A fully convergent joint blind equalization/phase recovery scheme for data communication systems

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A fully convergent joint blind equalization/phase recovery scheme for data communication systems *

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***Abstract*: We present a new adaptive algorithm for joint blind equalization and phase recovery in data communication systems. This algorithm is mainly based on the standard Decision Directed Algorithm (DDA) but the incorporation of both, a linear constraint in the filter coefficients and an adjustable complex parameter in the decision device, provide an unimodal error surface. This attractive performance is analyzed theoretically when faced with minimum phase channels. For the more realistic situation of non minimum phase channels exhaustive simulations show that a similar behavior can be expected.**

I. INTRODUCTION

Adaptive equalizers are currently the primary devices used by the receiver to combat the intersymbol interference (ISI). In addition to channel impairments of ISI, the receiver signal also suffer from other distortions such as frequency offset and phase jitter [1]. In the process of blind equalization, the treatment of these jointly detrimental effects becomes difficult since there is no training data available to serve as a reliable signal reference [2,3]. Specifically, the main drawback of the standard DDA is the multimodality of its error surface [4,5].

In a previous paper [6] we have proposed a new adaptive blind equalization scheme, labeled Modified Decision Directed Algorithm (MDDA), for minimum phase channels and binary transmission, which provides a globally convex error surface. Basically, our algorithm incorporates a linear constraint in the filter coefficients in order to preserve always the transmitted symbol: for a minimum phase channel we showed that the first coefficient is the responsible of the symbol transmitted and so that we keep this coefficient fixed. This idea is similar to the scheme proposed in [7] but we introduce the constraint in a DDA which provide much better performance that the constrained power minimization algorithm proposed. This result is due to the fact that the DDA improves the error reference avoiding the masking effect of the transmitting symbol. In addition, we have introduced an adjustable parameter in the decision device responsible of the equalizer gain which is shown to play an important role in the surface convexity.

This paper presents an unified analysis of adaptive blind equalization and phase recovery for QAM data. This scheme is an extension of [6] since it incorporates the recovery process into a decision device. To do that, we introduce a complex and adjustable factor capable of compensating dynamically both, the gain and rotation of the incoming signal.

With this proposal, our goal is to show analytically the convexity of the error surface. We have shown that for an arbitrary ISI distribution the MDDA may present local minima, and so that, no improvement is obtained. However, in [8] is given a new insight on the probability distribution of residual ISI, showing (for a channel with multipath fading and properly reception) that their curves approach more and more the bell-shaped characteristics with an increasing number of levels in the QAM data transmission. Identically in [9-11] is proposed a model assuming a gaussian distribution for the residual ISI; the theoretical results obtained for the Constant Modulus Algorithm (CMA) corroborate the utility of the conditioning approach. Following this model for a generic QAM transmission we are able to show that the MDDA provides a globally convex error surface for blind equalization and phase recovery.

Finally, some simulations support our proposal. However, since a minimum phase channel is not a very realistic assumption for many situations, we are involved now in the analysis of a more general kind of channel. Actually, many simulations in that way suggest that the error surface remains globally convex with independence of the phase channel characteristic for some ISI statistic. Therefore, in order to remark the improvement of the MDDA against the DDA, both algorithms will be simulated and compared in both environments: minimum and non-minimum phase channels. In the case of the standard DDA, a Phase-Locked Loop (PLL) based carrier tracking technique will be used to compensate any phase mismatching.

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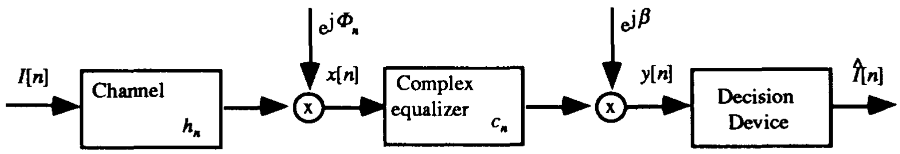

Fig. 1

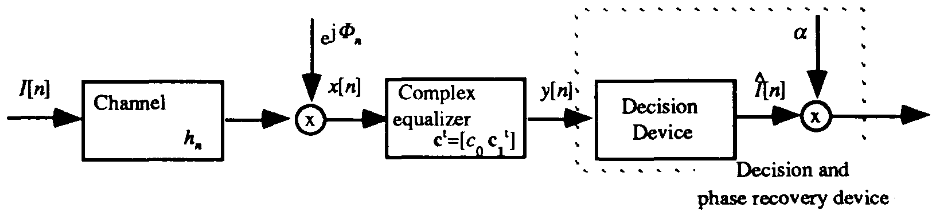

Fig. 2

II. DEFINITION OF THE ALGORITHM

We begin by introducing the generic block diagram of an equivalent low-pass standard communication system (see Fig. 1). A complex multilevel signal (QAM) is transmitted through a minimum phase channel where $h[n]$ is the equivalent low-pass impulse response. Here, Φ_n represents the phase impairment (in general this term can include a fixed phase shift, a fixed frequency offset and a random or quasi-periodic waveform that is a manifestation of phase jitter). In the receiver, a transversal complex equalizer deals with the compensation of the ISI and β is the phase estimate which intends the compensation of the phase shift, typically by means of a PLL. Finally, depending on the signal constellation, the corresponding decision device estimates the transmitted symbol.

The main idea of our modification to the standard scheme is straightforward. Considering a minimum phase channel and a fixed frequency offset Φ_0, let us assume (2.1) as the equalizer output

$$y[n] = \sum_{i=0}^{N} c_i \sum_{k=0}^{M} h_k I[n-i-k] \; e^{j(\Phi_0+\beta)} \qquad (2.1)$$

where N is the equalizer length (assumed long enough to implement the inverse channel) and M the channel impulse response length. If we split (2.1) into two parts, the transmitted symbol and the rest, we obtain:

$$y[n] = c_0 h_0 I[n] \; e^{j(\Phi_0+\beta)} + \\ + \left(c_0 \sum_{k=1}^{M} h_k I[n-k] + \sum_{i=1}^{N} c_i \sum_{k=0}^{M} h_k I[n-i-k] \right) e^{j(\Phi_0+\beta)} \qquad (2.2)$$

Observe that (2.2) is a convenient expression because we can separate what is desired signal and what is residual ISI. If we wish to preserve the presence of the transmitted symbol in the incoming signal we need to impose a simple linear constraint into the filter coefficients, that is, we just keep fixed the first coefficient, so that, the coefficients vector is now $\mathbf{c}=[c_0\ \mathbf{c}_1]$. where c_0 always remains fixed to any value.

$$y[n] = \left(c_0 h_0 I + c_0 x_0 + \mathbf{c}_1^t \mathbf{x}_1\right) e^{j(\Phi_0+\beta)} \qquad (2.3)$$

here I denotes the current transmitted symbol and x_0 and $\mathbf{x}_1$ are easily obtained splitting the last term in (2.2).

However, this constrained filter is able to deconvolve the received signal exactly except in a gain factor (this fact is also pointed out in [7]). Following that, we need to introduce a new parameter as a factor in the decision device which plays the role of an adjusting gain; extending this parameter to the complex field, the modulus can play that role and the phase should be capable of adjusting any synchronization or phase mismatching.

Summarizing, we propose a new algorithm for blind equalization very similar to the standard DDA, keeping fixed the first equalizer coefficient to preserve the transmitted symbol with the remainder coefficients vector implementing a scaled inverse channel. Additionally, another complex degree of freedom (labeled α) for scaling and rotating the estimated symbol, makes the algorithm capable to compensate any channel attenuation and phase shift. The algorithm proposed is depicted in Fig. 2.

Therefore, the new error cost function for this version of the MDDA is:

$$\mathcal{E}[n]=E\left\{\left|y[n]-\alpha\,\mathrm{dec}(y[n])\right|^2\right\}$$

$$\text{where}\begin{cases}\alpha=\alpha_r+j\alpha_i\\ \mathrm{dec}(y[n])=\mathrm{dec}(y_r[n])+j\,\mathrm{dec}(y_i[n])\end{cases}\qquad(2.4)$$

III. ANALYSIS OF CONVEXITY

The analysis of convexity requires to evaluate first the gradient of (2.4) to find the stability points and second the Hessian matrix to determine the kind of stability. Thus, taking derivatives in the expression of error performance surface given by (2.4) we obtain:

$$\frac{\partial\mathcal{E}}{\partial\mathbf{c}_1}=E\left\{\frac{\partial y}{\partial\mathbf{c}_1}\left[y-\alpha\,\mathrm{dec}(y)\right]^*\right\}\qquad(3.1)$$

$$\frac{\partial\mathcal{E}}{\partial\alpha}=E\left\{-\mathrm{dec}(y)\left[y-\alpha\,\mathrm{dec}(y)\right]^*\right\}\qquad(3.2)$$

where we have dropped the temporal dependence for simplicity. Substituting (3.2) into (3.1) we obtain the null gradient condition for the unconstrained equalizer coefficients:

$$E\left\{\frac{\partial y}{\partial\mathbf{c}_1}y^*\right\}-\frac{E\left\{\mathrm{dec}(y)y^*\right\}}{E\left\{\left|\mathrm{dec}(y)\right|^2\right\}}E\left\{\frac{\partial y}{\partial\mathbf{c}_1}\mathrm{dec}(y)^*\right\}=0\qquad(3.3)$$

The main difficulty for finding the equilibria points lies on the evaluation of the expected value involving a rather complicated nonlinear function. These moments are easily evaluated [12] if the involved random variables are gaussian: in section II we proposed a gaussian conditioned model for the equalizer output y; so that, by previous conditioning the observed signal to the transmitted symbol I, the conditioned random variable $\mathbf{Y}$ is gaussian. Moreover, taking derivatives in [2.3] we obtain:

$$\frac{\partial y}{\partial\mathbf{c}_1}=\mathbf{x}_1e^{j(\phi_0+\beta)}\quad\text{where}\quad x_1(i)=\sum_{k=0}^{M}h_kI[n-i-k]\qquad(3.4)$$

Regarding that every component of $\mathbf{x}_1$ can be considered as a linear combination of many i.i.d. random variables; so that, the Central Limit Theorem supports our proposal of the gaussian model so much the longer the channel is. We realize that this hypothesis may be weak. Other statistic models assuming a conditioned gaussian model to the symbols transmitted at each time may be more accurate, but the analysis become intractable and the results are acceptable for the unconditioned gaussian model.

Therefore, assuming a zero mean gaussian model for $\mathbf{x}_1$ and a conditioned gaussian model for the equalizer output, (3.3) can be expressed as:

$$E\left\{\frac{\partial y}{\partial\mathbf{c}_1}y^*\right\}=\frac{E\left\{\mathrm{dec}(y)y^*\right\}}{E\left\{\left|\mathrm{dec}(y)\right|^2\right\}}E_I\left\{\frac{E\left\{\frac{\partial y}{\partial\mathbf{c}_1}y^*|I\right\}E\left\{\mathrm{dec}(y)^*y|I\right\}}{E\left\{yy^*|I\right\}}\right\}\qquad(3.5)$$

where I denotes the transmitted symbol and E_I the corresponding expected value.

Developing (3.5), we can form an equivalent factored expression where every factor plays a different role:

$$E\left\{\frac{\partial y}{\partial\mathbf{c}_1}y^*\right\}\left[1-\frac{E\left\{\mathrm{dec}(y)y^*\right\}}{E\left\{\left|\mathrm{dec}(y)\right|^2\right\}}E_I\left\{\frac{E\left\{\mathrm{dec}(y)^*y|I\right\}}{E\left\{yy^*|I\right\}}\right\}\right]=0\qquad(3.6)$$

Paying attention to this new expression, it can be shown that the second factor only becomes zero for the condition of a perfect equalization, i.e., y=I=dec(y). Since this event can never occur for a real situation of signal plus noise and a finite length equalizer, the roots of (3.6) are just those corresponding to the first factor. The analysis of these roots follows by splitting the output channel autocorrelation matrix in the following way:

$$\mathbf{R}_\mathbf{x}=\begin{pmatrix}r_0&\mathbf{r}_1^H\\ \mathbf{r}_1&\mathbf{R}_2\end{pmatrix}\qquad(3.7)$$

It is then straightforward to verify that

$$E\left\{\frac{\partial y}{\partial\mathbf{c}_1}y^*\right\}=E\left\{\mathbf{x}_1y^*\right\}=0\ \Rightarrow\mathbf{c}_{1_{opt}}=-c_0\mathbf{R}_2^{-1}\mathbf{r}_1\qquad(3.8)$$

This result is a very interesting solution. Realize that (3.8) is the familiar solution of the linear prediction problem (Normal or Yule-Walker equations). So that, we can conclude that our algorithm can reach the same result as the linear prediction case. Furthermore, assuming that for $\mathbf{c}_1$=$\mathbf{c}_{1opt}$ we have compensated all the residual ISI, substituting (3.8) into (3.2) yields

$$\alpha_{opt}=c_0h_0e^{j\phi_0}\qquad(3.9)$$

a solution that deals with both, the attenuation and phase rotation compensation. Analyzing now the sign of the Hessian matrix of the active variables, that is, $[\mathbf{c}_1^t\ \ \alpha]$ at this point we determine the kind of stability. Therefore, taking second derivatives in (3.1) (3.2), we find:

$$\mathbf{H}=\begin{pmatrix}\mathbf{R}_2&0\\ 0&E\left\{\left|\mathrm{dec}(y)\right|^2\right\}\end{pmatrix}\qquad(3.10)$$

which obviously is definite positive and therefore, the stable point is a minimum.

IV. COMPUTER SIMULATIONS FOR MINIMUM PHASE CHANNELS.

In the theoretical analysis, a long enough equalizer and the absence of noise had to be assumed. In this section we now check the validity of the theory by presenting equalizer and phase tracking convergence results obtained with the stochastic gradient algorithms. Recall that we have presented our algorithm like an improved version of the standard DDA; so that, to compare their performances we will run both algorithms under the same simulation conditions.

The MDDA stochastic adaptive equations are obtained by the instantaneous estimation of the gradient vector in (2.4):

$$\begin{cases} \mathbf{c}_1[n+1] = \mathbf{c}_1[n] - \mu_1 \mathbf{x}_1[n] e^*[n] \\ \alpha[n+1] = \alpha[n] + \mu_2 \mathrm{dec}(y[n]) e^*[n] \end{cases} \tag{4.1}$$

We have consider a channel with a simple pole, so that a two taps equalizer can implement exactly the inverse filter.

$$H(z) = \frac{0.8}{1-0.5z^{-1}} \tag{4.2}$$

The signal is 4–QAM, the noise level is –30 dB, the phase rotation $\pi/8$ radians and the step sizes set to μ_1=0.005 and μ_2=0.01 for both algorithms The results obtained are depicted in Fig. 3-Fig. 5 (six simulation runs are averaged).

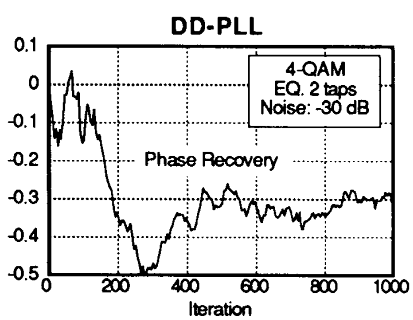

Fig. 3

Knowing that the convergence of the DDA is conditioned to the initialization settings whereas the MDDA will always converge to the desired solution, we have set an equivalent starting point regarding those situations. Fig. 3 shows the phase tracking of the DD-PLL scheme (note the lack of convergence), meanwhile in Fig. 4 the MDD reaches an stable point compensating the channel attenuation and phase rotation. Fig. 5 shows the corresponding learning curves. The DDA reaches a local minimum while the MDDA reaches the noise level.

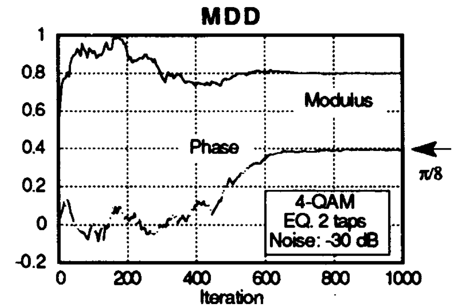

Fig. 4

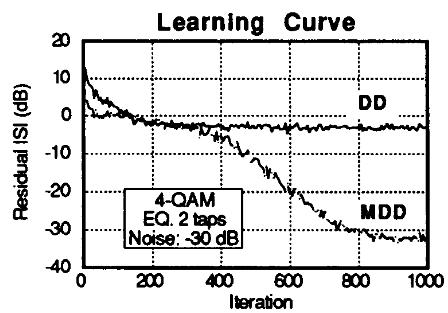

Fig. 5

V. COMPUTER SIMULATIONS FOR NON MINIMUM PHASE CHANNELS.

Our theoretical analysis deals with the equalization and phase recovery of minimum phase channels. Nevertheless, this model does not apply for the most of real channels. So that, it is fundamental to point out some results for non minimum phase channels. Presently, the lack of theoretical results is the main drawback for a generic channel analysis. Nevertheles, although we have not a similar analysis to show the global convexity, exhaustive simulations suggest that the performance of the MDDA will overcome the lack of convexity of DDA for any kind of channels. In the sequel we present some interesting computer simulations. The channel considered in the simulations is taken from [1] as a typical telephone channel which impulse response is

h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
.04	-.05	.07	-.21	-.5	.72	.36	0	.21	.03	.07

Tab. 1

Also, we have considered an attenuation factor (0.8) and a fixed rotation phase ($\pi/8$). The equalizer length is 31 taps and the noise level -30 dB. The step sizes are the same as the minimum phase channel and the initialization points are equivalent for both algorithms but chosen randomly. The results obtained are depicted in Fig.6-Fig. 8 (six simulation runs are averaged):

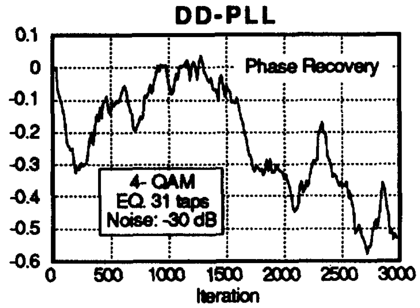

Fig. 6

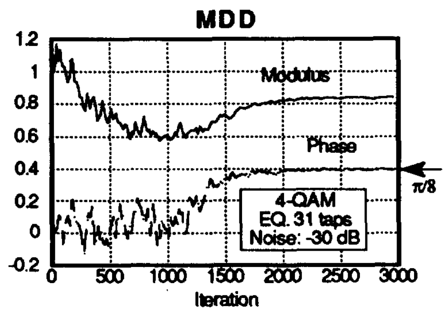

Fig. 7

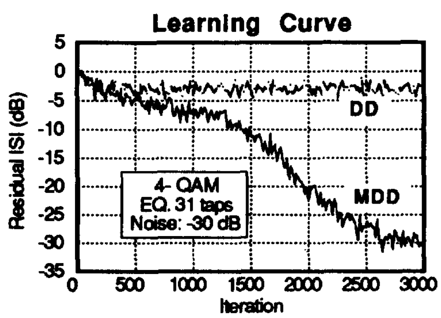

Fig. 8

Fig. 6 shows the evolution of the phase tracking of the DD-PLL scheme (note the lack of convergence), meanwhile in Fig. 7 the MDD reaches an stable point compensating the channel attenuation and phase rotation. Fig. 8 shows the convergence level for both algorithms.

VI. CONCLUSIONS

We have introduced in this paper a new algorithm for blind equalization, which is able to compensate the ISI and also recover any synchronization mismatch. We have shown analytically, for a minimum phase channel and a gaussian ISI, the ability of the MDDA to carry out this task properly overcoming the lack of convexity that characterizes the standard DDA. Finally many simulations dealing with a generic non minimum phase channel suggest that this attractive performance will remain independently of the channel phase characteristic. Additionally, as a practical consideration, we have observed that for a high order QAM signal constellations a biased convergence of parameter α is achieved with severe amplitude fading. This performance can be overcome by a right choice of the fixed coeffcient (usually we set it to one), as the inverse of the square root of the channel output power estimation.

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