

FINAL WORKSHOP - Ispra (IT), May 30th, 2013

Multi-Building Interactions and Site-City Effect

- an idealized experimental model -

L.Schwan (PhD student), C.Boutin, M.Dietz, L.A.Padron, P.-Y.Bard, S.Castellaro, E.Ibraim, O.Maeso, J.J.Aznárez and C.Taylor





A Site-City effect?

Context

- Structure-Soil-Structure Interactions not taken into account in usual engineering practice
- Records in urban area during Mexico earthquakes : long duration, beatings. Might they depend on the city ? (Wirgin & Bard, 1996, BSSA)
- Assumptions ? Methods ?



Horizontal accelerations recorded in Mexico on firm ground (top) and in the lake bed zone (bottom) during the 25th April 1989 event [Chavez-Garcia et al., 1994]

Objective

To identify, describe and quantify large scale multiple interactions phenomena through experimental, numerical and theoretical crossed analyses

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Interdisciplinary collaboration to approach the issue

Team members	Numerical Modelling	Theoretical Modelling	In-Situ Data
Pr. P.Y. Bard (lead user) ISTerre, University of Grenoble CNRS/IRD/IFSTTAR - France	Spectral Element Simulation		Structures and ground motion
Pr. C. Boutin, PhD student L. Schwan ENTPE, University of Lyon LGCB/CNRS - France		Homogenization of periodic systems	
Dr. L.A. Padròn, J.J. Aznàrez & O. Maeso University of Las Palmas de Gran Canaria - Spain	Boundary/Finite Element Methods		
Pr. S. Castellaro University of Bologna - Italy	Reverse analysis		Structures and ground motion

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and from the host facility *EQUALS* : M. Dietz, E. Ibraim, C. Taylor, *University of Bristol - U.K.*

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Institute	Modelling	Modelling	
Pr. P.Y. Bard (lead user)	Spectral		Structures and
ISTerre, University of Grenoble	Element		ground motion
CNRS/IRD/IFSTTAR - France	Simulation		
Pr. C. Boutin,		Homogenization	
PhD student L. Schwan		of periodic	
ENTPE, University of Lyon		systems	
LGCB/CNRS - France			
Dr. L.A. Padròn,	Boundary/Finite		
J.J. Aznàrez & O. Maeso	Element		
University of Las Palmas	Methods		
de Gran Canaria - Spain			
Pr. S. Castellaro	Reverse		Structures and
University of Bologna - Italy	analysis		ground motion

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Outline

Theoretical model : city impedance analysis

Design, shaking and instrumentation

Experimental results / city impedance analysis

Numerical simulations / city impedance analysis

L.A. Padròn, J.J. Aznàrez & O. Maeso

Conclusions



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Idealization of a city





Los Angeles - downtown



 Σ -periodic distribution of identical resonant structures

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Periodic surface under long wavelength



Σ-periodic surface

 $\Sigma = \ell^2$

 Scale separation : the wavelength is much larger than the width of the period

 $\Lambda \gg 2\pi\ell$

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Σ-periodic surface

 $\Sigma = \ell^2$

 Scale separation : the wavelength is much larger than the width of the period

 $\Lambda \gg 2\pi \ell$

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Isotropic :

Same resonant behaviour in all directions



Σ-periodic surface

 $\Sigma = \ell^2$

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 $\Lambda \gg 2\pi \ell$

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Anisotropic :

Resonant direction & Inert direction



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Anisotropic :

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Constructive interferences



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► Homogenization (Boutin & Roussillon, 2004, BSSA) : Surface stress= Force exerted by Resonator/|∑|



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Anisotropic :

Resonant direction & Inert direction

Constructive interferences

• Homogenization (Boutin & Roussillon, 2004, BSSA) : Surface stress= Force exerted by Resonator/ $|\Sigma|$

• Resonator exerts a force on surface because it is shaken **Resonator Force** = Resonator Impedance $\mathcal{Z} \times$ Surface velocity Periodic surface under long wavelength City Impedance



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• City impedance $Z_o = \mathcal{Z}/|\Sigma|$

Surface stress = City impedance $Z_o \times$ Surface velocity

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Ratio city impedance Z_o / soil impedance Z_S :

- parameter η
- resonator-dependent dynamical function

$$\frac{Z_o}{Z_S} = \eta \frac{i\frac{f}{f_o} + 2\xi_o \frac{f^2}{f_o^2}}{1 - i2\xi_o \frac{f}{f_o} - \frac{f^2}{f_o^2}}$$

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City resonance coincidences with layer resonance The layer would like to amplify the displacement imposed at its base



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BUT the resonant surface imposes

- ► a free-like condition in *Y* inert direction
- ► a rigid-like condition at resonance in *X* direction

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Layer



Design Layer

- Elastic, linear, isotropic
- Eigenfrequency $f_L < 15 Hz$

City

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► Aspect ratios > 2



- ▶ $h_L = 76 \ cm$
- $f_L = 9.36 H_z$ in X (9.11 H_z in Y)
- ► $\xi = 4.9 \%$

Design Layer

- Elastic, linear, isotropic
- Eigenfrequency $f_L < 15 Hz$
- Aspect ratios > 2



- $f_L = 9.36 H_z$ in X (9.11 H_z in Y)
- ► $\xi = 4.9 \%$

City

- ▶ Period width $\ell \ll \Lambda$ Wavelength
- Eigenfrequency $f_o \approx f_L$
- Modal mass m_o ~ Mass of layer under period



- Period width $\ell = 5 \ cm$
- $f_o \approx 8.4 \, Hz$ $\xi_o \approx 5 \, \%$
- $\eta \approx 13.6\%$ (Mexico : $\eta \sim 10\%$)

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Mock-up

Period $\ell = 5 \ cm$ 37 resonators





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in X



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in X resonant direction

Instrumentation







SETRA : 1D, 30 grams each, 8 *cm*-wide base plate



MEMS : 3D, 2 grams, 2 *cm* wide

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Drastic changes in spectrum Surface/Table



► 1 resonator : usual layer's resonance

Drastic changes in spectrum Surface/Table



- 1 resonator : usual layer's resonance
- 37 resonators : in X resonant direction : drastic change in layer's resonance

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37 resonators : in Y inert direction : usual resonance peak

Drastic changes in spectrum Surface/Table



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37 resonators : in Y inert direction : usual resonance peak
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- 1 resonator : usual layer's resonance
- 37 resonators : in X resonant direction : drastic change in layer's resonance

- 37 resonators : in Y inert direction : usual resonance peak
- City impedance analysis : qualitatively and quantitatively accurate



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1 sheet : usual temporal response of a layer



- 1 sheet : usual temporal response of a layer
- 37 sheets : drastic change in shape of records and lower amplitude

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- 1 sheet : usual temporal response of a layer
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City impedance analysis is accurate temporaly



- 1 sheet : usual temporal response of a layer
- 37 sheets : drastic change in shape of records and lower amplitude

- City impedance analysis is accurate temporaly
- Longer coda and clear beatings

Drastic changes in modal shapes



Depolarization









- X : antiresonance
- Y : resonance

X : resonance

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► Y : inertial

Depolarization



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Depolarization :

- frequency-dependent
- due to surface anisotropy
- Affects : direction, ellipticity, orientation

As the city gets denser



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As the city gets denser



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Already significant with 9 structures

• Model applies even for large period $(2\pi\ell/\Lambda \approx 0.4 \text{ for 9 sheets})$

As the city gets denser



As the city gets smaller



 $\begin{array}{l} \textbf{37 sheets} \\ \textbf{\textit{Gathered}} \\ \eta = 13.6\% \end{array}$

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As the city gets smaller



► The smaller the city, the smaller the effect

As the city gets smaller



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Still significant with only 5 structures

Various distributions of 9 resonators



Various distributions of 9 resonators



9 sheets random \approx 9 sheets gathered Periodic condition is not mandatory

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Numerical simulation with a BEM-FEM model

- ► The foam block is analyzed using a harmonic 2D direct boundary element formulation leading to a matrix form of the discretized boundary integral equation of the type Hu = Gt, where u and t are the vectors of displacements and tractions; and H and G are the coefficient matrices arising from the BEM.
- Quadratic 3-noded boundary elements are used in meshing the block
- Resonators are modelled using 2-node 6-dof beam finite elements
- Resonators and block are linked by perfectly-bonded rigid surfaces



Numerical simulation with a BEM-FEM model Considering equilibrium and compatibility conditions in the coupling, applying boundary conditions and reordering, the equations describing the dynamic response of the system can be written as

$$\begin{bmatrix} \mathbf{K}_{oo} & \mathbf{K}_{ob} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{bo} & \mathbf{K}_{bb} & \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{fc}\mathbf{D} & -\mathbf{G}_{fc} & \mathbf{A}_{ff} \\ \mathbf{0} & \mathbf{H}_{cc}\mathbf{D} & -\mathbf{G}_{cc} & \mathbf{A}_{cf} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{o} \\ \mathbf{u}^{b} \\ \mathbf{t}^{c} \\ \mathbf{x}^{f} \end{bmatrix} = \begin{cases} \mathbf{f}^{o} \\ \mathbf{0} \\ \mathbf{f}^{ff} \\ \mathbf{f}^{cf} \end{cases}$$
(1)

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where

- c and b refer to BEM and FEM nodes in the coupling, and f and o to the ones outside
- u and f are the vectors of displacements and external forces
- $\mathbf{F} \mathbf{K} = \mathbf{K}^* \omega^2 \mathbf{M}$
- D and C matrices defining the compatibility and equilibrium of the rigid interface
- and x^f the rest of BEM unknowns







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5 sheets All surface







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37 sheets All surface

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Conclusions

- Multiple interactions phenomena exist and can be significant
- Physical, theoretical and numerical models are validated
- Signatures of the phenomena are identified
- Phenomena are robust (only 5 structures seems sufficient)
- Theory gives the key parameters to quantify the effects
- Application to strongly non-linear soil is out of scope
- Other configurations have been tested within the project

Back to Experiment

Back to Design

Back to BEM-FEM

Appendix

Appendix

Cellular poylurethan foam Edge rods Modal shapes City gets denser City gets smaller Distributions of 9 resonators City with two types of resonators Other videos Other depolarizations

Material : Cellular polyurethan foam (Appendix)

- Homogeneous and light : $\rho = 49kg/m^3$
- Elastic linear, isotropic and soft : $E \approx 118 kPa$ $\nu \approx 0.1$
- No need for a container, clean, cheap, recyclable





 $\blacktriangleright L \times B \times H = 2.13 m \times 1.76 m \times 0.76 m$

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- Aspect ratios : L/H = 2.8 and B/H = 2.3
- Total mass : 140 kg (without the base plate assembly)

Design of the edge rods Appendix











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Design of the edge rods Appendix



3 mm-diameter 75 cm-long vertical steel rods are adhered at 35 cm centres around the periphery of the foam block

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Diameter and spacing designed to limit boundary effects

Design of the edge rods Appendix



- Eigenfrequency and damping remain unchanged
- The transfer function U_Γ/U_b is the one of a theoretical infinite layer
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Modal shapes Appendix



- At both resonance peaks : usual wave-quarter shape for layer
- 1st resonance peak : oscillators in phase with layer
- 2nd resonance peak : oscillators in phase opposition with layer

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At surface antiresonance : oscillators in resonance



1 sheet All surface $\eta = 0.2\%$









1 sheet All surface $\eta = 0.2\%$

5 sheets All surface $\eta = 1.4\%$













5 sheets All surface $\eta = 1.4\%$







9 sheets All surface $\eta = 2.8\%$







1 sheet All surface $\eta = 0.2\%$

5 sheets All surface $\eta = 1.4\%$



19 sheets All surface $\eta = 6.0\%$

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As the city gets denser Appendix









5 sheets All surface $\eta = 1.4\%$



19 sheets All surface $\eta = 6.0\%$

37 sheets All surface $\eta = 13.6\%$ 32/27







 $\begin{array}{l} \textbf{37 sheets} \\ \textbf{\textit{Gathered}} \\ \eta = 13.6\% \end{array}$

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 $\begin{array}{l} \textbf{37 sheets} \\ \textbf{\textit{Gathered}} \\ \eta = 13.6\% \end{array}$

19 sheets Gathered













19 sheets Gathered







9 sheets *Gathered*









 $\begin{array}{l} \textbf{37 sheets} \\ \textbf{\textit{Gathered}} \\ \eta = 13.6\% \end{array}$

19 sheets Gathered











5 sheets Gathered

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 $\begin{array}{l} \textbf{37 sheets} \\ \textbf{\textit{Gathered}} \\ \eta = 13.6\% \end{array}$

19 sheets Gathered

9 sheets Gathered

5 sheets Gathered

1 sheet











9 sheets All surface $\eta = 2.8\%$

9 sheets Gathered









10 11

10 11





9 sheets All surface $\eta = 2.8\%$

9 sheets Gathered

9 sheets *Randomly*

9 sheets + all angles All surface Theoric $\eta = 3.1 \%$





► \neq oscillators eigenfrequency $\Rightarrow \neq$ shapes



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35/27

• \neq oscillators eigenfrequency $\Rightarrow \neq$ shapes

2 ANTIRESONANCES (like theory)



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- 2 ANTIRESONANCES (like theory)
- Quite the same no matter the distribution



Same mass, different effects

 $h_o = 18.4 \ cm$ $\eta = 6.0\%$ 35/27







- X : Quasi-static
- Y : Quasi-static

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37/27



- X : Resonance
- Y : Near resonance

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X : Near anti-resonance

37/27

Y : Resonance



- X : Anti-resonance
- > Y : Not so far from resonance

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- X : Resonance
- Y : Inertial regime



- X : Inertial regime
- Y : Inertial regime

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- frequency-dependent
- due to surface anisotropy
- Affects : direction, ellipticity, orientation