Finite Formulation in Parallel Computation. Application to Electromagnetic Field in 3-D

Adrián de Pablo Sánchez, José Miguel Monzón-Verona, Leopoldo Simón Rodríguez, Pablo Gonzalez Domínguez
Departamento de Ingeniería Eléctrica
Instituto Universitario de Microelectrónica Aplicada
Las Palmas de Gran Canaria, Spain

***Abstract*—The Cell Method is a proposed numerical solution for electromagnetic field applications. It is based on Finite Formulation, so it is directly related to experimentation due to the global variables attached to the points, lines, surfaces and volumes of this theory. In this project, the Cell Method is used to simulate a real electromagnetic problem: a coil with a cylindrical metallic core and the air of their surroundings model. This paper shows the time consume problem faced during the pre-processing of the information obtained in the simulations and the proposed solution by the parallelization of the algorithms used. The preliminary results of the parallel pre-processing will be shown and compared with the serial procedure ones.**

***Keywords*—Cell Method; electromgnetic field; pre-processing; parallelization; MPI; speedup.**

I. Introduction

The Cell Method (CM) is a numerical method that uses discrete formulation of field equations [1]. It is based on the use of global variables and a pair of cell complexes, a primal and a dual, where each element has an orientation that can be internal or external.

This method associates physical properties of electromagnetism in form of global variables, commonly known as integral variables, with spatial and temporal elements such as points (P), lines (L), surfaces (S), volumes (V), instants (I) and time intervals (T), the basic elements of this theory [2].

Incidence matrices are built with the incidence numbers between two elements of different dimension of a cell complex. If the elements are not incident, the incidence number is zero. If they are incident and their orientations are compatible, the incidence number is 1. If they are incident but their orientations are not compatible, the incidence number is -1. Incidence matrices can be formed between lines and points (matrix G), between faces and lines (matrix C), and between volumes and faces (matrix D).

II. Field laws in finite form

Experimentation can lead to rewrite fundamental equations of electromagnetism in a finite form. By this, global variables are linked to physical and temporal elements of this theory.

Magnetic Gauss' law:

$$\phi[\partial V, I] = 0 \tag{1}$$

Faraday's electromagnetic induction law:

$$\varepsilon[\partial S, T] = \phi[S, I^-] - \phi[S, I^+] \tag{2}$$

Faraday's electrostatic induction law/electric Gauss' law:

$$\psi[\partial \tilde{V}, \tilde{I}] = Q^c[\tilde{V}, \tilde{I}] \tag{3}$$

Maxwell - Ampère's law:

$$F_m[\partial \tilde{S}, \tilde{T}] = \psi[\tilde{S}, \tilde{I}^+] - \psi[\tilde{S}, \tilde{I}^-] + Q^f[\tilde{S}, \tilde{T}] \tag{4}$$

Conservation of charge:

$$Q^f[\partial \tilde{V}, \tilde{I}] = Q^c[\tilde{V}, \tilde{I}^-] - Q^c[\tilde{V}, \tilde{I}^+] \tag{5}$$

III. Application

Finite formulation, and specifically CM, can be applied to solve physical problems based on electromagnetism. In this case, a coil model is being simulated to obtain the results of different electromagnetic equations form its entire domain.

The simulations have been realized with an Intel Core i7 - 3820, of 3.6 GHz frequency, 32 GB memory, 4 cores and 8 threads. It uses a NVIDIA GM107 (GeForce GTX 750 Ti) GPU with 2 GB.

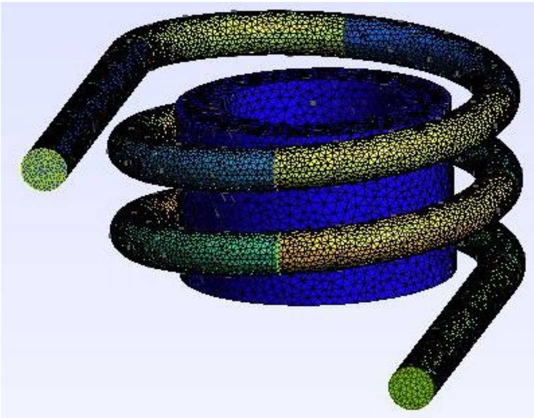

Fig. 1. 3D simulation of a coil with a cylindrical metallic core

Table 1. Speedup of parallel pre-processing algorithms using a serial algorithm as reference

Nodes	**Lines Generator**		**Surfaces Generator**		**C Matrix**		**D Matrix**	
	2 processors	**3 processors**	**2 processors**	**3 processors**	**2 processors**	**3 processors**	**2 processors**	**3 processors**
1,465	27.517%	46.309%	2.260%	27.119%	37.978%	56.459%	41.167%	62.002%
2,893	32.414%	39.310%	23.684%	43.421%	49.821%	64.686%	48.009%	64.760%
5,818	34.672%	43.066%	34.557%	49.541%	44.387%	61.708%	49.101%	64.949%
9,778	38.584%	49.855%	43.738%	51.963%	48.350%	63.015%	49.878%	65.741%
19,336	45.040%	48.922%	47.418%	54.484%	47.509%	65.197%	47.330%	64.751%
25,534	49.445%	50.264%	50.070%	54.008%	44.563%	67.330%	49.237%	65.467%
36,059	49.316%	51.882%	37.485%	52.337%	47.662%	63.823%	47.620%	63.904%

The 3D simulation of a coil model has been implemented in Gmsh, a three-dimensional finite element grid generator software with a build-in CAD engine and post-processor [3].

As it can be seen in Fig. 1, the simulation of the system includes a coil, a cylindrical metallic core and the surrounding air inside a cubic volume.

First time, this pre-processing was done following a serial philosophy. However, to improve the detail of the simulations, it is necessary to increase the number of points and physical elements in general. The number of nodes of the simulations varied from 1,465 to 36,059 nodes. This increase of elements leads to an exponential escalation of the pre-processing programs' execution times, from 10.423 seconds to 5329.586 seconds. As a result, a parallel adaption of the pre-processing algorithms was implemented using MPI (Message-Passing Interface) inside PETSc environment [4].

IV. Results

The objective of this study is to reduce the execution time of the pre-processing of a model by the parallelization of its algorithms.

The results of the parallel implementation of the 3D coil pre-processing algorithms can be seen in Table 1. The achieved speedup generally varies from 30% to 65%. Normally, speedup improves with the increase of the number of nodes and the number of processors.

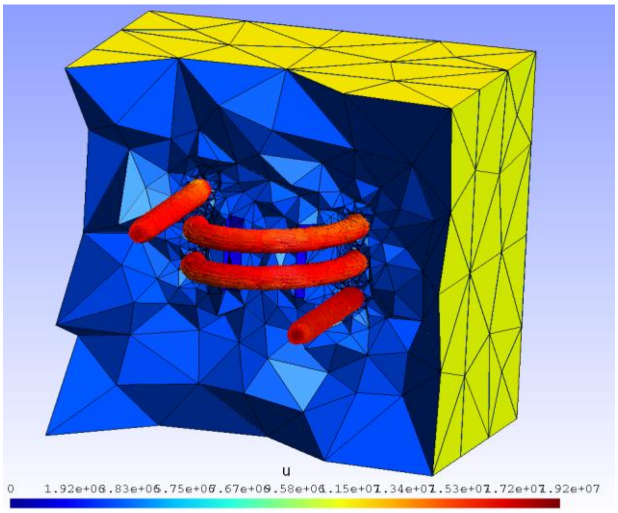

Fig. 2. Current density view obtained in post-processing

The final results of the simulations realized on the coil model can be seen in Fig. 2, a view of the current density along the coil when electric potential is applied to its extremes. This image is obtained from the post-processing of the model.

V. Conclusion

The parallelization of pre-processing algorithms for electromagnetic model simulations based on CM has been implemented.

This parallel philosophy has lead to speedups generally higher than 40%, up to 65%. This achievement allows simulations to progress in detail and in lower execution times. Future simulations on a computer with a higher number of processors will probably show even better results of parallelization. Further work will try to improve parallelization algorithms, what could reduce execution times even more.

References

[1] E. Tonti, "On the Geometrical Structure of Electromagnetism", Gravitation, Electromagnetism and Geometrical Structures, pp.281-308 1995.

[2] E. Tonti, "A Direct Discrete Formulation For The Wave Equation", International Conference on Theoretical and Computational Acoustics, May 1999.

[3] C. Geuzaine and J.-F. Remacle, " Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities", International Journal for Numerical Methods in Engineering 79(11), pp. 1309-1331, 2009.

[4] S. Balay, W. D. Gropp, L. C. McInnes and B. F. Smith, "Efficient Management of Parallelism in Object Oriented Numerical Software Libraries", Modern Software Tools in Scientific Computing, pp 163-202, 1997.

[5] M. Repetto, and F. Trevisan, "Global formulation for 3D magneto-static using flux and gauged potential approaches", International Journal for Numerical Methods in Engineering, 2002.

[6] M. Bullo, F. Dughiero, M. Guarnieri, and E. Tittonel, "Nonlinear Coupled Thermo-Electromagnetic Problems With the Cell Method", IEEE Transactions On Magnetics, Vol. 42, No. 4, April 2006.

[7] M. Bullo, V. D'Ambrosio, F. Dughiero, and M. Guarnieri, "A 3-D Cell Method Formulation for Coupled Electric and Thermal Problems", IEEE Transactions On Magnetics, Vol. 43, No. 4, April 2007.

[8] S. McFee, and D. Ma, "Generalized h-p Triangles and Tetrahedra for Adaptive Finite Element Analysis in Parallel Processing Environments", IEEE Transactions On Magnetics, Vol. 40, No. 2, March 2004.

[9] C. Fu, "Parallel Finite Element Computation Based On Coarse-Fine Mesh Formulation", International Conference on Computer Science and Software Engineering, 2008.