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Recovering Western On-line Signatures From Image-Based Specimens

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***Abstract*—This article propose a complete framework to recover the dynamic properties (i.e. velocity and pressure) of an on-line Western signature from an image-based signature. The framework is based on classical approaches to recover the writing order of the strokes and a novel process to recover the kinematic properties from thinned trajectories. In order to evaluate the quality of the recovered signatures and the impact of each stage of our framework, the performance of a signature verification system on obtained signatures in each stage are compared to the performance with real signatures. As a proof of concepts, in this study we use the first 50 users of BiosecurID signature database since they contain both the on-line and off-line version of Western signatures.**

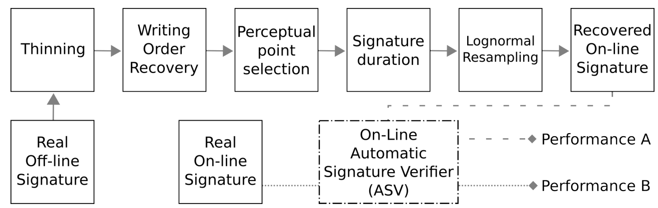


Fig. 1: Block diagram of the proposed Off-2-On framework and its performance-based validation.

I. Introduction

There are two modalities of handwriting signatures: *1) On-line or dynamic signatures*, which are commonly acquired with a digital tablet and contain the time functions of the pentip position and the pressure during their execution. *2) Off-line or static signatures*, which are obtained by scanning hard copy documents and stored as image templates, without temporal information.

According to the data format, signatures can be described by three kinds of features: static, pseudodynamic and dynamic features. Commonly static features involve geometric measures such as the calibre, proportion, spacing, or alignment to baseline [1]. Instead, pseudo-dynamic features try to infer dynamic properties from off-line signatures. They can be deduced from the High Pressure Points, the thickness of the pen strokes and its variations, distribution of pixels, progression, slant or form [1]. Dynamic features contain time functions, which allow to calculate the kinematic properties of signatures as their global parameters, among others. These differences could explain the bigger growth and performance improvements in on-line Automatic Signature Verifiers (ASVs) [2].

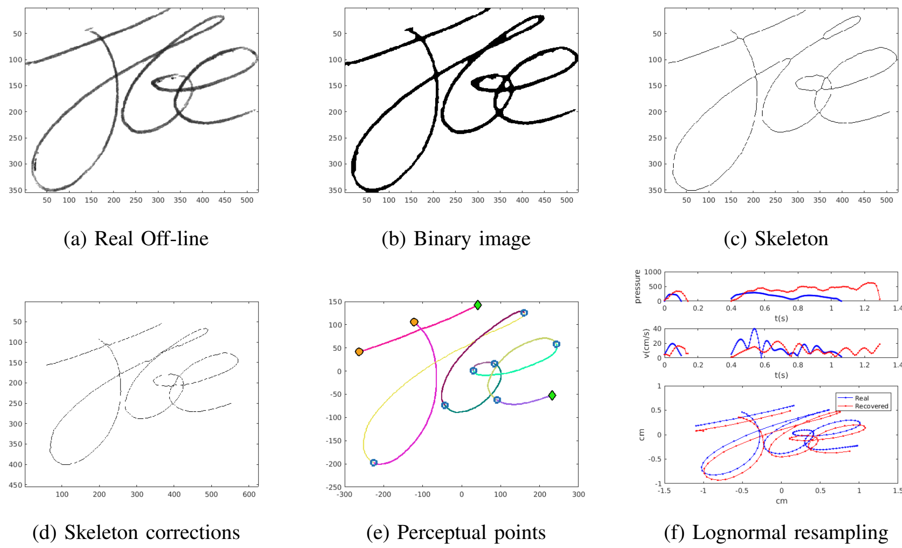
To take advantage of on-line ASVs, we wonder the following hypothesis: whether an on-line signature from a real off-line was recovered, could we use such recovered signature in on-line ASVs?

Despite being a classical pattern recognition problem [3], [4], recovering an on-line signature from its off-line counterpart (Off-2-On) has not been completely solved in the literature. Nevertheless, several genuine proposals and even competitions (e.g. [5]) have established genuine basis to solve some stages of this problem. One of the critical stage of the complete problem is the estimation of the writing order, which have been already approached in the literature. For instance, solving ambiguities in the loops [6], by the edge continuity relation concept [7], by the optimum skeleton path [8], among others. Another already approached stage is the stroke segmentation. Proposals are typically worked out the curvature [9], [10] of static handwriting specimens. Another critical stage is to infer the temporal properties from images, which have been also studied (e.g. [11]).

We propose a complete framework to recover on-line signatures from image-based specimens. This *Off-2-On* framework is based on several consecutive stages. One of the target of this paper is to quantify the influence of these stages in the recovering process. Since kinematic properties are derived by a real on-line signature, we also contribute with a novel lognormal resampling of an 8-connected trajectory to generate human-like dynamic profiles (i.e. velocity, acceleration and pressure).

To evaluate the quality of the recovered signature, we consider a state-of-the-art automatic signature verifier (ASV) and compare its performance when dealing with real on-line signatures and with recovered ones. Additionally, to mitigate the errors associated to writing order recovery stage, a modification of the used ASV is proposed in this article.

The reminder of this paper has the following organization. Section II describes the proposed framework. A modification of a state-of-the-art ASV is presented in Section III. A proof of the method is evaluated in Section IV and finally the paper closes with the conclusions in Section V.



(a) Real Off-line (b) Binary image (c) Skeleton

(d) Skeleton corrections (e) Perceptual points (f) Lognormal resampling

Fig. 2: Visual examples of the proposed framework to recover on-line signatures.

II. Proposed Framework

The proposed framework was divided into the following five stages, as it is shown in Fig. 1: A) Thinning, B) Writing order recovery, C) Perceptual point estimation, D) Duration of the signature and, E) Lognormal resampling. It could be said that this procedure shows a complete framework to estimate an on-line signature from an image-based specimen (Off-2-On). Visual details of the proposed framework are illustrated in Fig. 2.

A. Thinning

The goal of this stage is to create the skeleton from static signatures. Firstly, the signatures were binarized by using the Otsu method. Then a dilation was applied to the binary images as a morphological operation. A line was used as structural element, which changed its angle from 0 to 90 degrees. This preprocessing helped to fill some gaps due to incorrect ink depositions on the paper without adding too much unreal pixels to the images.

Then, the thinning was applied following a proposal method published in [12]. Such method is based on an augmented fast matching procedure, which computed the boundary location of each pixel. A thresholding phase was processed at the end in order to create the skeleton branches. An implementation of this contour-pruned skeletonization in Matlab was used in this work[1], setting 35 as the threshold value.

B. Writing order recovery

The tracing of the signature was carried out over an 8-connected skeleton. To obtain such 8-connected skeleton, two corrections were carried out to solve ambiguities in the branch points:

1) Let k be the number of lines, which converged in a branch point, where $k \in (3, 4, 5, 6, 7)$. More than 7 lines were never found during the experiments. In our implementation, each branch point was processed in order to join the lines in an 8-connected way. Moreover, the window size around the branch point would increase in order to find 8-connected branch reconstructions. To assign the pair of lines, the algorithm decided regarding a good continuity criterion, i.e. the less abrupt derivate trajectory after trying all pairs. Obviously, when k was not pair, the remained line was unpaired and therefore became either start or end point.
2) Since some small spurious branches still remained, they were also removed.

From the obtained image, it is extremely tough to guess how the signer traced his signature. In our implementation, to recover the writing order, we simplify the signer decisions in a twofold rule: Firstly he decides which is the next component to draw and secondly he selects the start and the end point of such component. To select the start point of the component, we follows this hypothesis: the signer started to write by selecting the free end-point closer to the top-right part of the image. It is worth mentioning that a real signer could modify these two rules in each execution of his signature as it is a behavioral action.

C. Perceptual point estimation

The requirement here consisted in segmenting an 8-connected trajectory generated from individual strokes[2]. In our implementation, we have followed a multi-scale method proposed in [10]. Briefly, let M be the total points of an 8-connected trajectory. Starting from 3 equidistant points until M, $M-3$ representations from a determined 8-connected component were created. Then, the local curvatures were calculated per each representation at each scale in order to

[1]The code used in the skeletonization is available in goo.gl/CAm8Bn

[2]In this work, *stroke* refers to a neuromuscular command to execute an elementary movement.

build a feature map. Such feature map required to normalize individual curves in the range $[0, 1]$. This way, a matrix was built by inserting in each row the local curvature values, which were interpolated to M values in the range $[0, 1]$. Finally, a saliency map was processed by summing the matrix in vertical. In the saliency map, the peaks corresponded to the perceptual points, which were selected in order to segment the 8-connected trajectories. In Fig. 2-e the perceptual points for the signature are highlighted.

D. Duration of the signature

What is the duration of a static line? This is one of the most crucial parts of this work, since it is by far one of the most discriminating properties in on-line ASVs. The time taken by each single stroke can be estimated from its existence. It is known that the so called Central Pattern Generator (CPG) produces rhythmic patterned outputs without sensory feedback. Moreover, it has been suggested that the mammalian locomotor CPG comprises a "timer" which generates step cycles of varying duration and a pattern formation layer which selects and grades the activation of motor pools. This time is usually compressed between 0.09 and 0.12 sec. Therefore, if the stroke generation was simulated by the CPG step cycle, the duration of each stroke could be established as 0.1 sec. Let $\tau_s + r$ the duration of a single stroke. Where $\tau_s = 0.1\, sec$ and r a random value that follows a normal distribution $N(0, 0.01)$ clipped in the range of ± 0.01, which comprises 68.2 % of the distribution. Finally, the duration of a component was determined by the sum of each stroke duration.

E. Lognormal resampling

The objective of this section is to generate a human-like on-line signature from an 8-connected signature.

Suggested by the kinematic theory of rapid human movements, the velocity profile of a handwriting signature could be interpreted as a vectorial summation of consecutive lognormals [13]. Being a single lognormal speed described as follows: $|v_i(t)| = \frac{D}{\sqrt{2\pi}\sigma_i (t - t_{0_i})} \exp\left(-\frac{(\ln(t - t_{0_i}) - \mu_i)^2}{2\sigma_i}\right)$.

1) Velocity profile: The velocity profile is designed by following the previous formulation. According to [14], it could be said that the distance e traveled at time t during the execution of a lognormal could be calculated through the lognormal cumulative function:

$$e(t) = \int_{-\infty}^{+\infty} v\, dt = \frac{D_i}{2}\left(1 + \text{erf}\left(\frac{(\ln(t - t_{0_i}) - \mu_i)^2}{\sqrt{2}\sigma_i}\right)\right) \quad (1)$$

From equation (1) it could be worked out the time and the velocity in terms of distances for each lognormal as follows:

$$t_{e_i} = t_0 + \exp\left(\sqrt{2}\sigma_i \text{erf}^{-1}(2\, e/D_i - 1) + \mu_i\right)^{0.5} \quad (2)$$

$$v_e = \frac{D_i \exp\left(-\sqrt{2}\sigma_i \text{erf}^{-1}(2\, e - 1)\right)}{\sigma_i \sqrt{2\pi} \exp\left(\sqrt{2}\sigma_i \text{erf}^{-1}(2\, e/D_i - 1) + \mu_i\right)} \quad (3)$$

It could be said that there exists a certain correspondence between the lognormals and perceptual points [9]. Accordingly, we have located a lognormal between two perceptual points.

Hence, the following step was to deduce the lognormal parameters from an 8-connected trajectory. Formally, we assumed that the distance traveled by a stroke in the signature l_{s_i} has a duration t_{s_i}. This way, equation (1) could be rewritten as follows:

$$l_{s_i} = \frac{D_i}{2}\left(1 + \text{erf}\left(\frac{(\ln(t_{s_i}) - \mu_i)^2}{\sqrt{2}\sigma_i}\right)\right) \quad (4)$$

Empirically, we studied the skewness and kurtosis of individual lognormals from handwriting signatures. For extracting the lognormal parameters, ScriptStudio software [13] was used. It was observed that the average skewness was 0.1301 and average kurtosis 3.082. So, a possible solution for equation (4) would assume that $\text{erf}(3) = 1$. Therefore:

$$l_{s_i} = D_i \quad (5)$$

$$\mu_i = \ln(t_{s_i}) - 3\sqrt{2}\sigma_i \quad (6)$$

On the other hand, the lognormal mode can be analytically defined by $e^{\mu_i - \sigma_i^2}$ and it is approximately $t_{s_i}/2$ with a slightly left skew. Numerically, we could said that:

$$\delta_k\, t_{s_i} = e^{\mu_i - \sigma_i^2} \quad (7)$$

δ_k being a uniform distribution heuristically defined in the range $[\delta_k^{min}, \delta_k^{max}] = [0.3, 0.4]$.

Then, combining equations (6) and (7) we could obtain the following relationship:

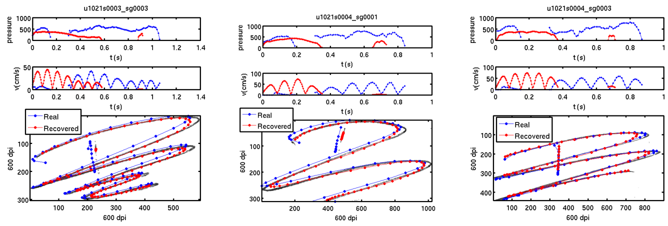
$$\sigma_i^2 + 3\sqrt{2}\sigma_i - \ln(\delta_k) = 0 \quad (8)$$

So, whether the length of a stroke (i.e. the space between two perceptual points) and its duration (see Sect. II-D) were known, the rest of lognormal parameters could be directly approximated, in the case of individual strokes. However, there exist certain synergies between the lognormals during the execution of a rapid movement like handwriting signatures [13]. Such synergies were approached by overlapping the individual lognormals and increasing heuristically their initial lengths to double.

Once the lognormal parameters have been estimated, the velocity profile was calculated in the spatial domain according to equation (3). It means the velocity value for each point of the 8-connected trajectory.

Let (x_c, y_c) be a pair of coordinates of an 8-connected trajectory. The distance of such trajectory, in centimeters, was determined by $d_e = \frac{2.54}{r}\sqrt{(x_c^k - x_c^{k-1})^2 + (y_c^k - y_c^{k-1})^2}$, r being the resolution of the skeleton. Then, at point level, it could be said that there exist a linear motion like $t_e = d_e/v_e$.

The time over each point was finally used to sample the continuous 8-connected trajectory. Points whose time was



(a) (b) (c)

Fig. 3: Example of recovered signatures with two components.

close to $1/f_m$ were selected, f_m being the sample frequency, 100 Hz in our case.

2) Pressure profile: Once the velocity profile was estimated, the pressure profile $p(t)$ was calculated. It is obtained by inverting the normalized A-law compressed velocity v_e as follows:

$$p(t) = 2\delta_u \frac{\max(v_e) - v_e}{\max(v_e)} + T + \delta_u \tag{9}$$

Where T is a scalar factor equal to 500 and δ_u a random value which follows a uniform distribution $[\delta_u^{min}, \delta_u^{max}] = [0, 150]$. Both of them considered the pressure margin from a commercial digitalized tablet. After observations of real pressure profiles, we found a linear transition of two or three samples long at the beginning and at the final of the pressure profile in a component. Obviously, equation (9) was only valid for pen-downs, in the case of pen-ups, $p(t) = 0$.

A visual example of the obtained dynamics can be seen in Fig. 2, last illustration. Also, Fig. 3 illustrates some examples of acceptable recovered on-line signatures for user #021 from BiosecurID. In the case of this illustration, our Off-2-On framework firstly detected the second executed component in all visual examples.

III. On-line Automatic Signature Verifier

We are aware of writing order recovery algorithm introduces mistakes to reproduce the real stroke orders. These mistakes are severely penalized by distance-based ASVs like dynamic time warping. For this reason, we propose to use a Manhattan-based on-line ASV [15]. This ASV is interesting for this problem since it is not strictly based on the writing order of the traces, but in the first and second derivate. However, we propose a slight modification of this original ASV for reducing some errors associated with the writing order recovery.

In particular, the modified ASV performs the verification by rearranging the components of the online signatures. Let $\{c_1, \ldots, c_i, \ldots, c_n\}$ be the n components randomly shuffled of an on-line signature (x_s, y_s), we worked out the sequence of coordinates of each component as described in Sect. II. Furthermore, in order to avoid abrupt changes in the derivate and guarantee the continuity, two consecutive components are connected in a way that the end point $(x^{c_{i-1}}[n], y^{c_{i-1}}[n])$ of the first component is the starting point $(x^{c_i}[0], y^{c_i}[0])$ of the following one. The sequence of coordinates obtained by connecting the coordinates of the shuffled components is then connected with its mirror version. Being $(\hat{x}^{c_i}, \hat{y}^{c_i})$ the mirror coordinates of (x^{c_i}, y^{c_i}). Formally, the modified coordinates of the signature (x_s, y_s) could be expressed as:

$$\hat{x}_s = [x^{c_1}, \ldots, x^{c_i}, \ldots, x^{c_n}, \hat{x}^{c_n}, \ldots, \hat{x}^{c_i}, \ldots, \hat{x}^{c_1}] \tag{10}$$

$$\hat{y}_s = [y^{c_1}, \ldots, y^{c_i}, \ldots, y^{c_n}, \hat{y}^{c_n}, \ldots, \hat{y}^{c_i}, \ldots, \hat{y}^{c_1}] \tag{11}$$

Finally, according to [15], the complete set of features were used to build the absolute and relative histograms. In our implementation, better results were obtained when the parameters ϵ_{abs} and ϵ_{rel} were set up to 0.4 and 0.004, respectively.

IV. Experimental Results

The experiments were conducted to compare the real and recovered on-line signatures through a performance-based evaluation. Recovered on-line signatures were created in order to quantify the errors produced in each stage of the proposed framework. For this purposes, the following five strategies were carried out to generate recovered on-line signatures:

1) The complete Off-2-On method was applied to real off-line signatures and the overall performance was evaluated.
2) The lines produced by the thinning stage were replaced by an 8-connected trajectory that was created by interpolating the real on-line signatures through the Bresenham algorithm. They were provided to the system in order to remove the thinning stage influence on the overall performance.
3) Same input as in strategy 2) and including the real tracing of the signature instead of the one produced by the writing order stage. This way, the influences of both thinning and writing order recovery stages were removed.
4) Same input as in strategy 3) and adding the real perceptual points, estimated from the minimum of the real velocity profile, so as to remove its influence on the overall performance of the first three stages.
5) On-line versions were created by using the same input as strategy 4) and including the real duration at each component. Therefore, only the influence of the lognormal resampling is evaluated in this last strategy.

Once the on-line signatures were recovered under these five strategies, both the original Manhattan-based ASV [15] and the modified one (Sect. III) were used in the experiments. As a proof of concepts, we used the first 50 users of the BiosecurID database [16], which comprises 800 genuine and 600 forged specimens. For training the system, five genuine signatures per user were randomly selected in each trial. The remaining genuine signatures were left for testing. In the Random Forgery (RF) experiments, the genuine signatures of each user were compared with the genuine signatures of the other users, while in the Skilled Forgery (SF) experiment the genuine signatures were compared with the corresponding

TABLE I: Performance evaluation in EER (%) of the proposed Off-2-On framework's stages - first 50 users from BiosecurID [16]

On-line ASV	Experiment	Real Off-line[1]	Thinning[2]	Writing Order[3]	Perceptual Points[4]	Duration[5]	Real On-line
Original Manhattan-based	Random Forgery	8.77	5.22	1.06	1.34	1.25	0.83
(own implementation [15])	Skilled Forgery	26.76	20.03	9.67	6.01	3.86	3.24
Modified Manhattan-based	Random Forgery	8.47	4.26	3.14	3.00	3.03	1.73
(Sect. III)	Skilled Forgery	27.83	19.23	16.31	10.78	5.80	5.01

[1] Known image-based signature (completed method, see Fig. 1) but unknown the rest
[2] Known real thinning but unknown real writing order, real perceptual points, real signature duration and real re-sampling
[3] Known real writing order but unknown the real perceptual points, signature duration and real re-sampling
[4] Known real writing order and the real perceptual points but unknown signature duration and real re-sampling
[5] Known real writing order, the real perceptual points and real signature duration but unknown real re-sampling

falsified ones. Each experiment was repeated ten times and results were given in terms of Equal Error Rate (EER, %).

Experimental results are given in Table I. The reference performance is given in the column shadowed in gray. It is worth pointing out that an ideal Off-2-On framework would achieve these performances.

A. Discussion

Recovering on-line signatures from real off-line specimens has several milestones. According to the proposed framework, some relevant properties of each stage could be quantified for both RF and SF.

On the one hand, in RF was observed that our *Thinning* stage produced a loss in the performance of 45 % and 63 % for both original and modified Manhattan-based ASVs, respectively. These losses were worked out by comparing the performances in *Real Off-line* and *Thinning* stages with respect to the *Real on-line*[3]. Comparing the *Writing Order* and *Thinning* performances with respect to the *Real on-line*, we could quantify a performance decrease of 95 % and 44 % for both verifiers, respectively. These results suggest that writing order is one of the most critical stage in the Off-2-On process. According to the experimental results, the following stages (i.e. *Perceptual points* and *Duration*) did not introduce any notorious improvement regarding *Real On-line* performances in both verifiers. More research in these stages would be necessary to highlight further difference between genuine signatures.

On the other hand, the outcome of SF exposed additional findings. On the *Thinning* stage, the comparison between the data in the *Real Off-line* and in the *Thinning* columns shown that, in case of the original Manhattan-based verifier, the thinning was responsible for more than 28 % of the decrease in performance with respect to the *Real on-line* case. It was also observed a performance decrease of 37 % for the modified verifier. It should be noted, however, that there were many external factors that limited the performance of the thinning stage, such as low resolution, dust in the images, type of sensor used to register them, blurred grayscale images and so on. Such drawbacks were especially incremented in Western signatures, which have a large quantity of crosses that hidden and blur relevant details.

On the *Writing Order* stage, the comparison between the data in the *Thinning* and in the *Writing Order* columns shown that this stage was responsible for 62 % and 21 % of performance decreases in both original and modified verifiers, respectively, with respect to the on-line case. If we compared the impact of both the *Thinning* and *Writing Order* with the *Real Off-line* effect regarding the *Real On-line* performances, we could observe 73 % and 50 % of the performance degradation. These results seems to confirm that these two stages were by far the most critical ones and, consequently, more attention should be given to them. A balanced view of the modified Manhattan-based system pointed that even though the final performance was not as competitive as the original verifier, the negative impact of these stages were slightly reduced.

One key to solve successfully the branch points would depend on differences between how human and machine see the signatures: While a human sees globally the shape of the signature and is able to solve correctly the branch points, algorithms typically see a small region around the branch point, losing the global information of the signature. Thus, the selection of the start and end points is usually a matter of empirical decision rules. Such rules try to approach the unpredictable decision of a signer to start writing a component. The sum of such negative effects during the *thinning* and *writing order* stages are attributed to be the main challenges in the performance degradation for both RF and SF. Moreover, it is even more complicated in Western signatures since the flourishes are usually quite longs and written over the text.

On the *Perceptual points* stage, the comparison between data in the *Writing Order* and in the *Perceptual points* columns shown that our perceptual points proposal was responsible for more than 57 % and 49 % of the decrease in performance for both ASVs. While an expected behavior was observed in the modified ASV, better performances were achieved in the original one.

In this case, two challenges were identified. Firstly, one difficulty is attributed to the estimation of extra points like turning points. Selecting extra points leads to unnatural rapid traces, whereas detecting a few points reduces the number of strokes in the recovered signatures. It is worth pointing out that in our approach the number of strokes were directly proportional to the number of perceptual points. Secondly,

[3] From Table I: (8.77-5.22)/(8.77-0.83)=0.4471 for the Original Manhattan-based ASV and (8.47-4.26)/(8.47-1.73)=0.6246 for the modified one. Accordingly, the quantification of each stage has been similarly measured along Sect. IV-A.

certain details in the trajectory of on-line signatures would give essential clues about the location of perceptual points in on-line signatures. However, in an Off-2-On process, many of these clues, which are essentials (e.g. [17]), are missing. Both limitations could explain that the results in column *Perceptual points* were not still comparable to the baseline (*Real On-line* column). It is worth taking into consideration that forgeries typically generate more perceptual points than genuine signers.

On the *Duration*, if we used our estimated duration, a degradation of 77 % and 86 % was obtained for both ASVs by comparing the *Perceptual points* and *Duration* columns. It should be noted that our approach simplified the estimation of the duration. We approach the temporal length of the signatures in the BiosecureID database by a Gaussian (mean, standard deviation) with $(3.61, 1.75)$ sec for genuine and $(9.07, 6.04)$ sec for forgeries. Beyond its enormous importance in on-line ASVs, these differences could explain the vital question of its correct estimation for SF.

On the *lognormal resampling* a direct comparison was carried out through column *Duration* and the baseline. The results were quite acceptable comparing with the real performance. Such results suggest a positive validation of the proposed lognormal resampling in SF. Note that SF is the most relevant experiment in signature verification.

V. Conclusion and Future Work Ideas

In this article a complete framework to recover on-line signatures from image-based specimens (Off-2-On) have been proposed. We have quantified the impact in the performance in several stages of such framework. As such, a detailed discussion is provided.

As on-line signatures are composed by time functions (velocity, acceleration, pressure, etc), a novel lognormal resampling of 8-connected trajectories was carried out. A modified Manhattan-based classifier is proposed to cope with errors associated with the writing order recovery algorithm. This modified ASV tries to mitigate the order selection of components and the assignation of the start and end points. However, the classifier does not solve errors in the branches when the trace is recovered.

As future work, applying this procedure to other scripts seems to be promising for signature verification. Western signatures contain a lot of crosses, ambiguities zones and longer components compared to Chinese or Bengali scripts, among others. Moreover, both thinning and writing order algorithms seem to be the major critical stages. So, several related algorithms would be tested and compared in a further study. Finally, provided that this complete Off-2-On framework reported comparable performance, the combination of recovered and real on-line signatures could lead to better improvements in on-line ASV.

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