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Finite Formulation in 2D for the Analysis of an Electrostatic Induction Micromotor

J. M. Monzón, F. J. Santana, S. García–Alonso, J. A. Montiel–Nelson, *Member, IEEE*

***Abstract*—An electrostatic induction micromotor has been simulated and analyzed using the Finite Formulation. An approach based on a primal–dual barycentric discretization of the 2D space is presented, considering the potential in each node on the primal mesh as unknown. We have introduced the analytical solution of the mathematical model for a simple geometry of the micromotor. The Finite Formulation solution has been compared with the analytical and FEM solution for verification purposes.**

I. Introduction

All existing numerical methods for the solution of the field equation have, in one way or another, a *differential formulation* as their starting point. A discrete formulation is then obtained by means of the many discretization methods, such as Finite Difference Methods, Finite Element Method, Boundary Element Method, etc.

As an alternative, most of the researchers reformulate field laws in *finite form* so that an algebraic system of equations is directly written to solve the field problem, avoiding the use of the discretization process applied to a differential equation. This approach is the Finite Formulation [1]–[3] and the corresponding numerical method is known as the Cell Method (CM) [4]–[6].

The present paper applies this method to the simulation and analysis of an electrostatic induction micromotor.

Currently, the design and implementation of a micromotor using MEMS technology is a great challenge [7]–[9]. For this purpose, we have developed some tools based on Finite Formulation to simulate the electromagnetic fields and torque of an electrostatic induction micromotor. The proposed analytical equations are compared with the obtained solutions provided by Finite Formulation tools. To our knowledge, there are not publications dealing with CM for micromotors. A generic study of other electrostatic applications is presented in [10].

The study has been carried out in a simple linear electrical induction micromachine constituted by two parallel plates —rotor and stator— isolated by a dielectric [8]. The distance between plates is 6μm. Fig. 1 summarizes the operation mode of the micromachine and Table 1 shows the nomenclature introduced.

This study is focused in the linear micromachine due to the greater simplicity of its analytical equations. The linear micromachine is the unfolding of a rotating electric micromachine and this is the reason why the conclusions obtained for the linear micromachine are easily generalized to the rotating one.

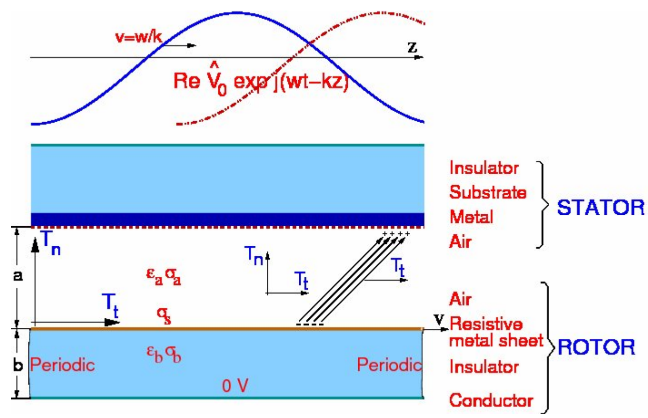

Fig. 1. Linear electrical induction micromachine.

II. Differential Formulation

The fundamental problem of a physical field can be stated as follows: first, we introduce the shape and the dimension of the field domain, second we study the spatial and temporal distribution of the field sources; third we present the nature of the material that fills the field domain, and finally we obtain the boundary condition that summarize the action of the external of the field domain [2].

To begin with, we present the following equations that are referenced in [11]-[13], and they have been taken as the base for this work.

As initial assumption we use Gauss's law:

$$\nabla \cdot \vec{D} = \rho_f \tag{1}$$

The charge conservation law says:

$$\nabla \cdot \vec{J}_f + \frac{\partial \rho_f}{\partial t} = 0 \tag{2}$$

As initial hypothesis we assume a quasi conservative electric field

$$\nabla \times \vec{E} \approx 0 \tag{3}$$

J. M. Monzón is in the Electrical Eng. Dept., University of Las Palmas de Gran Canaria, Spain (phone: +34 928-451-245; fax: +34 928-457-319; e-mail: jmonzon@die.ulpgc.es).

F. J. Santana, is in the Electrical Eng. Dept., University of Las Palmas de Gran Canaria, Spain (e-mail: fsantana@die.ulpgc.es).

S. García-Alonso is in the Electronic and Automation Eng. Dept., University of Las Palmas de Gran Canaria, Spain (e-mail: sgarcia@diea.ulpgc.es).

J. A. Montiel-Nelson is in the Electronic and Automation Eng. Dept., University of Las Palmas de Gran Canaria, Spain (montiel@iuma.ulpgc.es).

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TABLE I
NOMENCLATURE

Symbol	Name	Units
a	Height of dielectric 2	m
b	Height of dielectric 1	m
E	Electric field	V/m
D	Displacement current	C/m^2
k	Number of waves per metre	-
l^e	Element length	m
j	Imaginary unity	-
J_f	Volumetric current density	A/m^2
S	Slip	-
t^e	Element depth	m
v	Linear speed of mobile part	m/s
V	Interelectrodic voltage	V
V_0	Supply voltage	V
ε_a	Electric permittivity of the dielectric	F/m
ε_{eff}	Effective permittivity	F/m
φ	Electric scalar potential	V
ρ_f	Volumetric charge density	C/m^3
ω	Angular frequency of the signal	Hz
σ_a	Electric conductivity of the dielectric	S/m
σ_f	Superficial charge density	C/m^2
σ_{eff}	Effective Conductivity	S/m
σ_S	Superficial electric conductivity	1/Ω
Φ^b	Voltage in the interface	V

And the constitutive laws:

$$\sigma\vec{E} + \rho_f \vec{v} = \vec{J}_f \tag{4}$$

$$\varepsilon\vec{E} = \vec{D} \tag{5}$$

From (2) and (4), we obtain

$$\nabla \cdot \sigma\vec{E} + \nabla \cdot \rho_f \vec{v} + \frac{\partial \rho_f}{\partial t} = 0 \tag{6}$$

and from (1) and (5), we obtain

$$\nabla \cdot \sigma\vec{E} + \nabla \cdot \left(\vec{v}\nabla \cdot \varepsilon E\right) + \frac{\partial}{\partial t}\nabla \cdot \varepsilon\vec{E} = 0 \tag{7}$$

In the same way we can eliminate E from (7) and because

$$\vec{E} = -\nabla\varphi \tag{8}$$

we obtain

$$\nabla \cdot \sigma\nabla\varphi + \nabla \cdot \left(\vec{v}\nabla \cdot \varepsilon\nabla\varphi\right) = -\frac{\partial}{\partial t}\nabla \cdot \varepsilon\nabla\varphi \tag{9}$$

When the micromotor stars, then v=0, so

$$\nabla \cdot \sigma\nabla\varphi + \frac{\partial}{\partial t}\nabla \cdot \varepsilon\nabla\varphi = 0 \tag{10}$$

The previous equation is expressed in the time domain. In sinusoidal stationary regimen, the operator $\partial / \partial t$ equal to jω and it is expressed in the following form

$$\nabla \cdot \sigma\nabla\overline{\varphi} e^{jwt} + j\omega\nabla \cdot \varepsilon\nabla\overline{\varphi} e^{jwt} = 0 \tag{11}$$

where $\overline{\varphi}$ is an electric scalar potential complex distribution. Simplifying the time dependent term, we obtain the following equation (12). This is the field equation for the micromotor in the *differential formulation*

$$\nabla \cdot \sigma\nabla\overline{\varphi} + j\omega\nabla \cdot \varepsilon\nabla\overline{\varphi} = 0 \tag{12}$$

In this equation, the scalar potential $\overline{\varphi}$ is the unknown variable. Equation (12) shows that differential formulation imposes derivability conditions on field functions that are restrictive from the physical point of view.

This is a Poisson equation and requires that the domain contains a homogeneous material and that the potential admits second order partial derivatives. Alternatively, if the domain is composed of different materials, it must be subdivided into subdomains, which contain a homogeneous material. Then, the Poisson equation is applied to every subdomain and on the separation surfaces a jump condition must be satisfied. In other case, a Finite Formulation, based on global variables accepts material discontinuities.

III. Analytical Equations

From the basic physical principles that govern the micromotor behaviour, we determined an analytical equation for a planar elemental model [12]. It is very important to know the potential in the interface for verification purposes, and to calculate the torque through the electrostatic field and the induced charge in the mobile part of the micromachine. To accomplish this task we apply the charge conservation law in the interface.

We start from Laplace equation

$$\nabla^2\varphi = 0 \tag{13}$$

This equation has been particularized for the following boundary conditions — zero Volts for the inferior plate of the mobile part and V Volts for the fixed part.

From the charge conservation law in the interface, we write:

$$\frac{\partial \sigma_f}{\partial t} + \nabla_S \cdot \left(\sigma_S \cdot \vec{E}_Z + \vec{v}_Z \cdot \sigma_f\right) + \vec{n} \cdot \left\|\sigma\vec{E}\right\| = 0 \tag{14}$$

Once we have developed the terms of these equations we obtain:

$$\Phi^b = \frac{V_0}{\sinh(ka)} \frac{\dfrac{\sigma_a}{\sigma_{eff}} + \dfrac{\varepsilon_a}{\varepsilon_{eff}}\omega Sj}{(1 + \dfrac{\varepsilon_{eff}}{\sigma_{eff}}\omega Sj)} \tag{15}$$

where,

$$\sigma_{eff} = \sigma_a \coth(ka) + \coth(kb)\sigma_b + \sigma_S k \tag{16}$$

$$\varepsilon_{eff} = \varepsilon_a k\coth(ka) + \varepsilon_b k\coth(kb) \tag{17}$$

Equation (15) represents the analytical voltage in the interface of the micromotor and it will be the reference for our Finite Formulation.

TABLE II
PHYSICAL AND GEOMETRICAL PARAMETERS OF THE MICROMACHINE

Symbol	Name	Value	Unit
L	Length of the structure	44 e-6	m
hm	Height of the metallic plates	0.01 e-6	m
a	Height of dielectric 2	3 e-6	m
b	Height of dielectric 1	10 e-6	m
k	Number of waves per metre	2π/L	m^{-1}
v	Linear speed of mobile part	0	m/s
f	Temporal frequency of excitation	2.6 e6	Hz
V_0	Maximum value of excitation	200	V

IV. FINITE FORMULATION FOR THE MICROMOTOR

We begin using global variables for the Finite Formulation. The global variables refer to oriented geometrical elements like *points*, *lines, surfaces*, *volumes*, *instant*, and *interval*.

According to Finite Formulation, global variables can be also classified into *configuration*, *source*, and *energy* variables [1]. The configuration variables describe the configuration of the field without the intervention of the material parameters. The source variables describe the source of the field without involving the material parameters. The energy variables are the product between a configuration and source variables.

Cell Method requires the use of a pair of oriented cell complexes, one dual of the other, endowed with inner (i,j,k cell) and outer orientation (1,2,3,...,11 cell), respectively, as can be seen in Fig. 2.

According to the Finite Formulation of the electromagnetism, a first principle [3] says that the *configuration* variables are naturally associated with space and time elements of a *primal cell* complex endowed with *inner orientation*, while the *source* variables are associated with space and time elements of a *dual cell complex* endowed with *outer orientation*. The second principle says that in every physical theory there are physical laws that link global variables referred to an oriented space-time element with others referred to its oriented boundary.

The corresponding dual cell complexes, that we are going to follow are derived according to the barycentric subdivision [14], as can be seen in Fig. 2.

A. Topological equation of the micromotor in discrete form

The field equation of the micromotor can be enforced, on the cell complexes, in *exact discrete form* by using appropriated incidence matrices. They are called G, C and D and denote respectively the edges–node, faces–edges, and volumes–faces for the oriented primal cell complex.

Let matrices $\tilde{G}$, $\tilde{C}$ and $\tilde{D}$ denote, respectively, the edges–node, faces–edges, and volumes–faces for the oriented dual cell complex. These matrices above may be viewed as discrete counterparts of the differential operator gradient, curl, and divergence, respectively, [5] and [10].

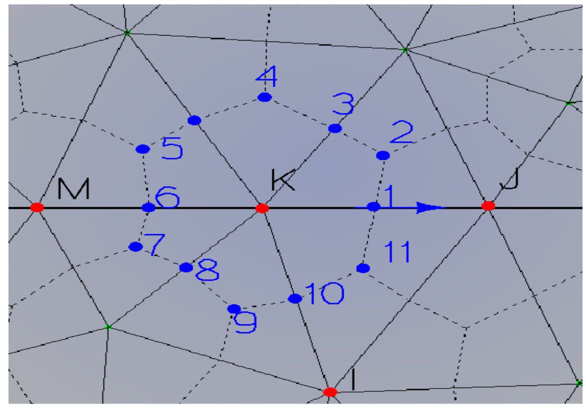

Fig. 2. Dual barycentric subdivision.

The following equations represent the counterparts of the differential Laws that we have seen in section II:

Gauss Law:

$$\tilde{D} \cdot \tilde{\psi} = \tilde{Q} \tag{18}$$

where $\tilde{\psi}$ is an electric flux vector associated to the dual faces and $\tilde{Q}$ an electric charge vector associated to the dual volumes.

Faraday Law (for Quasi-Electrostatic conditions):

$$C \cdot U \approx 0 \qquad U = -G \cdot V \tag{19}$$

where V is an electric potential vector associated to the primal nodes and U an electric potential vector associated to the primal edges.

Charge conservation:

$$\tilde{D} \cdot \tilde{I} + j \cdot w \cdot \tilde{D} \cdot \tilde{\psi} = 0 \tag{20}$$

where I is a current intensity vector associated to the dual faces.

The duality between the oriented primal and dual space cell complexes, along with some common convection, lead, in general to the following relationships [16]:

$$\tilde{D} = -G^T \qquad \tilde{G} = D^T \qquad \tilde{C} = C^T \tag{21}$$

B. Constitutive equation of the micromotor in discrete form

The approximation of the method itself begins when the integral voltage and flux state variables, that are allocated on two different cell complexes, are related to each other by the constitutive material equations. These equations are matrix equations. They contain the average information of the material and on the grid dimension [6], [10], [15] and [17].

Since the equations (18)-(20) are exact and contain only topological information, the discretization error is found to be located in the discrete constitutive material equations.

In the micromotor, we have volumetric and superficial properties (volumetric conductivity and permittivity, superficial conductivity), that is why we considerate two classes of cell for the discrete constitutive material equations. One is volumen cell and the other face cell. This corresponds in 2D to face and edge cell, respectively.

The constitutive equations for a simple primal-dual cell, see Fig. 3, are:

$$\widetilde{I}^{e} = M_{\sigma}^{e} \cdot U^{e} \tag{22}$$

$$\widetilde{\psi}^{e} = M_{\varepsilon}^{e} \cdot U^{e} \tag{23}$$

where we have for the face element the following expressions:

$$U^{e} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \qquad \widetilde{\psi}^{e} = \begin{pmatrix} \widetilde{\psi}_1 \\ \widetilde{\psi}_2 \\ \widetilde{\psi}_3 \end{pmatrix} \qquad \widetilde{I}^{e} = \begin{pmatrix} \widetilde{I}_1 \\ \widetilde{I}_2 \\ \widetilde{I}_3 \end{pmatrix} \tag{24}$$

The permittivity and conductivity tensors are

$$\varepsilon^{e} = \begin{pmatrix} \varepsilon_{11} & 0 \\ 0 & \varepsilon_{22} \end{pmatrix} \qquad \sigma^{e} = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix} \tag{25}$$

where U_1, U_2 and U_3 are the voltage associated to the edges I_1, I_2 and I_3 respectively (see Fig. 3), and $\widetilde{\psi}_1$, $\widetilde{\psi}_2$, $\widetilde{\psi}_3$ and $\widetilde{I}_1$, $\widetilde{I}_2$, $\widetilde{I}_3$ are the electric flow and the electric intensity associated to the surfaces $\widetilde{S}_1$, $\widetilde{S}_2$ and $\widetilde{S}_3$ respectively of the simple dual cell (see Fig. 3).

$M_{\sigma_S}^{e}$ is the superficial conductivity matrix

$$M_{\sigma_S}^{e} = \frac{t^{e}\sigma_S^{e}}{l^{e}} \tag{26}$$

And M_{σ}^{e} and M_{ε}^{e} are the volumetric conductivity and the permittivity matrix, respectively

$$M_{\sigma}^{e} = 1/3 \cdot \widetilde{S}^{e} \cdot \sigma^{e} \cdot (A+B+C) \tag{27}$$

$$M_{\varepsilon}^{e} = 1/3 \cdot \widetilde{S}^{e} \cdot \varepsilon^{e} \cdot (A+B+C) \tag{28}$$

Where A, B and C are dependent on geometry of the primal cell and $\widetilde{S}^{e}$ stands for

$$\widetilde{S}^{e} = \begin{pmatrix} \widetilde{S}_{1x} & \widetilde{S}_{1y} \\ \widetilde{S}_{2x} & \widetilde{S}_{2y} \\ \widetilde{S}_{3x} & \widetilde{S}_{3y} \end{pmatrix} \tag{29}$$

C. *Final global equation of the micromotor*

The local fundamental matrix can be derived by substituting in (20) the local constitutive (22) and (23) and Gauss Law (18) were U is expressed by means of (19), obtaining

$$G^{eT} M_{\sigma}^{e} G^{e} V^{e} + j \cdot w \cdot G^{eT} M_{\varepsilon}^{e} G^{e} V^{e} = 0 \tag{30}$$

where

$$G^{e} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \qquad V^{e} = \begin{pmatrix} V_i \\ V_j \\ V_k \end{pmatrix} \tag{31}$$

For computational purposes it is convenient to proceed with one cell at time. To obtain the global fundamental matrix we must assemble all the local fundamental matrices on the reference cell, see Fig. 3.

For a 2D, in case of triangular elements under the hypothesis of uniform field and using a dual mesh with barycentric subdivision, the resulting matrix for *one element* is symmetric. Moreover, this matrix is coincident with the element matrix obtained with Finite elements with affine approximation of the electric potential within of the triangle [4] and [10], so the resulting system of equations is coincident.

To solve equation (30), first we applied the following boundary conditions, one travelling wave on upper side, 0 V in the lower side, and periodic boundary conditions on the left and right side, see Fig. 1. It has been developed a program in Scilab language for the matrix calculus and the resolution of the equation system. Gmsh program has been used as automatic 2D finite element grid generator and advanced visualization capabilities [18].

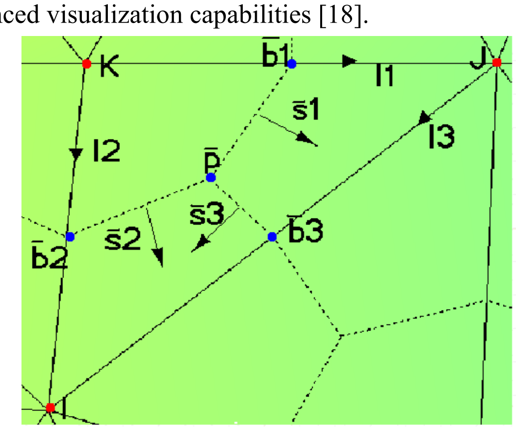

Fig. 3. Simple primal-dual cell for assemble process.

V. RESULTS

We have calculated the potential in the interface applying the Cell Method and the obtained analytical equations for five different values of the conductivity. The error between the results obtained using analytical equations and the Cell Method are neglected, as can be seen in Table II. Figure 4 and 5 show the Cell Method results for a conductivity of $1/(1800 \cdot 10^6)$ [1/Ω]. Figure 4 represents the imaginary part and Fig. 5 the real part.

Figure 6 represents the potential in the interface versus the conductivity.

Typical maximum discrepancies are lower than 0.1%.

We have also calculated the electric field in the interface. CM results and analytical solution results can be seen in Table III and in Fig. 7 for a conductivity of $1/(600 \cdot 10^6)$ [1/Ω].

The error between the results obtained using analytical equations and the CM are neglected.

CM convergence has been guaranteed with the refining of the meshes of the micromotor as can be seen in Table IV. The interfacial electrical voltage has been obtained for a conductivity of $1/(1800 \cdot 10^6)$ [1/Ω].

TABLE II
INTERFACE ELECTRICAL VOLTAGE

Conductivity (1/Ω)	Analytical	CM	Error (%)
$1/(50 \cdot 10^6)$	*21.6688*	*21.6947*	*-0.119*
$1/(100 \cdot 10^6)$	*37.7909*	*37.7259*	*0.172*
$1/(200 \cdot 10^6)$	*53.6311*	*53.5904*	*0.075*
$1/(600 \cdot 10^6)$	*64.2738*	*64.2748*	*-0.001*
$1/(1800 \cdot 10^6)$	*65.8906*	*65.9102*	*-0.029*

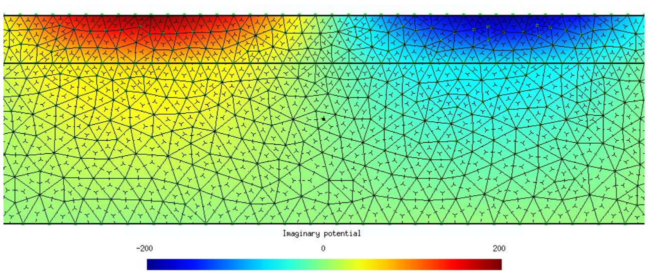

Fig. 4. Graphical representation of imaginary voltage (Cell Method).

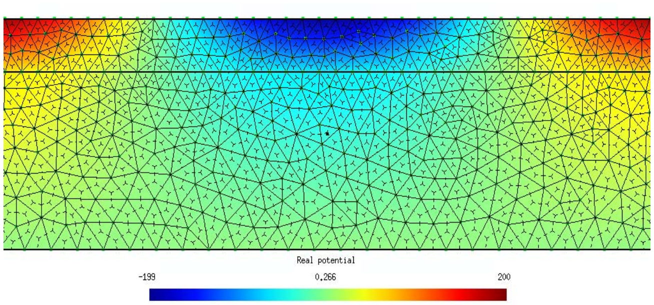

Fig. 5. Graphical representation of real voltage (Cell Method).

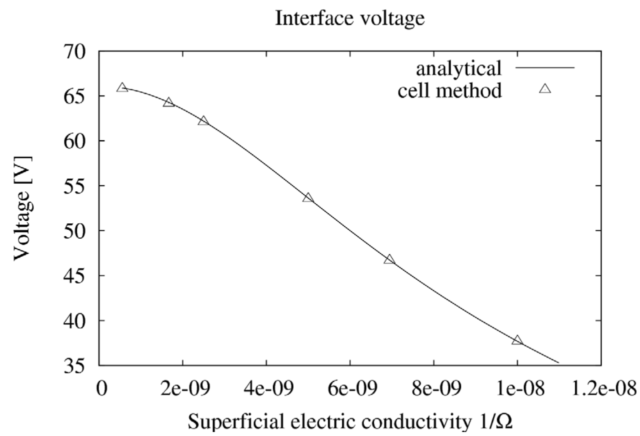

Fig. 6. Graphical representation of maximal voltage versus superficial electric conductivity.

TABLE III
ELECTRIC FIELD (V/m) IN THE STEADY STATE IN THE INTERFACE IN Z=0

Conductivity (1/Ω)	Analytical solution (V/m)	CM (V/m)	Error (%)
$1/(50 \cdot 10^6)$	3094307	3102000	-0.248
$1/(100 \cdot 10^6)$	5381641	5389700	-0.149
$1/(200 \cdot 10^6)$	7658503	7665400	-0.090
$1/(600 \cdot 10^6)$	9178278	9182800	-0.049
$1/(1800 \cdot 10^6)$	9409100	9419900	-0.114

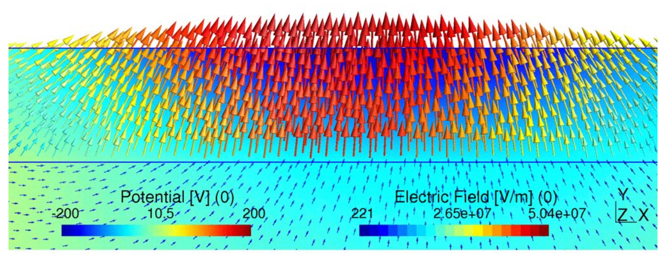

Fig. 7. Electric field for a superficial conductivity of $1/(600 \cdot 10^6)$.

TABLE IV
EFFECT OF THE MESH IN THE CONVERGENCE

Number of nodes	Number of elements	Analytical solution (V)	Numeric solution (V)	Error (%)
2353	4704	65.89	65.91	0.030
613	1224	65.89	66.02	0.197
284	566	65.89	66.20	0.470
170	338	65.89	66.40	0.774

VI. CONCLUSION

A mathematical model has been deduced for the induction electric lineal micromotor using the field equations in differential and finite form. An exact analytical equation has been found. Using this equation the potential and the electrical field has been determined.

Results have been compared and errors are neglected (lower than 0.1%). An analysis using CM and FEM has been carried out and the global matrices of both methods are equal. Hence, both results are coincident.

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