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A BEM–FEM MODEL FOR DYNAMIC SOIL–STRUCTURE AND STRUCTURE–SOIL–STRUCTURE PROBLEMS IN ELASTIC OR POROELASTIC SOILS

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Abstract. This work presents a time–harmonic BEM–FEM three–dimensional model for the dynamic analysis of structures founded on elastic and poroelastic soils. This model is suitable to study soil–structure and structure–soil–structure interaction problems, the effects of the torsional eccentricity of non–symmetrical structures or the influence in the response of the soil saturation level. In this paper, the procedure to incorporate the rigid body restrictions to the BEM model is explained. Also, the coupling of the superstructure, modelled using two–noded Timoshenko beam finite elements, and the extension of the excitation model, including elastic waves in poroelastic soils, are showed.

1 INTRODUCTION

The model presented herein is a development of a previous multi–domain BEM code (see for example [1, 2]). This new version is able to consider perfectly rigid domains, the modelling of the superstructure using two–noded Timoshenko beam finite elements and the extension of the excitation model including a field of vertically–incident planar waves in poroelastic domains. In case of viscoelastic media, the incident field can be defined as SH, SV, P or Rayleigh waves with a general angle of incidence. Figure 1 shows a sketch of a group of four nearby buildings as an example of what can be studied using this model.

The foundation and soil domains can be modelled as elastic or Biot poroelastic media using boundary elements. When the hypothesis of infinite rigidity is applicable, it is possible to consider the foundation as a perfectly rigid domain applying compatibility and equilibrium at the soil–foundation interfaces, yielding a considerable reduction in the number of degrees of freedom of the problem. The reference point of the rigid domain will be used for coupling the equations of motion of the superstructure to the system of equations that defines the behavior of the soil and the foundation. The superstructure shear deformation and torsional eccentricity for non–symmetrical cases are included in the elemental stiffness matrix.

As excitation, this model includes planar harmonic incident waves, that can be of SH, SV or P wave type with vertical and horizontal incident angles.

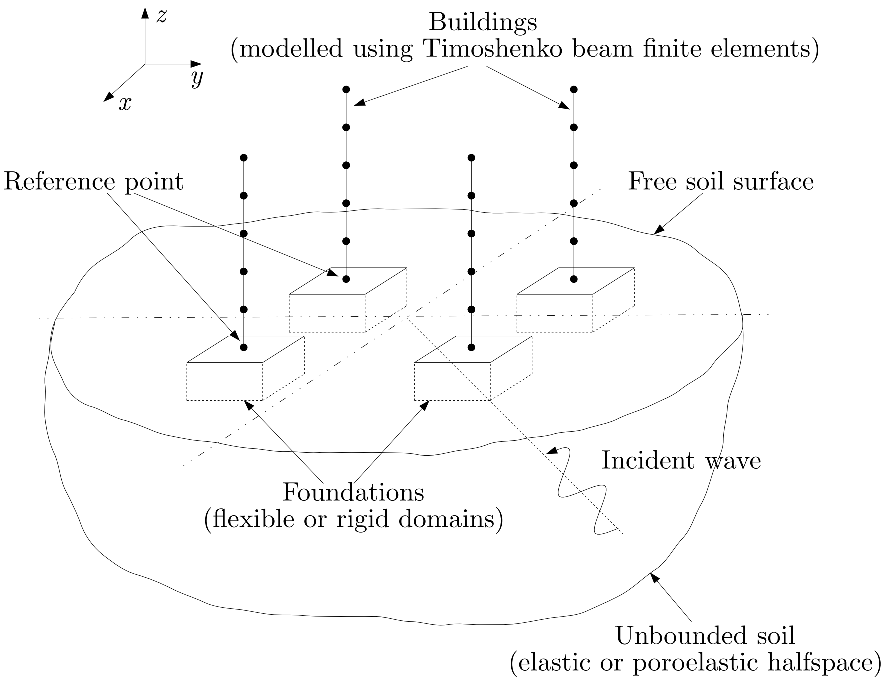

Figure 1: Example of application. Group of four nearby buildings founded on a halfspace.

In short, the model presented in this paper is able to rigorously represent the essential aspects of the problem while being, at the same time, more versatile and computationally efficient than the previous version of the code. The model could be used to address problems involving several buildings (figure 1), wind turbines or other type of structures.

2 Excitation model

The excitation is considered as a harmonic planar wave that comes from a far source and affects the site of the rigid foundation. The presence of the foundation makes disruptions in the incident wave field. Then, the field of the total displacement $\mathbf{u}$ is the superposition of the incident $\mathbf{u}_\mathrm{I}$ and refracted $\mathbf{u}_\mathrm{R}$ fields, $\mathbf{u} = \mathbf{u}_\mathrm{I} + \mathbf{u}_\mathrm{R}$, also the tractions field $\mathbf{p} = \mathbf{p}_\mathrm{I} + \mathbf{p}_\mathrm{R}$. Therefore, the algebraic BEM system of equations in the refracted field may be written as:

$$\mathbf{H}\left(\mathbf{u} - \mathbf{u}_\mathrm{I}\right) = \mathbf{G}\left(\mathbf{p} - \mathbf{p}_\mathrm{I}\right) \tag{1}$$

3 Rigid body restrictions

The strategy to include the rigid body restrictions is based on the traditional expansion method used in two–dimensional problems by Thomazo and Mesquita [3], but considering herein three–dimensional quadrilateral (9–noded) and triangular (6–noded) quadratic boundary elements. The process may be summarized as the task of incorporating kinematic restriction relation and equilibrium condition into the matrices of equation (1). The boundary Γ can be subdivided in two distinct boundaries, $\Gamma = \Gamma_s \cup \Gamma_r$, see figure 2. On the boundary Γ_s, regular displacements $\mathbf{u}_s$ and tractions $\mathbf{p}_s$ are prescribed. The boundary Γ_r is the interface between the soil domain Ω and the rigid foundation. The displacements and tractions over this boundary are defined, respectively as $\mathbf{u}_r$ and $\mathbf{p}_r$.

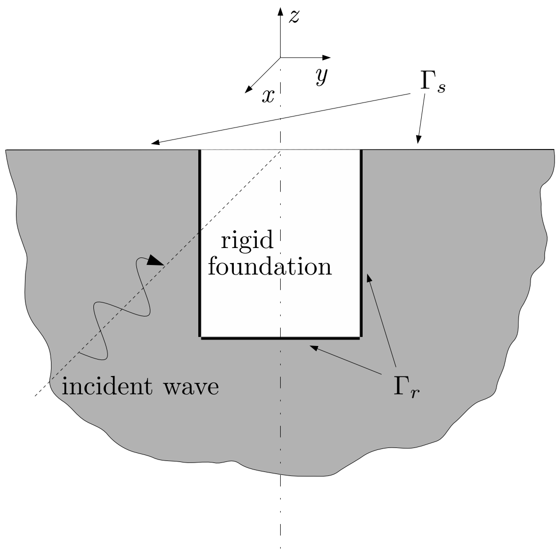

Figure 2: Foundation assuming rigid body restrictions. Free surface Γ_s ($\mathbf{p}_s = 0$) and rigid interface Γ_r.

Considering the boundary subdivision $\Gamma = \Gamma_s \cup \Gamma_r$, the system (1) may be partitioned as follows:

$$\begin{bmatrix} \mathbf{H}_{ss} & \mathbf{H}_{sr} \\ \mathbf{H}_{rs} & \mathbf{H}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{u}_s - \mathbf{u}_\mathrm{I} \\ \mathbf{u}_r - \mathbf{u}_\mathrm{I} \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{ss} & \mathbf{G}_{sr} \\ \mathbf{G}_{rs} & \mathbf{G}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{p}_s - \mathbf{p}_\mathrm{I} \\ \mathbf{p}_r - \mathbf{p}_\mathrm{I} \end{bmatrix} \tag{2}$$

The Γ_r boundary is discretized into n_e quadratic elements. The coordinates of the nodes of these elements are $(x_i,\, y_i,\, z_i)$, for $(i = 1$ to $n_r)$, being n_r the number of nodes at Γ_r. The three components of the displacements vector $\mathbf{u}_i$ at every node in the x, y and z direction are respectively u_i, v_i and w_i.

The six degrees of freedom of the rigid body (three displacements and three rotations) can be measured from an arbitrary point of reference with coordinates (x^{ref}, y^{ref}, z^{ref}) and may be organized in the rigid body displacements vector $\mathbf{u}^{\mathrm{ref}} = [u^{\mathrm{ref}}\ v^{\mathrm{ref}}\ w^{\mathrm{ref}}\ \theta_x^{\mathrm{ref}}\ \theta_y^{\mathrm{ref}}\ \theta_z^{\mathrm{ref}}]$.

The kinematic compatibility relations that exist between the rigid body degrees of freedom and the displacements of the i-node at the interface Γ_r can be written in matrix form as:

$$\begin{bmatrix} u_i^j \\ v_i^j \\ w_i^j \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & (z_i - z^{\text{ref}}) & (y^{\text{ref}} - y_i) \\ 0 & 1 & 0 & (z^{\text{ref}} - z_i) & 0 & (x_i - x^{\text{ref}}) \\ 0 & 0 & 1 & (y_i - y^{\text{ref}}) & (x^{\text{ref}} - x_i) & 0 \end{bmatrix} \begin{bmatrix} u^{\text{ref}} \\ v^{\text{ref}} \\ w^{\text{ref}} \\ \theta_x^{\text{ref}} \\ \theta_y^{\text{ref}} \\ \theta_z^{\text{ref}} \end{bmatrix} = \mathbf{C}_i \, \mathbf{u}^{\text{ref}} \tag{3}$$

The compact form of equation (3) extended to the n_r nodes in the Γ_r interface may be written as:

$$\mathbf{u}_r = \mathbf{C} \, \mathbf{u}^{\text{ref}} \tag{4}$$

being $\mathbf{u}_r = [\mathbf{u}_1, \dots, \mathbf{u}_{n_r}]$ and $\mathbf{C} = [\mathbf{C}_1, \dots, \mathbf{C}_{n_r}]$.

Also, equilibrium between the forces acting on the rigid foundation and the tractions on the soil–rigid body interface Γ_r is required. $\mathbf{p}_j(x, y, z) = (p_x^j, p_y^j, p_z^j)$ is the vector of tractions of the j–element over Γ_r. The resultants of the external forces are considered to be acting at the center of mass $(x^{\text{cg}}, y^{\text{cg}}, z^{\text{cg}})$ of the rigid body. Considering the inertial forces of the rigid foundation, the equilibrium relations can be expressed as:

$$\begin{aligned}
F_x^{\text{cg}} &= \sum_{j=1}^{n_e} \int_{\Gamma_r^j} p_x^j \, \mathrm{d}\Gamma_r^j - \omega^2 \, M \, u_x^{\text{cg}} \\
F_y^{\text{cg}} &= \sum_{j=1}^{n_e} \int_{\Gamma_r^j} p_y^j \, \mathrm{d}\Gamma_r^j - \omega^2 \, M \, u_y^{\text{cg}} \\
F_z^{\text{cg}} &= \sum_{j=1}^{n_e} \int_{\Gamma_r^j} p_z^j \, \mathrm{d}\Gamma_r^j - \omega^2 \, M \, u_z^{\text{cg}} \\
M_x^{\text{cg}} &= \sum_{j=1}^{n_e} \left(\int_{\Gamma_r^j} -p_y^j (z^j - z^{\text{cg}}) \, \mathrm{d}\Gamma_r^j + \int_{\Gamma_r^j} p_z^j (y^j - y^{\text{cg}}) \, \mathrm{d}\Gamma_r^j \right) - \omega^2 \, I_x^{\text{cg}} \, \theta_x^{\text{cg}} \\
M_y^{\text{cg}} &= \sum_{j=1}^{n_e} \left(\int_{\Gamma_r^j} p_x^j (z^j - z^{\text{cg}}) \, \mathrm{d}\Gamma_r^j + \int_{\Gamma_r^j} -p_z^j (x^j - x^{\text{cg}}) \, \mathrm{d}\Gamma_r^j \right) - \omega^2 \, I_y^{\text{cg}} \, \theta_x^{\text{cg}} \\
M_z^{\text{cg}} &= \sum_{j=1}^{n_e} \left(\int_{\Gamma_r^j} -p_x^j (y^j - y^{\text{cg}}) \, \mathrm{d}\Gamma_r^j + \int_{\Gamma_r^j} p_y^j (x^j - x^{\text{cg}}) \, \mathrm{d}\Gamma_r^j \right) - \omega^2 \, I_z^{\text{cg}} \, \theta_x^{\text{cg}}
\end{aligned} \tag{5}$$

being M the total mass, I_x^{cg}, I_y^{cg}, I_z^{cg} the inertia moments at the center of mass of the foundation, and ω the excitation frequency. The components of the tractions vector

$\mathbf{p}_j(x,\, y,\, z)$ can be expressed as a function of the natural coordinates of the boundary element $(\xi,\, \eta)$ as:

$$p_x^j = \sum_{k=1}^{n_j} (p_x^j)_k \, \phi_k^j(\xi,\, \eta) \quad ; \quad p_y^j = \sum_{k=1}^{n_j} (p_y^j)_k \, \phi_k^j(\xi,\, \eta) \quad ; \quad p_z^j = \sum_{k=1}^{n_j} (p_z^j)_k \, \phi_k^j(\xi,\, \eta) \qquad (6)$$

In the group of equations (6), n_j is the number of nodes of the j–element, ϕ_k^j is the interpolation function at the k–node of the j–element. In the same way the position vector of the j–element is:

$$x^j = \sum_{k=1}^{n_j} x_k^j \, \phi_k^j(\xi,\, \eta) \quad ; \quad y^j = \sum_{k=1}^{n_j} x_k^j \, \phi_k^j(\xi,\, \eta) \quad ; \quad y^j = \sum_{k=1}^{n_j} x_k^j \, \phi_k^j(\xi,\, \eta) \qquad (7)$$

Substituting equations (6) and (7) in (5) and reordering, the equilibrium relations can be written in a compact form as:

$$\mathbf{F}^{\mathrm{cg}} = \mathbf{E}\, \mathbf{p}_r - \omega^2\, \mathbf{M}\, \mathbf{u}^{\mathrm{cg}} \qquad (8)$$

being $\mathbf{E}$ the equilibrium matrix, where every matrix–element is the addition of the integrals of the traction at the i–node over the elements (i–node belongs) of the rigid interface mesh. $\mathbf{M}$ is the diagonal mass matrix.

The equilibrium can be defined at an arbitrary reference point considering in equation (8) the next equilibrium and kinematic relations between the center of mass and the point of reference:

$$\begin{bmatrix} F_x^{\mathrm{cg}} \\ F_y^{\mathrm{cg}} \\ F_z^{\mathrm{cg}} \\ M_x^{\mathrm{cg}} \\ M_y^{\mathrm{cg}} \\ M_z^{\mathrm{cg}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & (z^{\mathrm{cg}} - z^{\mathrm{ref}}) & (y^{\mathrm{ref}} - y^{\mathrm{cg}}) & 1 & 0 & 0 \\ (z^{\mathrm{ref}} - z^{\mathrm{cg}}) & 0 & (x^{\mathrm{cg}} - x^{\mathrm{ref}}) & 0 & 1 & 0 \\ (y^{\mathrm{cg}} - y^{\mathrm{ref}}) & (x^{\mathrm{ref}} - x^{\mathrm{cg}}) & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x^{\mathrm{ref}} \\ F_y^{\mathrm{ref}} \\ F_z^{\mathrm{ref}} \\ M_x^{\mathrm{ref}} \\ M_y^{\mathrm{ref}} \\ M_z^{\mathrm{ref}} \end{bmatrix} = \mathbf{T}\, \mathbf{F}^{\mathrm{ref}} \qquad (9)$$

$$\begin{bmatrix} u^{\mathrm{cg}} \\ v^{\mathrm{cg}} \\ w^{\mathrm{cg}} \\ \theta_x^{\mathrm{cg}} \\ \theta_y^{\mathrm{cg}} \\ \theta_z^{\mathrm{cg}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & (z^{\mathrm{cg}} - z^{\mathrm{ref}}) & (y^{\mathrm{ref}} - y^{\mathrm{cg}}) \\ 0 & 1 & 0 & (z^{\mathrm{ref}} - z^{\mathrm{cg}}) & 0 & (x^{\mathrm{cg}} - x^{\mathrm{ref}}) \\ 0 & 0 & 1 & (y^{\mathrm{cg}} - y^{\mathrm{ref}}) & (x^{\mathrm{ref}} - x^{\mathrm{cg}}) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{\mathrm{ref}} \\ v^{\mathrm{ref}} \\ w^{\mathrm{ref}} \\ \theta_x^{\mathrm{ref}} \\ \theta_y^{\mathrm{ref}} \\ \theta_z^{\mathrm{ref}} \end{bmatrix} = \mathbf{L}\, \mathbf{u}^{\mathrm{ref}} \qquad (10)$$

The methodology to incorporate the rigid body restrictions assumes that the prescribed boundary conditions on Γ_r are the external load vector $\mathbf{F}^{\mathrm{ref}}$, the traction vector $\mathbf{p}_s$ at the boundary Γ_s, and the displacements and tractions of the incident wave. This boundary conditions may be included in the right–hand side vector $\mathbf{B}$. The unknown quantities are the displacements $\mathbf{u}_r$ and tractions $\mathbf{p}_r$ over Γ_r and the displacements $\mathbf{u}_s$ over Γ_s. Then, the displacement vector $\mathbf{u}_r$ may be substituted considering the kinematic relation (4) and the equilibrium equations (8) are introduced into the system of equations (2), resulting:

$$\begin{bmatrix} \mathbf{H}_{ss} & -\mathbf{G}_{sr} & \mathbf{H}_{sr}\,\mathbf{C} \\ \mathbf{H}_{rs} & -\mathbf{G}_{rr} & \mathbf{H}_{rr}\,\mathbf{C} \\ 0 & \mathbf{E} & -\omega^2\,\mathbf{M}\,\mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{u}_s \\ \mathbf{p}_r \\ \mathbf{u}^{\text{ref}} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{T}\,\mathbf{F}^{\text{ref}} \end{bmatrix} \tag{11}$$

4 Modelling of the superstructures. Two–noded beam finite elements

The model presented above allows to consider the presence of superstructures, as buildings, coupled to the foundation rigid domain. The building is modelled using three–dimensional two–noded Timoshenko beam finite elements, as defined in [4], but considering three–dimensional element instead. Therefore, six degrees of freedom, three displacements and three rotations, are defined per node, see figure 3.

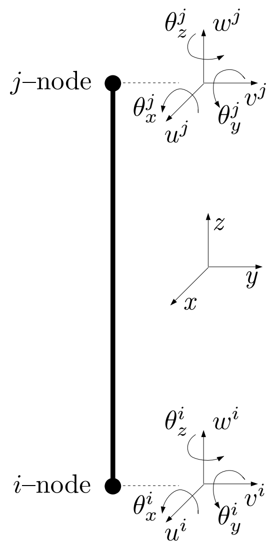

Figure 3: Two-noded beam finite element

The degrees of freedom of every node are defined at the center of gravity of the cross–section of the building. In the case of a completely symmetrical cross–section the center of gravity G and the shear–center C are located at the same point of the section. Therefore, there is no coupling between the torsional and bending forces at the center of gravity of the section. If the inertial forces are not considered, the equilibrium equations of the element are:

$$\begin{bmatrix} \mathbf{F}^i \\ \mathbf{F}^j \end{bmatrix}^{\mathrm{C}} = \mathbf{K}_{ij}^{\mathrm{C}} \begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{C}} \tag{12}$$

being $\mathbf{F}^i$ and $\mathbf{F}^j$ the vectors of nodal forces. The displacements vectors at every node of the element are respectively $\mathbf{u}^i$ and $\mathbf{u}^j$. The definition of the stiffness matrix $\mathbf{K}_{ij}^{\mathrm{C}}$ of the ij–element at the shear–center can be seen at [4], but herein axial and torsional degrees of freedom are taken into account.

In the case of buildings with non–symmetrical cross–section, the shear–center C and center of gravity G are not located at the same point, see figure 4. At the shear–center the torsional and bending forces are not coupled, and the stiffness matrix of the ij–element is $\mathbf{K}_{ij}^{\mathrm{C}}$. On the contrary, now the torsional and bending forces are coupled at the center of gravity. The stiffness matrix at the center of gravity can be obtained using the kinematic and equilibrium relations between G and C. These relations can be expressed in compact matrix form as:

$$\begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{C}} = \begin{bmatrix} \mathbf{S} & \\ & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{G}} \quad ; \quad \begin{bmatrix} \mathbf{F}^i \\ \mathbf{F}^j \end{bmatrix}^{\mathrm{C}} = \begin{bmatrix} \mathbf{Q} & \\ & \mathbf{Q} \end{bmatrix} \begin{bmatrix} \mathbf{F}^i \\ \mathbf{F}^j \end{bmatrix}^{\mathrm{G}} \tag{13}$$

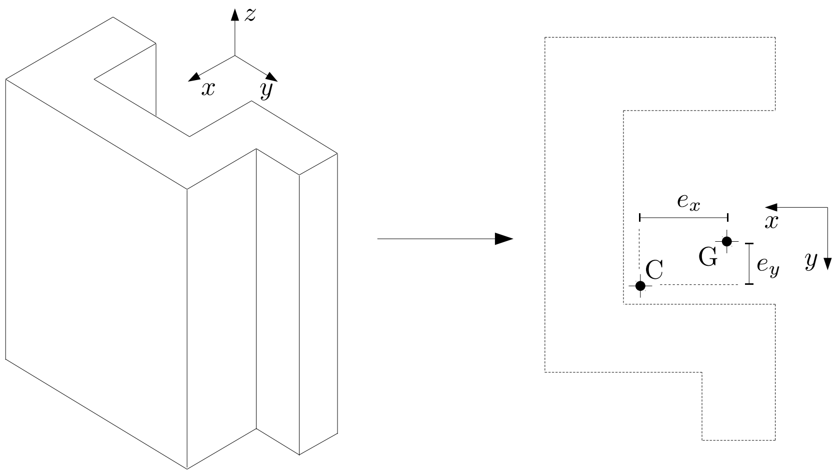

Figure 4: Building with non–symmetrical cross–sections.

where

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -e_y \\ 0 & 1 & 0 & 0 & 0 & e_x \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

and $\mathbf{Q}$ is the inverse of the transpose of $\mathbf{S}$; $\mathbf{Q} = (\mathbf{S}^{\mathrm{T}})^{-1}$. e_x and e_y represent the eccentricity of the cross–section in both directions x and y respectively.

Substituting equations (13) in equation (12) and taking into account the relation between $\mathbf{S}$ and $\mathbf{Q}$, the equilibrium equations defined at the center of gravity are:

$$\begin{bmatrix} \mathbf{F}^i \\ \mathbf{F}^j \end{bmatrix}^{\mathrm{G}} = \begin{bmatrix} \mathbf{S}^{\mathrm{T}} \\ \mathbf{S}^{\mathrm{T}} \end{bmatrix} \mathbf{K}_{ij}^{\mathrm{C}} \begin{bmatrix} \mathbf{S} \\ \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{G}} = \mathbf{K}_{ij}^{\mathrm{G}} \begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{G}} \tag{15}$$

The stiffness matrix $\mathbf{K}_{ij}^{\mathrm{G}}$ of the ij–element converges to $\mathbf{K}_{ij}^{\mathrm{C}}$ only when the cross–section is symmetrical ($e_x = e_y = 0$).

If the inertial forces are considered, they have to be added into the equation (15):

$$\begin{bmatrix} \mathbf{F}^i \\ \mathbf{F}^j \end{bmatrix}^{\mathrm{G}} = (\mathbf{K}_{ij}^{\mathrm{G}} - \omega^2 \mathbf{M}_{ij}^{\mathrm{G}}) \begin{bmatrix} \mathbf{u}^i \\ \mathbf{u}^j \end{bmatrix}^{\mathrm{G}} \tag{16}$$

The consistent mass–matrix $\mathbf{M}_{ij}^{\mathrm{G}}$ of the ij-element is defined at the center of gravity and is the addition of two matrices, the first one associated to the translational inertia and the second one associated to the rotatory inertia, see [4].

5 Coupling the rigid body foundation and the superstructure

The coupling between the rigid foundation and the superstructure is done through kinematic and equilibrium relations at the reference point. This arbitrary point of reference is considered located at the bottom point of the superstructure. Finally, the system of equations (16) of every ij–element needs to be introduced into (11).

6 Formulation of elastic waves in poroelastic media

The problem of wave propagation in elastodynamics is well known [5]. Also for poroelastic media, the integration of the Navier equations can be done in terms of dilatations (e, ε) and the rotation vectors (w, Ω). Therefore, both irrotational and equivolumial waves are possible in poroelastic domains. The formulation of the elastic waves with vertical incidence in poroelastic soils presented here is based on the model proposed by Biot [6]. Some properties need to be defined in order to explain this formulation easily, see table 1 and expressions (17).

$$\begin{gathered} \rho_1 = \rho_s\,(1-\phi) \quad ; \quad \rho_2 = \rho_f\,\phi \quad ; \quad \rho_{12} = -\rho_a \\ \rho_{11} = \rho_1 - \rho_{12} \quad ; \quad \rho_{22} = \rho_2 - \rho_{12} \end{gathered} \tag{17}$$

6.1 Irrotational wave. P–wave

Taking the divergence of the Navier equations, one obtains:

Table 1: Definition of the properties of a poroelastic soil

ϕ	porosity
ρ_s	density of the solid phase
ρ_f	density of the fluid phase
ρ_a	additional apparent density
b	coefficient of dissipation
Q, R	Biot constants
e, ε	volumetric dilatation of the solid and fluid respectively
w, Ω	rotation vectors of the solid and fluid respectively
τ_{ij}	stress tensor of the solid phase
τ	equivalent stress in the fluid phase

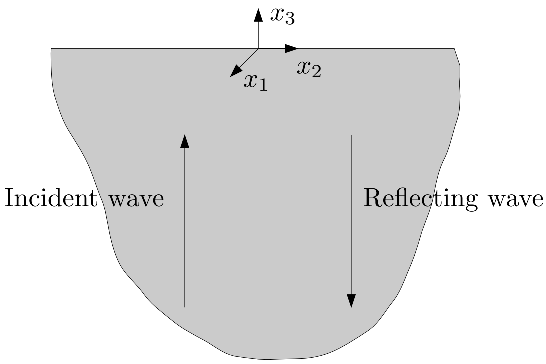

Figure 5: Incident and reflecting wave in poroelastic media.

$$\begin{aligned} \left(\lambda + 2\,\mu + \frac{Q^2}{R}\right)\nabla^2 e + Q\,\nabla^2\varepsilon &= -\omega^2(\rho_{11}\,e + \rho_{12}\,\varepsilon) + \mathrm{i}\,\omega\,b\,(e-\varepsilon) \\ Q\,\nabla^2 e + R\,\nabla^2\varepsilon &= -\omega^2(\rho_{12}\,e + \rho_{22}\,\varepsilon) - \mathrm{i}\,\omega\,b\,(e-\varepsilon) \end{aligned} \tag{18}$$

If a harmonic planar wave propagating in the positive x_3–axis direction is considered (figure 5), the volumetric dilatation of both solid and fluid phases can be written as:

$$e = D_s\,\mathrm{e}^{-\mathrm{i}\,k_{\mathrm{P}}\,x_3} \quad ; \quad \varepsilon = D_f\,\mathrm{e}^{-\mathrm{i}\,k_{\mathrm{P}}\,x_3} \tag{19}$$

being k_{P} the wave number, D_s and D_f are the amplitudes of the volumetric dilatation in the solid and fluid phase respectively. Now,substituting expressions (19) in (18), one obtains a system of equations with no trivial solution for specific values of k_{P}. The characteristic equation of this system may be expressed as:

$$\mathrm{A}\,(k_{\mathrm{P}}^2)^2 + \mathrm{B}\,k_{\mathrm{P}}^2 + \mathrm{C} = 0 \tag{20}$$

Two eigenvalues for the equation (20) are obtained, k^2_{P1} and k^2_{P2}. Therefore, there are two rotational waves (P1 and P2) with different propagation velocities:

$$c^2_{\mathrm{P1}} = \frac{\omega^2}{k^2_{\mathrm{P1}}} \qquad c^2_{\mathrm{P2}} = \frac{\omega^2}{k^2_{\mathrm{P2}}} \tag{21}$$

The eigenvector for P1–wave $[D^1_s\, D^1_f]$ is obtained using the eigenvalue k^2_{P1} and the eigenvector for P2–wave $[D^2_s\, D^2_f]$ through eigenvalue k^2_{P2}.

The displacements u_3 and U_3 in the x_3–axis direction induced by the irrotational incident waves (P1 and P2) in both solid and fluid phases of a poroelastic soil, taking into account the incident and reflected wave, can be expressed as:

$$\begin{bmatrix} u_3 \\ U_3 \end{bmatrix} = \begin{bmatrix} D^1_s \\ D^1_f \end{bmatrix} (D^{\mathrm{I}}_1\, \mathrm{e}^{-\mathrm{i}\, k_{\mathrm{P1}} x_3} + D^{\mathrm{R}}_1\, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{P1}} x_3}) + \begin{bmatrix} D^2_s \\ D^2_f \end{bmatrix} D^{\mathrm{R}}_2\, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{P2}} x_3} \tag{22}$$

The incident P2–wave is considered attenuated due to the high damping and is not included in the expression (22).

After deriving (22) respect x_3 to obtain e and ε, taking into account the constitutive law, the stresses are written as:

$$\begin{aligned} \tau_{11} &= \left(\lambda + \frac{Q^2}{R}\right) e + Q\,\varepsilon \\ \tau_{22} &= \left(\lambda + \frac{Q^2}{R}\right) e + Q\,\varepsilon \\ \tau_{33} &= \left(\lambda + \frac{Q^2}{R} + 2\,\mu\right) e + Q\,\varepsilon \\ \tau &= Q\,e + R\,\varepsilon \end{aligned} \tag{23}$$

The values D^{I}_1, D^{R}_1 and D^{R}_2 are obtained through the boundary conditions over the free surface at $x_3 = 0$:

$$u_3 = 1 \quad ; \quad \tau_{33} = 0 \quad ; \quad \tau = 0 \tag{24}$$

After applying this boundary conditions, $D^{\mathrm{R}}_2 = 0$ and $D^{\mathrm{I}}_1 = D^{\mathrm{R}}_1 = 0.5$. Also is possible to consider in (24) the average displacement $\bar{u}_3$ instead of u_3, being $\bar{u}_3 = (1-\phi)\, u_3 + \phi\, U_3$. If $\bar{u}_3 = 1$ is considered as boundary condition, then

$$D^{\mathrm{R}}_2 = 0 \quad ; \quad D^{\mathrm{I}}_1 = D^{\mathrm{R}}_1 = \frac{1}{2} \frac{1}{(1-\phi)\, D^1_s + \phi\, D^1_f}$$

6.2 Equivolumial wave. S–wave

Applying the curl operation to the equations of Navier, one obtains:

$$
\begin{aligned}
\mu\,\nabla^2 \mathrm{w} &= -\omega^2\,(\rho_{11}\,\mathrm{w} + \rho_{12}\,\Omega) + \mathrm{i}\,\omega\,b\,(\mathrm{w}-\Omega) \\
0 &= -\omega^2\,(\rho_{12}\,\mathrm{w} + \rho_{22}\,\Omega) - \mathrm{i}\,\omega\,b\,(\mathrm{w}-\Omega)
\end{aligned}
\tag{25}
$$

being w and Ω the rotation vector in the solid and fluid phase respectively. Considering an harmonic planar wave propagating in x_3 direction, the rotation vectors may be written as:

$$
\mathrm{w} = D_{\mathrm{w}}\,\mathrm{e}^{-\mathrm{i}\,k_{\mathrm{S}}\,x_3} \quad ; \quad \Omega = D_{\Omega}\,\mathrm{e}^{-\mathrm{i}\,k_{\mathrm{S}}\,x_3}
\tag{26}
$$

being k_{S} the wave number, D_{w} and D_{Ω} the amplitudes of the rotation vectors of the solid and fluid phases respectively. Replacing expressions (26) in the second equation (25) the next relation between the rotation vectors can be written:

$$
\Omega = \frac{\mathrm{i}\,\omega\,b + \omega^2\,\rho_{12}}{\mathrm{i}\,\omega\,b - \omega^2\,\rho_{22}}\,\mathrm{w} = \Lambda\,\mathrm{w}
\tag{27}
$$

Including the equation (27) and expressions (26) in the first equation of (25), the wave number k_{S} is obtained as:

$$
k_{\mathrm{S}} = \omega^2\,\frac{\rho}{\mu} \quad ; \quad \rho = \frac{\omega^2\,(\rho_{12}^2 - \rho_{11}\,\rho_{22}) + \mathrm{i}\,\omega\,b\,(\rho_{11} + 2\,\rho_{12} + \rho_{22})}{\mathrm{i}\,\omega\,b - \omega^2\,\rho_{22}}
\tag{28}
$$

Considering a rotational wave propagating in x_3 direction and producing displacements in the x_2 direction (figure5), the displacements field of the solid phase of the incident and reflected wave is:

$$
u_2 = D^{\mathrm{I}}\,\mathrm{e}^{-\mathrm{i}\,k_{\mathrm{S}}\,x_3} + D^{\mathrm{R}}\,\mathrm{e}^{\mathrm{i}\,k_{\mathrm{S}}\,x_3}
\tag{29}
$$

Using relation (27), the displacement of the fluid phase U_2 can be expressed as:

$$
U_2 = \Lambda\,u_2
\tag{30}
$$

The boundary conditions at the free surface at $x_3 = 0$ are $u_2 = 1$ and $\tau_{23} = 0$. Applying this conditions the values of the amplitude of the incident and reflecting wave are obtained, $D^{\mathrm{I}} = D^{\mathrm{R}} = 0.5$.

7 Conclusions

The formulation of a BEM-FEM model suitable to study dynamic soil–structure and structure–soil–structure interaction problems has been presented in this work. The procedure to incorporate the rigid body restrictions to the BEM model, the coupling of the superstructure, modelled using two–noded Timoshenko beam finite elements, and the extension of the excitation model, including elastic waves in poroelastic soils have been explained. The model can be applied to address problems involving several buildings, wind turbines or other type of structures founded on elastic or poroelastic soils.

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REFERENCES

[1] Maeso, O., Aznárez, J.J. and Domínguez, J. Effects of space distribution of excitation on seismic response of arch dams. *J. Eng. Mech.* ASCE (2002) **128(7)**:759–768.

[2] Maeso, O., Aznárez, J.J. and Domínguez, J. Three–dimensional models of reservoir sediment and effects on the seismic response of arch dams. *Earth. Eng. Struct. Dyn.* (2004) **33(10)**:1103–1123.

[3] Thomazo, L.H. and Mesquita, E. On strategies to incorporate rigid body conditions in an indirect version of the boundary element method. CMNE/CILAMCE, Portugal, (2007).

[4] Friedman, Z. and Kosmatka, J.B. An improved two–node Timoshenko beam finite element. *Computers & Structures* (1993) **47(3)**:473–481.

[5] Eringen, A.C. and Suhubi, E.S. *Elastodynamics* (Vol. 2. Linear Theory). New York. Academic Press. (1975)

[6] Biot, M.A. Theory of propagation of elastic waves in a fluid-saturated porous solid. I: Low frequency range. *Journal of the Acoustical Society of America* (1956) **28(2)**:168–178.