Fac. CC. Económicas y Empresariales
Universidad de La Laguna

Fac. CC. Económicas y Empresariales
Univ. de Las Palmas de Gran Canaria

Mergers and the limited liability effect

M. Pilar Socorro*

DOCUMENTO DE TRABAJO 2004-11

* Universidad de Las Palmas de Gran Canaria. Departamento de Análisis Económico Aplicado.

Mergers and the limited liability effect

M. Pilar Socorro*

Universidad de Las Palmas de Gran Canaria
Departamento de Análisis Económico Aplicado
35017 Las Palmas de Gran Canaria, Spain
Tel.: +34 928 45 9605 - Fax: +34 928 45 8183
E-mail: psocorro@daea.ulpgc.es

November 9, 2004

Abstract

This paper analyzes the effects of mergers in a homogenous product market with uncertainty over demand, fixed costs, and limited liability debt financing. On the one hand, given the limited liability effect, merging parties compete more aggressively and mergers that were unprofitable in absence of any debt obligation may become beneficial. On the other hand, socially advantageous mergers may be unprofitable for the colluding firms. In these cases, public intervention is needed. One possibility consists on subsidizing such mergers. However, it is proved that the combination of limited liability debt financing and an appropriate antitrust policy leads to higher social welfare than subsidies.

Keywords: merger, limited liability, debt
JEL Classification: G33, G34, L40

*I would like to thank Inés Macho-Stadler for advice and support. I am also very grateful to Albert Banal-Estañol, Javier Campos and David Pérez-Castrillo for helpful comments and suggestions. This research was undertaken with support from Generalitat de Catalunya, fellowship 2000FI 00481 and the research grant BEC 2003-01132. The responsibility for errors is solely mine.

Mergers and the limited liability effect

November 9, 2004

Abstract

This paper analyzes the effects of mergers in a homogenous product market with uncertainty over demand, fixed costs, and limited liability debt financing. On the one hand, given the limited liability effect, merging parties compete more aggressively and mergers that were unprofitable in absence of any debt obligation may become beneficial. On the other hand, socially advantageous mergers may be unprofitable for the colluding firms. In these cases, public intervention is needed. One possibility consists on subsidizing such mergers. However, it is proved that the combination of limited liability debt financing and an appropriate antitrust policy leads to higher social welfare than subsidies.

Keywords: merger, limited liability, debt

JEL Classification: G33, G34, L40

1 Introduction

Over the last twenty years, companies around the world have merged at an unprecedented scale. In 1999, the worldwide value of mergers rose to more than $3.4 trillion, and the number of merger deals announced was around thirty times what it was in 1981 (The Economist, 2000).

Given this considerable number of merger announcements, the role of antitrust authorities becomes crucial to guarantee that only mergers that increase social welfare are finally carried out. Moreover, mergers producing social gains should be encouraged even if they are not profitable for the merging parties. Policies promoting such mergers are quite common in Asian countries. A clear example is the Financial Institution Merger Law (FIML) implemented by the Chinese Government in 2000. The FIML encourages mergers among financial institutions by offering both tax and non-tax incentives. The Law also simplifies the procedures of mergers and acquisitions in order to reduce the costs arising from mergers. Other examples are the incentives offered by the Bangko Sentral in Philippines or the MITI in Japan. Spain has also provided fiscal advantages to mergers (Neven, Nuttall and Seabright, 1993), and in United States, the Pentagon has encouraged mergers within the Defence and pharmaceutical industries through a generous program to reimburse contractors for the costs of those mergers (Piachaud and Moustakis, 2000).

Most of the firms involved in mergers have limited liability, that is, if firms become insolvent, creditors are paid whatever operating profits are available. The purpose of this paper is to analyze the effects that mergers of limited liability companies have on firms' profits and social welfare, providing important implications to regulation policy.

The linkage between product and financial markets was first analyzed by Brander and Lewis (1986). They consider two symmetric firms competing à la Cournot in a homogenous output market under uncertainty over demand or costs. Under some assumptions (including linear demand and constant marginal costs), they find that the limited liability provisions of debt financing imply that changes in the financial structure alter the output market equilibrium. In particular, they show that, if marginal profits depend positively on the uncertainty term, the higher the debt level, the more aggressively firms compete.[1] This is what they call the limited liability effect of debt financing. Considering

[1] Other authors discussing the relationship between financial structure and output market performance are Glazer (1994), Poitevin (1989) and (1990), and Showalter (1995).

some conditions to guarantee uniqueness of equilibrium (see, for example, Kolstad and Mathiesen,1987), Brander and Lewis (1986) results can be easily generalized to a n-firms oligopoly.[2]

We consider n firms that compete in a Cournot market with uncertainty over demand, fixed costs and constant marginal costs. All firms enjoy a limited liability situation and own the same level of debt obligation. Output decisions are taken before the uncertainty over demand is realized.

Because financial structure influences the output market equilibrium, foresighted owners of the firms will have incentives to use financial structure to influence output in their favor. Given the behavior of the rival, a firm which ignored the strategic effect of financial decisions would have lower total value than a firm that took advantage of these limited liability effects. Mergers imply an increase in the new firm's debt level and hence a possible way of taking advantage of the limited liability effect to influence the equilibrium outputs.

We assume before operating in the market firms face a fixed cost. Although fixed cost savings may increase mergers profitability, they may be still injurious to the merging parties. Salant, Switzer and Reynolds (1983) demonstrate that, even with fixed cost savings, mergers may reduce the endogenous joint profits of the firms that are colluding in a complete information setting. Moreover, they show that privately unprofitable mergers may be socially gainful.

We use as a benchmark a situation in which firms have no debt obligation at all (and hence there is no limited liability effect). Privately and socially profitable mergers are analyzed. This benchmark slightly differs from the one of Salant, Switzer and Reynolds (1983) since there is uncertainty over the future demand. Given the uncertainty over demand, more mergers are privately profitable than in the case of Salant, Switzer and Reynolds (1983). It is also shown that in absence of any debt, mergers that increase social welfare might cause private losses to the colluding firms. In these circumstances, public intervention is needed. One possibility consists on subsidizing mergers. Faulí-Oller (2002) analyzes mergers between asymmetric firms. He finds the minimum subsidy required to guarantee that mergers that increase social welfare take place. In this paper,

[2]For empirical literature analyzing the links between financial structure and product market decisions, see Campos (2000), Chevalier (1995), Philips (1995), Showalter (1999), or Wanzenried (2000).

we propose another possible way of encouraging socially optimal mergers: limited liability debt financing.

As debt raises, given the limited liability, firms focus on good states of demand and compete more aggressively. The higher the debt level, the more likely is that the strategic effect of debt increases insiders' profits and the merger become privately advantageous. That is why some mergers that are not privately lucrative in absence of any debt may produce private gains for debt levels high enough. Moreover, it is shown that the combination of limited liability debt financing and a specific antitrust policy leads higher social welfare than a subsidy policy. On the one hand, the limited liability effect increases firms' incentives to merge and the society benefits from fixed costs savings. On the other hand, mergers imply a decrease in market quantity, but this reduction is lower given that firms compete more aggressively. This latter effect is not obtained through subsidies.

Even though limited liability debt financing has never been applied to mergers, it has already been used for regulatory purposes. Indeed, something similar is what Spanish Government does to finance certain R&D projects (*Proyectos Concertados*): firms are granted with free-of-interest-credits but if they are finally unable to reach their technical purposes, firms are not obliged to pay the whole debt. Although this kind of policy may involve incentives (moral hazard) problems, it encourages the execution of risky projects that would not be carried out without public intervention.

The introduction of limited liability debt financing changes firms' expected profits, and hence firms' attitude towards risk. Notice that limited liability firms only care about good states of the world, that is, firms are risk lovers. Indeed, the worst firms can do is going bankrupt, so as debt level increases, firms take chances they would not take in other situations. Although so far mergers between risk lovers firms have not been analyzed in the literature, there are some papers analyzing mergers between risk averter firms as a mean of better sharing and diversifying risk. Some empirical and theoretical contributions to this literature can be found in Amihud and Lev (1981), Banal-Estañol and Ottaviani (2003), Brown and Chiang (2002), and Levy and Sarnat (1970).

The rest of the paper is organized as follows. Section 2 presents the general model of limited liability. In section 3, the benchmark case in which firms do not have any debt obligation is considered. Both the private incentives and social welfare of mergers are studied. Section 4 is devoted to the analysis of the main results of the limited liability

model. We apply the limited liability effect to regulation policy in section 5. Finally, section 6 concludes.

2 The model

Consider n identical firms that produce in a homogenous output market with uncertainty over demand, with $n \geq 2$.[3] Let us compare the Cournot equilibrium of this industry with the Cournot equilibrium in which $m+1$ firms decide to merge, while the other firms remain independent (m being an integer between 0 and $n-1$). All decisions are taken before the uncertainty over demand is realized.

The demand function is given by $P = Max\{\theta - Q, 0\}$, where Q denotes the total market quantity and θ is a random variable that reflects the uncertainty that all firms have over the future demand.[4] The demand intercept θ is assumed to be uniformly distributed over the interval $[0, \overline{\theta}]$. Before operating in the market, all firms face a fixed cost F. Operating marginal costs are assumed to be constant and, for simplicity, are set to zero. Let D denote the initial debt obligation for every firm in the market. If $m+1$ firms merge, merged firm's debt level is the result of the sum of the debt obligations of its participants, $D_c = (m+1)D$.[5]

All firms enjoy a limited liability situation. We denote by $\widehat{\theta_k}$ firm k's critical bankruptcy threshold, reflecting the state of demand for which firm k can just meet its debt obligations with nothing left over. In bad states of demand ($\theta < \widehat{\theta_k}$), firm k is unable to pay debt claims and goes bankrupt. We assume that $0 < \widehat{\theta_k} < \overline{\theta}$ for every firm k, that is, firms never go bankrupt if the demand happens to be the highest one. Firm k's expected current-period profit is denoted by π^k:

$$\pi^k(q_k, q_{-k}, D_k) = \frac{1}{\overline{\theta}} \int_{\widehat{\theta_k}}^{\overline{\theta}} [(\theta - Q)q_k - D_k)]d\theta, \tag{1}$$

[3] Results do not change if, instead of uncertainty over demand, we consider uncertainty on firms' costs.

[4] As Salant, Switzer and Reynolds (1983) point out, the assumption of unitary slope is unrestrictive, since any linear demand curve can be expressed in this form if the output units are properly defined.

[5] We do not consider that merging firms increase financial leverage following mergers. Merging parties can increase financial leverage either because of an increase in debt capacity or because of unused debt capacity from pre-merger years (see for example, Ghosh and Jain, 2000, Kim and McConnell, 1977, or Lewellen, 1971). Assuming an increase in financial leverage following mergers reinforces even more our results.

where $\widehat{\theta_k}$ is defined as follows:

$$\widehat{\theta_k} = \frac{D_k}{q_k} + Q, \tag{2}$$

where $D_k = D$ if firm k is an independent firm and $D_k = (m+1)D$ if firm k is the coalition of $m+1$ firms, with m being an integer belonging to the interval $(0, n-1]$. Expression (1) represents firm k's expected current-period profits net of debt obligations in good states of demand ($\theta \geq \widehat{\theta_k}$). In bad states ($\theta < \widehat{\theta_k}$), firm k earns zero as all of its earnings are paid to debt-holders. Firm k's total expected profit Π^k is obtained through the sum of the current-period profit given by expression (1) and the level of debt obligation, D_k, that firm k borrowed in a previous period. Besides, firm k must pay a fixed cost F before operating in the market. So firm k's total expected profit Π^k is given by:

$$\Pi^k = \pi^k(q_k, q_{-k}, D_k) + D_k - F. \tag{3}$$

The timing of the game is as follows. In an initial stage, each firm borrows a debt level D. In the second stage of the game, $m+1$ firms decide to merge while the other firms remain independent. In next period, firms pay a fixed cost F and compete à la Cournot under uncertainty over demand. Finally, θ is realized and firms obtain their payoffs. Firms are obliged to meet their debt obligations out of operating profits, if possible. If operating profits are insufficient to pay creditors, the firm goes bankrupt.[6]

Parameters of the model are assumed to be such that we guarantee the existence of a unique equilibrium for every possible coalition structure. To better understand the main insights of the model let us start with the simple case in which firms do not have any debt obligation before deciding on production, that is $D = 0$. In this benchmark, even if a positive number of firms merge, all firms remain identical, though through mergers they save fixed costs.

[6]Note that firms are only obliged to pay debt out of operating profits. If the debt taken exceeds the fixed cost, the firm distributes the money to the shareholders before operating in the market. As Dastidar (2003) points out, this kind of leveraged recapitalisations are sometimes empirically observed, for example, as anti-takeover measures.

3 Benchmark case. Private incentives and social welfare of mergers

In this section, we consider that firms do not have any debt obligation before decisions on equilibrium outputs are taken, that is, $D = 0$ for every firm. In this framework, let us analyze the private incentives and social welfare of a merger of $m + 1$ firms. Denote by $\Pi^c(n, m, 0)$ the joint profits that these $m + 1$ firms obtain if they collude and debt obligation is zero. Denote by $\Pi^{nc}(n, m, 0)$ the joint profits that the $m + 1$ insiders would noncooperatively obtain if they do not merge and the level of debt obligation is zero. Let us denote by $g(n, m, 0)$ the increase in insiders' joint profits if $m+1$ firms decide to collude and debt obligation is zero:

$$g(n, m, 0) = \Pi^c(n, m, 0) - \Pi^{nc}(n, m, 0). \tag{4}$$

$\Pi^{nc}(n, m, 0)$ results just from the sum of the $m + 1$ profits obtained in an oligopoly of n symmetric firms, minus $m + 1$ fixed costs. Because the debt obligation is set to zero, once the insiders have merged, they behave exactly as the other $n-m-1$ symmetric firms in the industry, and the operating profits merged firm obtains are the operating profits a firm would obtain in an industry with $n - m$ identical firms, with fixed cost savings of mF. Thus, expression (4) can be rewritten as:

$$g(n, m, 0) = \pi(n - m) - (m + 1)\pi(n) + mF. \tag{5}$$

Each of the n symmetric firms of the industry maximizes its profits taken as given the other firms' outputs. In our case, firms decide quantities before the uncertainty over demand is realized so profits are taken in expected form. Prices cannot be negative so if the demand happens to be lower than expected, firms earn zero profits.[7] Every firm i solves the following maximization program:

$$\underset{q_i}{Max}\ \pi^i(q_i, q_{-i}, 0) = \underset{q_i}{Max} \int_Q^{\overline{\theta}} (\theta - Q) q_i \frac{1}{\overline{\theta}} d\theta. \tag{6}$$

The choice of output for firm i is obtained by setting the first derivative of $\pi^i(q_i, q_{-i}, 0)$ with respect to q_i equal to zero. Using subscripts to denote partial derivatives, the first

[7] This is the main difference between our benchmark case and the model of Salant, Switzer and Reynolds (1983).

order condition of the above maximization program can be written as:

$$\pi_i^i(q_i, q_{-i}, 0) = \frac{1}{\overline{\theta}} \int_Q^{\overline{\theta}} (\theta - Q - q_i) d\theta = 0. \tag{7}$$

The second order condition of firm i's maximization program is given by:

$$\pi_{ii}^i(q_i, q_{-i}, 0) = \frac{1}{\overline{\theta}} (q_i - \int_Q^{\overline{\theta}} 2 d\theta) < 0. \tag{8}$$

In a symmetric Nash equilibrium, outputs offered by all firms in the industry are identical, so the market quantity can be expressed as $Q = nq$. The unique symmetric Cournot equilibrium output that satisfies both the first and second order conditions is given by $q = \overline{\theta}/(n+2)$. Substituting equilibrium quantities in firms' benefits, we obtain equilibrium operating profits for each of the n identical firms of the industry, which are given by the following:

$$\pi(n) = \frac{2\overline{\theta}^2}{(n+2)^3}. \tag{9}$$

Thus, the change in insiders' profits due to a merger of $m+1$ firms when debt obligation is zero can be expressed as:

$$g(n, m, 0) = \frac{2\overline{\theta}^2}{(n-m+2)^3} - (m+1)\frac{2\overline{\theta}^2}{(n+2)^3} + mF. \tag{10}$$

For a merger of $m+1$ firms to be profitable, the change in insiders' profits must be positive, $g(n, m, 0) > 0$. The most interesting properties of this function are summarized in the following lemma.

Lemma 1 *The increase in insiders' joint profits if $m+1$ firms decide to collude and debt obligation is zero, $g(n, m, 0)$, is strictly convex in the number of colluding firms m. $g(n, m, 0)$ is zero for $m = 0$ and merger to monopoly is always profitable. If fixed cost F is not high enough, $g(n, m, 0)$ is initially negative and decreases in m, while every merger is profitable if fixed cost is sufficiently large. Finally, if there is no fixed cost, for any n, it is sufficient for a merger to be unprofitable that less than the 65.44 percent of the firms collude.*

The function $g(n, m, 0)$ is strictly convex in m. On the one hand, if fixed cost savings are not large enough, it is initially negative and decreasing in m. Thus, merger by a larger number of firms may cause a loss to the colluding firms. However, when all firms collude

joint profits are maximized and firms' profits are always higher than in the pre-merger equilibrium. On the other hand, if costs savings are large enough, every merger cause private gains to the colluding parties. The uncertainty over demand makes some mergers that would be unprofitable in a certain environment profitable in our model. In particular, we find that for any n and no saving costs, it is sufficient for a merger to be unprofitable that less than the 65.44 percent of the firms collude (instead of the 80 percent of Salant, Switzer and Reynolds, 1983).

Let us now analyze the social welfare of mergers. We are considering so far that firms produce and send their production to the market before the uncertainty over demand is resolved. Hereafter, if the demand happens to be lower than expected and prices inevitably fall to zero, consumers will acquire the goods at zero price.[8] The consumer surplus is obtained through the following formula:

$$CS = \frac{1}{\overline{\theta}}\left(\int_0^Q \frac{\theta^2}{2}d\theta + \int_Q^{\overline{\theta}} \frac{Q^2}{2}d\theta\right). \tag{11}$$

Social welfare is defined as the sum of consumer surplus and the profits of all firms competing in the market. The gain in social welfare due to the merger of $m+1$ firms in a $n-$firms oligopoly with zero debt obligation is given by:

$$S(n,m,0) = CS(n,m,0) + (n-m)\ \pi(n-m) - CS(n,0) - n\ \pi(n) + mF, \tag{12}$$

which in our benchmark model can be rewritten as:

$$\begin{aligned} S(n,m,0) = (n-m)\frac{2\overline{\theta}^2}{(n-m+2)^3} + \frac{\overline{\theta}^2}{2}\left[\frac{(n-m)^2}{(n-m+2)^2} - \frac{2}{3}\frac{(n-m)^3}{(n-m+2)^3}\right] \\ - n\frac{2\overline{\theta}^2}{(n+2)^3} - \frac{\overline{\theta}^2}{2}\left[\frac{n^2}{(n+2)^2} - \frac{2}{3}\frac{n^3}{(n+2)^3}\right] + mF. \end{aligned} \tag{13}$$

For small m, we can find values for the fixed cost F such that there exist social gains of mergers though they cause private losses to the merging firms, that is, $S(n,m,0) > 0 > g(n,m,0)$.[9] This is formally stated in the following proposition.

[8] For simplicity, we have considered the marginal cost to be zero. However, notice that, even if the marginal cost is strictly positive, firms may be interested in selling outputs at prices below the marginal cost, since production is sent to the market before the uncertainty over demand is realized.

[9] In absence of fixed cost savings, mergers are never socially advantageous. However, for small m the social loss is smaller than the loss to the merging parties. Fixed cost savings benefit both the merging parties and society in the same amount, so it is possible to select $F > 0$ so that $S(n,m,0) > 0 > g(n,m,0)$.

Proposition 1 *Mergers that increase social welfare might be unprofitable to the colluding parties.*

Many examples satisfy Proposition 1. For the sake of illustration, take for instance the case of ten firms, and suppose, for simplicity, that θ is uniformly distributed over the interval $[0, 100]$. Suppose also that fixed costs are $F = 4$. In this example, mergers of two, three and four firms are socially advantageous though none of these firms cause private profits to the merging firms. Indeed, in this example, for a merger to be privately profitable it is necessary that at least six firms decide to collude. Figure 1 plots both the gain in insiders' profits and social welfare if $m + 1$ firms merge in a ten-firms oligopoly.

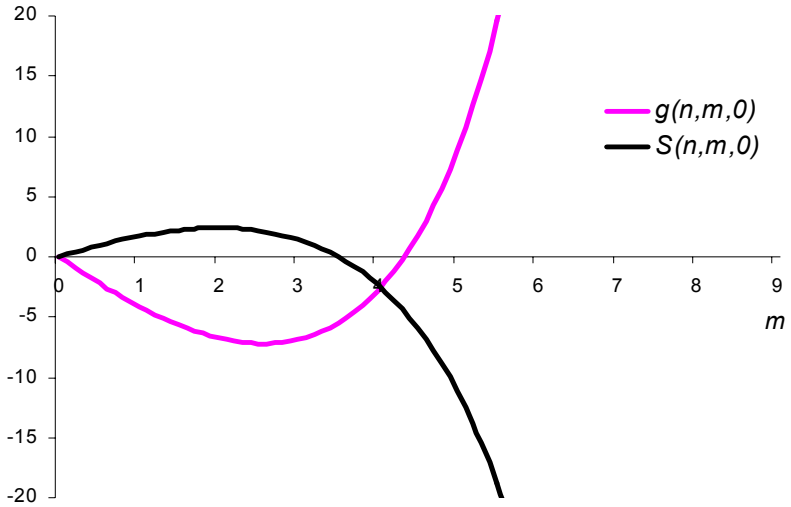

Figure 1: Social and private gains to insiders if $m + 1$ firms merge, for $n = 10$, $F = 4$ and $\overline{\theta}$= 100

So far we have considered that firms do not face any debt obligation before output decisions are taken. However, if firms are debt financed, a merger of $m+1$ firms increases the debt obligation of the resulting firm. Given that firms enjoy a limited liability situation, an increase in debt level makes merged firm to compete more aggressively than the other $n - m - 1$ symmetric firms in the industry. In next section, we discuss the consequences of the limited liability effect on the private incentives and social welfare of mergers.

4 The limited liability effect

This section examines how the limited liability aspects of debt financing affect the strategic output decisions of firms, and hence their incentives to merge. In order to make comparisons let us start with the case in which all firms remain independent. Taking existing debt level D as given, each of the n symmetric firms chooses output in order to maximize current-period expected profits:

$$\underset{q_i}{Max}\ \pi^i(q_i, q_{-i}, D) = \underset{q_i}{Max} \int_{\widehat{\theta}_i}^{\overline{\theta}} [(\theta - Q)q_i - D]\frac{1}{\overline{\theta}} d\theta, \tag{14}$$

where $\widehat{\theta}_i$ is defined by:

$$\widehat{\theta}_i = \frac{D}{q_i} + Q. \tag{15}$$

The first order condition of the above maximization program is obtained setting the first derivative of $\pi^i(q_i, q_{-i}, D)$ with respect to q_i equal to zero:

$$\pi_i^i(q_i, q_{-i}, D) = \int_{\widehat{\theta}_i}^{\overline{\theta}} (\theta - Q - q_i)\frac{1}{\overline{\theta}} d\theta = 0. \tag{16}$$

Besides the expression in (16), the derivative of $\pi^i(q_i, q_{-i}, D)$ includes another term, $-\frac{d\widehat{\theta}_i}{dq_i}\frac{1}{\overline{\theta}}[(\widehat{\theta}_i - Q)q_i - D]$, which is zero by definition.

The second order condition is given by:

$$\pi_{ii}^i(q_i, q_{-i}, D) = \frac{1}{\overline{\theta}}[-2\overline{\theta} + 2(\frac{D}{q_i} + Q) - (\frac{-D}{q_i^2} + 1)(\frac{D}{q_i} - q_i)] < 0. \tag{17}$$

All firms are identical so the Cournot equilibrium output is symmetric, $q_i = q_j = q$ for every firm $i \neq j$. Market quantity can be expressed as $Q = nq$. The symmetric Nash equilibrium quantity satisfies both the first and second order conditions given by expressions (16) and (17).[10]

In bad states of demand, firms' profits are not sufficient to meet debt obligations and firms go bankrupt. The limited liability aspects of debt financing implies that firms are only concerned about good states of demand, $\theta \in \left[\widehat{\theta}_i, \overline{\theta}\right]$. Increases in the level of debt obligation raises the critical bankruptcy threshold $\widehat{\theta}_i$ and forces firms to take output decisions they would not take in other circumstances. As debt raises firms compete more

[10] In general, the symmetric solution of expression (16) is not unique. However, we can always impose restrictions on the debt level D such that there exists a solution that satisfies the second order condition given by expression (17), and it is unique.

aggressively. The worst the firms can do is going bankrupt, and if they do not compete more aggressively they will go bankrupt anyway. In other words, limited liability implies a change in firms' expected profits and firms become risk lovers. As a consequence, the symmetric Cournot equilibrium output increases as debt level raises.

Lemma 2 *The unique symmetric Cournot equilibrium output for the case in which all firms remain independent q is strictly increasing in the initial level of debt obligation D.*

The most interesting results of the limited liability model arise when there is an unilateral increase in the level of debt obligation due to a merger of firms. The Cournot equilibrium output in which $m+1$ firms merge is not longer purely symmetric- for m belonging to the interval $(0, n-1)$. The coalition of these $m+1$ firms faces a level of debt higher than the other $n-m-1$ symmetric firms in the industry and merged firm will not exactly behave as the others. Denote by q_c and q_i the quantity offer by the coalition of firms and each of the other $n-m-1$ firms in the market, respectively. The coalition of $m+1$ firms solves the following maximization program:

$$\underset{q_c}{Max}\ \pi^c(q_c, q_i, D) = \underset{q_c}{Max} \int_{\widehat{\theta}_c}^{\overline{\theta}} [(\theta - Q)q_c - (m+1)D] \frac{1}{\overline{\theta}} d\theta, \tag{18}$$

where $\widehat{\theta}_c$ is defined by:

$$\widehat{\theta}_c = \frac{(m+1)D}{q_c} + Q. \tag{19}$$

Each of the $n-m-1$ symmetric firms that remain independent solves:

$$\underset{q_i}{Max}\ \pi^i(q_c, q_i, D) = \underset{q_i}{Max} \int_{\widehat{\theta}_i}^{\overline{\theta}} [(\theta - Q)q_i - D] \frac{1}{\overline{\theta}} d\theta, \tag{20}$$

where $\widehat{\theta}_i$ is defined by:

$$\widehat{\theta}_i = \frac{D}{q_i} + Q. \tag{21}$$

The $n-m-1$ outsiders act in the same way so the market quantity can be expressed as $Q = q_c + (n-m-1)q_i$. The Nash equilibrium outputs q_c and q_i are obtained from the simultaneous solution of the first order conditions of the above maximization programs. These first order conditions can be written as:

$$\frac{\overline{\theta}^2}{2} - \overline{\theta}(Q + q_c) - \frac{1}{2}(\frac{(m+1)D}{q_c} + Q)^2 + (Q + q_c)(\frac{(m+1)D}{q_c} + Q) = 0. \tag{22a}$$

$$\frac{\overline{\theta}^2}{2} - \overline{\theta}(Q + q_i) - \frac{1}{2}(\frac{D}{q_i} + Q)^2 + (Q + q_i)(\frac{D}{q_i} + Q) = 0. \tag{22b}$$

The simultaneous Nash solution of expressions (22a) and (22b) must also satisfy the following second order conditions:[11]

$$3q_c^4 + 2[(n-m-1)q_i - \overline{\theta}]q_c^3 + (m+1)^2 D^2 < 0. \quad (23a)$$

$$[2(n-m)-1]q_i^4 + 2(q_c - \overline{\theta})q_i^3 + D^2 < 0. \quad (23b)$$

As shown in Lemma 2, firms are only concerned about good states of demand and as debt raises, firms take more aggressive output decisions. The merger of $m+1$ firms implies a unilateral increase in debt. Hence, mergers will not only affect the number of firms competing in the market but also merged firm's behavior. Indeed, the increase in the debt level serves as a commitment of the merged firm to compete more aggressively.

Lemma 3 *In the unique Cournot equilibrium, merged firm competes more aggressively than the other* $n-m-1$ *symmetric firms in the industry:* $q_c > q_i$.

In absence of any debt obligation, once insiders have merged, they behave exactly as the other $n-m-1$ firms in the industry. However, the strategic effect of debt in a limited liability context affects merged firm's behavior. Merged firm has to pay a higher level of debt. If it does not take more risk, merged firm is more likely to bankrupt.

Let us now analyze firms' private incentives to merge. Recall firm k's total expected profit Π^k is the result of firm i's current-period profit π^k, plus the level of debt firm k borrowed in a previous period D_k, minus the fixed cost firm k paid before operating in the market F. Denote by $\Pi^c(q_c, q_i, D)$ total expected profits if $m+1$ firms with initial level of debt D collude, where q_c and q_i are the Cournot equilibrium outputs resulting from the simultaneous solution of expressions (22a) and (22b). Denote by $\Pi(q, D)$ the joint profits that the $m+1$ insiders would noncooperatively obtain if they do not merge, where q is the symmetric Cournot output given by expression (16). Let $g(n, m, D)$ denote the increase in insiders' joint profits if $m+1$ firms decide to collude with initial debt obligation D.

$$g(n, m, D) = \Pi^c(q_c, q_i, D) - (m+1)\Pi(q, D) = \pi^c(q_c, q_i, D) - (m+1)\pi(q, D) + mF. \quad (24)$$

The higher the initial level of debt obligation, the higher the limited liability effect of debt financing. The higher the limited liability effect, the more aggressive merged firm

[11] Again, the solution of expressions (22a) and (22b) is not generally unique but we can always impose restrictions on the debt level D such that there exists a solution that satisfies the second order conditions given by expressions (23a) and (23b), and it is unique.

competes and the larger the difference between the cooperative and noncooperative profits of insiders. As initial debt raises, mergers that are not privately profitable may become beneficial.

Proposition 2 *If the initial level of debt obligation D is high enough, the limited liability effect makes some mergers that are not privately profitable with zero debt obligation $g(n, m, 0) < 0$, privately profitable, $g(n, m, D) > 0$.*

In order to see that the limited liability effect increases mergers profitability, take again the case of ten firms, with θ uniformly distributed over the interval $[0, 100]$ and $F = 4$. We know in absence of any debt obligation, merger of three firms is not privately profitable: $g(n, m, 0) = -6.722$, for $n = 10$ and $m = 2$. However, it is easy to show that if the initial level of debt obligation is high enough, for example $D = 40$, merger of three firms becomes privately beneficial: $g(n, m, 40) = 2.1874$, for $n = 10$ and $m = 2$.

Social welfare is the result of the sum of consumer surplus, firms' total expected profits and creditors' profits. Creditors' profits are equal to their revenues minus the level of debt they lent to each firm in a previous period. In good states of demand creditors are paid the whole debt. However, if firms become insolvent creditors can only collect firms' current-period operating profits.[12] Figure 2 plots the revenues creditors obtained from firm k as function of the state of the demand.

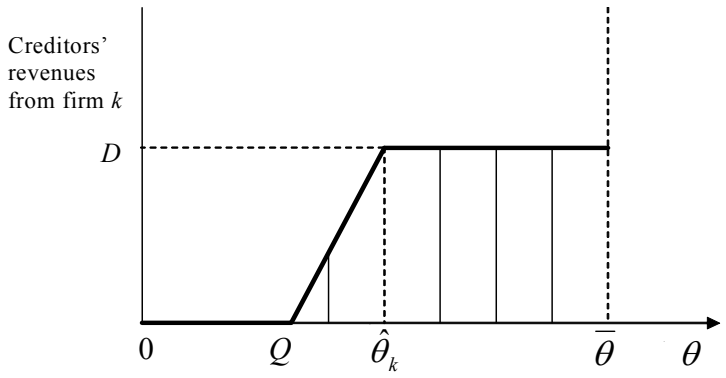

Figure 2: Creditors' revenues from firm k as function of the state of demand

[12]For the sake of simplicity and following Brander and Lewis (1986), we assume that the asset value of firms is zero (as if assets are completely used up in the production of output). Therefore, creditors can only collect current operating profits if firms go bankrupt.

If all firms remain independent in a $n-$firms oligopoly, creditors' profits are given by the following expression:

$$Cr(n,D) = -nD + n\int_{Q}^{\widehat{\theta}}(\theta - Q)q\frac{1}{\overline{\theta}}d\theta + n\int_{\widehat{\theta}}^{\overline{\theta}} D\frac{1}{\overline{\theta}}d\theta, \tag{25}$$

where q is the symmetric Cournot solution of expression (16), $Q = nq$ is the market quantity, and $\widehat{\theta} = \frac{D}{q} + nq$, the critical bankruptcy threshold of each of the symmetric firms in the market.

On the other hand, if $m+1$ firms in an industry of n firms collude, creditors' profits can be expressed as:

$$\begin{aligned} Cr(m,n,D) = -nD + \int_{Q}^{\widehat{\theta}_c}(\theta - Q)q_c\frac{1}{\overline{\theta}}d\theta + (m+1)\int_{\widehat{\theta}_c}^{\overline{\theta}} D\frac{1}{\overline{\theta}}d\theta \\ +(n-m-1)\int_{Q}^{\widehat{\theta}_i}(\theta - Q)q_i\frac{1}{\overline{\theta}}d\theta + (n-m-1)\int_{\widehat{\theta}_i}^{\overline{\theta}} D\frac{1}{\overline{\theta}}d\theta, \end{aligned} \tag{26}$$

where q_c and q_i are the Cournot equilibrium outputs resulting from the simultaneous solution of expressions (22a) and (22b) and the market quantity is given by $Q = q_c + (n - m - 1)q_i$.

The gain in social welfare due to the merger of $m+1$ firms in an $n-$firms oligopoly with initial debt obligation D is the result of the difference between the social welfare if $m+1$ firms collude and the social welfare if all firms remain independent in the market. Formally, the gain in social welfare can be obtained through the following expression:

$$\begin{aligned} S(n,m,D) = CS(n,m,D) + Cr(n,m,D) + \pi^c(q_c,q_i,D) \\ +(n-m-1)\pi^i(q_c,q_i,D) - CS(n,D) - Cr(n,D) - n\pi(q,D) + mF. \end{aligned} \tag{27}$$

5 Policy implications

Important results of the limited liability model can be applied to regulation policy. In absence of any debt, mergers may increase social welfare. However, socially optimal mergers may be privately injurious to the colluding parties. We will show that, in these circumstances, a social planner may be interested in financing firms in a limited liability situation in order to increase their incentives to merge.

To demonstrate that this possibility can indeed arise, let us consider once again the simple case of ten firms. We know that, if θ is uniformly distributed over the interval

$[0, 100]$ and $F = 4$, in absence of any debt, for mergers to be privately profitable it is necessary that at least six firms decide to merge. However, any merger between more than four firms causes social losses and should be forbidden by antitrust authorities. On the other hand, mergers up to four firms increase social welfare and should be encouraged. One possibility is through the limited liability model.

If the initial level of debt obligation is high enough, any merger becomes privately beneficial. Table 1 compares the gain in insiders' profits and social welfare in the cases of $D = 0$ and $D = 40$ for the ten-firms example.

$n = 10,\ \overline{\theta} = 100,\ F = 4$	**D = 0**	**D = 40**		
	m = 2	***m = 1***	***m = 2***	***m = 3***
Gain in insiders' profits if *m*+1 firms merge	**-6.722**	**0.7619**	**2.1874**	**4.5629**
Social welfare if *m*+1 firms merge	**1621.4**	**1625.7**	**1627.5**	**1627.9**
Consumer surplus	1493.4	1573.0	1548.3	1514.9
Firms' profits	128.0	404.9	425.15	451.54
Creditors' profits	0	-352.2	-345.95	-338.54
Social welfare if firms remain independent	**1619.0**	**1623.1**	**1623.1**	**1623.1**
Consumer surplus	1543.2	1591.3	1591.3	1591.3
Firms' profits	75.8	389.61	389.61	389.61
Creditors' profits	0	-357.81	-357.81	-357.81
Gain in social welfare if *m*+1 firms merge	**2.4**	**2.6**	**4.4**	**4.8**

Table 1: Gain in insiders' profits and social welfare if $m + 1$ firms merge, for $n = 10$, $\overline{\theta} = 100$ and $F = 4$

In absence of debt and given that mergers between more than four firms are not allowed by the antitrust authority, all firms remain independent. In this case, social welfare is $SW(n, 0) = 1619$, for $n = 10$.

However, if the social planner finance limited liability firms with initial debt level $D = 40$, any merger becomes privately profitable. Social welfare is maximized if four firms merge. Allowing mergers up to four firms, the minimum social welfare is obtained for the case in which none of the firms merge, and even in this case, social welfare is higher than in the case in which firms are not financed and remain independent: $SW(n, m, 40) = 1623.1 > 1619$, for $n = 10$ and $m = 1$. It is obvious that in this simple example, it is advantageous for the social planner to finance firms with limited liability.

Claim 1 *In order to induce a certain market structure, it might be optimal for the social planner to finance firms in a limited liability situation.*

Mergers that create social benefits may be detrimental for the colluding firms. In these cases, it may be optimal for the social planner to finance firms with limited liability. Given the limited liability aspects of debt financing, merging parties compete more aggressively and privately injurious mergers may become beneficial.

From the data provided in Table 1, we can also see the advantage of the limited liability debt financing over subsidies. In absence of any debt, social welfare is maximized if three firms merge. However, we know such merger is not privately profitable and it will not occur without public intervention. One possibility is subsidizing the merger, and the social welfare will be $S(n, m, 0) = 1621.4$. It is obvious that financing firms with initial debt level $D = 40$ and forbidding any merger larger than four firms yields higher social welfare.

Proposition 3 *The combination of limited liability debt financing and an appropriate antitrust policy leads higher social welfare than a subsidy policy.*

On the one hand, mergers reduce the market quantity and hence they have a negative effect on consumers' surplus. On the other hand, mergers imply fixed costs savings. Subsidizing mergers, the society will benefit from fixed costs savings, but consumers will be surely worse off. Limited liability debt financing also increase firms' incentives to merge, so that fixed cost savings are achieved. Besides, it also affects firms' aggressiveness and the reduction in market quantity is mitigated.

Social planner's intervention not only affect the total value of welfare but also its distribution among agents. As debt raises firms compete more aggressively, so the higher the debt and level of competition, the better consumers are. For creditors, it is the other way around. The higher the debt and level of competition, the more likely firms go bankrupt, and the lower creditor's profits are.

In the ten-firms example, if few firms merge ($m = 1$ and $m = 2$) and $D = 40$, consumers are better off than in the case in which no firm merges and there is not debt at all. The reason is that, though there is less level of competition, the limited liability effect forces firms to compete more aggressively and the market quantity is not finally reduced.

6 Conclusions

The search of firms' optimal size is the main reason for the processes of mergers and acquisitions in which many firms have been involved in recent years. Most firms that try to restructure their businesses enjoy a limited liability situation, that is, if the firm is unable to meet its debt obligations, its creditors are paid whatever operating profits are available. This paper analyzes firms' private incentives to merge in a homogenous product market with uncertainty over demand, fixed costs, and limited liability debt financing. It also studies the gain in social welfare due to mergers and some applications to regulatory policy.

It has been argued that increases in the debt level raise the output market equilibrium. This is what Brander and Lewis (1986) call the limited liability effect of debt financing: as firms take on more debt, they will have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that firms will ignore reductions in returns in bankrupt states.

A unilateral increase in firm's debt causes that the firm competes more aggressively and obtains higher returns. Given that the level of debt obligation is assumed to be exogenous, the only way firms have to take advantage of the strategic effect of financial decisions is through a merger.

The first part of the paper analyzes the private and social benefits of mergers in absence of any debt obligation. If fixed cost savings are not high enough, the gain in insiders' profits initially decreases in the number of colluding parties and then it increases. Therefore, if fixed cost savings and the number of merging firms are not sufficiently large, mergers are not privately profitable. Those mergers may be socially advantageous, though they will not occur without public intervention.

If firms own an initial level of debt obligation, mergers cause an unilaterally increase of debt and merged firm competes more aggressively. If the strategic effect of debt is large enough to guarantee large returns to the merger, mergers that were unprofitable in absence of debt might become privately beneficial.

This result has important policy implications since socially beneficial mergers may be unprofitable to the merging parties. In these cases, it may be optimal for the social planner to finance limited liability firms and increase their incentives to merge. Combining this way of intervention with a precise antitrust policy, social welfare is shown to be larger

than if just subsidies are used. The reason is that the limited liability effect increases firms' aggressiveness so the reduction in market quantity due to mergers is alleviated.

In the analysis of this paper, no bankrupt costs have been considered. However, bankrupts may have a considerable cost associated, including legal fees, court costs, consultant fees, documents, and the social problems arising from firing large numbers of workers. As an example of the importance of such costs, we can look at the case of Enron, whose bankruptcy costs are expected to hit $1 billion by the end of 2006.

If firm's have debt obligations and bankruptcy costs are large enough, it might be optimal to encourage firms' mergers in order to decrease the probability of bankruptcy. In this case, the combination of public limited liability debt financing and an antitrust policy might be again a suitable way of intervention, since the probability of bankruptcy is reduced without reducing too much the quantity offered in the market.

Appendix

Proof of Lemma 1. Most properties stated in Lemma 1 can be easily checked computing the first and second derivatives of $g(n,m,0)$ and verifying that, for $n \geq 2$, $\frac{\partial g(n,m,0)}{\partial m} \mid_{m=0} = F - 2\overline{\theta}^2 \frac{n-1}{(n+2)^4} < 0$ iff $F < 2\overline{\theta}^2 \frac{n-1}{(n+2)^4}$, $\frac{\partial^2 g(n,m,0)}{\partial m^2} = \frac{24\overline{\theta}^2}{(n-m+2)^5} > 0$, and $g(n,n-1,0) = n(n^2+6n-15)+8+(n-1)F > 0$. To prove that for $F=0$ and any n, it is sufficient for a merger to be unprofitable that less than the 65.44 percent collude, denote by $m^*(n)+1$ the number of firms such that $g(n,m^*,0)=0$ and by $(m^*(n)+1)/n$ the fraction of those firms in the industry. Then ,it is easy to check that this fraction reaches its minimum value of 0.6544 for $n=7$. ■

Proof of Lemma 2. We can exploit the symmetry of this case, $q_i = q_j = q$ for every firm $i \neq j$, and totally differentiate just one of the first-order conditions:

$$\pi^i_{ii} dq + (n-1)\pi^i_{ij} dq + \pi^i_{iD} dD = 0.$$

Then, the derivative of q with respect to D can be written as:

$$\frac{dq}{dD} = -\frac{\pi^i_{iD}}{\pi^i_{ii} + (n-1)\pi^i_{ij}} > 0,$$

since $\pi^i_{ii} < 0$ by definition, $\pi^i_{iD} = -\frac{d\widehat{\theta}_i}{dD}(\widehat{\theta}_i - Q - q)\frac{1}{\overline{\theta}} = -\frac{1}{q\overline{\theta}}(\widehat{\theta}_i - Q - q) > 0$ and $\pi^i_{ij} = \frac{1}{\overline{\theta}}[-\int_{\widehat{\theta}_i}^{\overline{\theta}} d\theta - \frac{d\widehat{\theta}_i}{dq_j}(\widehat{\theta}_i - Q - q)] = -\frac{1}{\overline{\theta}}(\overline{\theta} - Q - q) < 0$, by first-order condition. ■

Proof of Lemma 3. In order to prove that merged firm competes more aggressively, let us start supposing that $D_c = D_i$. In this case, we would have that $q_c = q_i$. Suppose now that D_c unilaterally increases. To obtain the change in Cournot outputs following the unilateral increase in D_c, we have to totally differentiate the first order conditions for the merged firm and the other $n-m-1$ independent firms, which are given by:

$$\begin{aligned} \pi_c^c &= \frac{1}{\overline{\theta}} \int_{\widehat{\theta}_c}^{\overline{\theta}} (\theta - Q - q_c) d\theta = 0, \\ \pi_i^i &= \frac{1}{\overline{\theta}} \int_{\widehat{\theta}_i}^{\overline{\theta}} (\theta - Q - q_i) d\theta = 0. \end{aligned}$$

Exploiting the symmetry for the $n-m-1$ firms, totally differentiation of the above first order conditions generates the following system:

$$\begin{aligned} \pi_{cc}^c dq_c + (n-m-1)\pi_{ci}^c dq_i + \pi_{cD_c}^c dD_c &= 0, \\ \pi_{ic}^i dq_c + \pi_{ii}^i dq_i + \pi_{iD_c}^i dD_c &= 0. \end{aligned}$$

Firstly, note that π_i^i does not depend on D_c. The solution of the system yields the following comparative static effects:

$$\begin{aligned} \frac{dq_c}{dD_c} &= -\frac{\pi_{ii}^i \pi_{cD_c}^c}{\pi_{cc}^c \pi_{ii}^i - (n-m-1)\pi_{ci}^c \pi_{ic}^i}, \\ \frac{dq_i}{dD_c} &= \frac{\pi_{ic}^i \pi_{cD_c}^c}{\pi_{cc}^c \pi_{ii}^i - (n-m-1)\pi_{ci}^c \pi_{ic}^i}. \end{aligned}$$

Notice that $\pi_{cc}^c \pi_{ii}^i - (n-m-1)\pi_{ci}^c \pi_{ic}^i > 0$ since the Cournot output is assumed to be unique (see Kolstad and Mathiesen, 1987), and $\pi_{ii}^i < 0$, by definition. Moreover, $\pi_{ic}^i = -\frac{1}{\overline{\theta}}(\overline{\theta} - Q - q_i) < 0$, and $\pi_{cD_c}^c = -\frac{1}{q_c \overline{\theta}}(\widehat{\theta}_c - Q - q_c) > 0$, by first order conditions. Then, we can conclude that $dq_c/dD_c > 0$ and $dq_i/dD_c < 0$.

We know mergers imply a unilateral increase in debt, $D_c = (m+1)D_i$, so $q_c > q_i$, as we wanted to prove. ■

References

[1] Amihud, Y. and B. Lev (1981): "Risk reduction as a managerial motive for conglomerate mergers", *Bell Journal of Economics* 12(2), 605-617.

[2] Banal-Estañol, A. and M. Ottaviani (2003): "Mergers with product market risk", mimeo, Universitat Autònoma de Barcelona.

[3] Brander, J. and T. Lewis (1986): "Oligopoly and financial structure: the limited liability effect", *American Economic Review* 76(5), 956-970.

[4] Brown, M. and S.-H. Chiang (2002): "Unsystematic risk and coalition formation in product markets", *International Journal of Industrial Organization* 20(3), 313-338.

[5] Campos, J. (2000): "Responsabilidad limitada, estructura financiera y comportamiento de las empresas españolas", *Investigaciones Económicas* XXIV (3), 585-610.

[6] Chevalier, J. (1995): "Capital structure and product-market competition: empirical evidence from the supermarket industry", *American Economic Review* 85(3), 415-435.

[7] Dastidar, K. (2003): "Oligopoly and financial structure revisited", *Economics Bulletin* 12(3), 1-12.

[8] The Economist (2000): "How mergers go wrong", vol. 356, no 8180, 17.

[9] Faulí-Oller, R. (2002): "Mergers between asymmetric firms: profitability and welfare", *The Manchester School* 70(1), 77-87.

[10] Ghosh, A. and P.C. Jain (2000): "Financial leverage changes associated with corporate mergers", *Journal of Corporate Finance* 6, 377-402.

[11] Glazer, J. (1994): "The strategic effects of long-term debt in imperfect competition", *Journal of Economic Theory* 62, 428-443.

[12] Kim, H.E. and J.J. McConnell (1977): "Conglomerate mergers and the co-insurance of corporate debt", *Journal of Finance* 32, 349-363.

[13] Kolstad, C.D. and L. Mathiesen (1987): "Necessary and sufficient conditions for uniqueness of a Cournot equilibrium", *Review of Economic Studies* LIV, 681-690.

[14] Levy, H. and M. Sarnat (1970): "Diversification, portfolio analysis, and the uneasy case for conglomerate mergers", *Journal of Finance* 25, 795-802.

[15] Lewellen, W.G. (1971): "A pure financial rationale for the conglomerate merger", *Journal of Finance* 26, 521-537.

[16] Neven, D., R. Nuttall and P. Seabright (1993): *Merger in Daylight*, CEPR, London.

[17] Phillips, G.M. (1995): "Increased debt and industry products markets. An empirical analysis, *Journal of Financial Economics* 37, 189-238.

[18] Piachaud, B.S. and F. Moustakis (2000): "Is there a valid case for mergers within the Defense and pharmaceutical industries? A qualitative analysis, *The Journal of World Affairs & New Technology* 3(4), 1-23.

[19] Poitevin, M. (1989): "Financial signalling and the deep pocket argument", *Rand Journal of Economics* 20, 26-40.

[20] Poitevin, M. (1990): "Strategic financial signaling", *International Journal of Industrial Organization* 8, 499-518.

[21] Salant, S. W., S. Switzer, and R.J. Reynolds (1983): "Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot- Nash equilibrium", *Quarterly Journal of Economics* 98, 185-199.

[22] Showalter, D. (1995): "Oligopoly and financial structure: comment", *American Economic Review* 85, 647-653.

[23] Showalter, D. (1999): "Strategic debt: evidence in manufacturing", *International Journal of Industrial Organization* 17, 319-333.

[24] Wanzenried, G. (2000): "On the strategic use of corporate debt: empirical evidence from Europe", mimeo, University of Bern.