EXPERIMENTAL METHODS

TO MEASURE

NUCLEAR RADII

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ABSTRACT:

In the following text I have tried to describe approximately the more used methods to measure nuclear radii. After reading several pages of different books, (named in the Bibliography), I explained the extracted information with a more useful technical language which combines elemental concepts about atoms, with graphics and tables in order to teach the reader and avoiding being boring.

INTRODUCTION:

Nuclei have a well-defined size. In order to show a difference between the structure of atoms and nuclei, let's say that in an atom the probability of the presence of an electron gradually goes to zero with increasing distance from the center. The structure of nuclei is different in contrast with the atom structure, the probability of the presence of nuclear constituents is high in the surface and diminishes to zero outside in an interval which in heavy nuclei is small compared to the radius. This makes possible an accurate definition of the nuclear radius. This concept is aplicable to medium heavy nuclei, not to very light nuclei (A<20). (Evans, 1977)

The experimental studies demonstrate that the volume of nuclei is approximately proportional to the number of A constituents. The shape is, most probably, spherical. The radius R of the nucleus is approximately proportional to $A^{1/3}$, and the relation that fits the experimental data best is:

$$R = r_0 A^{1/3}$$

The constant r_o is equal to $1.5 \cdot 10^{-13}$ cm. However, the value of r_o variates within an interval depending on the element, and on the way the nuclear radius was measured. Some measurements add an additional constant term in the expression above. There are used several methods to measure the nuclear radius:

A. SCATTERING OF HIGH-ENERGY NEUTRONS BY NUCLEI:

The comparison of experiments and theory therefore provides a determination of the radius. From the wave theory of neutron scattering the expected cross section can be calculated more accurately is a function of the radius, although the total cross section of the very fast neutrons reaches the value $2 \cdot \pi \cdot R^2$

A1.HIGH ENERGY, HEAVY AND INTERMEDIATE NUCLEI:

When the energy of the incident neutrons is of the order of 1 Mev. or higher, there are inelastic scattering and reactions in which charged particles are emitted, but because of the existence of the Coulomb barrier, the emission of neutrons is more probable than the emission of charged particles. The re-emitted neutrons can leave the residual nucleus in an excited state, so that there is obtained an inelastic neutron scattering.

Since the re-emission of neutrons is the most probable process $\sigma_c(n)$(Fig.1) also represents to a good approximation the cross section for inelastic scattering of neutrons at energies E larger than several Mev.

The energy distribution of the inelastically scattered neutrons reflects the level spectrum of the target nucleus. There is known experimentally very little about the energy distribution of the re-emitted neutrons. The experimental results obtained so far seem to be in agreement with the theoretical predictions.

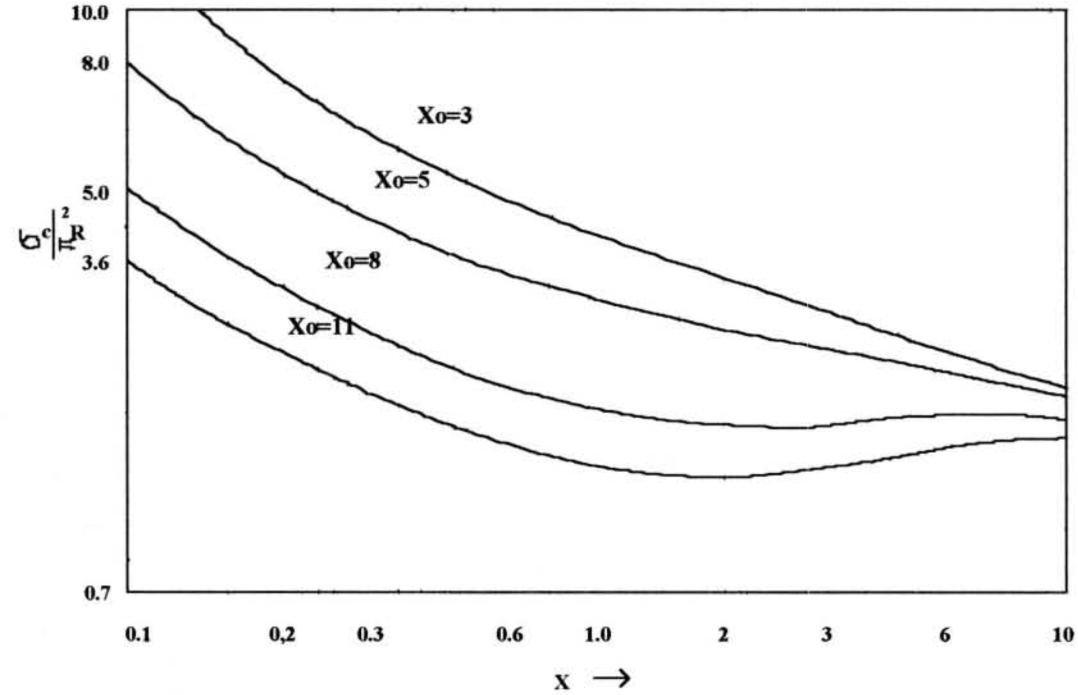


fig.1(σ)

The abscissa is $X=KR=0'218e^{1/2}R$ if e is given in Mev, R in 10^{-13} cm.

$X_o=K_oR$ and is roughly equal to the nuclear radius in units of 10^{-13} cm.

A2. VERY HIGH ENERGY, HEAVY AND INTERMEDIATE NUCLEI:

The neutron cross sections in the very high-energy region have very simple features. The total cross section shouldn't depend on special properties of the individual nuclei. The total cross section shows the very-high region is characterized by the fact that its value lies near its asymptotic value $2 \cdot \pi \cdot R^2$ shows a number of nuclear radii determined from such measurements at 14 Mev. and 25 Mev (table 1).

Table 1. Nuclear radii

Element	Energy (Mev)	R (10^{-13} cm.)	r_0 (10^{-13} cm.)
Be	14	2.4	1.17
B	14	3.4	1.54
C	25	3.8	1.65
O	25	4.3	1.71

Element	Energy (Mev)	R (10^{-13} cm.)	r_0 (10^{-13} cm.)
Mg	14	4.5	1.57
Al	14	4.6	1.53
	25	4.6	1.52
S	14	4.1	1.30
Cl	25	4.7	1.44
Fe	14	5.6	1.46
Cu	25	5.5	1.38
Zn	14	5.9	1.48
Se	14	6.3	1.46
Ag	14	6.8	1.44
	25	6.9	1.46
Cd	14	7.2	1.48
Sn	14	7.4	1.52
Sb	14	7.3	1.46

Element	Energy (Mev)	R (10^{-13} cm.)	r_0 (10^{-13} cm.)
Au	14	7.5	1.33
Hg	14	8.3	1.42
Hg	25	8.4	1.44
Pb	14	7.8	1.32
Bi	14	7.9	1.34

The radius R may depend on the energy as well as on the nature of the incident particle. R represents the distance at which the nuclear forces begin to act. It's defined more exactly as the distance at which the wave number of the incident particle assumes the value K which is characteristic of the value numbers in the interior of the nucleus.

R may fluctuate by amounts of the order of $K^{-1} = 10^{-13}$ cm. In the three elements Al, Ag, Hg for which cross sections have been measured both at 14 Mev. and at 25 Mev, the implied values of R are very nearly equal to each other (table 2). The reaction cross section σ_r is the sum of the cross sections of all reactions except the elastic scattering and can be measured by determining how many neutrons are removed from a neutron beam, not counting those that are scattered without change of energy.

Table 2. Neutron reaction cross sections at 14 Mev.

($\sigma_r \cdot 10^{24}$ cm^2)

Element	Experimental Measurement	Theoretical Estimate
Al	+0.90 1.06	1.0
Fe	1.43 1.45	1.4
Cd	1.89	2.0

Element	Experimental Measurement	Theoretical Estimate
Au	2.51	2.2
Hg	2.47	2.5
Bi	2.56	2.3
Pb	2.22 2.29 2.56	2.3

The very high region is characterized by the fact that the incident particle possesses enough energy to initiate secondary or tertiary reactions.

B. THE YIELD OF NUCLEAR REACTIONS INITIATED BY PROTONS OR α-PARTICLES:

B1. HIGH ENERGY, BELOW NEUTRON REACTION THRESHOLD:

The cross sections of nuclear reactions initiated by protons or α-particles are inmeasurably small for incident energies below 0'1 Mev, with a few exceptions among the very light nuclei. The Coulomb barrier prevents any appreciable interaction with the nucleus at low energies. The character of the reactions initiated by protons or α-particles is quite different below and above the threshold energy of the reactions in which neutrons are emitted, the (p,n) or (α,n) reactions. The (p,n) threshold energies are very large for the light nuclei and the lighter of the intermediate nuclei.

They are of the order of the proton Coulomb barrier energies for all nuclei for which Z=N-1, Z=N, Z=N-2. In the first case, the (p,n) reaction leads to a mirror nucleus, whose energy differs from that of the target nucleus just by the Coulomb energy, in the positive direction. In the second and third cases, the replacement of a neutron by proton destroys the symmetry and adds Coulomb energy. Both factors increase the energy of the nucleus.

Then, there is expected large (p,n) threshold for nuclei up to A=40. Protons which have an energy below this threshold can give rise only to elastic scattering, inelastic scattering, radiactive capture, and perhaps (p,α) reactions, which are very much weaker than the others.

The lifetimes of the compound states are relatively long and sharp resonances may be expected. They were observed in many cases by measuring the yield of the radiactive capture process as a function of the proton energy.

Many sharp resonances have been found with this method by bombarding the following nuclei with: Na^{23},Mg^{25},Al^{27},P^{31},Cl^{37}.

The inelastic scattering of protons is an important mean to determine the level spectrum of the target nucleus. This method is practically restricted to the energy region near or below the (p,n) threshold, if the neutrons can leave the nucleus with energies above 1 Mev.

The energy of the α-particles is too small to penetrate the barrier. A good method of studying resonances with charged particles is the observation of the elastic scattering near resonance. The deviations from pure Rutherford scattering are of two kinds:

(1) Off resonance the nucleus acts not like a point charge, but like a charged impenetrable sphere of radius R.

(2) In resonance there is a specifically nuclear contribution to the scattering.

The analysis of the energy and angular dependence of the deviations from Rutherford scattering off resonance allows in principle a very accurate measurement of the effective nuclear radius R. These analyses have been developed far only for a few resonances in light nuclei.

B2. HIGH ENERGY, ABOVE NEUTRON REACTION THRESHOLD:

The neutron reaction becomes the principal reaction if the compound nucleus has enough excitation energy to emit a neutron of 1 Mev.or more. The high probability of neutron emission also makes bigger the levels of the compound nucleus. Since the neutron reaction is dominant, its cross section is almost equal to the cross section for formation of the compound nucleus:

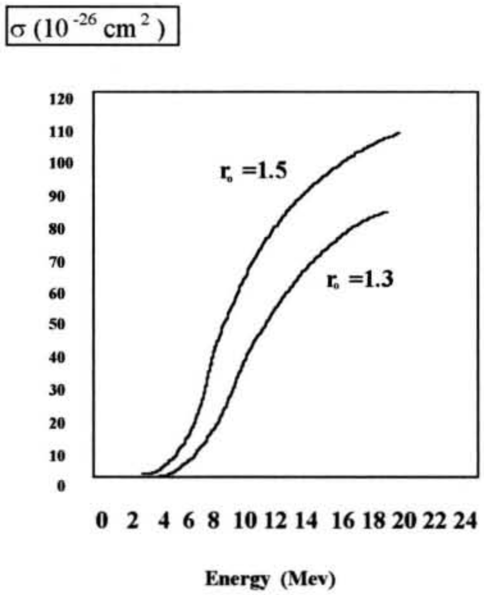
$$\sigma(p,n)=\sigma_e(p)$$

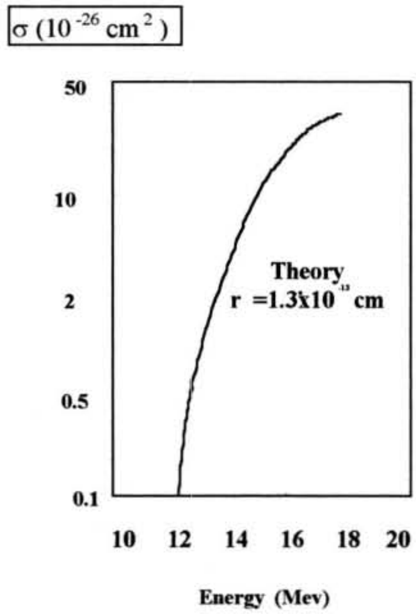
$$\sigma(\alpha,n)=\sigma_c(\alpha)$$

The comparison with experimental results is satisfactory in most cases as the examples in figs. 2 and 3. In order to be according to the shape of the excitation functions, nuclear radii chosen are somewhat smaller than those deduced from neutron experiments. Their value is represented by:

$$R = 1'3 \cdot 10^{-13} \cdot A^{1/3}$$

Recent measurements by scientists suggest nuclear radii which are more in conformity with those found in neutron experiments. There is not enough experimental material available to test the theory.





$\sigma_c(p)$ for Cu^{63} $\sigma_c(\alpha)$ for Rh^{103}

fig 2,3

B3. VERY HIGH ENERGY:

The very high-energy region is characterized by the appearance of secondary reactions. The compound nucleus decays mostly by the emission of a neutron, but the residual nucleus still possess enough energy to emit the second particle. This secondary reaction is the (p,2n) or (α,2n) reaction.

The higher-order reactions with charged particles have shown good agreement with the theoretical predictions. Goshal made a very instructive series of measurements with protons bombarding Cu^{63} and α-particles bombarding Ni^{60}. Both bombardments produce the compound nucleus Zn^{64}, in the same state of excitation, but both particles having different values of energy.

Then, according to the Bohr assumption, the processes following the two bombardments must be equal. This is due to the accidental fact that the cross section σ_c(proton) at an energy E_p is almost equal to σ_c(α-particle) at the energy E_p+7 Mev in the energy of the region observed.

C. α-DECAY LIFETIMES

DISCUSSION OF EXPERIMENTAL DATA:

According to the theoretical results of the previous section, the probability of the emission of an α-particle is a very sensitive function of the energy $E\alpha$ of the α-particle. The general trend of the dependence on $E\alpha$ is the same for all values of Z and R of importance for naturally radioactive nuclei.

Table 3 collects important data of a number of α-emitting natural nuclei. The most important result of this table is the list of channel radii R determined from the natural lifetimes and energies. The first column lists the parent nucleus, the second the daughter nucleus. The third column gives the observed lifetime. The fourth column gives the α-energy in Mev. The channel radius R was calculated by using some complicated formulas which depend on many parameters. The radius R_x is the one of the residual daughter nucleus X, this radius is the most important data, it was calculated from formula $R=R_x+r_0\alpha$. The last column shows how closely these radii approximate the simple law $R_x=r_0 \cdot A^{1/3}$, where r_0 is a constant close to $1'4 \cdot 10^{-13}$ cm.

Table 3.

Parent	Daughter	Half-life	α-energy (eMv)	R (10^{-13} cm)	R_x (10^{-13} cm)	$r_o=R_xA^{-1/3}$ (10^{-13} cm)
$_{83}Bi^{212}{}_{129}$	$_{81}Tl^{208}{}_{127}$	11h	6.20	8.0	6.8	1.14
$_{83}Bi^{214}{}_{131}$	$_{81}Tl^{210}{}_{129}$	76d	5.61	9.2	8.0	1.34
$_{84}Po^{210}{}_{126}$	$_{82}Pb^{206}{}_{124}$	138.3d	5.40	8.4	7.2	1.21
$_{84}Po^{212}{}_{128}$	$_{82}Pb^{208}{}_{126}$	$3.0 \cdot 10^{-7}$s	8.95	9.0	7.8	1.31
$_{84}Po^{214}{}_{130}$	$_{82}Pb^{210}{}_{128}$	$1.5 \cdot 10^{-4}$s	7.83	9.3	8.1	1.35
$_{84}Po^{215}{}_{131}$	$_{82}Pb^{211}{}_{129}$	$1.8 \cdot 10^{-3}$s	7.50	9.5	8.3	1.40
$_{84}Po^{216}{}_{132}$	$_{82}Pb^{212}{}_{130}$	0.158	6.89	9.3	8.1	1.35
$_{84}Po^{218}{}_{134}$	$_{82}Pb^{214}{}_{132}$	3.05m	6.12	9.6	8.4	1.40
$_{85}At^{215}{}_{130}$	$_{83}Bi^{211}{}_{128}$	10^{-4}s	8.15	9.3	8.1	1.35

Parent	Daughter	Half-life	α-energy (eMv)	R (10^{-13} cm)	R_x (10^{-13} cm)	$r_o=R_xA^{-1/3}$ (10^{-13} cm)
${}_{86}Em^{219}{}_{133}$	${}_{84}Po^{215}{}_{131}$	4.7s	6.94	9.6	8.4	1.39
${}_{86}Em^{220}{}_{134}$	${}_{84}Po^{216}{}_{132}$	54.5s	6.39	9.7	8.5	1.40
${}_{86}Em^{222}{}_{136}$	${}_{84}Po^{218}{}_{134}$	3.83d	5.59	9.7	8.5	1.40
${}_{88}Ra^{223}{}_{135}$	${}_{86}Em^{219}{}_{133}$	20.2d	5.82	9.7	8.5	1.40
${}_{88}Ra^{224}{}_{136}$	${}_{86}Em^{220}{}_{134}$	3.8d	5.78	9.8	8.6	1.42
${}_{88}Ra_{138}$	${}_{86}Em^{222}{}_{136}$	1700y	4.88	9.9	8.7	1.44
${}_{89}Ac^{227}{}_{138}$	${}_{87}Fr^{223}{}_{136}$	1810y	5.04	9.6	8.4	1.39
${}_{90}Th^{227}{}_{137}$	${}_{88}Ra^{223}{}_{135}$	93d	6.16	8.8	7.6	1.26
${}_{90}Th^{228}{}_{138}$	${}_{88}Ra^{224}{}_{136}$	2.64y	5.52	9.5	8.3	1.36
${}_{90}Th^{230}{}_{140}$	${}_{88}Ra^{226}{}_{138}$	$1*10^{5}$y	4.76	9.5	8.3	1.36
${}_{90}Th^{232}{}_{142}$	${}_{88}Ra^{228}{}_{140}$	$1.39*10^{10}$y	4.05	9.8	8.6	1.43
${}_{92}U^{234}{}_{142}$	${}_{90}Th^{230}{}_{140}$	$2.35*10^{5}$y	4.84	9.5	8.3	1.35
${}_{92}U^{238}{}_{146}$	${}_{90}Th^{234}{}_{144}$	$4.51*10^{9}$y	4.25	9.6	8.4	1.36

D. CLASSICAL THEORY OF THE COULOMB-ENERGY RADIUS, MIRROR NUCLEI:

This method lead to the evidence that the nuclear volume is substancially proportional to the number of nucleons in a determined nucleus. This means that nuclear matter is essentially incompressible and has a constant density for all nuclei. The variations from constant density, due to the nuclear compressibility, appear to be only 10 per cent. The method involve the combined effects of nuclear charge and nuclear size.

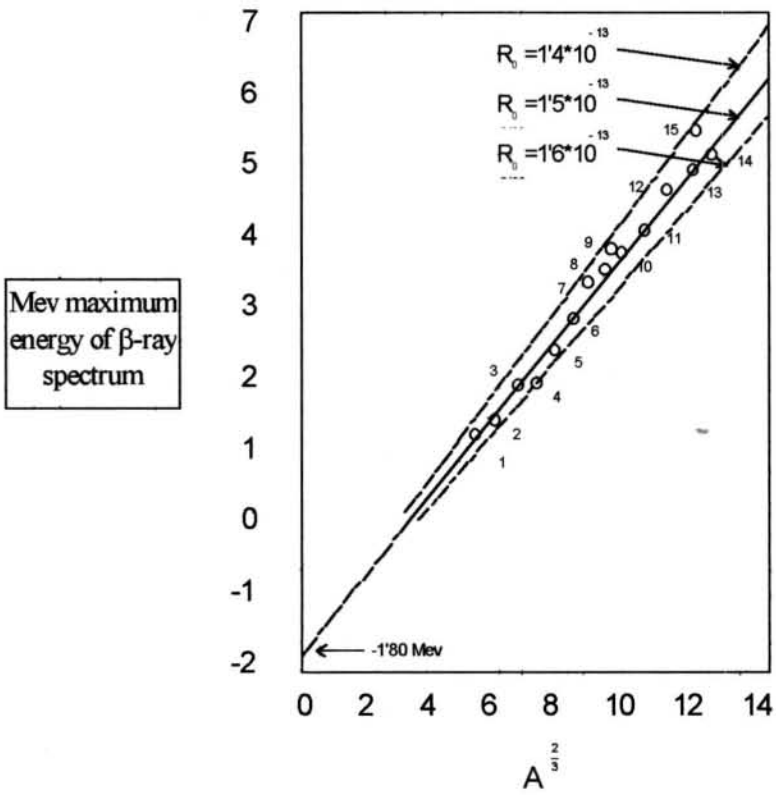
The coulomb energy is a measurable quantity in some nuclei which suffer radioactive β-decay. For these nuclei, the expression of the coulomb energy is $W_{COUL}=3/5 \cdot e^2/R \cdot Z^2$

This equation represents a definition of the nuclear radius. In β-decay, the mass number A doesn't change, and R doesn't change. In positron β-decay, the nuclear charge Z of the parent decreases to Z-1 for the decay product, one proton in the parent nucleus changes into a neutron in the product nucleus.

Simultaneously a neutrino and a positron (the β-ray) are expelled from the nucleus. There are a group of nuclei, called mirror nuclei, which undergo β-decay. These mirror nuclei are any pair of nuclei which can be made from each other by interchanging all protons and neutrons. Their stable decay products each contain just one more neutron than the number of protons.

In each of these nuclei, the mass number is A=2Z-1, Z is the atomic number of the parent positron β-decaying nucleus. From experimental evidence analyzing mirror nuclei, we know that nuclear forces are symmetrical in neutrons and protons and that nuclear binding between two neutrons is the same as that between two protons.

In the figure below the fact that the experimental values tend to lie on a straight line indicates that these nuclei have coulomb-energy radii which correspond to a constant-density model $R_{coul}=R_0 \cdot A^{1/3}$, with the slope of the data giving the particular value $R_0=1'45 \cdot 10^{-13}$ cm for the nuclear unit radius. Dotted lines for $R_0=1'4$ and $1'6 \cdot 10^{-3}$ cm clearly constitute an interval for the coulomb-energy unit radius.



The coulomb energy radius $R_0=1'45\cdot 10^{-13}$ cm is in agreement with the nuclear radii obtained for these same nuclei by other methods. As a conclusion, we may say that the coulomb-energy radii of nuclei having $A\leq 41$ follow the constant-density model $R_{coul}=R_0\cdot A^{1/3}$, and have a unit radius of $R_0=1'45\cdot 10^{-13}$ cm.

❋ <u>Let's show a practical example:</u>

Calculate the value of the β^+ transition energy between the mirror nuclei ^{13}N and ^{13}C. From this result, evaluate the r_0 parameter of the nuclear radii. (Mattauch et al.,1965)

The atomic mass of the nuclei are:

$$M(^{13}N)=13'005738 \text{ u}$$

$$M(^{13}C)=13'0033544 \text{ u}$$

The excess of mass is the fractional part of the atomic mass converted into Kev, so the respective excesses of mass are:

^{13}N: 5342'2 Kev

^{13}C: 3124'6 Kev

The disintegration scheme of ^{13}N is:

$${}_{7}^{13}N \rightarrow {}_{6}^{13}C + \beta^{+} + \nu$$

The expression of the β^{+} disintegration energy is:

$E(\beta^{+}) = [\ M({}_{7}^{13}N) - M({}_{6}^{13}C) - 2me\ ]\ c^{2} - \Delta E_{enl}\ (e^{-})$

the last term of the equation is unappreciable compared to the others.

$E(\beta^{+})$ = 5'3452-3'1246-2·0'05110 = 1'1986 Mev

If we take a look to a chart, we'll see that ^{13}N is a β^{+} emitter, with a semidisintegration period of 9'97 minutes and a maximum energy of 1'20 Mev, which shows that calculations are well made.

The difference between the atomic mass of both mirror nuclei, M(A,Z) and M(A,Z+1) is given by the equation:

$[\, M(A,A\text{-}Z) - M(A,Z)\,]\, c^2 = \Delta U_c - (\, m_n - M_H\,)\, c^2$

Taking as $M(A,Z) = M_H Z + M_n(A\text{-}Z) - a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4(A\text{-}2Z)^2 A^{-1} + a_5(A,Z)$

a_i are semi-empirical parameters. Some values have been found by several authors.

ΔUc represents the difference between both Coulomb energies.

Comparing this last expression with the one that gives the β^+ disintegration energy, we obtain:

$\Delta U_c = E(\beta\ t\ p^+) - (m_n - M_H\,)\, c^2 + 2m_e\, c^2$

ΔU_c = 1'1986 + 0'7825 + 2*0'5110 = 3'003 Mev = $4'81 \cdot 10^{-13}$ Joule

The Coulomb energy of both mirror nuclei, is:

$$X^Z_A \rightarrow U_c = \frac{3Z(Z-1)\, e^2}{20\pi\, \varepsilon_0\, R}$$

$$Y^A_{Z+1} \rightarrow U_{c'} = \frac{3(Z+1)\, Ze^2}{20\pi\, \varepsilon_0\, R}$$

From these equations we can obtain the difference:

$$\Delta U_c (Y - X) = \frac{3\, e^2}{20\pi\, \varepsilon_0\, R}\, [(Z+1)Z - Z(Z-1)] = \frac{3\, Ze^2}{10\pi\, \varepsilon_0\, R}$$

In all the expressions, R represents the nuclear radii, which is the same for each pair of nuclear radii:

$$R = r_0\, A^{1/3}$$

Introducing R in the last expression, we have:

$$r_0 = \frac{3\, Ze^2}{10\pi\, \varepsilon_0\, A^{1/3}\, \Delta U_c}$$

Putting numerical values, the final result is:

$$r_0 = \frac{3x6x(1'6.\, 10^{-19}\ )^2}{10x3'1416x8'85.\, 10^{-12}\ x\ 13^{1/3}\ x4'81.\, 10^{-13}} = 1'47.\, 10^{-13}\ m$$

CONCLUSIONS:

Many of the methods explained here, are historically important experiments that demonstrated the limitations of classical mechanics, so the wave mechanics gave a good interpretation of the observations.

The nuclear radii are known with much greater accuracy than the radii of the whole atoms, although there is no single definition of nuclear radius which can be applied to all nuclear situations.

All methods are involved with nuclear charge and size. I have explained the more extended and useful, but there are others such as:

Isotope shift in line spectra, Energy of radioactive β-ray decay, Elastic scattering of fast electrons by nuclei, Characteristic electromagnetic radiations from μ-mesonic atoms, Fine-structure splitting of ordinary electronic X-ray levels in heavy atoms.

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