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High-resolution frequency determination of tidal components in coastal currents

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Abstract

The ability to resolve closely spaced frequencies of two high-resolution AR spectral methods, the Burg´s and Marple´s approaches, is examined by using time series of coastal currents measured in waters of Canary Islands. We emphasise their usefulness to resolve tidal harmonic components with close frequencies and low frequency components.

1 Introduction

In physical oceanography, like in many other branches of science and engineering, it is very common to handle time series data from field observations. A basic procedure for extracting information from experimental records is to transform the sequences into the frequency domain and make use of the resultant spectrum, to search for hidden periodicities in time series and to investigate the physics of the underlying phenomena generating the observed data.

The widespread application of spectral analysis has given rise to several spectral estimation methods. Each one of these methods has its own advantages, drawbacks and uncertainties, in terms of various properties of the spectrum estimator, such as the consistency (variance and bias) and the ability to resolve closely spaced frequency components (spectral resolution), among others. Furthermore, each basic procedure has various different specific techniques.

Thus, an important question arises: What is the best spectral estimation procedure we can select for a given application? The answer is not easy and is often provided by experience. The spectral estimates of a given process are

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usually computed by using the well known conventional, or non-parametric, techniques. That is, by applying the Blackman-Tukey, or the Fast Fourier Transform methods. However, due to the inherent variance of the raw spectral density function computed by these methods, it is often prescribed to smooth the resulting spectral estimates by applying some arbitrarily chosen spectral window or averaging procedure. The frequency resolution of the resulting spectral density function is thus drastically reduced. On the other hand, the frequency resolution of these methods is critically dependent on the time duration of the measured time series.

Coastal currents often result from the effect of various physical forces, which set the sea in motion. These forces cover a very broad band of the frequency spectrum and may be divided in two main groups. One, including terms which often produce non-oscillatory motions, such as the drag of the wind on the sea surface, changes in atmospheric pressure, and density gradients due to non-uniform salinity or temperature distributions. In contrast, another set of physical phenomena produces oscillatory motions. This second group includes gravitational tides, caused by the regular movements of the Earth-Moon-Sun system, the meteorological tides, also named as radiational tides because their periods are directly related to the solar day, and the shallow water tides, generated by non-linear hydrodynamic effects in waters of finite depth. Nevertheless, currents in most coastal regions are dominated by astronomical tides, which energy is split among several frequencies but is usually dominated by diurnal and semidiurnal periods in a relative proportion varying with the local tidal and meteorological conditions.

So, in analysing coastal current records, an important problem emerges when it result necessary to extract with high accuracy some spectral components close in frequency, such as in the case of some tidal components which can be very close in frequency. Furthermore, as stated above, this problem is enhanced when the observed time series are extended over short periods of time, which is the normal case when working with the available records of coastal currents from a given location for practical objectives. This fact is particularly important in the low frequency band, which is normally of great interest in coastal engineering.

Due to the above mentioned drawbacks the Blackman-Tukey and FFT methods often result impractical for this and many other applications. To overcome these restrictions presented by the non-parametric spectral methods, many parametric methods have been proposed, which are very effective for extracting frequency components from relatively short time series, without zero padding techniques, and eliminate the need of windowing or smoothing procedures to stabilize the spectral density estimates. However, these methods have also its own advantages and disadvantages. The parametric methods can be classified as autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA), in terms of the linear model used to represent the time series being studied. Among these the AR spectral methods, sometimes

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referenced as maximum entropy spectral methods or high-frequency resolution estimators, are the most used in practice.

2 Autoregressive Spectral Methods

The main step in developing an AR model for a given time series is the determination of the AR parameters. Various techniques have been proposed to reach this goal, giving rise to different AR spectral methods. In this study, we investigate the effectiveness of two parametric (AR) approaches to obtain highly resolved and stable spectral estimates from short time series of coastal currents. These are the Burg method, (Burg, [1]) which is, probably, the most popular one among the AR procedures, and the Marple approach, (Marple, [2]). Both methods are based on the assumption that an AR model may be adequately fitted to data. The spectrum of this AR model is considered as the spectrum of the data.

The principle of AR methods is to fit the observed time series $\{x_t\}$ to a P order *AR* model, *AR (p)*, represented by

$$x_t = -\sum_{m=1}^{p} a_m x_{t-m} + w_t \tag{1}$$

where a_m are the AR coefficients and w_t is the input to the *AR* linear model, generally a white noise with variance σ_w^2. Multiplying each term of eq. (1) by x_{t-k} and taking expectations of each term we may write

$$E[x_t x_{t-k}] = -\sum_{m=1}^{p} a_m E[x_{t-m} x_{t-k}] + E[w_t x_{t-k}] \tag{2}$$

Thus, assuming that $\{x_t\}$ has a zero mean value, we obtain the following relationship between the autocorrelation sequence and the AR parameters

$$R(k) + \sum_{m=1}^{p} a_m R(k-m) = \sigma_w^2 \delta_k \tag{3}$$

where δ_k is the Kronecker delta. This equation is often termed the extended AR Yule-Walker equation and can be expressed in matrix form as.

$$\begin{bmatrix} R_0 & R_1 & \cdots & R_p \\ R_1 & R_0 & \cdots & R_{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ R_p & R_{p-1} & \cdots & R_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_w^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

The most obvious procedure to estimate the parameters a_m and σ_w^2 is to substitute the true unknown autocovariances by their biased estimates. This approach, referenced as the Yule-Walker (YW) estimation method or as the autocorrelation method, results very appealing due to the Toeplitz form of the

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autocovariance matrix, which makes possible the efficient estimation of the AR parameters by using the Levinson´s algorithm. Unfortunately, the use of the biased or unbiased autocorrelation estimates gives rise to problems. Thus, unbiased autocorrelations may produce non positive definite covariance matrices so that the matrix inversion can not be done. On the another hand, biased autocorrelations eliminate this risk, but at expenses of a degradation of the AR spectral resolution and a shifting of spectral peaks from their true location (Marple, [3]). Furthermore, this method assumes a zero value for the data outside of the observed sample. Then, the spectral resolution is drastically reduced for short data records. Besides, spectral estimations obtained through this approach can produce spectral line splitting (Kay and Marple, [4]). These drawbacks have induced the development of alternative techniques to estimate the AR parameters.

2.1 Burg´s method

The most popular procedure to estimate the AR parameter is that introduced by Burg [1]. This method is often named as the maximum entropy (ME) method because it makes use of the maximum entropy principle to extrapolate the autocorrelation function for lags $m>p$. In other words, given a finite sample of a random process, the extrapolated autocorrelation function is consistent with the observed data and maximizes the randomness of the process. Thus, the Burg´s method do not consider the time series information to be zero outside the interval in which it was measured, such as is done in conventional and YW methods. As a consequence, this approach provides a much higher spectral resolution. In fact, the ME procedure has no limit on spectral resolution other than that imposed by the signal/noise constraints, Marple [3].

Using the Yule-Walker equation (4) carries out extrapolation but in contrast to the YW method, the AR coefficients are not estimated directly from the data. Burg assumed that x_t can be estimated by a weighted sum of m previous observations and a weighted sum of m future observations, using the same weights a_m in both directions. That is, he considers the following forward and backward linear predictors

$$\hat{x}_t = \sum_{m=1}^{p} \hat{a}_m \, x_{t-m} \qquad t = m+1, m+2, \;\cdots\; , N$$

$$\bar{x}_t = \sum_{m=1}^{p} \bar{a}_m x_{t+m} \qquad t = 1,2, \;\cdots\; , N-m$$

Then, the AR parameters are estimated by minimizing the sum of squares of the forward and backward prediction errors with the constraint that the entropy in the data is maximum (see, e.g., Ulrych and Bishop [5]). The solution to this constrained maximization problem is a spectrum, which correspond to the most random time series whose autocorrelation function is consistent with the observed values.

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The ME method produces more spectral estimations with higher frequency resolution than the conventional and the YW approaches. However, various authors (see, e.g., Kay and Marple [4]) have observed shortcomings such as frequency shifts of the spectral peaks and spectral line splitting.

2.2 Marple´s method

Another method to estimate the AR parameters was proposed, independently, by Ulrych and Clayton [6] and Nutall [7]. This approach, often known as the least squares (LS) algorithm, may be considered as an improvement of the Burg´s method, which seems to remove the above commented drawbacks.

In a similar way to the ME technique, in the LS method the AR parameters are estimated by means of a least squares minimization procedure which considers a criterion involving both forward and backward prediction errors minimization. However, in contrast with ME, the minimization procedure is not subjected to the constraint imposed by the Levinson´s recursion, which is equivalent to impose a Toeplitz structure for the autocovariance matrix.

Since in the LS procedure the autocovariance matrix adopts a non Toepltiz form the Levinson´s algorithm is not valid. Marple [2] derived a recursive algorithm by taking into account the special symmetric structure of the correlation matrix resulting in the LS approach, which can be decomposed into products of Toeplitz matrices.

The computational efficiency of the Marple´s algorithm is comparable to that of the Levinson´s algorithm. Furthermore, it has been observed that this method have less frequency bias and slightly better frequency resolution than the ME spectral method. Besides, it has not been observed evidence of spectral line splitting.

Once we get the AR coefficients, by applying one of the above outlined procedures, the power spectral density can be computed from

$$S(f) = \frac{2\sigma_w^2 \Delta t}{\left|1 + \sum_{m=1}^{p} a_m \exp(-i 2\pi \ f\Delta t)\right|^2} \qquad 0 \le f \le \frac{1}{2\Delta t}$$

3 Coastal Current Time Series

Coastal current time series used in this study were recorder by using an Aanderaa current meter anchored at 20 meters depth, in a place of 50 meters total depth, at the East coast (27°59'20"N, 15°21'30"W) of Gran Canaria island. The measurement period extended from 23 June (12:45h) to 22 July (13:05h), with a sampling period of 10 minutes. The complete time series for the study is shown in Fig.1 as a vector stick diagram. It has been decomposed into E-W and N-S directions, assuming the positive northward and eastward convention. The

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analysis has been developed by estimating the spectrum corresponding to each one of the resulting sequences, denoted as $u(t)$ and $v(t)$, respectively.

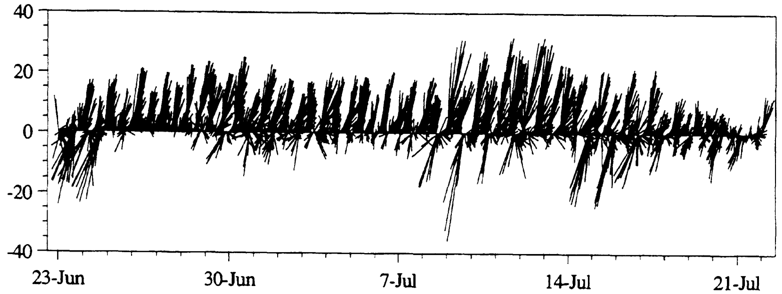

Fig.1. Vector stick diagram for the measured time series. The E-W and N-S components are the stick projections on the x and y-axis, respectively.

4 Determination of the AR Model Order

The most important problem in AR spectral analysis is the determination of the order of the model to be fitted to data. Many different order determination rules based on the error prediction variance have been suggested. However, the experimental results given by a host of authors indicate that the model order criteria do not yield definitive results. In other words, there is not a single rule, which works adequately under all conditions. So, it results apparent that in the absence of any solid criteria one should try different model orders and different criteria to look for the better criterion to select the order model in each case.

Criterion	Author/s	Expression
Final prediction error	Akaike, 1969	$FPE(m) = \frac{N+(m+1)}{N-(m+1)} S_m^2$
Akaike information criterion	Akaike, 1974	$AIC(m) = \ln(S_m^2) + \frac{2m}{N}$
Criterion autoregressive transfer	Parzen, 1974	$CAT(m) = \frac{1}{N}\sum_{k=1}^{m}\frac{N-k}{NS_k^2} - \frac{N-m}{NS_m^2}$
Bayesian criterion	Kashyap, 1977	$BC(m) = N\ln(S_m^2) + m\ln(N)$
Minimum description length	Schwartz, 1978 Rissanen, 1978	$MDL(m) = \ln(S_m^2) + m\frac{\ln(N)}{N}$
Hannan & Quinn criterion	Hannan and Quinn, 1979	$HQC(m) = \ln(S_m^2) + \frac{4m}{N}\ln(\ln(N))$

Table 1. Criteria used to estimate the order of the AR model fitted to the observed data.

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Thus, in this study, we check the ability of some commonly used criteria, given in Table 1, to select the adequate order to fit an AR model to coastal current time series. In the expressions given in table 1, S_m^2 is the estimated prediction error variance, N is the total number of data in the sample and m is the number of parameters in the *m-th* order model. In these criteria, the order that minimize the criterion is selected as adequate. Details on these criteria and references can be found in Kay [8].

5 Results and Discussion

It was mentioned in the previous section that the selection of the AR model order is a critical problem in AR modelling to estimate the spectrum of measured time series. Several authors have used the AR spectral techniques for oceanographic time series (see, e.g., Holm and Hovem [9]) and have concluded that the criteria based on the prediction error variance underestimate the order. Our results, shown in Figure 2, present the same drawbacks, but enhanced for various reasons later discussed.

The error prediction variance was estimated, for each one of the current velocity components, by using the Burg and Marple approaches. It can be seen that FPE, AIC and CAT criteria present a similar behaviour with a local minimum close to 150, for both components. Thenceforth there is a slow but progressive increase. In contrast, the another three criteria present a practically monotonic increase with a small downward jump near to 150. Clearly, a model order of 150 is a very large value. The explanation of these results is that to identify spectral components in a process with very broad band characteristics, as in this case, a very high model order is needed.

Unfortunately, such as expected, this order model do not give the desired results. Thus, as shown in Figure 3, the ME and LS methods can only resolve the more energetic tidal frequency bands, that is, the diurnal and semidiurnal bands, denoted by 1 and 2, respectively, and some constituents of higher frequency, such as the third-diurnal, M_3, forth-diurnal, M_4, etc. However, they are not able to split off the different constituents included in the diurnal and semidiurnal bands and the low frequency range appears as a broad band with a significant energy content, but without resolving any spectral peak.

The low frequency resolution reached with this large model order, relatively better but not too higher than that obtained with the non-parametric methods for the used record length, may be explained by taking into account the following considerations. First, if very closely spaced components are present in a given process, a high model order is needed for its identification. Second, it is also necessary to increase the model order to split off very low frequency spectral components. These two troubles are always present in the spectra of coastal currents, which have components of extremely low frequency. Furthermore, in addition to the nearness of the tidal constituents in a given frequency band, some

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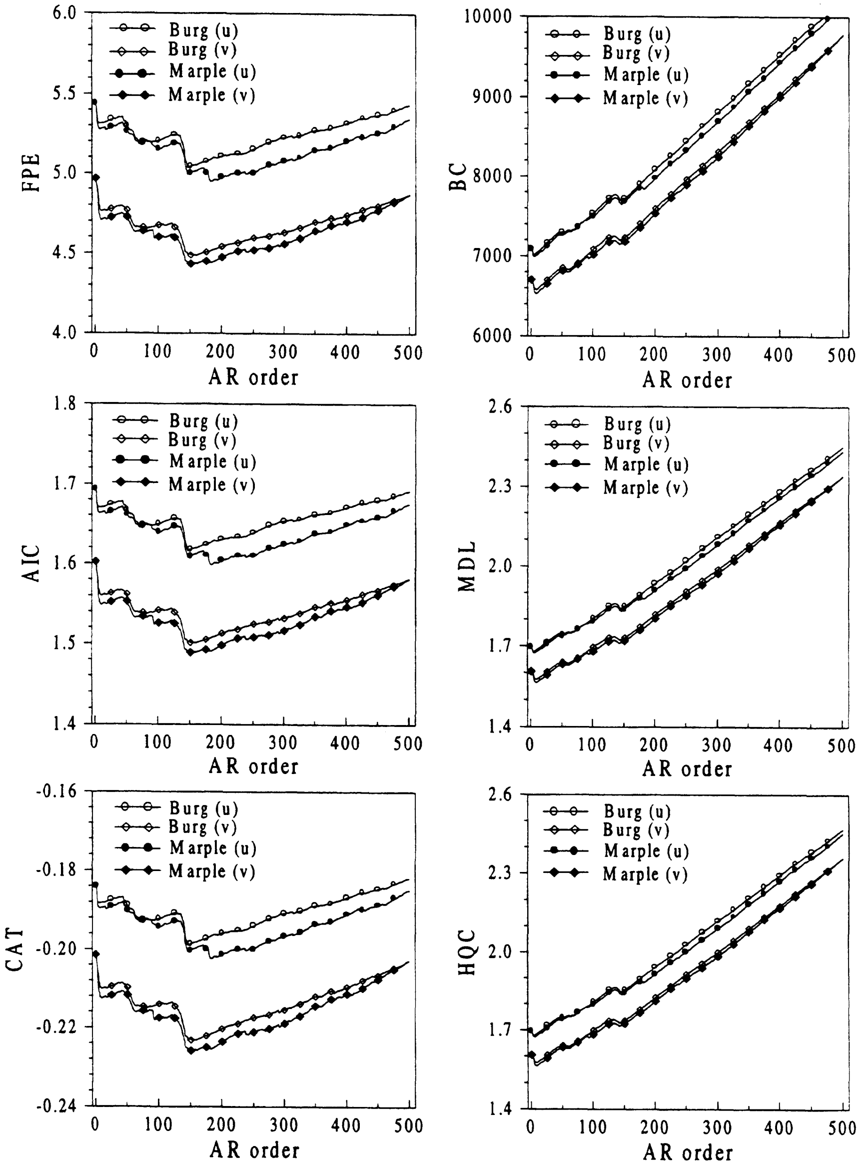

Figure 2. Criteria examined to determine the optimal order of the AR model fitted to the coastal current time series analysed.

non-tidal components may be practically overlapped, as is the case of the diurnal tidal constituents and the inertial period in the zone of study. Moreover, the low frequency band holds a host of very closely spaced spectral components mainly caused by tidal and meteorological phenomena. All this gives rise to a very complex spectral structure and makes necessary a very high order model to characterise coastal currents by means of an autoregressive model.

In an attempt to obtain a better frequency resolution, we performed many trials increasing the AR order model, taking care of possible line splitting, mainly in the Burg's method. Thus, we observed that more and more spectral components could be identified as the order increases. We stopped this procedure for order values near to one thousand. Naturally, this is an extremely large order but only with a so high order was possible to identify clearly the lunar fortnightly component, M_f, which can be guessed by observing the semi-monthly modulation present in the amplitude of the stick vectors shown in Figure 1. This fact becomes clearer by representing each velocity component independently. These graphs (not shown) reflect an evident modulation, which is stronger for the u component than for the v component. The reduction in the fourteen days modulation for the u component is probably due to the alongshore trade wind, which was blowing in the south and southwestward direction during the measurement period. Then, the u component results less affected by the wind induced stress on the sea surface, but similarly affected by the wind driven pressure fluctuations.

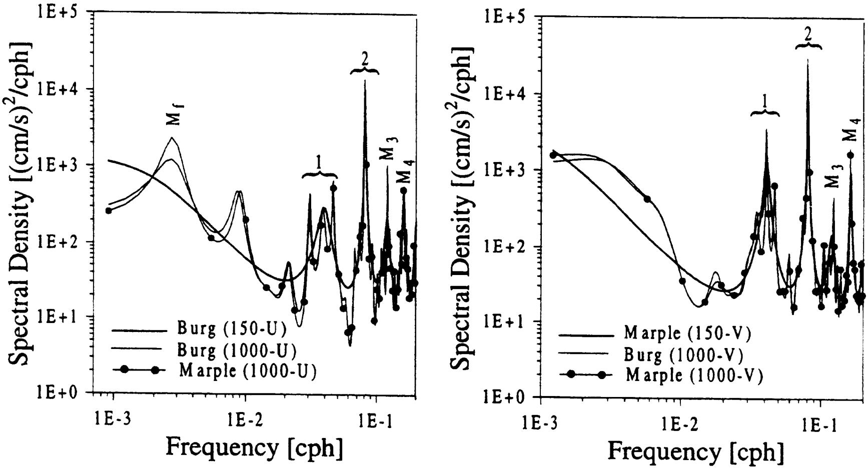

Figure 3. AR spectral estimates for the u component (a), and the v component (b) of the current velocity. Dashed lines represent the ME and LS estimations with AR models of order $p=150$. Solid line shows the ME estimations for $p=1000$, and solid dotted line stands for the LS estimate with $p=1000$.

These facts seem to be the cause of the peak observed in the spectrum of the u component with a period near to five days. This peak, and the M_f constituent, can not be resolved in the v component because, probably, they are masked

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together with the low frequency wind drag, resulting in the low frequency broad band observed in figure 3 (b). It results interesting to indicate that for a very large order model, around 500, both spectral, estimation methods are able to split off the lunar, M_2, solar, S_2, and lunar elliptic, N_2, constituents in the semidiurnal band. This fact is also true for the diurnal band, which split up in two peaks associated to the luni-solar, K_1, and lunar, O_1, diurnal constituents, and a third peak likely due to an inertial oscillation. Besides, although both methods resolve peaks successfully, by inspecting the low frequency spectral estimations for the u component, it may be noted that the Marple's method shows a slightly higher frequency resolution.

6 Summary

It has been observed that the criteria for order model selection examined do not give adequate results. This may be due to the high complexity of coastal current time series, which includes very close spectral components and considerable energy content in very low frequency ranges. On the another hand, the ME and the LS spectral methods permit to obtain high frequency resolution estimates of the coastal current sequences, particularly the second one, but very large order models are needed to reach adequate results.

References

1. Burg, J.P. Maximum Entropy Spectral Analysis *Proc. 37th Meeting, Soc. of Exploration Geophysics,* 1967.
2. Marple, S.L. A new autoregressive spectrum analysis algorithm, *IEEE Trans. ASSP,* 1980, **28**, 441-454.
3. Marple, S.L. Frequency resolution of Fourier and maximum entropy spectral estimates, *Geophysics,* 1982, **47**, 1303-1307
4. Kay, S.M. and Marple, S.L. Spectrum analysis- A modern perspective, *Proceedings of the IEEE*, 1981, **69** 1380-1419.
5. Ulrych, T.J.and Bishop, T.N. Maximum entropy spectral analysis and autoregressive decomposition, *Reviews of Geophysics and Space Physics*, 1975, **13**, 183-200.
6. Ulrych, T.J. and Clayton, R.W. Time series modelling and maximum entropy, *Physics of the Earth and Planetary Interiors*, 1976, **12**, 188-200.
7. Nutall, A.H. Spectral analysis of a univariate process with bad data points, via maximum entropy and linear predictive techniques, Naval Underwater Systems Center, New London, CT, *Technical Report* 5303, 1976.
8. Kay, S.M., *Modern Spectral Estimation*, Prentice-Hall, N.J., 1988.
9. Holm, S. and Hovem, J. Estimation of scalar ocean wave spectra by the maximum entropy method, *IEEE J. Oceanic Eng.*, 1979, **4**, 76-83.