ON THE TEMPORAL MEMORY OF TURBULENCE

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INTRODUCTION

Most atmospheric and oceanographic models parameterize the role of turbulence in transferring properties in terms of diffusion coefficients that are instantaneous functions of the forcing mechanisms. According to these common parameterizations the diffusion coefficients will increase or decrease instantaneously, as forcing does. The reality, however, is that turbulence has a memory of its prior history which causes that the generation and occurrence of turbulence is usually out of phase: turbulence may peak earlier or later than maximum forcing and last for a considerable period of time after forcing has already ceased.

The importance of these issues has been illustrated with simple models that predict the formation of layered type structures in stratified flows when the temporal memory of turbulence is taken into account (Barenblatt *et al.*, 1993; Kranenburg, 1996; Pelegrí and Sangrà, 1998; Pelegrí *et al.*, 1998). In Pelegrí and Sangrà (1998), for example, a high value in the vertical density flux is maintained during a characteristic time interval through a Langevin type formulation for the vertical density diffusivity. The result is that turbulence, expressed in terms of the density diffusivity, lasts considerably longer than those subcritical conditions that lead to the generation of instability, and an initially highly stratified region becomes unstable and mixes up rather than smoothes down.

Here we are interested in studying how turbulence, expressed in terms of a diffusion coefficient (e.g., the vertical one), is the cumulative result of its previous history and its current forcing. We examine this relation using two different approaches, both requiring an understanding of the physical processes that determine the natural temporal scale of turbulence. In one approach the diffusion coefficient is the result of some effective forcing, calculated as the cumulative effect of all previous forcing through an appropriate time-weighting curve. In the other approach the diffusion coefficient is obtained with a Langevin type equation.

In order to assess the temporal lag between forcing and actual diffusion we calculate, for the two above cases, the diffusion coefficients for different periodic forcing functions and temporal scales of turbulence. In particular, we examine the cross-correlation between the forcing function and the resulting diffusion coefficient. The results illustrate that the time lag between forcing and actual diffusion is a function of both the forcing function period and the temporal scale of turbulence.

LANGEVIN APPROACH

A simple way to appeciate the importance of the temporal memory of turbulence is specifying the density diffusivity K with a one-term Taylor expansion. In this manner the vertical density diffusion at time t is related with those conditions that took place some time δt ago, and leads to the following relation expression for the evolution of K:

$$\left(\frac{\partial K}{\partial t}\right)_{t-\delta t} = \frac{-K(t)+K(t-\delta t)}{\delta t} \qquad (1)$$

with the derivative evaluated at time t-δt.

The choice of δt is critical since a too large value would lead to very inaccurate results but a value much smaller than the natural scale of turbulence would cause the diffusion coefficient to change unrealistically rapid. Provided that we determine the natural scale of turbulence, τ, then equation (1) suggests a Langevin equation:

$$\frac{\partial K_t}{\partial t}\tau = -K_t + K_f \qquad (2)$$

with the forcing diffusion coefficient given by $K_f \equiv K(t)$ and the actual diffusion coeffcient given by $K_t \equiv K(t-\tau)$; note, however, that the derivative is now evaluated at time t.

The Langevin equation has the appeal of incorporating a slowly decaying field (the turbulence with its own characteristic temporal scale) which is continuously changing due to external forcing (e.g., the local stratification and vertical shear). If there was no external forcing the diffusion coefficient would decay exponentially with a temporal scale given by τ. All what is required is to specify a proper value for τ, which will depend on the particular problem under consideration. For shear-induced turbulence in stratified flows, for example, the temporal lag τ is proportional to the inverse buoyancy frequency.

We now illustrate the results obtained using a sinusoidal forcing function that resembles the intermittent character of frontal structures in geophysical flows (Rodriguez-Santana *et al.*, 1999). The magnitude of forcing varies between zero and some maximum value, and introduces the forcing period as a new parameter. Figure 1 shows the cross-correlation between the forcing function and the actual diffusion coefficient as a function of time t and time lag τ.

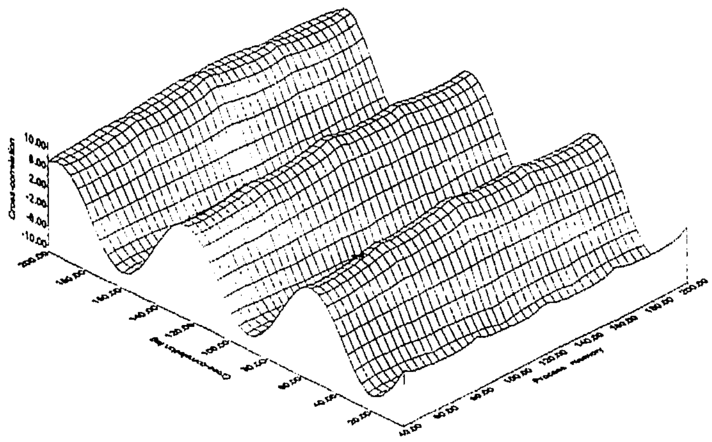

Figure 1

EFFECTIVE FORCING

One mathematical difficulty with the above derivation of Langevin equation's is that in equation (1) the derivative of K, rather than K itself, is evaluated at time $t-\tau$. The result is that the diffusion coefficient at time $t-\tau$ is modified by the forcing function at time t, rather than by some time averaged value between $t-\tau$ and t. One possibility is to modify the Langevin equation as follows

$$\frac{\partial K_t}{\partial t}\tau = -K_t + F(K) \qquad (3)$$

where $F(K) = \int_{t-\tau}^{t} w(t-\tau)K(t-\tau)d\tau$. In this definition $w(t-\tau)$ is some normalized weighting function. In this manner $F(K)$ provides a temporally weighted value of the forcing function, the integral over a time interval characteristic of the turbulence.

Figure 2 illustrates the cross-correlation between actual diffusion and forcing function, as a function of time t and time lag τ, resulting from the solution of equation (3). The forcing function is the same sinusoidal as above and the weighting function is a Gaussian.

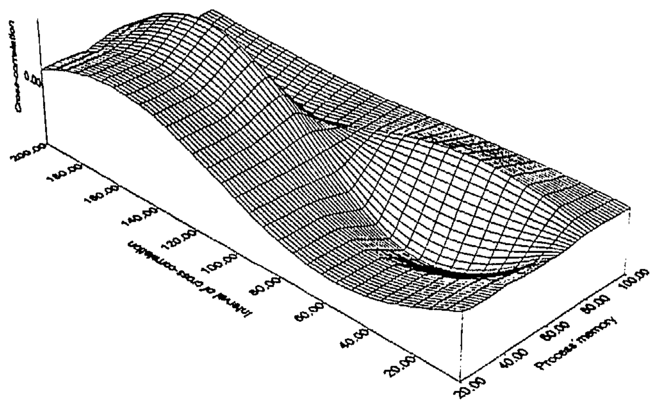

Figure 2

Figure 3 corresponds to the same case as in Figure 2 but without the first term (the temporal decay term) in the right hand side of equation (3).

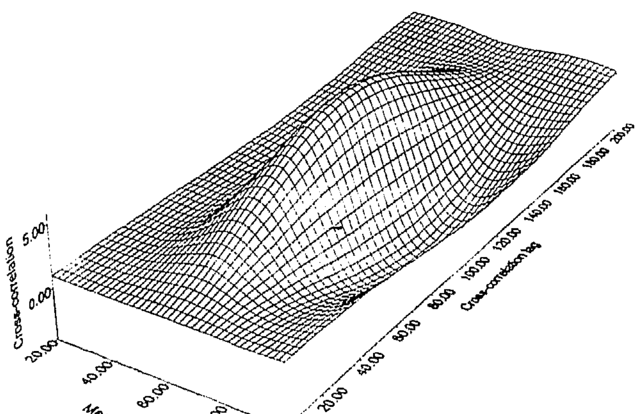

Figure 3

CONCLUSIONS

In the Langevin approach, with a periodic forcing function, the results indicate that the diffusion coefficient and the forcing function are periodically correlated, independently of the size of the temporal memory (Figure 1). On the other hand, when the forcing is obtained as an integrated value over some characteristic time lag, the cross-correlation shows a strong dependence with both time and time lag (Figures 2 and 3). The behavior of this second case appears rather realistic since diffusion depends strongly both on the natural response time of the system and the forcing history (i.e., it is a hysterical process).

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