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Some Bayesian Credibility Premiums Obtained by Using Posterior Regret Γ-Minimax Methodology

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Abstract. In credibility theory, the premium charged to a policyholder is computed on the basis of his/her own past claims and the accumulated past claims of the corresponding portfolio of policyholders. In order to obtain an appropriate formula for this, various methodologies have been proposed in actuarial literature, most of them in the field of Bayesian decision methodology.

In this paper, following the robust Bayesian paradigm, a procedure based on the posterior regret Γ-minimax principle is applied to derive, in a straightforward way, new credibility formula, making use of simple classes of distributions. This methodology is applied to the most commonly used premium calculation principles in insurance, namely the net, Esscher and variance principles.

Keywords: Classes of distributions, Credibility, Minimax, Premium, Posterior regret, Robustness

1 Introduction

Credibility theory is an experience rating technique that was developed in actuarial science and is frequently used in assessing automobile insurance, workers' compensation premium, loss reserving and IBNR-Incurred But Not Reported claims. Under this theory, premiums are established according to the accumulated past claims in a portfolio. As a result, the premium for a policyholder in class j, $j = 1, \ldots, l$, is computed by combining the experience of the individual (contract or policyholder) with the experience of a collective (portfolio) by using the expression

$$P_j = (1 - Z_j)m + Z_j M_j,\ j = 1, 2, \ldots, l. \tag{1}$$

where P_j is the credibility adjusted premium, m is the overall mean (the expected claim size for the whole portfolio), M_j is the mean for individual risk j and Z_j is the credibility factor, a number between 0 and 1. This expression was suggested by Whitney (1918) in a heuristic form, but can also be obtained under different Bayesian methodologies. In this regard, credibility theory is used to determine the expected claims experience of an individual risk when risks are not homogeneous, since the individual risk belongs to a heterogeneous collective. The main purpose of credibility theory is to calculate the weight to be assigned to individual risk data in order to determine a fair premium.

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Various useful results, not shown in this work, have been proposed regarding how to choose the credibility factor. These include limited fluctuation credibility, the Hachemeister random coefficient regression model, multi-dimensional credibility and Hilbert spaces methods. The most important contribution to date was proposed by Bühlmann (1967, 1969), who, in a simple and elegant form, derived a general result under the squared-error loss function without assuming any probability distribution for modelling risk. This result is known as Bühlmann's classical model (see Bühlmann (1967, 1969); and see also Bühlmann and Gisler (1967) for recent contributions in this field).

Obviously expression (1) can also be thought of as a compromise between the mean of the current observations, the data, and the prior mean, an estimate based on the actuary's prior opinion. This expression includes the concept of prior knowledge, in the spirit of the Bayesian paradigm. Firstly, Bailey (1945) and later other authors (Jewell (1974); Gerber and Arbor (1980); Heilmann (1989); Landsman and Makov (1998, 2000); among others) obtained some approaches to the credibility problem computed under standard Bayesian methodologies and robust ones (Eichenauer et al. (1988) and Gómez et al. (2006)).

In this paper, we apply the idea proposed in Zen and DasGupta (1993) and Ros Insua et al. (1995) to the problem of selecting a prior distribution in a class of possible prior distributions when global Bayesian robustness analysis has been completed. The decision-maker is interested in choosing a single action from the set of actions provided by a global procedure. The idea of this is to select the posterior regret Γ-minimax action (see Zen and DasGupta (1993); Ros Insua et al. (1995) and Gómez et al. (2006)). This procedure has the advantage, with respect to the Γ-minimax action (also called conditional Γ-minimax), that the estimator is always Bayesian when a parametric class is used, with the parameters varying over a connected set of the real line (Zen and DasGupta (1993) and Ros Insua et al. (1995)).

This relatively recent methodology consists of choosing an estimate which lies between the Bayes action and the Bayes robust methodology. By using this procedure (see Gómez et al. (2006)), we have derived new credibility expressions under the net premium principle and the gamma-gamma model as the likelihood and the prior, respectively. A generalization of the results obtained in the earlier study is completed in the present paper to obtain new credibility premiums under the exponential dispersion family of distributions and the squared-error loss function. The procedure set out here is carried out by choosing the weighted squared-error loss function.

The paper is organized as follows. In Section 2, the methodology to be applied to obtain premiums and to implement the posterior regret Γ-minimax procedure is presented. In Section 3, we derive credibility premiums under this procedure, addressing the squared-error loss function and the exponential dispersion family of distributions. The latter includes, as a particular case, the natural exponential family of distributions. In Section 4, an extension is obtained by using the weighted squared-error loss function. Section 5 contains a numerical application under the classical Poisson model with gamma prior distribution when the weighted squared-error loss is used. Finally, some concluding remarks are made in Section 6.

2 The methodology

In risk theory, the procedure of premium calculation is modelled as follows. The number of claims of one contract in one period is specified by a random variable $X \in \mathcal{X}$ following a probability density function $f(x|\theta)$ depending on an unknown risk parameter $\theta \in \Theta$. A premium calculation principle (e.g. Eichenauer et al. (1988) and Heilmann (1989)) assigns to each risk parameter θ a premium within the set $\mathcal{P} \in \mathbb{R}$, the action space. Let $L : \Theta \times \mathcal{P} \to \mathbb{R}$ be a loss function that assigns to any $(\theta, P) \in \Theta \times \mathcal{P}$ the loss sustained by a decision-maker who takes the action P and is faced with the outcome θ of a random experience. The premium must be determined such that the expected loss is minimized.

From this parameter, the unknown premium $P \equiv P(\theta)$, called the risk premium, can be obtained by minimizing the expected loss $E_f\left[L(\theta, P)\right]$. L is usually taken as the weighted squared-error loss function, i.e. $L(a, x) = h(x)(x-a)^2$. Using different functional forms for $h(x)$ we have different premium principles. For example, for $h(x) = 1$, $h(x) = \exp\{cx\}, c > 0$ and $h(x) = x$ we obtain the net, Esscher and variance premium principles, respectively (Heilmann (1989) and Gómez et al. (2006); among others).

If experience is not available, the actuary computes the collective premium, $P(\pi)$, which is given by minimizing the risk function, i.e. minimizing $E_\pi\left[L(P(\theta), \theta)\right]$, where π is the prior distribution on the unknown parameter $\theta \in \Theta$. On the other hand, if experience is available, the actuary takes a sample $\mathbf{x}$ from the random variables $X_i,\ i = 1, 2, \ldots, t$, assuming X_i i.i.d., and uses this information to estimate the unknown risk premium $P(\theta)$, through the Bayes premium $P(\pi_{\mathbf{x}})$, obtained by minimizing the Bayes risk, i.e. minimizing $E_{\pi_{\mathbf{x}}}\left[L(P(\theta), \theta)\right]$. Here, $\pi_{\mathbf{x}}$ is the posterior distribution of the risk parameter, θ, given the sample information $\mathbf{x}$.

Another approach to the Bayes setup analyzed above is to be found when practitioners suppose that a correct prior π exists, but they are unable to apply the pure Bayesian assumption, perhaps because they are not confident enough to specify it completely. Thus, a prior π is assigned to the risk parameter θ, which is a good approach for the true prior. A similar situation arises when a problem must be solved by two or more decision-makers and they do not agree on the prior distribution to be used. A common approach to prior uncertainty in Bayesian analysis is to choose a class Γ of prior distributions and then to calculate the range of Bayes actions as the prior ranges over Γ. This is known as the robust Bayesian methodology (see Berger (1994) and Ros Insua and Ruggeri (2000), for a review of this question). An alternative to this approach consists of choosing a procedure which lies between the Bayes action and the robust Bayesian methodology. Such a hybrid approach is known as the posterior regret Γ-minimax principle.

If $\rho(\pi_{\mathbf{x}}, P)$ is the posterior expected loss of an action P under $\pi_{\mathbf{x}}$, the posterior regret of P is defined as (see Zen and DasGupta (1993) and Ros Insua et al. (1995))

$$r(\pi_{\mathbf{x}}, P) = \rho(\pi_{\mathbf{x}}, P) - \rho(\pi_{\mathbf{x}}, P(\pi_{\mathbf{x}})), \tag{2}$$

which measures the loss of optimality incurred when P is chosen instead of the optimal action $P(\pi_{\mathbf{x}})$.

$RP(\pi_{\mathbf{x}}) \in \mathcal{P}$ is the posterior regret Γ-minimax action if

$$\inf_{P\in\mathcal{P}} \sup_{\pi\in\Gamma} r(\pi_{\mathbf{x}}, P) = \sup_{\pi\in\Gamma} r(\pi_{\mathbf{x}}, RP(\pi_{\mathbf{x}})). \tag{3}$$

This methodology is based on the idea that the optimal action minimizes the supremum of the function over distributions in class Γ. Therefore, in insurance settings, the actuary should seek to ensure that the largest possible increase in risk, when the wrong choice of prior distribution is made, should be kept as small as possible.

It is easy to show (see Ros Insua et al. (1995) and Zen and DasGupta (1993)) that if we choose $h(x) = 1$, i.e. the premium considered is the net premium principle, and that the posterior regret Γ-minimax action is the midpoint of the interval $[\inf_{\pi\in\Gamma} P(\pi_{\mathbf{x}}), \sup_{\pi\in\Gamma} P(\pi_{\mathbf{x}})]$.

In the following sections we provide some classes of distributions for which the posterior regret Γ-minimax actions are Bayes actions and can be written as a credibility formula, as in (1). The idea of obtaining credibility expressions from the standard Bayes premiums is based on the correct choice of the likelihood and its conjugate prior distributions.

3 Regret credibility premiums under the squared-error loss function

When the weighted squared-error loss function with $h(x) = 1$ is used, this is the simple squared-error loss function. The action that minimizes the expected loss $E_\pi[L(\theta, P)]$ is called the net premium principle. In this case, the risk and collective net premiums are given by (see Eichenauer et al. (1988); Heilmann (1989) and Gómez et al. (2006); among others):

$$\begin{aligned} P(\theta) &= E_f\left(X|\theta\right) = \int_{\mathcal{X}} x f(x|\theta)dx, \\ P(\pi) &= E_\pi\left[E_f\left(X|\theta\right)\right] = \int_{\Theta} P(\theta)\pi(\theta)d\theta, \end{aligned} \tag{4}$$

respectively. The Bayes premium, $P(\pi^{\mathbf{x}})$, is obtained by replacing $\pi(\theta)$ in (4) by $\pi^{\mathbf{x}}(\theta)$.

Bailey (1945) showed that if the likelihood is the binomial distribution and the prior is the Beta distribution, then credibility occurs under the net premium principle. In the same paper, the credibility expression in the Poisson-gamma case was obtained. Similar results were later obtained by Mayerson (1964). Jewell (1974) generalized these results and showed that for the natural exponential family of distributions (NEF, henceforth) and its conjugate priors, exact credibility premiums were derived. Finally, Landsman and Makov (1998, 2000) obtained a more general result by using the exponential dispersion family of distributions (EDF, henceforth) (see also Jorgensen (1986)). In this work, the EDF family with parametrization

$$f(x|\theta,\lambda) = \exp\{\lambda(x\theta - k(\theta))\}q(x|\lambda), \quad \theta \in \Theta \subset \mathbb{R},\ \lambda \in \mathbb{R}^+ \tag{5}$$

and a conjugate prior distribution given by the density

$$\pi(\theta) \propto \exp\{x_0\theta - t_0 k(\theta)\}, \tag{6}$$

are used.

Landsman and Makov (1998, 2000) proved that given t years of individual experience $x_1, x_2, \ldots, x_t$, the Bayesian net premium is given by,

$$P(\pi_{\mathbf{x}}) = \frac{x_0 + \lambda\sum_{i=1}^{t} x_i}{t_0 + t\lambda} = Z(t)\bar{\mathbf{x}} + (1 - Z(t))P(\pi), \quad \bar{\mathbf{x}} = \frac{1}{t}\sum_{i=1}^{t} x_i. \tag{7}$$

Here, $Z(t) = t\lambda/(t_0 + t\lambda)$, with $Z(t) \to 0$ when $t \to 0$ and $Z(t) \to 1$ when $t \to \infty$, while $P(\pi) = \int_\Theta P(\theta)\pi(\theta)d\theta = x_0/t_0$, with $P(\theta) = E_f(X|\theta) = k'(\theta)$.

By taking $\lambda = 1$, the EDF reduces to the NEF and obviously (7) coincides with the result in Jewell (1974). It is easy to show (see Jewell (1974)) that this credibility factor admits the same formulation as in Bühlmann (1967, 1969), which is:

$$Z(t) = \frac{t\mathrm{Var}_\pi(E_f(X|\theta))}{t\mathrm{Var}_\pi(E_f(X|\theta)) + E_\pi(\mathrm{Var}_f(X|\theta))}, \tag{8}$$

Example 1. In Table 1 some natural likelihoods and their prior distributions for NEF are shown. The credibility expression and the corresponding credibility factor are also shown.

The Bayesian methodology requires specification of the prior distribution. Thus, after an elicitation procedure by which a prior π is obtained, any prior close to π could also constitute a good representation of prior beliefs. In accordance with the notion of Bayesian robustness, we will consider the practitioner to be unwilling or unable to choose a functional form for the prior distribution π. Total unawareness of the prior distribution could be solved by choosing the following class of prior distributions

$$\mathcal{Q}_1 = \{\text{All probability distributions}\}.$$

It is well known (see Sivaganesan and Berger (1989) and Gómez et al. (2000); among others) that under a global Bayesian methodology, the range of variation of the Bayes net premium when the prior distribution enters $\mathcal{Q}_1$ is given by $[\inf_{\theta\in\mathbf{R}^+} k'(\theta), \sup_{\theta\in\mathbf{R}^+} k'(\theta)]$ and therefore the regret gamma-minimax net premium is

$$RP(\pi_{\mathbf{x}}; \mathcal{Q}_1) = \frac{1}{2}\left[\inf_{\theta\in\mathbf{R}^+} k'(\theta) + \sup_{\theta\in\mathbf{R}^+} k'(\theta)\right],$$

where it is assumed that Θ is restricted to take values in $(0, +\infty)$.

Likelihood Prior	Posterior	$P(\pi)$	$P(\pi_{\mathbf{x}})$	$Z(t)$
$X \sim \mathcal{P}o(\theta)$ $\theta \sim \mathcal{G}(a,b)$	$\mathcal{G}(a+t, b+t\bar{\mathbf{x}})$	$\frac{a}{b}$	$\frac{a+t\bar{\mathbf{x}}}{b+t}$	$\frac{t}{b+t}$
$X \sim \mathcal{NB}(r,\theta)$ $\theta \sim \mathcal{B}(a,b)$	$\mathcal{B}(a+tr, b+t\bar{\mathbf{x}})$	$\frac{rb}{a-1}$	$\frac{r(b+t\bar{\mathbf{x}})}{a+tr-1}$	$\frac{rt}{a+tr-1}$
$X \sim \mathcal{B}i(m,\theta)$ $\theta \sim \mathcal{B}(a,b)$	$\mathcal{B}(a+t\bar{\mathbf{x}}, b+mt-t\bar{\mathbf{x}})$	$\frac{ma}{a+b}$	$\frac{m(a+t\bar{\mathbf{x}})}{a+b+tm}$	$\frac{mt}{a+b+mt}$
$X \sim \mathcal{G}(\theta,\nu)$ $\theta \sim \mathcal{G}(a,b)$	$\mathcal{G}(a+t\bar{\mathbf{x}}, b+t\nu)$	$\frac{\nu a}{b-1}$	$\frac{\nu(a+t\bar{\mathbf{x}})}{b+t\nu-1}$	$\frac{t\nu}{b+t\nu-1}$
$X \sim \mathcal{N}(\theta,\sigma^2)$ $\theta \sim \mathcal{N}(a,\tau^2)$	$\mathcal{N}\left(\frac{a\sigma^2+t\bar{\mathbf{x}}\tau^2}{\sigma^2+t\tau^2}, \frac{\sigma^2\tau^2}{\sigma^2+t\tau^2}\right)$	a	$\frac{a\sigma^2+t\bar{\mathbf{x}}\tau^2}{\sigma^2+t\tau^2}$	$\frac{t\tau^2}{\sigma^2+t\tau^2}$

$\mathcal{P}o$: Poisson, $\mathcal{G}$: Gamma, $\mathcal{NB}$: Negative binomial,
$\mathcal{B}i$: Binomial, $\mathcal{B}$: Beta, $\mathcal{N}$: Normal

Table 1: Credibility premiums under NEF and the net premium principle

If class $\mathcal{Q}_1$ is used and similar conclusions are obtained, then no additional information is required. However, if the range of variation is very large, more information is needed. In this case, we might acquire partial information about the prior (for example, the mode) and consider all prior distributions that are compatible with this information, using

$$\mathcal{Q}_2 = \{\text{All distributions with a given mode, } \theta_0\}.$$

The range of variation of the Bayes net premium when the prior distribution enters $\mathcal{Q}_2$ is given by (see Sivaganesan (1991) and Gómez et al. (2000)) $[\mathcal{M}_1, \mathcal{M}_2]$, where $\mathcal{M}_1 = \inf_{z\in\mathbf{R}^+} \mathcal{R}(z)$ and $\mathcal{M}_2 = \sup_{z\in\mathbf{R}^+} \mathcal{R}(z)$, where

$$\mathcal{R}(z) = \frac{\int_{\theta_0}^{\theta_0+z} k'(\theta) f(\mathbf{x}|\theta,\lambda)d\theta}{\int_{\theta_0}^{\theta_0+z} f(\mathbf{x}|\theta,\lambda)d\theta}, \quad z > 0 \tag{9}$$

$$\mathcal{R}(z) = k'(\theta_0), \quad z = 0 \tag{10}$$

and therefore, the regret gamma-minimax net premium is $RP(\pi_{\mathbf{x}}; \mathcal{Q}_2) = \frac{1}{2}(\mathcal{M}_1 + \mathcal{M}_2)$.

As might be expected, other classes of prior distributions can also be considered. For example, the class defined by the quantile (Lavine (1991) and Sivaganesan (1991)) or the class given by generalized moment conditions Betrò et al. (1994) and Goutis (1994); among others). Excellent surveys of this topic can be found in Berger (1994) and Ros Insua and Ruggeri (2000).

Obviously, this methodology does not provide closed expressions for premiums that are rewritten as a credibility formula. For this purpose, henceforth, we will assume that the practitioner can assert that the prior distribution is an element of the family defined by (6), but that he is unaware of the simultaneous values of the parameters x_0 and t_0. Accordingly, the following classes of prior distributions will be used:

$$
\begin{aligned}
\Gamma_1 &= \left\{\pi(\theta) : x_0^{(1)} \leq x_0 \leq x_0^{(2)},\ t_0 \ \text{fixed}\right\}, \\
\Gamma_2 &= \left\{\pi(\theta) : t_0^{(1)} \leq t_0 \leq t_0^{(2)},\ x_0 \ \text{fixed}\right\}, \\
\Gamma_3 &= \left\{\pi(\theta) : \gamma_1 \leq P(\pi) \leq \gamma_2,\ t_0 \ \text{fixed}\right\}.
\end{aligned}
$$

Observe that Γ_3 has the feature of moment specification, in particular a generalized moment condition; a similar class appears in Eichenauer et al. (1988). If subjective information about the distribution of the parameter θ is available, then it seems reasonable to consider that information to be valid for the risk premium, which is a characteristic of the prior distribution.

The posterior regret Γ-minimax net premiums for the classes $\Gamma_j,\ j = 1, 2, 3$ are given in the following result.

Theorem 1. Consider the EDF in (5) and the conjugate prior distribution (6), then the posterior regret Γ-minimax net premiums for $\Gamma_i,\ \ i = 1, 2, 3$, classes are given by:

$$
RP(\pi_{\mathbf{x}}; \Gamma_j) = \frac{X_j + t\lambda\bar{\mathbf{x}}}{T_j + t\lambda}, \quad j = 1, 2, 3, \tag{11}
$$

where

$$
\begin{aligned}
X_1 &= \frac{1}{2}\left(x_0^{(1)} + x_0^{(2)}\right), \quad T_1 = t_0, \\
X_2 &= x_0, \quad T_2 = \frac{t\lambda(t_0^{(1)} + t_0^{(2)}) + 2t_0^{(1)}t_0^{(2)}}{2t\lambda + t_0^{(1)} + t_0^{(2)}}, \\
X_3 &= \frac{1}{2}\left(\gamma_1 + \gamma_2\right) t_0, \quad T_3 = t_0,
\end{aligned}
$$

Proof: The Bayes premium is given by

$$
P(\pi_{\mathbf{x}}) = \frac{x_0 + \lambda t\bar{\mathbf{x}}}{t_0 + t\lambda}. \tag{12}
$$

Then, for class Γ_1 the infima and suprema of (12) are given by:

$$
\inf_{\pi \in \Gamma_1} P(\pi_{\mathbf{x}}) = \frac{x_0^{(1)} + \lambda t\bar{\mathbf{x}}}{t_0 + t\lambda}, \quad \sup_{\pi \in \Gamma_1} P(\pi_{\mathbf{x}}) = \frac{x_0^{(2)} + \lambda t\bar{\mathbf{x}}}{t_0 + t\lambda},
$$

while for class Γ_2 they are:

$$
\inf_{\pi \in \Gamma_2} P(\pi_{\mathbf{x}}) = \frac{x_0 + \lambda t\bar{\mathbf{x}}}{t_0^{(2)} + t\lambda}, \quad \sup_{\pi \in \Gamma_2} P(\pi_{\mathbf{x}}) = \frac{x_0 + \lambda t\bar{\mathbf{x}}}{t_0^{(1)} + t\lambda}.
$$

For the class Γ_3, the restriction $\gamma_1 \le P(\pi) \le \gamma_2$ is equivalent to $\gamma_1 t_0 \le x_0 \le t_0\gamma_2$ and, therefore, the infima and suprema are given by:

$$\inf_{\pi\in\Gamma_3} P(\pi_{\mathbf{x}}) = \frac{\gamma_1 t_0 + \lambda t\bar{\mathbf{x}}}{t_0 + t\lambda}, \quad \sup_{\pi\in\Gamma_3} P(\pi_{\mathbf{x}}) = \frac{\gamma_2 t_0 + \lambda t\bar{\mathbf{x}}}{t_0 + t\lambda}.$$

Finally, after some algebra, it is straightforward to obtain the desired result by choosing

$$RP(\pi;\Gamma_j) = \frac{1}{2}\left(\inf_{\pi\in\Gamma_j} P(\pi_{\mathbf{x}}) + \sup_{\pi\in\Gamma_j} P(\pi_{\mathbf{x}})\right), \quad j = 1,2,3. \qquad \diamond$$

Since closed intervals on the real line are connected sets, by using Proposition 3.2 in Ros Insua et al. (1995), we can conclude that $RP(\pi;\Gamma_j)$, $j = 1,2,3$, are Bayes premiums under the prior (6) with parameters (X_j, T_j), $j = 1,2,3$. Observe that the corresponding prior in Γ_2 depends on the sample size but not on the actual observations.

The next result shows that expression (11) can be written as a credibility formula.

Corollary 1. The posterior regret Γ-minimax net premiums in (11) can be rewritten as a credibility formula

$$Z_i(t)g(\bar{\mathbf{x}}) + (1 - Z_i(t))P_i(\pi), \quad i = 1,2,3,$$

where $g(\bar{\mathbf{x}}) = \bar{\mathbf{x}}$,

$$\begin{aligned} P_1(\pi) &= \frac{x_0^{(1)} + x_0^{(2)}}{2t_0}, \\ P_2(\pi) &= \frac{x_0(2t\lambda + t_0^{(1)} + t_0^{(2)})}{t\lambda(t_0^{(1)} + t_0^{(2)}) + 2t_0^{(1)}t_0^{(2)}}, \\ P_3(\pi) &= \frac{\gamma_1 + \gamma_2}{2}, \end{aligned}$$

and the credibility factor is given by:

$$\begin{aligned} Z_i(t) &= \frac{t\lambda}{t_0 + t\lambda}, \quad \text{for } i = 1,3, \\ Z_i(t) &= \frac{t\lambda(2t\lambda + t_0^{(1)} + t_0^{(2)})}{2t\lambda(t_0^{(1)} + t_0^{(2)}) + 2t_0^{(1)}t_0^{(2)} + 2t^2\lambda^2}, \quad \text{for } i = 2. \end{aligned}$$

Proof: The proof is readily apparent. $\diamond$

4 Regret credibility premiums under the weighted squared-error loss function

In Bayesian robustness literature, little attention has been paid to the general weighted squared-error loss function. Under the posterior regret methodology, Boratyńska (2008) studied its effect on insurance premiums in the collective risk model. Usually, in Bayesian robustness analysis, the principal goal concerns the posterior mean of a function of the unknown parameter θ, i.e. $P = \int_\Theta m(\theta)\pi_{\mathbf{x}}(\theta)d\theta$. This posterior mean is obtained under the squared–error loss function, i.e. $h(x) = 1$.

If we use the weighted squared-error loss function with an arbitrary weight function $h(x)$, by minimizing the expected loss $E[L(\theta, P)]$, then the risk and collective premiums are given by (see Eichenauer et al. (1988); Heilmann (1989) and Gómez et al. (2006)):

$$\begin{aligned} P(\theta) &= \frac{\displaystyle\int_{\mathcal{X}} xh(x)f(x|\theta)dx}{\displaystyle\int_{\mathcal{X}} h(x)f(x|\theta)dx}, \\ P(\pi) &= \frac{\displaystyle\int_\Theta P(\theta)h(P(\theta))\pi(\theta)d\theta}{\displaystyle\int_\Theta h(P(\theta))\pi(\theta)d\theta}, \end{aligned} \tag{13}$$

respectively. Again, the Bayes premium, $P(\pi^{\mathbf{x}})$, is obtained by replacing $\pi(\theta)$ in (13) by $\pi^{\mathbf{x}}(\theta)$.

Note that the collective premium can be expressed as:

$$P(\pi) = \frac{\displaystyle\int_\Theta P(\theta)h(P(\theta))\pi(\theta)d\theta}{\displaystyle\int_\Theta h(P(\theta))\pi(\theta)d\theta} = \int_\Theta P(\theta)\pi^*(\theta)d\theta,$$

which can be rewritten as an expectation of $P(\theta)$ with respect to the probability density function

$$\pi^*(\theta) = \frac{h(P(\theta))\pi(\theta)}{\displaystyle\int_\Theta h(P(\theta))\pi(\theta)d\theta}, \tag{14}$$

and the Bayes premium is also obtained by replacing π and π^* by $\pi_{\mathbf{x}}$ and $\pi^*_{\mathbf{x}}$, respectively. Then, the Bayes premium can be seen as a posterior expectation, i.e. the net premium, with respect to the new prior distribution $\pi^*(\theta)$. Therefore, the posterior regret of P and the posterior regret Γ-minimax actions are as in (2) and in (3), by replacing π by π^*, respectively. Now, we have the following result.

Proposition 1. Suppose that the risk X follows a distribution as in (5) and that θ follows a prior distribution as in (14), where π is as in (6) and that the risk premium is given by $P(\theta) = mk'(\theta)$, $m \in \mathbb{R}^+$. Then, the collective and Bayes premiums are given by

$$\begin{aligned} P(\pi^*) &= m\frac{\alpha m + x_0}{t_0}, \\ P(\pi^*_{\mathbf{x}}) &= m\frac{\alpha m + x_0 + \lambda t\bar{\mathbf{x}}}{\lambda t + t_0}, \end{aligned} \tag{15}$$

respectively.

Proof: It is easy to see that

$$\pi^*(\theta) \propto \exp\left\{(x_0 + \alpha m)\theta - t_0 k(\theta)\right\},$$

while the posterior distribution is given by

$$\pi^*(\theta|\mathbf{x}) \propto \exp\left\{(x_0 + \alpha m + \lambda t \bar{\mathbf{x}})\theta - (t_0 + \lambda t)k(\theta)\right\}.$$

Now, following a reasoning similar to expressed in Jewell (1974) and Landsman and Makov (1998), we obtain the result. $\diamond$

Observe that expression (15) can be written as a credibility formula

$$Z(t) = Z(t)g(\bar{\mathbf{x}}) + [1 - Z(t)]\, P(\pi^*),$$

where $g(\mathbf{x}) = m\bar{\mathbf{x}}$ and $Z(t) = t_0/(t_0 + \lambda t)$.

The next result is a consequence of Proposition 1 and is similar to that set out in Theorem 1 and, therefore, the proof will be omitted.

Theorem 2. Under the assumptions of Proposition 1 the posterior regret Γ-minimax net premiums for $\Gamma_i, \quad i = 1, 2, 3$, classes are given by:

$$RP(\pi^*_{\mathbf{x}}; \Gamma_j) = m\frac{\alpha m + X_j + t\lambda\bar{\mathbf{x}}}{T_j + t\lambda}, \quad j = 1, 2, 3, \tag{16}$$

where

$$\begin{aligned}
X_1 &= \frac{1}{2}\left(x_0^{(1)} + x_0^{(2)}\right), \quad T_1 = t_0, \\
X_2 &= x_0, \quad T_2 = \frac{t\lambda(t_0^{(1)} + t_0^{(2)}) + 2t_0^{(1)}t_0^{(2)}}{2t\lambda + t_0^{(1)} + t_0^{(2)}}, \\
X_3 &= \frac{1}{2m}\left(\gamma_1 + \gamma_2\right)t_0, \quad T_3 = t_0,
\end{aligned}$$

It is straightforward to prove that expression (16) can be rewritten as a credibility formula.

4.1 The Esscher principle

Let us suppose that the practitioner decides to compute the premium by using the Esscher premium principle (see Gerber and Arbor (1980), Heilmann (1989) and Zehnwirth (1981)). In this case $h(x) = e^{\alpha x}$, where the parameter $\alpha > 0$ is known as the safety loading.

Since the Esscher premium principle tends to the net premium principle when $\alpha \to 0$ (see Zehnwirth (1981)), the results in Theorem 2 coincide with those in Theorem 1 when α is chosen to be close to 0 and $m = 1$.

Let us now consider the particular case where the risk X follows a Poisson distribution with parameter $\theta > 0$ and the prior distribution is a gamma distribution with parameters $a > 0$ and $b > 0$. After some simple computations, it is straightforward to obtain that $\pi^*(\theta)$ is a gamma distribution with parameters a and $b - \alpha e^{\alpha},\ b > \alpha e^{\alpha}$. Then, the risk, collective and Bayes premiums are given by:

$$\begin{aligned} P(\theta) &= \theta e^{\alpha}, \\ P(\pi^*) &= \frac{a e^{\alpha}}{b - \alpha e^{\alpha}}, \quad b > \alpha e^{\alpha}, && (17) \\ P(\pi^*_{\mathbf{x}}) &= \frac{(a + t\bar{\mathbf{x}})e^{\alpha}}{b + t - \alpha e^{\alpha}}, \quad b + t > \alpha e^{\alpha}. && (18) \end{aligned}$$

Observe that expression (18) can be rewritten as:

$$Z(t)g(\bar{\mathbf{x}}) + (1 - Z(t))P(\pi^*),$$

where

$$Z(t) = \frac{t}{b + t - \alpha e^{\alpha}} \tag{19}$$

is the credibility factor,

$$g(\bar{\mathbf{x}}) = e^{\alpha}\bar{\mathbf{x}} \tag{20}$$

and $P(\pi)$ is the collective premium given by (17). This corresponds to the premium charged to a policyholder in the portfolio, regardless of the sample information about him. Therefore, the Bayes premium is a credibility expression with the same formulation as in (8) as can be easily proved.

Following the Bayesian robustness paradigm, we will assume a class of prior distributions instead of a single prior, in the following way:

$$\begin{aligned} \Gamma_1 &= \left\{\pi^*(\theta) = \mathcal{G}(a, b - \alpha e^{\alpha}) : a_1 \le a \le a_2,\ b \ \text{ fixed}\right\}, && (21) \\ \Gamma_2 &= \left\{\pi^*(\theta) = \mathcal{G}(a, b - \alpha e^{\alpha}) : b_1 \le b \le b_2,\ a \ \text{ fixed}\right\}, && (22) \\ \Gamma_3 &= \left\{\pi^*(\theta) = \mathcal{G}(a, b - \alpha e^{\alpha}) : \gamma_1 \le P(\pi) \le \gamma_2,\ b \ \text{ fixed}\right\}. && (23) \end{aligned}$$

The next result provides a guide to reaching the posterior regret Γ-minimax action in the Poisson-gamma model under the Esscher premium principle.

Proposition 2. Under the Poisson-gamma model and classes $\Gamma_i,\ i = 1, 2, 2$ the posterior regret-Γ-minimax Esscher premiums are given by

$$RP(\pi_{\mathbf{x}}; \Gamma_j) = \frac{(\delta_i + t\bar{\mathbf{x}})e^{\alpha}}{\beta_i + t - \alpha e^{\alpha}},\ i = 1, 2, 3, \tag{24}$$

where

$$\delta_1 = \frac{a_1 + a_2}{2}, \quad \beta_1 = b,$$

$$\delta_2 = a, \quad \beta_2 = \frac{2b_1 b_2 + (b_1 + b_2)(t - \alpha e^{\alpha})}{b_1 b_2 + 2(t - \alpha e^{\alpha})},$$

$$\delta_3 = \frac{\gamma_1 + \gamma_2}{2} \frac{b - \alpha e^{\alpha}}{e^{\alpha}}, \quad \beta_3 = b.$$

Proof: The proof is a direct consequence of applying the results of Theorem 2. $\diamond$

Again, the posterior regret Γ-minimax Esscher premiums under $\Gamma_i,\ i = 1, 2, 3$, are Bayes premiums with respect to the priors $\mathcal{G}(\delta_i, \beta_i)$, for $i = 1, 2, 3$, and the corresponding prior in Γ_2 depends on the sample size but not on the actual observations.

Corollary 2. The posterior regret Γ-minimax Esscher premiums in (24) can be rewritten as a credibility formula:

$$Z_i(t) g(\bar{\mathbf{x}}) + (1 - Z_i(t)) P_i(\pi^*), \quad i = 1, 2, 3,$$

where $g(\bar{\mathbf{x}}) = e^{\alpha} \bar{\mathbf{x}}$,

$$\begin{aligned} P_1(\pi^*) &= \frac{(a_1 + a_2) e^{\alpha}}{2(b - \alpha e^{\alpha})}, \\ P_2(\pi^*) &= \frac{a e^{\alpha}}{\frac{2b_1 b_2 + (b_1 + b_2)(t - \alpha e^{\alpha})}{b_1 b_2 + 2(t - \alpha e^{\alpha})} - \alpha e^{\alpha}}, \\ P_3(\pi^*) &= \frac{\gamma_1 + \gamma_2}{2}, \end{aligned}$$

and the corresponding credibility factors are given by:

$$\begin{aligned} Z_i(t) &= \frac{t}{b + t - \alpha e^{\alpha}}, \quad \text{for } \ i = 1, 3, \\ Z_i(t) &= \frac{t}{\frac{2b_1 b_2 + (b_1 + b_2)(t - \alpha e^{\alpha})}{b_1 b_2 + 2(t - \alpha e^{\alpha})} + t - \alpha e^{\alpha}}, \quad \text{for } \ i = 2. \end{aligned}$$

Proof: The proof is obvious. $\diamond$

4.2 The variance principle

Let us suppose that the practitioner wishes to compute the premium by using the variance premium principle $h(x) = x$ (see Heilmann (1989) and Caldern et al. (2006); among others). An extra effort will be needed to obtain a general result for NEF similar to the one completed under the Esscher premium principle. For this reason, we will assume that the risk X follows a gamma distribution with parameters $\nu > 0$ and

$\theta > 0$ and that the prior distribution is a gamma distribution with parameters $a > 0$ and $b > 0$. After some simple computation it is straightforward to obtain that $\pi^*(\theta)$ is a gamma distribution with parameters $a-1,\ a>1$ and b. Then, it is easy to prove that the risk, collective and Bayes premiums are given by:

$$
\begin{aligned}
P(\theta) &= \frac{\nu+1}{\theta}, \\
P(\pi^*) &= (\nu+1)\frac{b}{a-2}, \quad a>2, && (25)\\
P(\pi^*_{\mathbf{x}}) &= (\nu+1)\frac{b+t\bar{\mathbf{x}}}{a+t\nu-2}, \quad a+t\nu>2. && (26)
\end{aligned}
$$

Observe that expression (26) can be rewritten as:

$$Z(t)g(\bar{\mathbf{x}}) + (1-Z(t))P(\pi^*),$$

where $Z(t) = t\nu/(a+t\nu-2)$ is the credibility factor, $g(\bar{\mathbf{x}}) = (\nu+1)\bar{\mathbf{x}}/\nu$ and $P(\pi^*)$ is the collective premium given by (25). Therefore, the Bayes premium is a credibility expression with the same formulation as in (8).

Assuming that the prior distribution lies in classes (21), (22) and (23), posterior regret Γ-minimax variance premiums are obtained in the next result.

Theorem 3. Under the gamma-gamma model and classes $\Gamma_i,\ i=1,2,3$, the posterior regret-Γ-minimax variance premiums are given by

$$RP(\pi_{\mathbf{x}};\Gamma_j) = (\nu+1)\frac{\beta_i+t\bar{\mathbf{x}}}{\delta_i+t\nu-2},\ i=1,2,3, \tag{27}$$

where

$$\delta_1 = \frac{2a_1a_2+(a_1+a_2)(t\nu-2)}{a_1+a_2+2t\nu-4}, \quad \beta_1 = b,$$

$$\delta_2 = a, \quad \beta_2 = \frac{b_1+b_2}{2},$$

$$\delta_3 = a, \quad \beta_3 = \frac{\gamma_1+\gamma_2}{2}\frac{a-2}{\nu+1}.$$

Proof: The proof is similar to that of Theorem 1. $\diamond$

Again, the posterior regret Γ-minimax variance premiums for $\Gamma_i,\ i=1,2,3$, are Bayes premiums with respect to the priors $\mathcal{G}(\delta_i,\beta_i)$, for $i=1,2,3$, and the corresponding prior in Γ_1 depends on the sample size but not on the actual observations.

Corollary 3. The posterior regret Γ-minimax variance premiums in (27) can be rewritten as a credibility formula

$$Z_i(t)g(\bar{\mathbf{x}}) + (1-Z_i(t))P_i(\pi^*), \quad i=1,2,3,$$

where $g(\bar{\mathbf{x}}) = (\nu+1)\bar{\mathbf{x}}/\nu$,

$$\begin{aligned} P_1(\pi^*) &= \frac{(\nu+1)b}{\frac{2a_1a_2+(a_1+a_2)(t\nu-2)}{a_1a_2+2t\nu-4}-2}, \\ P_2(\pi^*) &= \frac{(\nu+1)(b_1+b_2)}{2(a-2)}, \\ P_3(\pi^*) &= \frac{\gamma_1+\gamma_2}{2}, \end{aligned}$$

and the corresponding credibility factors are given by:

$$\begin{aligned} Z_i(t) &= \frac{t\nu}{\frac{2a_1a_2+(a_1+a_2)(t\nu-2)}{a_1a_2+2t\nu-4}+t\nu-2}, \quad \text{for } i=1, \\ Z_i(t) &= \frac{t\nu}{a+t\nu-2}, \quad \text{for } i=2,3. \end{aligned}$$

Proof: The proof is obvious. $\diamond$

Remark 1. Observe that all the new credibility factors obtained, when the prior distributions π or π^* are considered, have the same form as (8).

5 Numerical application

In this section, an application to the Esscher premium principle is examined to demonstrate how the methodology works.

Let us consider a Poisson model where the Poisson parameter θ represents a driver's propensity to make a claim and the prior indicates how that propensity is distributed throughout the population of insured drivers. This pattern has been used successfully to model the number of vehicle motor accidents (see for example Lemaire (1979)).

Let us also assume that the practitioner accepts that the expected frequency is $E(\theta) = 0.4$. Thus, the company can expect about two claims every five years with this policy. According to Scollnik (1995), the prior information available for this parameter could be well modelled by a gamma distribution with parameters a and b. This is reasonable, since the shape of the gamma density is very flexible.

Since the mode is a very intuitive statistical concept, a well-prepared actuary should assess the unimodality of the risk parameter and its numerical value, based on historical data. This phenomenon, in the insurance context, is commonly found in vehicle motor accidents. Let us suppose now that the practitioner accepts that the mode of the prior distribution is 0.2, i.e. $\theta_0 = 0.2$. If we have limited prior information available on θ, then the choice $a = 2$, $b = 5$ will result in a fairly satisfactory and relatively diffuse prior for θ.

Table 2 shows the credibility factor, $g(\bar{\mathbf{x}})$, collective and Bayesian premiums obtained by using expressions (19), (20), (17) and (18), respectively. The loading was taken as $\alpha = 0.4$.

$t=1$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$P(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.185	0.000	0.677	0.552
$\bar{\mathbf{x}}=2$	0.185	2.983	0.677	1.104
$\bar{\mathbf{x}}=4$	0.185	5.967	0.677	1.656
$t=5$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$P(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.531	0.000	0.677	0.317
$\bar{\mathbf{x}}=2$	0.531	2.983	0.677	1.903
$\bar{\mathbf{x}}=4$	0.531	5.967	0.677	3.490
$t=10$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$P(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.694	0.000	0.677	0.207
$\bar{\mathbf{x}}=2$	0.694	2.983	0.677	2.278
$\bar{\mathbf{x}}=4$	0.694	5.967	0.677	4.350

Table 2: Credibility factor, $g(\bar{\mathbf{x}})$, collective premium and Bayes premium. Esscher principle

We have computed now the posterior regret Γ-minimax Esscher premium for the class $\mathcal{Q}_2$. The values of $\mathcal{M}_1$ and $\mathcal{M}_2$ were computed by substituting $k'(\theta)e^{k'(\theta)}$ in (9) and (10) for $k'(\theta)$ (see Table 3). It can be seen that the larger the range of variation, the larger the regret Bayes premium computed for this class. Therefore, larger premiums than the standard Bayesian ones shown in Table 2 are now obtained, except for $t=10$ with $\bar{\mathbf{x}}=2$ and $\bar{\mathbf{x}}=4$.

Table 3: Posterior regret Γ-minimax premium and range of variation of the premium under the class $\mathcal{Q}_2$. Esscher principle

$t=1$	$\mathcal{M}_1$	$\mathcal{M}_2$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.298	3.997	2.147
$\bar{\mathbf{x}}=2$	0.300	11.098	5.699
$\bar{\mathbf{x}}=4$	0.298	18.496	9.397
$t=5$	$\mathcal{M}_1$	$\mathcal{M}_2$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.298	0.637	0.467
$\bar{\mathbf{x}}=2$	0.298	3.726	2.010
$\bar{\mathbf{x}}=4$	0.298	7.114	3.706
$t=10$	$\mathcal{M}_1$	$\mathcal{M}_2$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.200	0.457	0.328
$\bar{\mathbf{x}}=2$	0.298	3.338	1.818
$\bar{\mathbf{x}}=4$	0.298	6.504	3.401

Using the results in Theorem 2 and Corollary 2, the posterior regret Γ-minimax Esscher premiums were computed by taking classes in (21), (22) and (23) with the following parameter bounds: $a \in [1,3]$, $b \in [3,8]$ and $P(\pi) \in [0.1, 0.6]$. The results are shown in Tables 4, 5 and 6.

$t=1$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.185	0.000	1.355	1.104
$\bar{\mathbf{x}}=2$	0.185	2.983	1.355	1.656
$\bar{\mathbf{x}}=4$	0.185	5.967	1.355	2.208
$t=5$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.531	0.000	1.355	0.634
$\bar{\mathbf{x}}=2$	0.531	2.983	1.355	2.221
$\bar{\mathbf{x}}=4$	0.531	5.967	1.355	3.807
$t=10$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.694	0.000	1.355	0.414
$\bar{\mathbf{x}}=2$	0.694	2.983	1.355	2.485
$\bar{\mathbf{x}}=4$	0.694	5.967	1.355	4.557

Table 4: Credibility factor, $g(\bar{\mathbf{x}})$, collective premium and posterior regret Γ-minimax premium. Esscher principle. Class Γ_1

$t=1$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.342	0.000	1.553	1.021
$\bar{\mathbf{x}}=2$	0.342	2.983	1.553	2.043
$\bar{\mathbf{x}}=4$	0.342	5.967	1.553	3.065
$t=5$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.425	0.000	0.442	0.254
$\bar{\mathbf{x}}=2$	0.425	2.983	0.442	1.524
$\bar{\mathbf{x}}=4$	0.425	5.967	0.442	2.794
$t=10$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.447	0.000	0.241	0.133
$\bar{\mathbf{x}}=2$	0.447	2.983	0.241	1.468
$\bar{\mathbf{x}}=4$	0.447	5.967	0.241	2.804

Table 5: Credibility factor, $g(\bar{\mathbf{x}})$, collective premium and posterior regret Γ-minimax premium. Esscher principle. Class Γ_2

Thus, the collective and posterior regret Γ-minimax Esscher premiums take values which are always between those obtained for classes Γ_1 and Γ_3, that is, $RP(\pi^*_{\mathbf{x}};\Gamma_3) < P(\pi^*_{\mathbf{x}}) < RP(\pi^*_{\mathbf{x}};\Gamma_1)$. It also seems that the larger the value of t, the larger the difference between the standard Bayesian premium and the posterior regret Γ-minimax Esscher premium. A graphical illustration of these comments is shown in Figure 1.

Therefore, a more stable situation arises when classes Γ_1 and Γ_3 are used. This is surely provoked by the fact that the values of the prior distribution, when the class Γ_2 is used, depend on t, as is shown in Theorem 2. Since the variation range of the Bayes premium for classes Γ_1 and Γ_3 (this is easily computed using the results in Theorem 2)

$t=1$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.185	0.000	0.3150	0.285
$\bar{\mathbf{x}}=2$	0.185	2.983	0.350	0.837
$\bar{\mathbf{x}}=4$	0.185	5.967	0.350	1.389
$t=5$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.531	0.000	0.350	0.161
$\bar{\mathbf{x}}=2$	0.531	2.983	0.350	1.750
$\bar{\mathbf{x}}=4$	0.531	5.967	0.350	3.336
$t=10$	$Z(t)$	$g(\bar{\mathbf{x}})$	$P(\pi^*)$	$RP(\pi^*_{\mathbf{x}})$
$\bar{\mathbf{x}}=0$	0.694	0.000	0.350	0.107
$\bar{\mathbf{x}}=2$	0.694	2.983	0.350	2.178
$\bar{\mathbf{x}}=4$	0.694	5.967	0.350	4.250

Table 6: Credibility factor, $g(\bar{\mathbf{x}})$, collective premium and posterior regret Γ-minimax premium. Esscher principle. Class Γ_3

is lower than that computed for the class $\mathcal{Q}_2$, the practitioner is able to choose the Bayes premium with a certain security level and, as might be expected, either the Bayes or the posterior regret Γ-minimax premium could be chosen, depending on the practitioner's preferences.

Since, as mentioned above, the Esscher premium principle tends to the net premium principle when $\alpha \to 0$ (see Zehnwirth (1981)), expression (24) coincides with expression (11) when α is chosen to be close to 0 and $\lambda = 1$. As a result, the posterior regret Γ-minimax net premiums can be easily computed.

In robust Bayesian analysis, the practitioner assumes a prior distribution belonging to a class of prior distributions instead of a single prior. The robust Bayesian methodology usually provides a variation range of the Bayes premium in an interval form, but it does not indicate how to choose the correct action. As Zen and DasGupta (1993) say "it is clearly essential to be able to recommend one action from this set to the user". The methodology developed in this work seems to go in that direction, providing the practitioner with an action, i.e. a selection criterion. Furthermore, all the premiums proposed under this methodology can be written as credibility expressions, a convex sum of the collective premium and the sample information. These expressions, which are attractive for the actuarial community, have been obtained under different Bayesian methodologies, including standard Bayesian methods, the Γ-minimax approach and the methodology used in this work, the posterior regret Γ-minimax.

6 Closing comments

The analysis proposed in this paper has been used to derive a number of new Bayesian premiums that can be expressed as a credibility formula. These expressions have been found to be rewarding in actuarial practice when experience rating, via Bayesian anal-

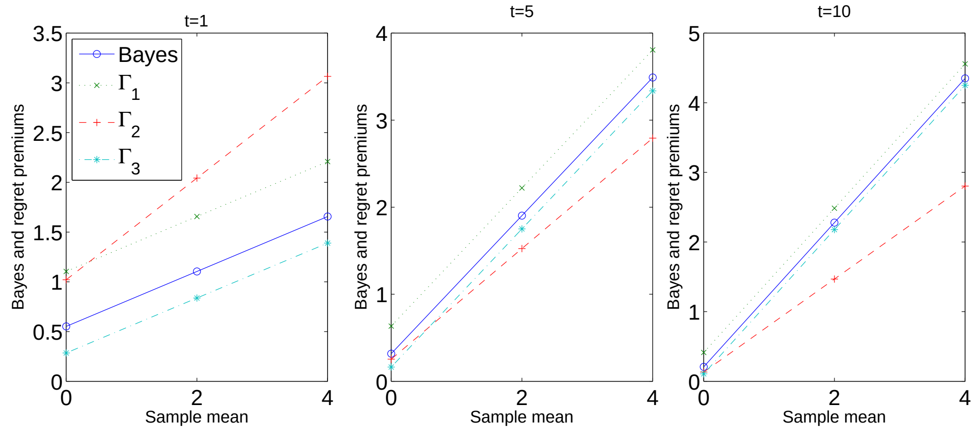

Figure 1: Bayes and posterior regret Γ-minimax premiums under the different classes considered

ysis, is used to compute premiums expressed as a compromise between the past claims of the policyholder that belongs to a portfolio, and the past claims of this portfolio as a whole.

The methodology proposed is uncomplicated and the credibility formulae are straightforwardly obtained. Moreover, this technique has advantages over other Bayesian robustness methodologies, i.e. the local (Caldern et al. (2006)), global (Ros Insua and Ruggeri (2000)) and the Γ-minimax approaches (Eichenauer et al. (1988)), since more basic and plausible classes of distributions can be used.

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