Structural Robust Design Optimization of Steel Frames with Engineering Knowledge-based Variance-reduction Simulation

David Greiner, José M. Emperador, Blas Galván and Gabriel Winter

Abstract**—** Structural frames robust optimum design under uncertain loads is handled simultaneously minimizing the constrained mass (adding structural mass and constraint average distribution), as well as the constraint violation distribution standard deviation, using the non-dominated sorting genetic algorithm NSGA-II. The consideration of external loads as random variables is handled by the use of Monte-Carlo simulations for each structural candidate solution. A variance-reduction inspired simulation procedure based in engineering design knowledge is proposed and applied in a test case, allowing a high computational cost reduction without harming the non-dominated front quality. Results obtain a solution set that allow selecting minimum mass optimum designs and maximum robustness for external load uncertainty.

Keywords**—** Frames, Monte Carlo Simulation, Multiobjective Optimization, Steel Structures, Structural Optimization, Robust Design.

I. Introduction

THE consideration of uncertainties in optimum design has been a focus on the research activities of a wide range of engineering fields, such as aeronautic or structural optimum design applications, see e.g. [1].

When dealing with optimum design constraint problems, as is often usual in the structural engineering field (constraints in terms of stresses, displacements or buckling considerations), the optimum solutions lie in the border of the feasible region, which is limited by the constraints. That is, those solutions that fit more accurately the constraints are the optimum ones. Here, the deterministic optimum design (where no uncertainties are taken into account) serves as reference for the robust optimum design (where some parameters are random variables). From an engineering interest point of view, the solutions set of robust design are related with the deterministic optimum design. It is taken advantage of this engineering knowledge-based information, proposing a variance-reduction Monte Carlo simulation technique. It allows reducing the number of structural fitness function evaluations.

The multiobjective optimization of the constrained mass and the constraint violation standard deviation is performed using evolutionary multiobjective algorithms [2], [3], [4], concretely, the non dominated sorting genetic algorithm, NSGA-II [5], is used.

In section 2, the handled structural problem is described, both in terms of the deterministic and robust design optimization. Section 3 shows the frame test case used in this paper. Then section 4 describes the standard and reduced procedure results, showing the advantages of our proposal. Finally, section 5 ends with the conclusions.

Manuscript received October 31, 2010. This work was supported in part by the research project ULPGC08-009, and UNLP08-3E.2010 of Secretaría de Estado de Universidades e Investigación, Ministerio de Ciencia e Innovación (Spain) and FEDER.

The authors are with the Institute of Intelligent Systems and Numerical Applications in Engineering (SIANI), Universidad de Las Palmas de Gran Canaria, 35017, Spain (phone: 928-451925; fax: 928-451921; e-mail: dgreiner@iusiani.ulpgc.es).

II. The Structural Problem

A. Deterministic Design

Discrete optimization of bar structures using evolutionary algorithms was introduced in [6] and the first application of multiobjective evolutionary algorithms in structural engineering was in [7].

No uncertainties (that is, no random variable consideration) are taken into account in the deterministic design problem. The fitness function, in order to perform the constrained mass minimization, has to consider the proper requirements of the bar structure to fulfil its function. Its value is directly related with the acquisition cost of raw material of the metallic frame. The information needed by the fitness function is obtained through a finite element code and the applied constraints in order to guarantee the appropriate functionality of the structure are defined in terms of stresses, compressive slenderness and displacements:

Stresses of the bars, where the limit stress depends on the frame material and the comparing stress takes into account the axial and shearing stresses by the shear effort, and also the bending effort. For each bar, (1) has to be accomplished.

$$\sigma_{co} - \sigma_{\lim} \leq 0 \qquad (1)$$

Compressive slenderness limit, for each bar where the buckling effect is considered (depending on the code used it could have different values); (2) has to be satisfied.

$$\lambda - \lambda_{\lim} \leq 0 \qquad (2)$$

Displacements of joints or middle points of bars are also a possible requirement, as observed in (3).

$$u_{co} - u_{\lim} \leq 0 \quad (3)$$

With these constraints, the fitness function *constrained mass*, which integrates the constraints violations as mass penalties, is shown in (4).

$$FitnessFunction = \left[\sum_{i=1}^{Nbars} A_i \cdot \rho_i \cdot l_i \right] [1 + k \cdot \sum_{j=1}^{Nviols} (viol_j - 1)] \quad (4)$$

Where:
A_i = area of the section type of bar i; ρ_i = density of bar i; l_i = length of bar i; k = constant that regulates the equivalence between mass and restriction (suitable values around the unity order); $viol_j$ = for each violated restriction j, is the quotient between the violated restriction value (stress, displacement or slenderness) and its reference limit.

B. Design including Uncertainties

The deterministic optimum design of a bar structure is defined frequently by the imposed constraints in terms of stress, displacement or buckling, which are conducted to their limit values, without surpassing them. The variation condition in loads is in real structures frequent, and it is considered in the design codes. So, a deterministic optimized structure, due to the fact that has their constraints near the limit values is expected to be more sensitive to those random variations. An analysis of those uncertainties is required to guarantee a robust design. The principal objective of robust design is to find a solution with less sensitive structural performance to the fluctuations of parameters without eliminating their variation.

The variation of the load actions which act over a structure from the viewpoint of the probabilistic or semi-probabilistic safety criteria, is associated with considering the loads as stochastic variables and to the existence of some limit ultimate states that guide to the total or partial ruin of the structure and limit service states that when achieved produce its malfunctioning. Here, in order to define the actions, it is assumed that their variation follows a Gaussian probability density function. The characteristic value of an action is defined as the value that belongs to the 95% percentile, that is, a probability of 0.05 to be surpassed, which is considered as the deterministic value in no uncertainty consideration case.

In this paper, in order to model the stochasticity of the actions, standard Monte Carlo simulations are performed considering variable distribution of the external loads. In addition, an improved strategy is proposed, and detailed in section IV.B, called reduced procedure, allowing a high reduction of the required structural fitness function evaluations.

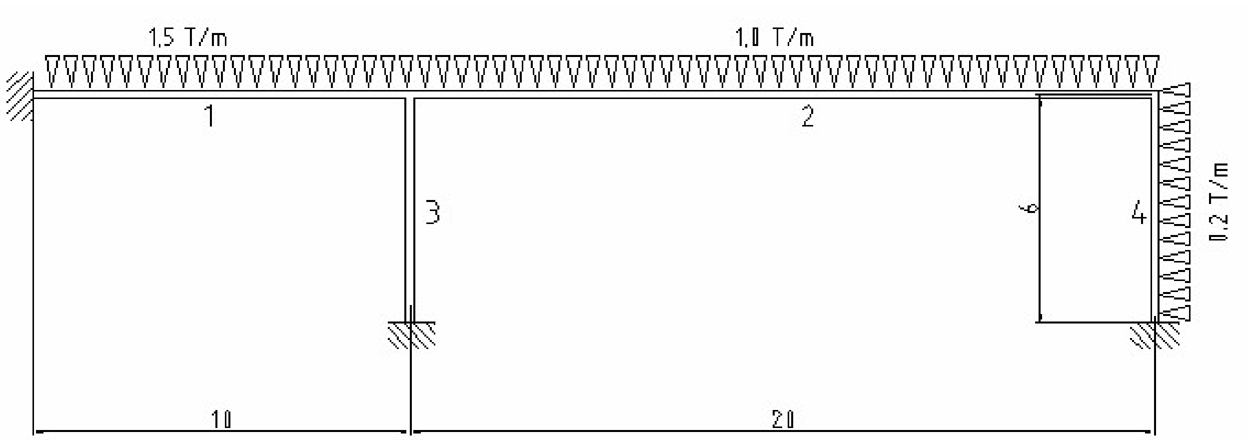

Fig.1 Structural Test Case

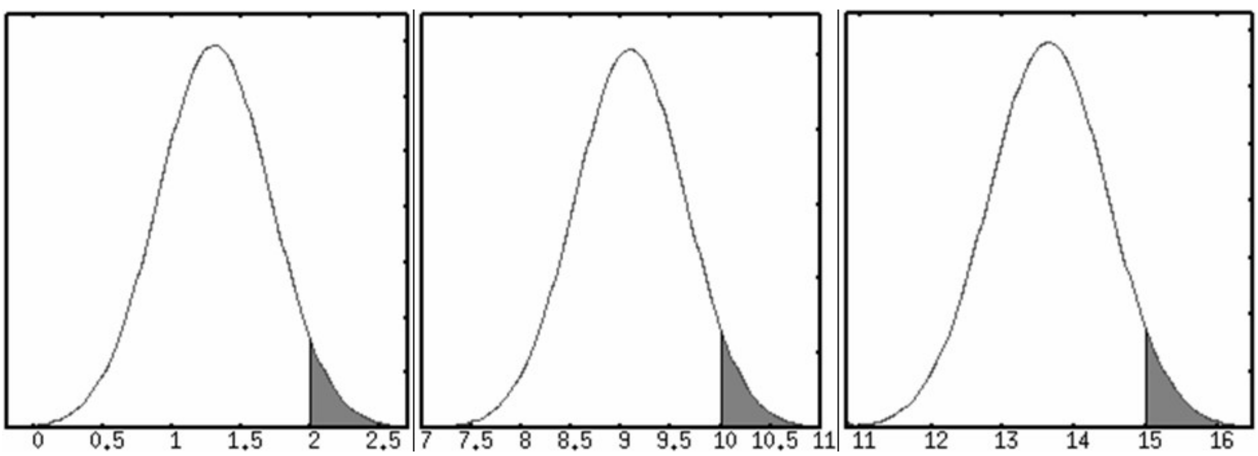

Fig. 2 External Load Distributions (Gaussian); Bars 4, 2 and 1, respectively

III. Test Case

The considered reference test case is based on a problem taken from Hernández Ibáñez [8] for single objective mass minimization using continuous variables. The solution reported in the previous reference using classical optimization methods was improved using evolutionary algorithms in Greiner et al. [9]. This last deterministic evolutionary optimum design is taken as reference in this work and compared with the robust optimum design non-dominated front.

Fig. 1 shows the test case, where lengths (10 and 20) and height (6) are in meters and the loads in T/m. (1.5, 1.0 and 0.2). There is a constraint of maximum displacement of middle point of bar 2 equal to length/300, that is 6.67 cm. It is a discrete domain problem, belonging the cross-section types to the IPE class (16 different types per bar). It has been taken into account the buckling effect, and also its own gravitational load. The considered density (7.85 T/m^3) and Young modulus (2100 T/cm^2) are common steel values and the yield stress is 235.2 Mpa.

The Monte Carlo simulation has been performed considering 30^N simulations per structural design in order to construct its constraint violation distribution, being N the number of different variables considered. Here the simulated variables correspond to the linear uniform loads of the frame structure which are three, belonging to each loaded bar (1, 2 and 4).

The distribution of each linear uniform load is simulated through a Gaussian distribution, which is calculated considering the test case load value as the characteristic value and its coefficient of variation being 6,1% for the vertical loads (bars 1 and 2) and 30,5% for the lateral load (bar 4). Their distributions are graphically represented in Fig. 2.

IV. Results and Discussion

A. Standard Procedure

Results based in this standard procedure are reported successfully in [10]. Ten independent executions were performed for each multiobjective evolutionary algorithm. A population size of 200 individuals, uniform crossover, uniform mutation rate of 0.06, and a stop criterion of 100 generations were considered in all cases. Results are graphically represented in Fig. 3, using the NSGA-II algorithm. The x-axis belongs to the constrained mass value (in kg), obtained by adding the mass of the particular structural design and the average of the constraints violation distribution in terms of mass. The y-axis belongs to the standard deviation of the constraints violation distribution in terms of mass. In Fig. 3a, the accumulated final non-dominated fronts of the ten executions are represented; on the other hand, in the right part, the total corresponding non-dominated front of each algorithm is depicted. Fig. 3b shows the final non-dominated front evaluated from the accumulated total number of executions performed.

A total of twelve different frame structural designs compose the obtained Pareto optimal front. Due to the stochastic modeling of Monte Carlo simulations, it is possible to achieve different values of the objective functions for a single design. However, the differences among them are minor, indicating the suitable performance of the stochastic load simulation.

NSGA-II is capable to locate the extreme frame structural design solutions. The number of final Pareto front designs located considering the non-dominated front obtained by accumulating the whole solution set is 14. The algorithm also found all the twelve design solutions (as can be seen in Fig. 3b).

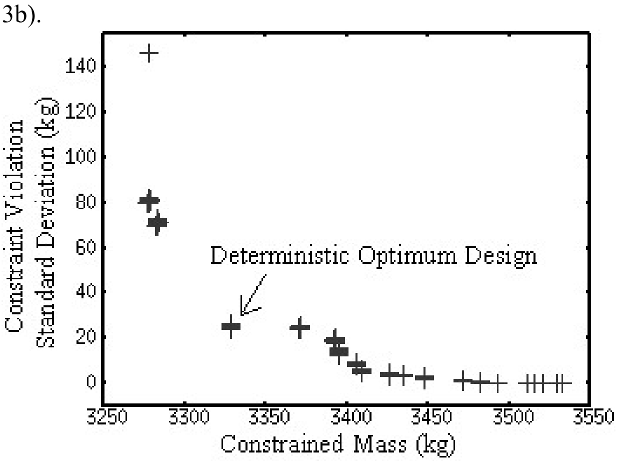

Fig 3a Accumulated Non-Dominated Fronts of each of the ten independent executions by NSGA-II

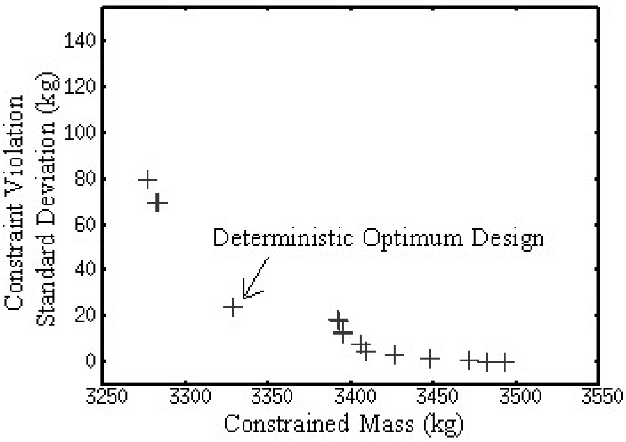

Fig 3b Non-dominated solutions of accumulated Non-Dominated Fronts of each of the ten independent executions by NSGA-II

The deterministic optimum design, whose loads have the characteristic value of the imposed Gaussian distributions, has no constraint violations and it has a mass of 3324.3 kg. Its correspondent design under uncertain loads is highlighted in bold type in Table 1. When the robust design is considered including the load variations, it is observed, that this design violates the constraints in certain occasions, being the standard deviation of its distribution of 24.1 kg. and its mean of 3.8 kg. Therefore, the engineer or decision-maker, should select an individual among this deterministic optimum design and the most right solution of the front, which has no constraint violations at all, even in the stochastic case.

B. Reduced Procedure

The standard simulation procedure described in the previous section implies the calculation of 30^3=27000 structures for each evolutionary computation solution candidate in the handled test case of this paper. In this section, a variance reduction procedure inspired in the well known Importance Sampling Monte Carlo variance reduction technique [11] and based in engineering design knowledge is introduced to diminish the computational effort without losing solution quality.

Each structural evaluation is necessary in order to calculate the possible constraints produced by each load case and therefore, their contribution value to the constraint distribution, whose standard deviation is chosen as structural robustness measure of each structural design: the structure with the lowest mass that has zero standard deviation corresponds to the most right solution of the Pareto Front; on the contrary, the structure with the highest standard deviation and lowest mass corresponds to the most left solution of the Pareto Front.

Considering the constant load distribution as the characteristic value (that is, the value belonging to the 95% percentile) of the Gaussian model, it is proposed for each structural design to evaluate only those load cases that surpass at least one of its characteristic values (1.96, 9.8 and 14.7 kN/m, as shown in Fig. 2. Therefore, only 1-0.95^3 = 0.142625, equal to 14.2625% of 27000 (3850 structures) are needed to be evaluated with this reduced simulation procedure. The benefit in terms of computational cost is of 16% in NSGA-II (slightly higher compared versus the fitness function saving due to the inherent cost of the evolutionary algorithm). The rest of cases are estimated to have null contribution to the constraint distribution. With this assumption, the obtained results are described as follows.

A total of thirty-three different frame structural designs compose the obtained Pareto optimal front. They are detailed graphically in Fig. 4. The number of final Pareto front designs located considering the non-dominated front obtained by accumulating the whole solution set is 33 (as can be seen in Fig. 4b).

Comparing these results with the standard procedure ones, we can observe that the reduced simulation procedure achieves a wider front. It can be seen both in terms of number of obtained structural designs (33 versus 12) as well as in terms of numerical values (the left non-dominated solutions reach 781.7 kg and 2999.9 kg in terms of standard deviation and constrained mass average, respectively, versus the standard distribution most left values of 79.5 kg. and 3277.1 kg, respectively). As the contribution to the constraint distribution is limited to the cases where the load characteristic value is surpassed, there are structures that have more reduced constraint average (it is added to the structural mass and considered in the x-axis fitness function value) than in the standard simulation (where they were dominated designs) and therefore they appear as new non-dominated solutions in the front.

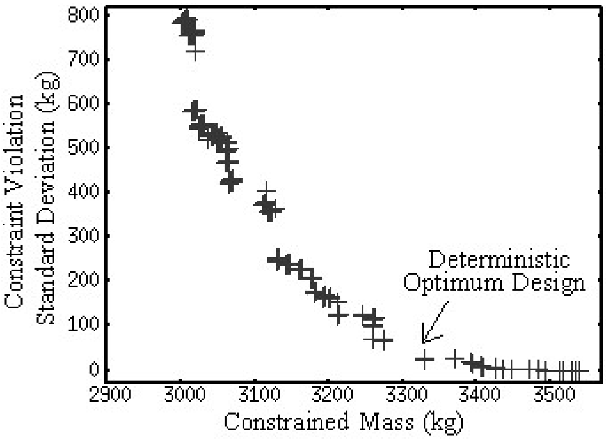

Fig. 4a Accumulated Non-Dominated Fronts of each of the ten independent executions by NSGA-II. Reduced Distribution.

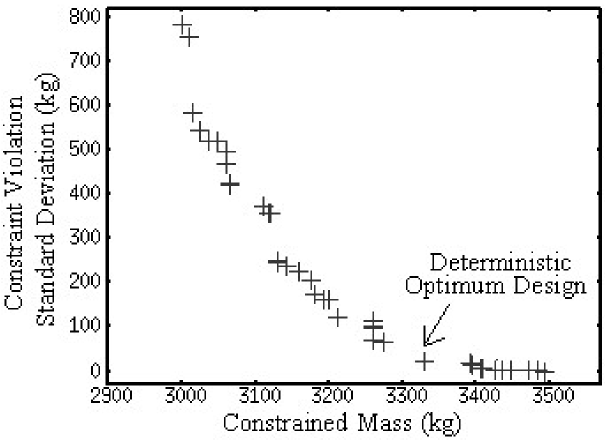

Fig. 4b Non-dominated solutions of accumulated Non-Dominated Fronts of each of the ten independent executions by NSGA-II. Reduced Distribution.

However, if the engineer or decision-maker preferences are taken appropriately into account in this structural problem, both procedures obtain equivalent non-dominated frame designs. He has to choose a structural design between two extremes:

1) The most right extreme: The solution of the front (highest constrained mass and null standard deviation) that represents the structural design which despite of the uncertainty of the loads, has none constraints violation.

2) The most left extreme: The solution of the front that in the case there were no load uncertainties, has the lowest mass and no constraints violation, which corresponds to the structural design coincident with the deterministic optimum.

Considering this, both procedures produce identical non-dominated fronts when the useful functional space is restricted to those abovementioned extremes, as can be seen in Fig. 5.

This is explained because both simulation procedures (standard and reduced) produce identical constraint distributions in those non-dominated structural designs where the characteristic load values are surpassed. Indeed, the first design where both non-dominated front procedures are coincident, becomes this deterministic optimum design solution; so, comparing both distributions could lead to a method that achieves the deterministic optimum design in case of no load uncertainties. The slight variations in the depicted solutions are due to the stochastic nature of the Monte Carlo simulation, but the proximate points represent the same structural design.

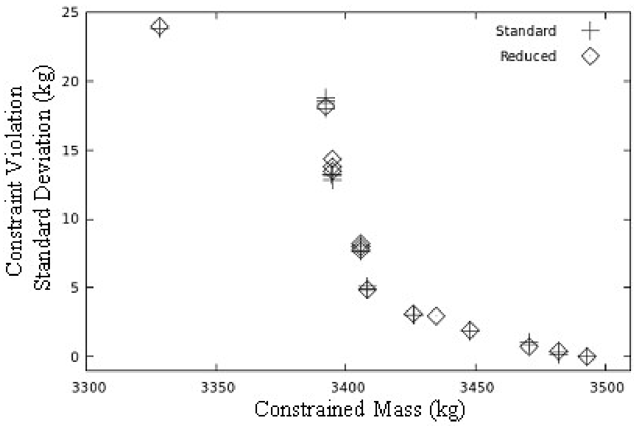

Fig. 5 Final Non-Dominated Fronts obtained by the Standard (crosses) and Reduced (rhombus) distributions, starting with the deterministic optimum solution at top left. Both are coincident.

The detailed numerical values of the non-dominated structural frame designs corresponding to the reduced procedure shown in Fig. 5, are in Table 1.

V. Conclusion

Robust optimum design of frame structures with real discrete cross-section types has been handled successfully in this paper, having considered the modeling of uncertain loads by Monte Carlo simulation and the multiobjective optimization using NSGA-II. Moreover, a variance-reduction inspired simulation procedure is proposed, which allows reducing the structural evaluations required and indeed the computational cost, drastically (85% in the handled test case), obtaining equivalent non-dominated solutions considering appropriately the engineer or decision-maker preferences, showing additionally that significantly computational cost gains can be achieved by the use of appropriate Monte Carlo variance reduction techniques, which constitutes a promising research line for the near future.

Dealing with uncertainty has been performed including all the possible factors of constraints violation (stresses, displacements and buckling effect) into the final designs. Two objectives were simultaneously minimized: first, the constrained mass, by adding the mass of the structure and the average of constraints violation penalty distribution; and second, the standard deviation of the constraints violation

TABLE I
Detailed Non-Dominated Structural Frame Designs of Fig. 5 (Reduced Procedure)

Constrained Mass (kg)	Constraint Violation Standard Deviation (kg)	Constraint Violation Average (kg)	Cross Section Type Bar 1	Cross Section Type Bar 2	Cross Section Type Bar 3	Cross Section Type Bar 4
3328.26	**24.002**	**3.946**	**IPE330**	**IPE500**	**IPE450**	**IPE500**
3328.27	**23.806**	**3.955**	“	“	“	“
3392.12	18.823	2.335	IPE400	IPE550	IPE220	IPE450
3392.13	18.182	2.347	“	“	“	“
3392.21	18.018	2.427	“	“	“	“
3394.53	13.249	1.285	IPE360	IPE550	IPE300	IPE450
3394.55	13.202	1.305	“	“	“	“
3394.59	13.162	1.346	“	“	“	“
3394.61	12.869	1.364	“	“	“	“
3405.85	8.056	0.516	IPE330	IPE500	IPE500	IPE500
3405.87	7.635	0.536	“	“	“	“
3408.38	4.867	0.222	IPE400	IPE550	IPE160	IPE500
3408.41	4.832	0.252	“	“	“	“
3426.17	3.047	0.114	IPE400	IPE550	IPE180	IPE500
3426.18	3.002	0.123	“	“	“	“
3434.8	2.984	0.106	IPE360	IPE550	IPE330	IPE450
3447.76	1.861	0.041	IPE400	IPE550	IPE200	IPE500
3470.81	0.692	0.014	IPE400	IPE550	IPE220	IPE500
3482.26	0.202	0.003	IPE360	IPE550	IPE360	IPE450
3492.94	0	0	IPE400	IPE550	IPE160	IPE550

ISBN: 978-960-474-251-6

penalty distribution.

A well known frame test case has been solved, obtaining a non-dominated final front, where an optimum design (minimal mass) without any constraint violation despite the uncertainty of the external loads is achieved. Their solution designs have been also compared with the deterministic optimum design, which is also included in the final front. In this test case, the consideration of the five percent excess over the characteristic load has guided to an increment of 5% in structural mass. It is an indicator of the failure probability allowed in the used limit state theory.

REFERENCES

[1] Y. Tsompanakis, N.D. Lagaros, M. Papadrakakis, "*Structural Design Optimization Considering Uncertainties*", Structures & Infrastructures Series, Vol. 1, Series Ed: Frangopol D, Taylor & Francis, USA, 2008

[2] C.A. Coello Coello, "Evolutionary Multi-Objective Optimization: A Historical View of the Field", *IEEE Computat. Intellig. Mag*., 1, 28-36, 2006.

[3] C.A. Coello Coello, D. Van Veldhuizen, G. Lamont, *"Evolutionary Algorithms for solving multi-objective problems"*. Kluwer Academic Publishers - GENA Series, 2nd Edition, 2008.

[4] K. Deb, *Multiobjective Optimization using Evolutionary Algorithms*. John Wiley & Sons -Series in Systems and Optimization-, 2001.

[5] K. Deb, A. Pratap, S. Agrawal, T. Meyarivan, "A fast and elitist multiobjective genetic algorithm NSGA-II", *IEEE Transactions on Evolutionary Computation*, Vol. 6(2), 2002, pp. 182-197.

[6] S. Rajeev, C.S. Krishnamoorthy, "Discrete Optimization of Structures using Genetic Algorithms", *Journal of Structural Engineering*, 118, 5, 1233-1250, 1992.

[7] P. Hajela, CY. Lin, "Genetic search strategies in multicriterion optimal design", *Structural Optimization*, Vol. 4, 99-107, 1992.

[8] S. Hernández Ibáñez, "*Structural Optimal Design Methods*", Seinor Collection, Col. Ingenieros Caminos, Canales y Puertos, Madrid, 1990 (in Spanish).

[9] D. Greiner, G. Winter, J.M. Emperador, "Optimising Frame Structures by different strategies of genetic algorithms", *Finite Elements in Analysis and Design*, Vol. 37(5), pp. 381-402, 2001.

[10] D. Greiner, J.M. Emperador, B. Galván, G. Winter, "Robust Design of Frames under Uncertain Loads by Multiobjective Genetic Algorithms", *The Eighth International Conference on Computational Structures Technology*, Civil-Comp-Press, 2006.

[11] G.S. Fishman, "*Monte Carlo: Concepts, Algorithms and Applications*", Springer Series in Operations Research, New York, Inc, 1996.

ISBN: 978-960-474-251-6