computational load of the algorithm.

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