# Clustering and Competitive Balance in NBA and ACB Professional Basketball 

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#### Abstract

Some basketball leagues are more competitive than others. The level of uncertainty in the final standings is closely related to the league's appeal. A team's effectiveness has a reciprocal relationship with the emerging and critical environment: competition. Teams are affected by their surrounding environment. The competitive model directly impacts competition which means that small changes can dramatically alter the outcome. We compared two different sports models to determine the degree of hierarchy in these competitions. We studied the results of two professional basketball leagues: 18 NBA seasons (USA) and 14 ACB seasons (Spain). We found that there are three performance levels in ACB teams (ratio $0.15 \pm 0.05 ; 0.45 \pm 0.15 ; 0.8 \pm 0.1$ ). However, NBA data are less scattered and more Gaussian (peak ratio 0.5). General analysis (Shannon entropy) shows that competitive balance is not stable (mean NBA $\mathrm{Sn}=0.9842 \pm 0.0037$; mean $\mathrm{ACB} \mathrm{Sn}=0.9793 \pm 0.0053$ ). More detailed study (cluster analysis) shows that there are teams in the ACB which are clearly rooted in a particular area of the competition. Most NBA teams have reached the playoffs. There is no consensus in studying competitive balance. We propose using a number of methodologies in order to determine the competitiveness of a given league. The sports model has a significant impact on levels of competitive balance. Both the ACB and the NBA have high competitive balance. The NBA has specific mechanisms to ensure high competitiveness while the ACB does not meet the absence of long-run domination requirement.


Keywords: basketball, competitiveness, sports organisations, complex systems, NBA, ACB

## Introduction

The conditions for the emergence and maintenance of cooperation in evolving populations have been extensively studied in biological and social sciences (Guimera, Uzzi, Spiro, \& Amaral, 2005; Riolo, Cohen, \& Aselrod, 2001). Basketball is a collaboration-opposition sport. The collaboration of the players creates an emerging structure: the team. A league is a structure which emerges from the opposition between them and also selects the teams with players who collaborate. The main purpose of a sports league is to eliminate the sports gradients that are constantly being created in order to ensure energy intake and
also to be able to compete with other sports for resources (fans, sponsors, television contracts, etc.).

Competitiveness reflects the ability of teams to strive for a goal. The more balanced the competition, the greater the degree of competitiveness and vice versa. The degree of competitiveness is the level of equality of the playing strengths of the teams. Hence a higher degree of competitiveness should lead to increased demand (Goossens, 2006; Quirk \& Fort, 1997). The most competitive leagues tend to be more attractive (Szymanski, 2003) and generate more energy and material for the system (better players, better coaches, better facilities,
revenues, ticket sales, sponsors, TV, etc.), something which is closely related to the sports model (Ribeiro, Mendes, Malacarne, \& Santoro, 2010). However, when a competitor achieves a very high level of dominance competitive balance breaks down, meaning that uncertainty is significantly reduced (Goossens, 2006; Quirk \& Fort, 1997). In these situations when uncertainty of outcome diminishes, interest in the competition may be considerably reduced. This may lead to a fall in fan attendance and consequently access to energy resources may be compromised (Berri, Brook, Frick, Fenn, \& Vicente-Mayoral, 2005; Kesenne, 2010). As a result sports organisations designing sports competition models (leagues) seek to develop structures and rules that will militate against a decline in competitiveness in a championship. A certain level of competitive balance seems reasonable in order to maintain the interest of spectators and sponsors of all teams, yet determining its optimal level is extremely complex.

The sports model used directly impacts competition. Sport cannot be understood by separating out the factors in its relationship with its environment and there is a duality between the competitive model and its environment (social, cultural, economic, political, organisational, etc.). The final table is the direct result of the encounters between the teams and so their close relationship means slight alterations can significantly alter the final result (Lebed, 2006).

The US professional basketball league (the NBA) is a franchise competition. The teams are divided into two conferences (east and west) which in turn are divided into three divisions per conference with five teams each. When the regular season finishes, the top teams meet in a playoff for the title. The NBA is a closed model in which there is no promotion or relegation.

By contrast, the Spanish professional basketball league (the ACB) is an open league whose teams change each season due to promotion and relegation. The eight top-ranked teams go into the playoffs to decide which one will be the league champion. The purpose of our study is to examine the competitive balance of the ACB and the NBA and to compare two different professional basketball sports models.

## Competitive balance

Competitiveness has been investigated by a number of sports researchers (Quirk \& Fort, 1997; Yilmaz \& Chatterjee, 2000; Zimbalist, 2002; Sanderson, 2002;

Humphreys, 2002; Smith \& Stewart, 2010; Fort, 2010; de Saá Guerra et al., 2012). Competitiveness establishes relationships between teams that are predetermined (the league calendar) and not predetermined (game results, the final table). One of the most widespread ideas used to explain equality between competitors is the concept of competitive balance. This concept has frequently been used in the field of sports economics (Schmidt \& Berri, 2001; Fort \& Maxcy, 2003; Rhoads, 2004; Goossens, 2006) in order to measure the degree of competitiveness of the leagues in a range of sports including baseball (Scully, 1989; Owen, Ryan, \& Weatherston, 2007; Wenz, 2012), American football (Humphreys, 2002), basketball (Noll, 1988; Berri et al., 2005), ice hockey (Richardson, 2000), soccer (Halicioglu, 2006) and golf (Rhoads, 2005).

In general, some authors accept that a competition with high competitive balance is more attractive (Quirk \& Fort, 1997; Goossens, 2006). The more competitive the league, the more revenue it generates (ticket sales, sponsors, TV, etc.) and the more attractive it is for fans and the media (Soebbing, 2008; Ribeiro et al., 2010; Watanabe, 2012).

Cairns, Jennett and Sloane (1986) introduced the various dimensions of competitive balance. They proposed a number of uses of what they called "uncertainty of outcome" and considered four types of competitive balance. The first is the uncertainty of a particular match. The second is seasonal uncertainty which covers uncertainty within a single season. The third is the predominance of some teams over several seasons called uncertainty of championship. Fourthly and finally there is uncertainty of outcome. The absence or presence of long-run domination by one club can lead to a decrease in the interest of fans and even sponsors. This may depend on the levels of uncertainty of the season with which it is associated.

1. Match uncertainty.
2. Seasonal uncertainty.
3. Championship uncertainty.
4. Absence of long-run domination.

Szymanski (2003) also uses the same classification but only mentions the first three factors. Berri et al. (Berri et al., 2005) note that every time a competitor reaches a level of dominance, uncertainty of outcome has been compromised and demand for this industry's product is likely to decline. Some authors (Gould, 1989; Knowles, Sherony, \& Haupert, 1992; Rascher, 1999)
point out that crowds for major leagues are highest when the probability that the home team wins is about 0.6 . If the home team has a greater chance of success, the number of fans coming to the game will probably fall. Consequently, and given the importance of spectator attendance for a league's financial success, leagues are likely to implement rules and institutions to design measures that address the relative strength of the teams in competitions.

## Competition (sports model)

Sports organisations can be viewed as emerging structures which seek to regulate competition between teams. Leagues operate by creating a confrontational format (tournament, league, etc.), competition calendar, scoring and formal requirements (stadiums, stadium capacity, materials, regulations, league rules, etc.) and hence leagues can be considered as an environment.

Professional leagues, or federations in some cases, are also involved in issues that have a direct impact on games: the number of referees in a game, rules (such as rules for defending, time rules, rules for space, etc.), rules for spatial dimension, sports facilities, etc., and for other general aspects such as a salary cap, player recruitment, the draft, participating teams, designing the competition's format, the competition system (open or closed), etc. The overall trend for sports leagues should be to maintain or increase their competitive balance, and in the case of professional leagues also to maintain and improve the profitability of the companies involved.

However, these emerging structures also seek to address the appearance of gradients in sport: economic gradients, sports gradients, gradients related to players, etc. Consequently a league can be considered as a complex adaptive system consisting of multiple stakeholders who interact nonlinearly (De Saá Guerra et al., 2012; García Manso et al., 2008; McGarry, Anderson, Wallace, Hughes, \& Franks, 2002).

## Methods. Basketball analysis

In this paper we measure the competitive balance of two of the best professional basketball leagues: the ACB and the NBA. We examined 14 ACB regular seasons (1996-97 to 2009-10) and 18 NBA seasons (1992-93 to

2009-10). Most authors use a number of methodologies such as Shannon entropy, probability distributions, etc. in order to determine the dimension of equality of sports models. This approach is appropriate when two or more seasons or leagues are compared. However, if the idea is to conduct thorough analysis of the internal dynamics of a single league and then compare it with other league models we suggest using cluster analysis (as we show at the end of the results section). This analysis enables us to accurately determine the competitive balance of a league and hence improve the mechanism that enhances this process over time.

We can use a protocol to determine competitive balance by obtaining the normalised Shannon entropy value ( $S$ ), which is an average measure of uncertainty and refers to the average amount of information contained in a variable (De Saá Guerra et al., 2012). If we define balance as the situation of maximum competitiveness (maximum competitive balance), $S$ provides a numerical competitiveness value for a given season:

$$
S=\sum_{i=1}^{N}\left(p_{i} \log 1 / p_{i}\right)
$$

The value of $S$ changes along with the value of $N$, and if $p$ is the probability distribution obtained from a given result matrix $A$ for $N$ teams, we would not be able to compare different seasons in the year if the number of teams changes. Hence it is preferable to use normalised entropy (Sn):

$$
S n=\frac{S}{\log (N)}
$$

Table 1 shows the evolution of competitiveness measured by normalised entropy to compare both leagues. Thus the value of Sn is bounded between 0 and 1, where 1 is the situation where all $p$ values are equal. If we define balance as the situation of maximum competitiveness, Sn provides a numerical competitiveness value for a given season. From this standpoint, if a competition is less random then the degree of competitiveness is lower, which means we have a competition with less uncertainty about the final outcome.

This proposal discriminates well between leagues yet only provides coarse analysis of competitive balance without taking into account the league standings (the team's position in the regular season). Consequently we

| Season | ACB entropy <br> values | NBA entropy <br> values |
| :---: | :---: | :---: |
| $1992-93$ | - | 0,9838 |
| $1993-94$ | - | 0,9799 |
| $1994-95$ | - | 0,9840 |
| $1995-96$ | - | 0,9825 |
| $1996-97$ | 0,9730 | 0,9778 |
| $1997-98$ | 0,9803 | 0,9771 |
| $1998-99$ | 0,9772 | 0,9839 |
| $1999-2000$ | 0,9860 | 0,9836 |
| $2000-2001$ | 0,9737 | 0,9843 |
| $2001-02$ | 0,9756 | 0,9886 |
| $2002-03$ | 0,9789 | 0,9871 |
| $2003-04$ | 0,9871 | 0,9892 |
| $2004-05$ | 0,9782 | 0,9851 |
| $2005-06$ | 0,9874 | 0,9894 |
| $2006-07$ | 0,9849 | 0,9902 |
| $2007-08$ | 0,9809 | 0,9828 |
| $2008-09$ | 0,9723 | 0,9824 |
| $2009-10$ | 0,9744 | 0,9834 |

Table 1. Normalised Shannon entropy values for the ACB and NBA leagues
compared the values of the probability distributions. The vector $R$ (vector score) represents the results obtained by every single team in each season analysed. Historical values of $R$ or for previous seasons divided by the sum of all the games can be viewed as a discrete probability distribution:

$$
p_{i}=\frac{R_{i}}{\sum_{j=1}^{N} R_{j}}
$$

If the distribution is uniform, all $p_{i}$ values are equal or similar, which means that all the teams have about the same chance of winning. This is the case in which it is most difficult to predict the final outcome and can be considered the one with highest competitive balance. In terms of statistical mechanics, these distributions are related to balance situations in which all structures and gradients have been removed.

By contrast, if there are pi values higher than the rest, this means some teams perform better than others. This shows the relevance of what we call 'sports gradients':

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Figure 1. Histogram of win probability (games won vs. games played). The histogram shows the ratio values for the entire $A C B$ and NBA sample. The asymmetry in the distribution is around the 0.4 ratio in the ACB and 0.5 in wins vs. games played in the NBA
differences between teams (budgets, player quality, organisational structure, etc.) which set the internal dynamics of the league.

## Results and discussion

The win probabilities of both leagues are not uniform as can be seen in the figure 1.

Both leagues appear to follow a similar pattern. They both have a peak ratio value and some marked differences in terms of probability values. These differences indicate the competitive balance in the league. Competitiveness, like the probability of winning, is not uniform and


Figure 2. Box plot of the ratio of all wins of each team and the number of games played (wins/games played) in the sample. In the top graph (ACB results) there are approximately three clusters or three areas. NBA teams (bottom graph) seem to have values that are more similar to each other. This suggests a higher degree of competitiveness, as virtually any team can achieve a high performance level
there appear to be some values which are more common than others. However, the trend in both leagues is located around a mean value, which is good for the balance of the competition since it provides an appropriate degree of uncertainty. It should be borne in mind that a very high or very low probability value is associated with a fall in fan attendance and a decline in the league's appeal.

Figure 2 shows the box plot of the $R$ result of all participating teams through the seasons analysed in normalised values (wins/games played).

Figure 2 shows that the ACB data appear to be nested by performance level. There are about three areas. In the first, with the highest performance level, the data (data cloud, means, interquartile ranges and confidence intervals) for the top three teams are clearly above the rest. They compete for first place in the table (very high degree of competitiveness). The same applies to the second cluster in which the next four teams have a similar performance level (very high competitiveness). The teams in the middle sector (third cluster) also compete with


Figure 3. $A C B$ and NBA means and medians. Clusters are clearly visible in the ACB while the trend in the NBA is much more compact
each other. In addition they can be viewed as "transition teams" because their performance puts them in a boundary position between the other two performance regions. In the bottom part of this cluster the data have little statistical value as this area experiences more change due to promotion and relegation.

The NBA data show much more homogeneous behaviour. Most of the data cloud and the medians are around the mean ratio values, indicating high competitive balance. Sometimes teams reach values which are unusually high (highly established teams) or low (teams which are not well established) and some of them have significant data scattering, suggesting that they are teams with good sports results and a decline in performance or vice versa.

Figure 3 shows the means and medians of all the ACB and NBA values by way of clarification of the above.

In the ACB groups can be clearly distinguished based on their performance by clustering the means and medians. The data cover a broader segment of the ratio ( 0.75 to 0.25 approximately). By contrast, the NBA does not present these clusters and has a smaller dispersion range (approximately 0.70 to 0.35 ).

In order to find out whether teams are grouped by their performance, we carried out non-hierarchical partition assignment cluster analysis (k-means analysis) (Figure 4) which places the points on a plane for clustering. These points are assigned to the group that is closest to their centroid. This is a cluster analysis method which aims to partition $n$ observations into $k$ clusters in which each observation belongs to the cluster with the nearest mean. This results in a partitioning of the data space into Voronoi cells.

A Voronoi diagram is a special kind of partitioning of a plane based on the distance to points in a specific subset of the plane. In other words, it partitions the plane into as many regions as there are points, so that for each point there is a corresponding region consisting of all points closer to it than to any other.

In the ACB cluster (Figure 4) there are up to five regions clearly determined by performance level. The centroids are located hierarchically, indicating stratification. The two lower regions (* and $\nabla$ ) clearly consist of teams that once achieved good results but for the most part visibly belong to these regions. The other region that is clearly separated from the rest is the one marked by • and the teams in this area are markedly better than the rest.. Teams in the $\square$ and + areas are transition teams because they sometimes achieve better or worse results than the mean results for their region. Hence these two regions could be viewed as a single region in terms of the behaviour of the teams it contains.

The NBA cluster (Figure 4) contains six regions. The +, $\nabla$, and * regions show a similar level of performance but it is unclear which teams belong to each one. This might mean that after a poor season NBA teams can be competitive in the following one as a result of their internal mechanisms, while teams with good results have to restructure their roster season after season in order to maintain them. Indeed, some teams sometimes achieve very good results ( $\bullet$ and $\square$ ) and not such good ones in other seasons (some of them with a very marked data spread). This might suggest dynasties; for example, as long as Michael Jordan was with the Chicago Bulls the


Figure 4. $A C B$ and NBA cluster analysis. The upper panel shows $A C B$ clustering. There are five regions which are clearly related with the team's performance. There are some teams which are clearly located in one region (•, $\nabla$ and * areas) and occasionally achieve a different result. In other words, they undoubtedly belong to a region. Teams in $\square$ and + areas can be considered transition teams because they sometimes achieve better or worse results than in other seasons. We might even consider these regions to be a single region with respect to the behaviour of the teams in it. The lower panel shows NBA clustering. It is completely different to the ACB. The results indicate four regions (+, $\nabla$, and *) with similar performance. This means that a team can be on top in one or more seasons and in the following seasons in a lower position or vice versa. Moreover, there is an elite located in its own region due to its results ( $\bullet$ ), and other teams seldom manage to achieve these positions ( $\square$ ).
team was successful, but after he retired the Bulls suffered a run of bad results.

## ACB

The ACB has a hierarchical structure. Its teams are clustered by themselves in relation to their performance level (Figure 4) and this creates frequency barriers (win
frequency) for the less powerful teams. The ACB's peak is around a 0.40 ratio (Figures 1 and 2). Teams below this point are very erratic and cannot achieve the performance level required to compete in the middle area of the league table. This aspect seems to function as a barrier, where this means a significantly higher frequency. It is noteworthy that most teams are in the intermediate regions (Figures 2, 3 and 4) and only a few are beyond the
second barrier ( 0.80 ratio), which could be considered the area with greatest competition.

The teams above the barriers are always the same save on rare occasions. Hence the highly competitive area is always occupied by the same teams (Figure 4). In other words, the teams are clustered by their performance level and they have to overcome certain barriers if they want to achieve higher performance levels.

The best outcomes in the ACB are indeed attained by teams which are well-established in this competition and with high performance in European leagues. The lowest outcomes in the ACB data correspond to teams which were poorly established in the seasons we examined (Figures 2, 3 and 4).

These different performance regions (Figure 4) could be the result of the ACB's competition model of an open league in which teams change as a result of promotion and relegation (from or to a lower division) and where the eight top-ranked teams go into the playoffs. Teams build their rosters based on their budgets, and the latter are almost always based on the outcomes achieved. The higher the budget, the better the players, coaches and staff they can sign or vice versa. In principle, newly promoted teams have less competitive rosters and also tighter budgets.

Due to its open structure, some teams (and their underlying structures, such as economic networks, executive committee, players, youth academies, etc.) become more experienced. The existence of these teams has an impact on the others and especially the less experienced teams. Consequently the teams placed at each end are closely related: if the differences between the lowest ranked teams and top teams are very large, it may be that the advantage at the top is more evident, since there is a high probability that the top ranked teams will defeat the bottom teams. This means that the best teams can improve their chances of winning.

It can be inferred that there is a different level of criticality for each area. This sports potential gradient is maintained by energy (players, coaches, money, etc.) which means that the performance differences of some ACB teams are insurmountable, especially for newly promoted ones whose budget and rosters are tight. The sports model greatly influences the market.

The fact that teams tend to cluster in areas is not random but rather the result of something called preferential attachment (Barabási \& Albert, 1999), also called the Matthew effect (Bunge, 2001; García Manso \& Martín González, 2008), by which the strong teams achieve more successes and the weaker teams become less wealthy Another mechanism that causes this behaviour is the
"memory effect", what systems have. Teams are tied to an attractor, such as some areas of the table (Figure 4).

Other explanations for these differences might be the teams' sports planning for each season, their roster, budget, external competitions (European League, King's Cup, tournaments or players turning out for national teams), etc., all of which can have a substantial impact.

## NBA

In general the NBA has a greater degree of uncertainty than the ACB (de Saá Guerra et al., 2012) and its structure and dynamics are completely different (Figure 4). Figure 1 shows that most of the data are near the 0.5 ratio and how the teams are scattered across several regions (Figure 4).

There is no relegation or promotion. In fact, the worst outcomes (ratio <0.15) (Figures 1 and 2) have an advantage for the next season as these teams get higher picks in the NBA draft, which means they can strengthen their rosters.

Achieving the best results is also very unlikely (Figures 1, 2 and 4). NBA seasons are very long (82 games) and playoff classification is extremely hard fought. Attaining a win ratio higher than 0.70 is rare. The most probable outcome is that most teams are in the middle areas (Figures 1,2 and 4 ); in other words reaching values above 0.70 or lower than 0.25 is unlikely for most of the teams.

A team can be fighting for playoff positions in one season and then be much lower in the table in the following one (Figures 1, 2 and 4). However, almost all teams have similar performance levels (Figure 4). These are the possible reasons why the cluster analysis shows similar regions in which the teams in them change from one season to another.

The existence of this performance dynamic could also be due to the sports model used by the NBA. There are more teams in the NBA than in the ACB (30 vs. 18) and they play many more games (82 vs. 34). Furthermore, the competitive structure is the diametric opposite. The NBA has also mechanisms designed to prevent monopoly by a team (the draft, salary cap, reserve clause, etc.) whose purpose is to ensure competitive balance. Hence it may be that the most critical parts of the competition are at the two ends, because these are the areas which reward the teams in them (playoff and draft). Perhaps because of its competitive dynamics, the NBA is a good example of the Red Queen principle proposed by Van Valen (1973): "For an evolutionary system, continuing development is
needed just to maintain its fitness relative to the systems it is co-evolving with." In other words, an endless race in which all competitors need to improve just to continue competing.

## Comparison

The ACB and the NBA seem to behave in opposite ways. In the ACB, the most competitive area is the middle ratio area (lower differences) and $\square$ and + regions (Figure 4). In the NBA, the most competitive area is the top and the bottom of the table. This explains why teams are so scattered in the cluster analysis (Figure 4). It should be stressed that both leagues are very competitive but complete opposites: the ACB is an open model in which the bottom teams are relegated which leads to a high level of competitiveness, whereas in the NBA the goal is to qualify for the title playoffs or get a good spot in the draft lottery.

In the ACB the teams are clustered around their level of performance. There are teams which are clearly positioned in a particular area of the competition, and this might indicate the level of competitiveness of the team. The first four positions are occupied almost entirely by the same three teams and occasionally a team has been able to break into this elite group (Figures 2 and 4). There is a similar pattern in the playoff spots (the top eight) which clearly cover the data cloud and confidence intervals of several teams in this area (Figure 2). The bottom positions are the most atypical, since the last two teams drop down to a lower league and are replaced by two different teams. Newly promoted teams do not in principle have the same level of performance as the teams in the middle area.

In the NBA almost all the teams have reached the playoff positions although there are some teams that do so more often (Figures 3 and 4). Their data are less scattered in the cluster analysis (Figure 4) and they are more firmly established in this area. Virtually all teams have made it into the top five of the table.

At the same time, and notwithstanding the chaotic behaviour of the competition, the teams always tend towards an attractor (team clustering). Hence competitiveness can be seen as an attractor in itself.

## Conclusions

There is no consensus about certain aspects of competitive balance theory. Accordingly we propose using
a number of methods to determine the degree of competitiveness of a given league. Some simple techniques such as box plots can give us clues about the level of competitive balance. In this case the ACB has some features that lead to less competitive balance. This is related to its open model which fosters the domination of the highly established teams. By contrast the NBA has put in place mechanisms to avoid this situation. Their purpose is to preserve the uncertainty of the championship which makes it more attractive.

At all events, both the ACB and the NBA are very competitive leagues with high competitive balance. The sports model has a major impact on levels of competitive balance. The fact that the ACB is an open league means the less powerful teams make it less competitive as a whole. We need to devise strategies to maintain or even increase the league's level of global competitiveness just as the NBA does. In spite of these issues, the Spanish ACB basketball league can be considered a very competitive league.

As noted above, the NBA has specific mechanisms in place to ensure high competitiveness, such as the draft, the salary cap, reserve clause, etc. Their purpose is to preserve the competitive balance of the competition. It is a league with a high degree of uncertainty about the final outcome, and consequently all teams have a real chance of qualifying for the playoffs.

In short, the ACB does not meet the fourth item described by Cairns, Jennett and Sloane (1986), the absence of long-run domination, while the NBA does.

## Conflict of interests

The authors declares that there is no conflict of interests.

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