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Implementation of the consistent lumped-parameter model for the computation of the seismic response of nonlinear piled structures

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Abstract

The computation of the nonlinear response of piled structures taking soil-structure interaction into account is tackled in this paper. The structure is assumed to be founded on a group of piles, whose impedance and kinematic interaction functions are computed, in the frequency domain, through a boundary elements-finite elements coupled formulation. On the other hand, the response of the system is computed using a time-stepping procedure for the integration of the equations of motion for an inelastic superstructure, which requires a time-domain representation of the above-mentioned impedance and kinematic interaction functions. This can be achieved by the use of different methodologies, such as standard lumped-parameter models, higher-order consistent lumped-parameter models, or hidden state variable models, among others. In this paper, the use of high-order consistent lumped-parameter models for the deterministic representation of the impedance functions of deep foundations will be discussed together with different aspects of the resulting equivalent systems such as degree of accuracy, physical representation of parameters or arising numerical stability issues. Then, the use of this approach for the computation of the inelastic response of piled structures subject to seismic excitation will be explored.

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Keywords: dynamic soil-structure interaction; lumped-parameter model; non-linear behaviour of structures; piles

1. Introduction

There exist many problems related to the fields of Structural Dynamics and Earthquake Engineering where the soil-structure interaction phenomena must be taken into account. When this is realized by substructuring methods, it is very common to characterize the dynamic response of the piled foundations by means of impedance functions that are usually obtained directly in frequency domain. These functions are useful in many situations, but must be adapted when used for problems involving non-linearities where it is necessary to solve the problem directly in time-domain.

One of the ways to use these functions directly in the time domain is through the construction of equivalent lumped-parameter models (LPMs). These models were extensively developed in the field of structural dynamics by Wolf [1–5] and Wolf and Paronesso [6–8] in the early nineties. Based on these models, Wu and Lee [9,10] tackled the problem from a more systematic point of view, by fitting impedance curves through the concept of polynomial-fraction function

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form. Some recent applications of this methodology for SSI problems are those of Andersen [11] for the assessment of lumped-parameter models in rigid footings, and Damgaard et al. [12] for the study of real time dynamic responses of offshore wind turbines on monopiles. An extended review on this topic can be found in Andersen [13].

This work presents, very briefly, preliminary results concerning the implementation and use of a consistent LPM approach for the time-domain characterization of pile foundation impedance functions. Impedance functions of piles and groups of piles were calculated by a BEM-FEM coupled model previously developed by the group [14,15]. This LPM implementation allows the study of the non-linear behaviour of a superestructure, using a substructure approach like the one shown in [16]. An example of it will be also shown.

2. Methodology

2.1. Substructure approach of the superstructure under study

The superstructure analysis can be studied by means of a substructure approach in which the system is subdivided into building-cap structure and soil-foundation stiffness and damping, represented by means of springs and dashpots, as shown in figure 1(a). The superstructure can be defined by the mass of the building cap m, the mass of the pile cap m_0, the moment of inertia of the pile cap I_0, the height of the building cap h, the viscous damping ratio ξ and the structural stiffness k that could have a non-linear law of behaviour. Note that the horizontal u_f and rocking φ_f motions of the foundation are here defined as relative to the foundation input motions.

The complex-valued frequency-dependent functions $\tilde{Z}_{xx} = k_{xx} + ia_0c_{xx}$, $\tilde{Z}_{\phi\phi} = k_{\phi\phi} + ia_0c_{\phi\phi}$ and $\tilde{Z}_{x\phi} = k_{x\phi} + ia_0c_{x\phi}$ represent the stiffness and damping of the foundation in the horizontal, rocking and cross-coupled horizontal-rocking vibration modes, respectively. The cross-coupled rocking-horizontal vibration mode has been considered identical to the cross-coupled horizontal-rocking vibration mode in this approach. Hence, the equations of motion of the system shown in figure 1(a), assuming small displacements, can be written in terms of relative motions as shown in equations (1) to (3) [16], where equation (1) represents the horizontal force equilibrium of the structure, equation (2) the horizontal force equilibrium of the structure-foundation system and equation (3) the moment equilibrium of the structure-foundation system about a horizontal axis passing through the centre of gravity of the pile cap.

$$m(\ddot{u} + \ddot{u}_f + \ddot{u}_g + h(\ddot{\varphi}_f + \ddot{\varphi}_g)) + ku + c\dot{u} = 0 \tag{1}$$

$$m(\ddot{u} + \ddot{u}_f + \ddot{u}_g + h(\ddot{\varphi}_f + \ddot{\varphi}_g)) + k_{xx}u_f + c_{xx}\dot{u}_f + k_{x\phi}\varphi_f + c_{x\phi}\dot{\varphi}_f + m_0(\ddot{u}_f + \ddot{u}_g) = 0 \tag{2}$$

$$hm(\ddot{u} + \ddot{u}_f + \ddot{u}_g + h(\ddot{\varphi}_f + \ddot{\varphi}_g)) + k_{\phi x}u_f + c_{\phi x}\dot{u}_f + k_{\phi\phi}\varphi_f + c_{\phi\phi}\dot{\varphi}_f + I_0(\ddot{\varphi}_f + \ddot{\varphi}_g) = 0 \tag{3}$$

In order to solve the system under study in time domain, the step-by-step Newmark linear acceleration method has been implemented [17]. But first, it's necessary to transform the frequency-dependent impedance functions in a proper combination of discrete-elements (masses, dampings and springs) whose response is equivalent, as detailed in the following section. A time step short enough has to be ensure in order to obtain stability conditions in time domain. The condition of $\Delta t < 0.551T_N$, where T_N is the shortest natural period of the system, has to be accomplished [17].

2.2. Impedance curves fitting by optimization of the poles and LPM schemes of different orders used in this study

As stated by Andersen [13], the impedance functions, defined in the frequency domain, can be expressed as $\tilde{Z}(a_0) = Z^0S(a_0)$ where Z^0 denotes the static stiffness $Z(0)$ and a_0 denotes the dimensionless frequency, typically $a_0 = \omega d/c_s$ in the case of piles and groups of piles where d denotes the pile diameter and c_s is the soil shear-wave velocity. The frequency-dependent stiffness dimensionless coefficient $S(a_0)$ is then decomposed into a singular part $S_s(a_0)$, and a regular part $S_r(a_0)$: $S(a_0) = S_s(a_0) + S_r(a_0) = k^\infty + ia_0c^\infty + S_r(a_0)$. In this expression, k^∞ and c^∞ are two real constants which are selected so that $Z^0S_s(a_0)$ provides the entire stiffness in the high-frequency limit $a_0 \to \infty$.

The regular part $S_r(a_0)$ is the remaining part of the stiffness. Generally, a closed expression for $S_r(a_0)$ is unavailable. Hence, the regular part of complex stiffness is usually obtained by fitting a rational filter to the results obtained in a numerical or semi-analytical model, here the BEM-FEM model previously mentioned [14,15]. Once numerical solution provided by the BEM-FEM simulation is obtained, $\tilde{Z}(a_0)$, is taken as the "exact" solution, and regular part of the stiffness coefficient is found as $S_r(a_0) = \tilde{Z}(a_0)/Z^0 - S_s(a_0)$. A rational approximation can be written as [13]:

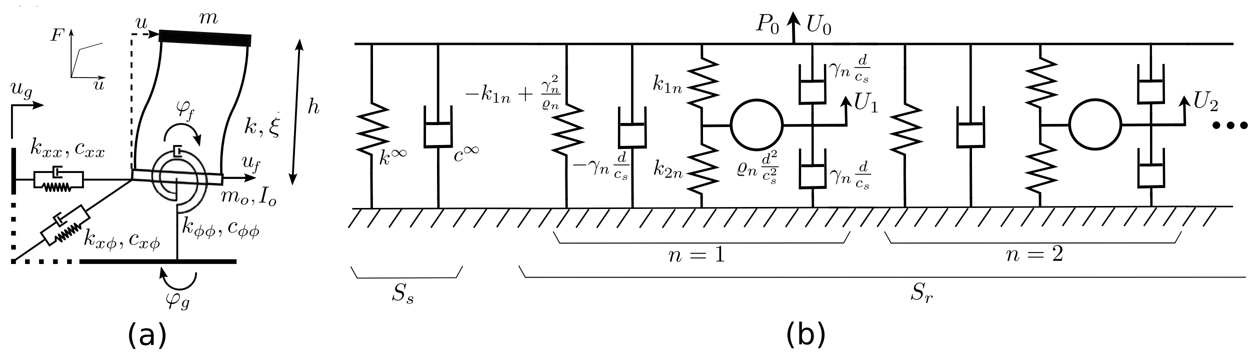

Fig. 1. (a) superstructure model under study [16]; (b) discrete-element scheme used.

$$S_r(a_0) \approx \widehat{S}_r(ia_0) = \frac{1 - k^{\infty} + p_1(ia_0) + p_2(ia_0)^2 + ... + p_{M-1}(ia_0)^{M-1}}{\prod_{n=1}^{N}(ia_0 - s_n)(ia_0 - s_n^*)\prod_{n=N+1}^{M-N}(ia_0 - s_n)}, \qquad 2N \leq M \tag{4}$$

The curve-fitting of the rational filter to the regular part of the dynamic stiffness is made by a least-squares technique, where the roots s_n are identified as the optimization variables in addition to the coefficients of the numerator polynomial. In order to get a stable solution in the time domain, the poles of $\widehat{S}_r(ia_0)$ should all reside in the second and third quadrant of the complex plane, i.e. the real parts of the poles must all be negative.

On equation (4), an asterisk (*) denotes the complex conjugate. Experience shows that as many as possible of the roots should appear as complex conjugates [13]; e.g. if M is even, $N = M/2$ should be used. This provides a good fit in most situations and may, at the same time, generate the LPM with fewest possible internal degrees of freedom.

Once calculated, the poles and the coefficients in the numerator have little insight into the physics of the problem. In order to obtain physical meaningful coefficients, a recasting of equation (4) into partial-fraction form should be carried out: $\widehat{S}_r(ia_0) = \sum_{m=1}^{M} R_m/(ia_0 - s_m)$, where s_m are the poles of $\widehat{S}_r(ia_0)$, and R_m are the corresponding residues. If the conjugate complex and the corresponding residues are added together, a second-order term with real coefficients appears, so the partial-fraction form can be rewritten as shows equation (5). The real coefficients α_{0n}, α_{1n}, β_{0n} and β_{1n} are given by simple mathematical operations between poles and residues of the complex conjugate terms [13].

$$\widehat{S}_r(ia_0) = \sum_{n=1}^{N} \frac{\beta_{0n} + \beta_{1n} ia_0}{\alpha_{0n} + \alpha_{1n} ia_0 + (ia_0)^2} \tag{5}$$

The approximation of the impedance functions consists of the sum of the singular part of the stiffness, and N second-order terms (the regular part of the stiffness, equation (5)) as shows figure 1(b). The second-order discrete-element model chosen has only one internal degree of freedom, and the four parameters (k_{1n}, k_{2n}, γ_n and ϱ_n) can be determined from the equilibrium formulation for its own node and pile cap (node 0), in comparison with the second-order term shown in equation (5). In order to calculate ϱ_n, a quadratic equation has to be solved: $a\varrho_n^2 + b\varrho_n + c = 0$ where $a = \alpha_{1n}^4 - 4\alpha_{0n}\alpha_{1n}^2$, $b = -8\alpha_{1n}\beta_{1n} + 16\beta_{0n}$ and $c = 16\beta_{1n}^2/\alpha_{1n}^2$. This equation results in two solutions for ϱ_n, for which real values must be ensured. Deciding which one of the two values of ϱ_n to take is critical: a positive value is beneficial because the majority of the discrete parameters will then be also positive which will be better to ensure stability in time domain. But, if the value of ϱ_n results in a mass much bigger than others corresponding to others degrees of freedom, contemplating the negative value could be beneficial. When ϱ_n has been determined, the three remaining coefficients can be calculated using the following expressions: $k_{1n} = \varrho_n\alpha_{1n}^2/4 - \beta_{1n}/\alpha_{1n}$, $k_{2n} = \varrho_n\alpha_{0n} - k_{1n}$, $\gamma_n = \varrho_n\alpha_{0n}/2$.

3. Results

A particular case will be introduced for the study of the influence of the foundation type on the response of the superstructure introduced in section 2.1, with the aim of validating the proposed LPM and checking the influence of the quality of the adjustment of the impedance curves. As introduced previously, all the possible roots will be forced

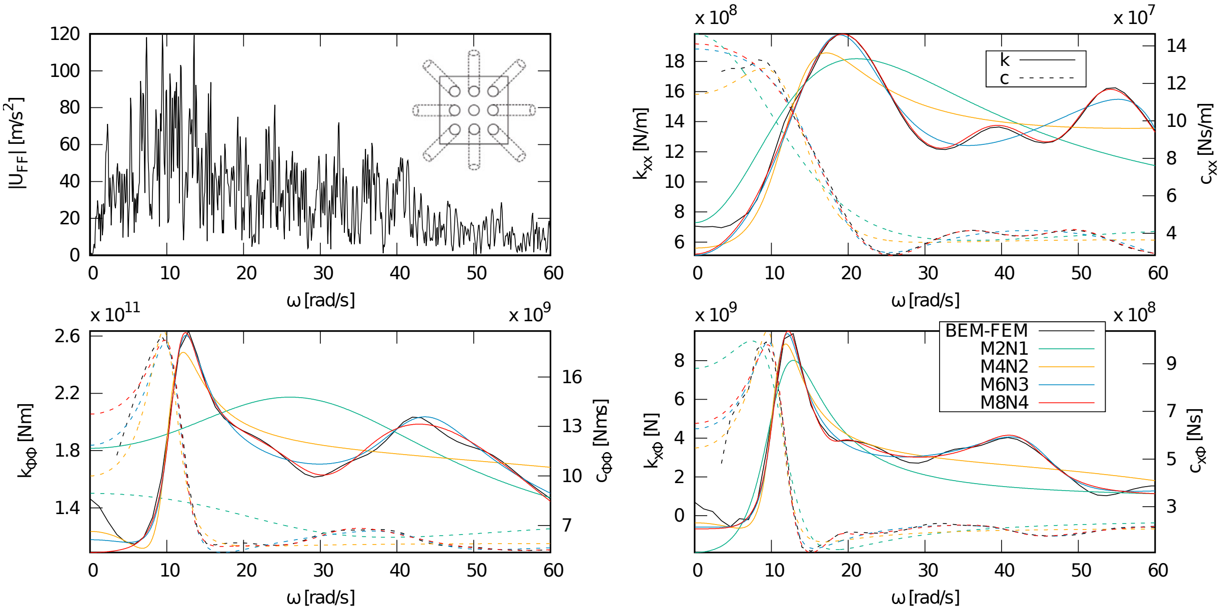

Fig. 2. "El Centro" earthquake spectrum and fitting of impedance curves with different LPM orders (M2N1, M4N2, M6N3 and M8N4).

to be complex conjugates ($N = M/2$) in the fitting process of impedance curves (horizontal, rocking and crossed), therefore in this paper only the cases of $M = 2$ and $N = 1$ (M2N1), $M = 4$ and $N = 2$ (M4N2), $M = 6$ and $N = 3$ (M6N3), $M = 8$ and $N = 4$ (M8N4) are studied.

One of the main objectives of this work is to show the potentiality of this model for the analysis of any type of nonlinearity in piled superstructures. As an example, the elastoplastic response of the superstructure columns will be shown in a second part of this section.

3.1. Validation

The results obtained by the step-by-step procedure in the time domain are compared with those obtained by the direct method in the frequency domain, since for the validation of the method, the superstructure works within the linear range. The impedance curves of the piles foundation were obtained using the BEM-FEM model previously developed by the research group [14,15], and have been taken as the exact ones in order to compare results. These curves correspond to a group of 3x3 inclined piles, whose rake angle is 20° following a distribution shown schematically into figure 2, and the properties used in order to obtain the curves are: $d = 1.5$ m, $L = 20$ m, where L is the pile length, $E_p/E_s = 1000$ (soft soil), where E_p and E_s are the pile and soil Young's modulus respectively, $\nu_s = 0.4$, where ν_s is the soil Poisson's ratio, $\rho_p/\rho_s = 1.6$, where ρ_p and ρ_s are the pile and soil densities, $s = 9$ m, where s is the pile spacing, $\xi_s = 5\%$, where ξ_s is the soil internal hysteretic damping coefficient, $E_s = 2.25 \cdot 10^7$ Pa and $c_s = 70$ m/s.

For the superstructure model (figure 1(a)) the following values of the properties have been chosen: $m = 10^6$ kg, $m_0 = 2.5{\cdot}10^5$ kg, $h = 4$ m, $I_0 = 2{\cdot}10^5$ kg/m^2, $\xi = 5\%$ and $k = 10^8$ N/m. As excitation, "El Centro" earthquake has been selected, whose frequency spectrum is shown in figure 2 and as can be seen from it, most of energy of this particular earthquake is concentrated around 10 rad/s. The kinematic interaction factors of the pile foundation, obtained through BEM-FEM [14,15], have been taken into account in order to obtain displacements u_g and rotations φ_g at the base of the structure (figure 1(a)), transforming free field spectrum U_{FF} through Fast Fourier Transform algorithms.

In view of the resulting displacements and rotations of the superstructure (figure 3), it is verified that the degree of freedom corresponding to the rotation φ_f is the most sensitive of the three that make up the superstructure to the order of the fitting of impedance curves (figure 2). If the temporal evolution of this degree of freedom is observed (φ_f), the use of an LPM of order M2N1 is insufficient because the response separates much more than the others from the reference one, by 4.32% (the others: M4N2 = 1.42%, M6N3 = 1.39%, M8N4 = 1.37%); the main reason why it occurs it's a bad fitting of impedance curves for this LPM order, mainly the rocking ones. On the other hand it can

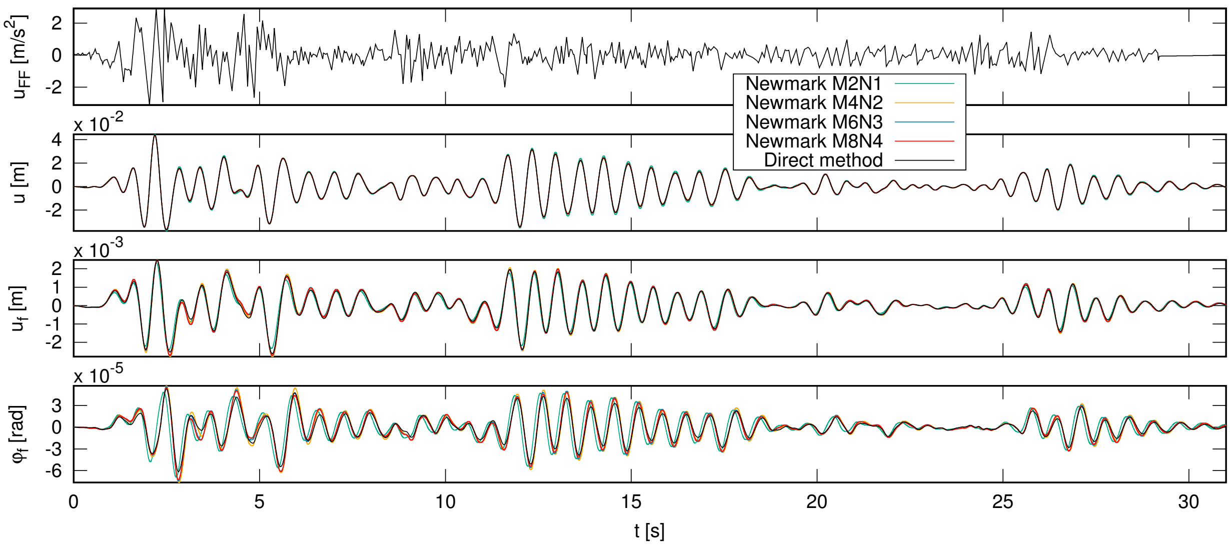

Fig. 3. Free field response of "El Centro" earthquake (top) and comparison of the response of superstructure degrees of freedom, computed by direct method in the frequency domain (reference solution) or through the step-by-step in the time domain with different orders LPM.

also be observed in a qualitative way that the response for an LPM of order M4N2 is not accurate in some maximum values of the response, but could be valid in most study cases.

Based on the above it may be concluded that for this particular case an optimal order of LPM is M6N3 because it employs the minimum amount of internal degrees of freedom and a high quality in the response is obtained. Note that same order of LPM has been used for various impedances, i.e. for the optimum LPM chosen (M6N3), there are three internal degrees of freedom for $\tilde{Z}_{xx} = k_{xx} + ia_0c_{xx}$, three for $\tilde{Z}_{x\phi} = k_{x\phi} + ia_0c_{x\phi}$, three for $\tilde{Z}_{\phi x} = k_{\phi x} + ia_0c_{\phi x}$ and three for $\tilde{Z}_{\phi\phi} = k_{\phi\phi} + ia_0c_{\phi\phi}$, which together with three of the superstructure yields a system of fifteen degrees of freedom. This wouldn't have to be like that, as different orders of LPM for different impedance curves could have been chosen.

3.2. Inelastic response of the superstructure

Finally, the use of this approach for the computation of the inelastic response of the superstructure subjected to seismic excitation is explored. The same parameters as in the previous validation section have been used, and only the optimal LPM order (M6N3) has been studied. With the aim of illustrating the use of the methodology when non-linearities arise in the superstructure, a bilinear force-deformation relationship with initial stiffness $k = 10^8$ N/m, post-yield stiffness ratio $\alpha = 0.20$ and yield deformation $u_f = 0.02$ m has been considered for the columns. Figure 4 illustrates the response obtained when the system is subjected to the "El Centro" Earthquake.

As a result of this type of elastoplastic behaviour, systems with lower yield strength yield more frequently and for long intervals, and with more yielding, the permanent deformation of the structure after the ground stops shaking tends to increase. Part of the input energy imparted to an inelastic system by an earthquake is dissipated by yielding [18]. More detailed results will be shown in future works.

4. Conclusions

In this paper, a consistent Lumped Parameter Model has been used in order to take into account the frequency-dependency of pile foundations when computing the seismic response of nonlinear piled structures in the time domain. To do so, the LPM is organized using a combination of parameters with physical meaning (masses, dampers and springs). This meaningful parameters and systematic structure of constants allows an easy control over the numerical instabilities in time domain as will be shown in future works. Validation tests in the linear range show that the numerical scheme can be used to study the problem at hand. Besides, initial tests in the non-linear range suggest that the numerical scheme is also able to handle superstructure nonlinearities. It is shown that the implemented LPM

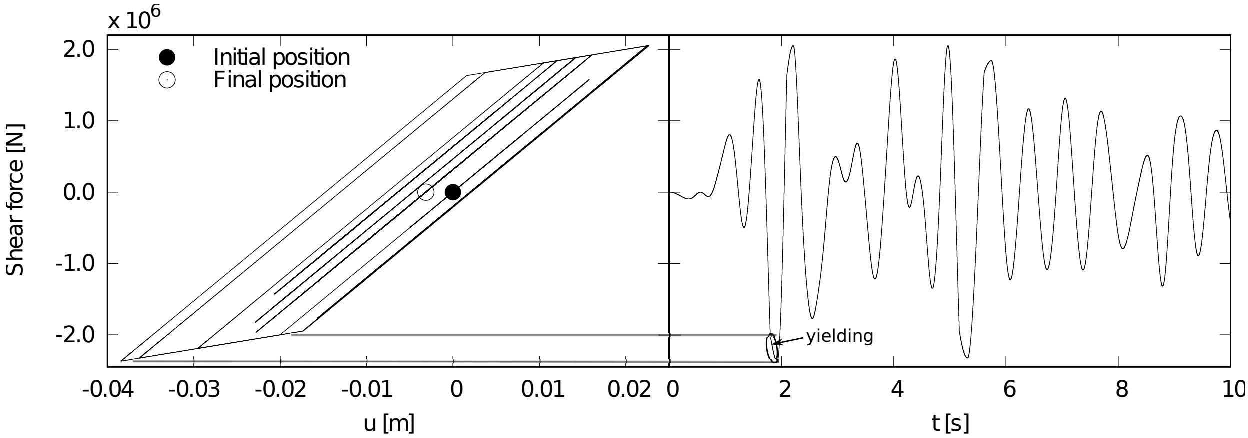

Fig. 4. Elasto-plastic behaviour of the columns for “El Centro” earthquake.

schemes are quite robust, as accurate results are obtained even with a low order scheme, and as the order increases, the accuracy increases, as might be expected. As a practical example of application, the non-linear behaviour of the columns of the superstructure has been studied following a perfect elasto-plastic model with hardening. A more detailed study of non-linear behaviour of other interesting types of structures will be shown in future works.

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