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Fixed point theory for cyclic (φ - ψ)-contractions

Erdal Karapinar[1*] and Kishin Sadarangani[2]

* Correspondence: erdalkarapinar@yahoo.com
[1]Department of Mathematics, Atilim University, 06836, Incek, Ankara, Turkey
Full list of author information is available at the end of the article

Abstract

In this article, the concept of cyclic (φ - ψ)-contraction and a fixed point theorem for this type of mappings in the context of complete metric spaces have been presented. The results of this study extend some fixed point theorems in literature.

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1. Introduction and preliminaries

One of the most important results used in nonlinear analysis is the well-known Banach's contraction principle. Generalization of the above principle has been a heavily investigated branch research. Particularly, in [1] the authors introduced the following definition.

Definition 1. *Let X be a nonempty set, m a positive integer, and T: $X \to X$ a mapping. $X = \cup_{i=1}^{m} A_i$ is said to be a cyclic representation of X with respect to T if*

(i) *A_i, $i = 1, 2, ..., m$ are nonempty sets.*
(ii) *$T(A_1) \subset A_2, ..., T(A_{m-1}) \subset A_m, T(Am) \subset A_1$.*

Recently, fixed point theorems for operators T defined on a complete metric space X with a cyclic representation of X with respect to T have appeared in the literature (see, e.g., [2-5]). Now, we present the main result of [5]. Previously, we need the following definition.

Definition 2. *Let (X, d) be a metric space, m a positive integer $A_1, A_2, ..., A_m$ nonempty closed subsets of X and $X = \cup_{i=1}^{m} A_i$. An operator T: $X \to X$ is said to be a cyclic weak φ-contraction if*

(i) *$X = \cup_{i=1}^{m} A_i$ is a cyclic representation of X with respect to T.*
(ii) *$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y))$, for any $X \in A_i$, $y \in A_{i+1}$, $i = 1, 2, ..., m$, where $A_{m+1} = A_1$ and φ: $[0, \infty) \to [0, \infty)$ is a nondecreasing and continuous function satisfying $\varphi(t) > 0$ for $t \in (0, \infty)$ and $\varphi(0) = 0$*

Remark 3. *For convenience, we denote by **F** the class of functions φ: $[0, \infty) \to [0, \infty)$ nondecreasing and continuous satisfying $\varphi(t) > 0$ for $t \in (0, \infty)$ and $\varphi(0) = 0$.*

The main result of [5] is the following.

Theorem 4. [[5], Theorem 6] *Let (X, d) be a complete metric space, m a positive integer, $A_1, A_2, \ldots, A_m$ nonempty closed subsets of X and $X = \cup_{i=1}^m A_i$. Let $T: X \to X$ be a cyclic weak φ-contraction with $\varphi \in \boldsymbol{F}$. Then T has a unique fixed point $z \in \cap_{i=1}^m A_i$.*

The main purpose of this article is to present a generalization of Theorem 4.

2. Main results

First, we present the following definition.

Definition 5. *Let (X, d) be a metric space, m a positive integer, $A_1, A_2, \ldots, A_m$ nonempty subsets of X and $X = \cup_{i=1}^m A_i$. An operator $T: X \to X$ is a cyclic weak $(\varphi - \psi)$-contraction if*

(i) *$X = \cup_{i=1}^m A_i$ is a cyclic representation of X with respect to T,*
(ii) *$\varphi(d(Tx, Ty)) \leq \varphi(d(x, y)) - \psi(d(x, y))$, for any $X \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \ldots, m$, where $A_{m+1} = A_1$ and $\varphi, \psi \in \boldsymbol{F}$.*

Our main result is the following.

Theorem 6. *Let (X, d) be a complete metric space, m a positive integer, $A_1, A_2, \ldots, A_m$ nonempty subsets of X and $X = \cup_{i=1}^m A_i$. Let $T: X \to X$ be a cyclic $(\varphi - \psi)$-contraction. Then T has a unique fixed point $z \in \cap_{i=1}^m A_i$.*

Proof. Take $x_0 \in X$ and consider the sequence given by

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \ldots.$$

If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$ then, since $x_{n_0+1} = Tx_{n_0} = x_{n_0}$, the part of existence of the fixed point is proved. Suppose that $x_{n+1} \neq x_n$ for any $n = 0, 1, 2, \ldots$. Then, since $X = \cup_{i=1}^m A_i$, for any $n > 0$ there exists $i_n \in \{1, 2, \ldots, m\}$ such that $x_{n-1} \in A_{i_n}$ and $x_n \in A_{i_{n+1}}$. Since T is a cyclic $(\varphi - \psi)$-contraction, we have

$$\phi(d(x_n, x_{n+1})) = \phi(d(Tx_{n-1}, Tx_n)) \leq \phi(d(x_{n-1}, x_n)) - \psi(d(x_{n-1}, x_n)) \leq \phi(d(x_{n-1}, x_n)) \quad (2.1)$$

From 2.1 and taking into account that φ is nondecreasing we obtain

$$d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n) \text{ for any } n = 1, 2, \ldots$$

Thus $\{d(x_n, x_{n+1})\}$ is a nondecreasing sequence of nonnegative real numbers. Consequently, there exists $\gamma \geq 0$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = \gamma$. Taking $n \to \infty$ in (2.1) and using the continuity of φ and ψ, we have

$$\phi(\gamma) \leq \phi(\gamma) - \psi(\gamma) \leq \phi(\gamma),$$

and, therefore, $\psi(\gamma) = 0$. Since $\psi \in \mathbf{F}$, $\gamma = 0$, that is,

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \quad (2.2)$$

In the sequel, we will prove that $\{x_n\}$ is a Cauchy sequence.

First, we prove the following claim.

Claim: For every $\varepsilon > 0$ there exists $n \in \mathbb{N}$ such that if $p, q \geq n$ with $p - q \equiv 1(m)$ then $d(x_p, x_q) < \varepsilon$.

In fact, suppose the contrary case. This means that there exists $\varepsilon > 0$ such that for any $n \in \mathbb{N}$ we can find $p_n > q_n \geq n$ with $p_n - q_n \equiv 1(m)$ satisfying

$$d(x_{q_n}, x_{p_n}) \geq \varepsilon. \tag{2.3}$$

Now, we take $n > 2m$. Then, corresponding to $q_n \geq n$ use can choose p_n in such a way that it is the smallest integer with $p_n > q_n$ satisfying $p_n - q_n \equiv 1(m)$ and $d(x_{q_n}, x_{p_n}) \geq \varepsilon$. Therefore, $d(x_{q_n}, x_{p_{n-m}}) \leq \varepsilon$. Using the triangular inequality

$$\begin{aligned}\varepsilon \leq d(x_{q_n}, x_{p_n}) &\leq d(x_{q_n}, x_{p_{n-m}}) + \sum_{i=1}^{m} d(x_{p_{n-i}}, x_{p_{n-i+1}}) \\ &< \varepsilon + \sum_{i=1}^{m} d(x_{p_{n-i}}, x_{p_{n-i+1}}).\end{aligned}$$

Letting $n \to \infty$ in the last inequality and taking into account that $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$, we obtain

$$\lim_{n\to\infty} d(x_{q_n}, x_{p_n}) = \varepsilon \tag{2.4}$$

Again, by the triangular inequality

$$\begin{aligned}\varepsilon \leq d(x_{q_n}, x_{p_n}) &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{p_{n+1}}) + d(x_{p_{n+1}}, x_{p_n}) \\ &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{q_n}) + d(x_{q_n}, x_{p_n}) + d(x_{p_n}, x_{p_{n+1}}) + d(x_{p_{n+1}}, x_{p_n}) \\ &= 2d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_n}, x_{p_n}) + 2d(x_{p_n}, x_{p_{n+1}})\end{aligned} \tag{2.5}$$

Letting $n \to \infty$ in (2.4) and taking into account that $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ and (2.4), we get

$$\lim_{n\to\infty} d(x_{q_{n+1}}, x_{p_{n+1}}) = \varepsilon. \tag{2.6}$$

Since x_{q_n} and x_{p_n} lie in different adjacently labelled sets A_i and A_{i+1} for certain $1 \leq i \leq m$, using the fact that T is a cyclic $(\varphi - \psi)$-contraction, we have

$$\phi(d(x_{q_{n+1}}, x_{p_{n+1}})) = \phi(d(Tx_{q_n}, Tx_{q_n}) \leq \phi(d(x_{q_n}, x_{p_n})) - \psi(d(x_{q_n}, x_{p_n})) \leq \phi(d(x_{q_n}, x_{p_n}))$$

Taking into account (2.4) and (2.6) and the continuity of φ and ψ, letting $n \to \infty$ in the last inequality, we obtain

$$\phi(\varepsilon) \leq \phi(\varepsilon) - \psi(\varepsilon) \leq \phi(\varepsilon)$$

and consequently, $\psi(\varepsilon) = 0$. Since $\psi \in \mathbf{F}$, then $\varepsilon = 0$ which is contradiction.

Therefore, our claim is proved.

In the sequel, we will prove that (X, d) is a Cauchy sequence. Fix $\varepsilon > 0$. By the claim, we find $n_0 \in \mathbb{N}$ such that if $p, q \geq n_0$ with $p - q \equiv 1(m)$

$$d(x_p, x_q) \leq \frac{\varepsilon}{2} \tag{2.7}$$

Since $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ we also find $n_1 \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) \leq \frac{\varepsilon}{2m} \tag{2.8}$$

for any $n \geq n_1$.

Suppose that $r, s \geq \max\{n_0, n_1\}$ and $s > r$. Then there exists $k \in \{1, 2, ..., m\}$ such that $s - r \equiv k(m)$. Therefore, $s - r + j \equiv 1(m)$ for $j = m - k + 1$. So, we have $d(x_r, x_s) \leq d(x_r, x_{s+j}) + d(x_{s+j}, x_{s+j-1}) + ... + d(x_{s+1}, x_s)$. By (2.7) and (2.8) and from the last inequality, we get

$$d(x_r, x_s) \leq \frac{\varepsilon}{2} + j\frac{\varepsilon}{2m} \leq \frac{\varepsilon}{2} + m\frac{\varepsilon}{2m} = \varepsilon$$

This proves that (x_n) is a Cauchy sequence. Since X is a complete metric space, there exists $x \in X$ such that $\lim_{n\to\infty} x_n = x$. In what follows, we prove that x is a fixed point of T. In fact, since $\lim_{n\to\infty} x_n = x$ and, as $X = \cup_{i=1}^{m} A_i$ is a cyclic representation of X with respect to T, the sequence (x_n) has infinite terms in each A_i for $i \in \{\in 1, 2, ..., m\}$.

Suppose that $x \in A_i$, $Tx \in A_{i+1}$ and we take a subsequence x_{n_k} of (x_n) with $x_{n_k} \in A_{i-1}$ (the existence of this subsequence is guaranteed by the above-mentioned comment). Using the contractive condition, we can obtain

$$\phi(d(x_{n_{k+1}}, Tx)) = \phi(d(Tx_{n_k}, Tx)) \leq \phi(d(Tx_{n_k}, x)) - \psi(d(x_{n_k}, x)) \leq \phi(d(x_{n_k}, x))$$

and since $x_{n_k} \to x$ and φ and ψ belong to **F**, letting $k \to \infty$ in the last inequality, we have

$$\phi(d(x, Tx)) \leq \phi(d(x, x)) = \phi(0) = 0$$

or, equivalently, $\varphi(d(x, Tx)) = 0$. Since $\varphi \in$ **F**, then $d(x, Tx) = 0$ and, therefore, x is a fixed point of T.

Finally, to prove the uniqueness of the fixed point, we have $y, z \in X$ with y and z fixed points of T. The cyclic character of T and the fact that $y, z \in X$ are fixed points of T, imply that $y, z \in \cap_{i=1}^{m} A_i$. Using the contractive condition we obtain

$$\phi(d(y, z)) = \phi(d(Ty, Tx)) \leq \phi(d(y, z)) - \psi(d(y, z)) \leq \phi(d(y, z))$$

and from the last inequality

$$\psi(d(y, z)) = 0$$

Since $\psi \in$ **F**, $d(y, z) = 0$ and, consequently, $y = z$. This finishes the proof.

In the sequel, we will show that Theorem 6 extends some recent results.

If in Theorem 6 we take as φ the identity mapping on $[0, \infty)$ (which we denote by $Id_{[0, \infty)}$), we obtain the following corollary.

Corollary 7. *Let (X, d) be a complete metric space m a positive integer, $A_1, A_2, ..., A_m$ nonempty subsets of X and $X = \cup_{i=1}^{m} A_i$. Let $T: X \to X$ be a cyclic $(Id_{[0, \infty)} - \psi)$ contraction. Then T has a unique fixed point $z \in \cap_{i=1}^{m} A_i$.*

Corollary 7 is a generalization of the main result of [5] (see [[5], Theorem 6]) because we do not impose that the sets A_i are closed.

If in Theorem 6 we consider $\varphi = Id_{[0, \infty)}$ and $\psi = (1 - k)Id_{[0, \infty)}$ for $k \in [0, 1)$ (obviously, $\varphi, \psi \in$ **F**), we have the following corollary.

Corollary 8. *Let (X, d) be a complete metric space m a positive integer, $A_1, A_2, ..., A_m$ nonempty subsets of X and $X = \cup_{i=1}^{m} A_i$. Let $T: X \to X$ be a cyclic $(Id_{[0, \infty)} - (1 - k)Id_{[0, \infty)})$ contraction, where $k \in [0, 1)$. Then T has a unique fixed point $z \in \cap_{i=1}^{m} A_i$.*

Corollary 8 is Theorem 1.3 of [1].

The following corollary gives us a fixed point theorem with a contractive condition of integral type for cyclic contractions.

Corollary 9. *Let* (X, d) *be a complete metric space,* m *a positive integer,* $A_1, A_2, \ldots, A_m$ *nonempty closed subsets of* X *and* $X = \cup_{i=1}^{m} A_i$. *Let* $T: X \to X$ *be an operator such that*

(i) $X = \cup_{i=1}^{m} A_i$ *is a cyclic representation of* X *with respect to* T .
(ii) *There exists* $k \in [0, 1)$ *such that*

$$\int_0^{d(Tx,Ty)} \rho(t)dt \le k \int_0^{d(x,y)} \rho(t)dt$$

for any $X \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \ldots, m$ *where* $A_{m+1} = A_1$, *and* $\rho: [0, \infty) \to [0, \infty)$ *is a Lebesgue-integrable mapping satisfying* $\int_0^{\varepsilon} \rho(t)dt$ *for* $\varepsilon > 0$.

Then T *has unique fixed point* $z \in \cap_{i=1}^{m} A_i$.

Proof. It is easily proved that the function $\varphi: [0, \infty) \to [0, \infty)$ given by $\varphi(t) = \int_0^t p(s)ds$ satisfies that $\varphi \in \mathbf{F}$. Therefore, Corollary 9 is obtained from Theorem 6, taking as φ the above-defined function and as ψ the function $\psi(t) = (1 - k)\varphi(t)$.

If in Corollary 9, we take $A_i = X$ for $i = 1, 2, \ldots, m$ we obtain the following result.

Corollary 10. *Let* (X, d) *be a complete metric space and* $T: X \to X$ *a mapping such that for* $x, y \in X$,

$$\int_0^{d(Tx,Ty)} \rho t dt \le k \int_0^{d(x,y)} \rho t dt$$

where $\rho: [0, \infty) \to [0, \infty)$ *is a Lebesgue-integrable mapping satisfying* $\int_0^{\varepsilon} \rho(t)dt$*for* $\varepsilon > 0$ *and the constant* $k \in [0, 1)$. *Then* T *has a unique fixed point.*

Notice that this is the main result of [6]. If in Theorem 6 we put $A_i = X$ for $i = 1, 2, \ldots, m$ we have the result.

Corollary 11. *Let* (X, d) *be a complete metric space and* $T: X \to X$ *an operator such that for* $x, y \in X$,

$$\phi(d(Tx, Ty)) \le \phi(d(x, y)) - \psi(d(x, y)),$$

where $\varphi, \psi \in \boldsymbol{F}$. *Then* T *has a unique fixed point.*

This result appears in [7].

3. Example and remark

In this section, we present an example which illustrates our results. Throughout the article, we let $\mathbb{N}^* = \mathbb{N}\backslash\{0\}$.

Example 12. *Consider* $X = \left\{\frac{1}{n} : n \in \mathbb{N}^*\right\} \cup \{0\}$ *with the metric induced by the usual distance in* $\mathbb{R}$, *i.e.,* $d(x, y) = |x - y|$. *Since* X *is a closed subset of* $\mathbb{R}$, *it is a complete metric space. We consider the following subsets of* X:

$$A_1 = \{\frac{1}{n} : n \text{ odd}\} \cup \{0\}$$

$$A_2 = \{\frac{1}{n} : n \text{ even}\} \cup \{0\}$$

Obviously, $X = A_1 \cup A_2$. Let $T: X \to X$ be the mapping defined by

$$Tx = \begin{cases} \frac{1}{n+1} & if\ \ x = \frac{1}{n} \\ 0 & if\ \ x = 0 \end{cases}$$

It is easily seen that $X = A_1 \cup A_2$ is a cyclic representation of X with respect to T. Now we consider the function ρ: $[0, \infty) \to [0, \infty)$ defined by

$$\rho(t) = \begin{cases} 0 & if\ \ t = 0 \\ t^{\frac{1}{t}-2}[1 - \ln t] & if\ \ 0 < t < e \\ 0 & if\ \ t \geq e \end{cases}$$

It is easily proved that $\int_0^t \rho(s)ds = t^{1/t}$ for $t \leq 1$.

In what follows, we prove that T satisfies condition (ii) of Corollary 9.

In fact, notice that the function $\rho(t)$ is a Lebesgue-integrable mapping satisfying $\displaystyle\int_0^{\varepsilon} \rho(t)dt > 0$ for $\varepsilon > 0$. We take $m, n \in \mathbb{N}^*$ with $m \geq n$ and we will prove

$$\int_0^{d(T(\frac{1}{n}),T(\frac{1}{m}))} \rho(s)ds \leq \frac{1}{2}\int_0^{d(\frac{1}{n},\frac{1}{m})} \rho(s)ds$$

Since $\int_0^t \rho(s)ds = t^{1/t}$ for $t \leq 1$ and, as diam$(X) \leq 1$, the last inequality can be written as

$$d(T(\frac{1}{n}),T(\frac{1}{m}))^{\overline{\frac{1}{d(T(\frac{1}{n}),T(\frac{1}{m}))}}} \leq \frac{1}{2}d\left(\frac{1}{n},\frac{1}{m}\right)^{\overline{\frac{1}{d\left(\frac{1}{n},\frac{1}{m}\right)}}}$$

or equivalently,

$$\left|\frac{1}{n+1} - \frac{1}{m+1}\right|^{\overline{\frac{1}{\left|\frac{1}{n+1} - \frac{1}{m+1}\right|}}} \leq \frac{1}{2}\left|\frac{1}{n} - \frac{1}{m}\right|^{\overline{\frac{1}{\left|\frac{1}{n} - \frac{1}{m}\right|}}}$$

or equivalently,

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\left(\frac{(n+1)(m+1)}{m-n}\right)} \leq \frac{1}{2}\left(\frac{m-n}{nm}\right)^{\left(\frac{nm}{m-n}\right)}$$

or equivalently,

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{n+m+1}{m-n}} \times \left(\frac{nm}{(n+1)(m+1)}\right)^{\frac{nm}{m-n}} \leq \frac{1}{2} \tag{3.1}$$

In order to prove that this last inequality is true, notice that

$$\frac{nm}{(n+1)(m+1)} < 1 \tag{3.2}$$

and, therefore,

$$\left(\frac{nm}{(n+1)(m+1)}\right)^{\frac{nm}{m-n}} \leq 1$$

On the other hand, from

$$m - n \le m.n$$
$$m - n \le m + n$$

we obtain

$$2(m-n) \le (n+1)(m+1)$$

and, thus,

$$\left(\frac{m-n}{(n+1)(m+1)}\right) \le \frac{1}{2}.$$

Since $\dfrac{n+m+1}{m-n} \ge 1$,

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{n+m+1}{m-n}} \le \frac{1}{2}. \tag{3.3}$$

Finally, (3.2) *and* (3.3) *give us* (3.1).

Now we take $x = \frac{1}{n}$, $n \in \mathbb{N}^*$ and $y = 0$. In this case, condition (*ii*) of Corollary 9 for $k = \frac{1}{2}$ has the form

$$\begin{aligned} d\left(T\left(\frac{1}{n}\right), T(0)\right)^{\frac{1}{d(T(\frac{1}{n}),T(0))}} &= \left(\frac{1}{n+1}\right)^{n+1} \\ &\le \frac{1}{2} d\left(\frac{1}{n}, 0\right)^{\frac{1}{d(\frac{1}{n},0)}} \\ &= \frac{1}{2}\left(\frac{1}{n}\right)^{n} \end{aligned}$$

The last inequality is true since

$$\left(\frac{1}{n+1}\right)^{n} < \left(\frac{1}{n}\right)^{n}$$

and, then,

$$\left(\frac{1}{n+1}\right)^{n+1} = \left(\frac{1}{n+1}\right)^{n} \frac{1}{n+1} \le \frac{1}{2}\left(\frac{1}{n+1}\right)^{n} < \frac{1}{2}\left(\frac{1}{n}\right)^{n}.$$

Consequently, since assumptions of Corollary 9 are satisfied, this corollary gives us the existence of a unique fixed point (which is obviously $x = 0$).

This example appears in [6].

Now, we connect our results with the ones appearing in [3]. Previously, we need the following definition.

Definition 13. *A function* ϕ*:* $[0, \infty) \to [0, \infty)$ *is a (c)-comparison function if* $\sum_{k=0}^{\infty} \varphi^k(t)$ *converges for any* $t \in [0, \infty)$*. The main result of* [3]*is the following.*

Theorem 14. *Let* (X, d) *be a complete metric space, m a positive integer,* $A_1, A_2, \ldots, A_m$ *nonempty subsets of X,* $X = \cup_{i=1}^{m} A_i$*and* ϕ*:* $[0, \infty) \to [0, \infty)$ *a (c)-comparison function.*

Let $T: X \to X$ be an operator and we assume that

(i) $X = \cup_{i=1}^{m} A_i$ *is a cyclic representation of X with respect to T.*
(ii) $d(Tx, Ty) \leq \phi(d(x, y))$, *for any $X \in A_i$ and $y \in A_{i+1}$, where $A_{m+1} = A$.*

Then T has a unique fixed point $z \in \cap_{i=1}^{m} A_i$.

Now, the contractive condition of Theorem 6 can be written as

$$\phi(d(Tx, Ty)) \leq \phi(d(x, y)) - \varphi(d(x, y)) = (\phi - \varphi)(d(x, y))$$

for any $x \in A_i$, $y \in A_{i+1}$, where $A_{m+1} = A_1$, and $\varphi, \phi \in \mathbf{F}$.

Particularly, if we take $\varphi = Id_{[0,\ \infty)}$ and $\varphi(t) = \frac{t^2}{1+t}$, it is easily seen that $\varphi, \phi \in \mathbf{F}$. On the other hand,

$$(\phi - \varphi)(t) = t - \frac{t^2}{1+t} = \frac{t}{1+t}$$

and

$$(\phi - \varphi)^{(n)}(t) = \frac{t}{1+nt}$$

Moreover, for every $t \in (0, \infty)$, $\sum_{k=0}^{\infty} (\phi - \varphi)^{(k)}(t)$ diverges. Therefore, φ - ϕ is not a (c)-comparison function. Consequently, our Theorem 6 can be applied to cases which cannot treated by Theorem 14.

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Author details
[1]Department of Mathematics, Atilim University, 06836, Incek, Ankara, Turkey [2]Department of Mathematics, University of Las Palmas de Gran Canaria, Campus Universitario de Tafira, 35017 Las Palmas de Gran Canaria, Spain

Authors' contributions
The authors have contributed in obtaining the new results presented in this article. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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