ANALYSIS OF IRREGULAR MICROSTRIP STRUCTURES USING A FULL WAVE MoM SCHEME *

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Abstract

This article presents the results obtained in the analysis of irregular microstrip structures using a full wave method of moments scheme. The irregular microstrip structures are divided into rectangular subdomains. The EFIE is discretized an solved over the subdomains using a Galerkin type scheme. Base and weight functions are piece wise sinusoidals (PWS) or triangular. Delta gap voltage generators are used as sources [3]. Green functions are computed using a freely available library developed by our research group [4]. All the calculations are carried out in the so called ”spatial domain” so there is no need of using regular grids during the discretization process. Usefull information for engineers such us scattering parameters, field distributions and so on is extracted from the computed current distribution. Some results are presented showing predicted and measured data. The prototypes (hybrids, meander lines and patch antennae) has been built and measured with our ”in house” equipment. Other results are presented and compared with those available from other authors. Sure, there is no ”rocket science” nor ”state of the art” in our research, but we want to remark that our code, as that in [4], is freely available for everybody. Everybody is invited to get it, to test it and, of course, to improve it.

INTRODUCTION

The moment method is a numerical method to compute the current distribution of any arbitrary microstrip structure. This is accomplished transforming the EFIE ’Electrical Field Integral Equation’ (1) into a linear algebraic equations system.

$$\overline{E}^i + j\omega\mu_0 L(\overline{J}_s) = 0 \tag{1}$$

Here L is a known linear (integral) operator, $\overline{E}^i$ is the incident field in the boundary and $\overline{J}_s$ is the surface current density to compute. The method of the moments solves this equation dividing the structure into M different entities called *patches*. Each $patch$ is composed of four rectangular subdomains. The current distribution is expanded as a linear combination of two-dimensional *basis* functions which are defined over each subdomain.

$$\overline{J}_s = \sum_{i=1}^{M} I_{x_i} J_{x_i}(x', y')\hat{x} + \sum_{i=1}^{M} I_{y_i} J_{y_i}(x', y')\hat{y} \tag{2}$$

The *basis* function can be decomposed into two indepent variable functions $J_{x_i}(x', y') = T_i(x')Q_i(y')$. The longitudinal component has a piecewise sinusoidal or triangular behaviour (3) while the transversal component has a constant distribution (4). This function is an arch shape one like showed in the figure 1. The descomposition of $J_{y_i}(x', y')$ is similar.

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$$T_i(x') = \begin{cases} \dfrac{\sin[k_e(l_{1i} + x' - x_i)]}{\sin(k_e l_{1i})} & \text{si } x_i - l_{1i} \leq x' \leq x_i \\ \dfrac{\sin[k_e(l_{2i} - x' + x_i)]}{\sin(k_e l_{2i})} & \text{si } x_i \leq x' \leq x_i + l_{2i} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

$$Q_i(y') = \begin{cases} \dfrac{1}{w_{1i} + w_{2i}} & \text{si } y_i - w_{1i} \leq y' \leq y_i + w_{2i} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

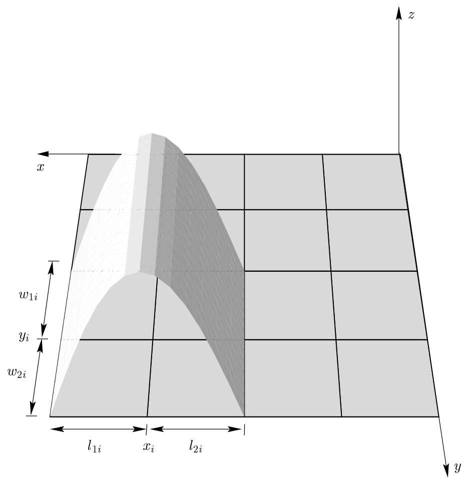


Figure 1: Arch shape in the structure.

Here k_e is a propagation effective constant which value is usually taken as $k_e = \frac{2\pi}{\lambda_0 \sqrt{\frac{\epsilon_r + 1}{2}}}$.

The EFIE has two scalar functional equations that have M unknowns each one. These equations aren't suficient to determine the $2M$ unknokn constants $[I_{x_i}, I_{y_i}]_{i=1,2,\ldots,M}$. In order to solve the $2M$ constants, it is necesary to compute $2M$ linearly independent equations. This can be accomplished by evaluating the EFIE at M different points (Point Matching). That's the same thing that weighting the EFIE using Dirac delta functions. However, to improve the solution and minimize the residual, the *weight* and *basis* functions are the same. This technique is known as Galerkin's method [1]. The final system of equations has $2M$ equations and 2M unknowns, where $1 \leq i, j \leq M$ and can be expressed:

$$\begin{pmatrix} Z_{xx}^{ij} & Z_{xy}^{ij} \\ Z_{yx}^{ij} & Z_{yy}^{ij} \end{pmatrix}_{2M \times 2M} \begin{pmatrix} I_x^i \\ I_y^i \end{pmatrix}_{2M \times 1} = \begin{pmatrix} V_x^j \\ V_y^j \end{pmatrix}_{2M \times 1} \qquad \begin{matrix} i = 1..M \\ j = 1..M \end{matrix} \tag{5}$$

The elements of the matrix Z_{xx}^{ij} and Z_{yy}^{ij} are the autoimpedances due to the couplings of the equal current densities. Z_{xy}^{ij} and Z_{yx}^{ij} are the couplings due to distinct components. The elements of the matrix $[Z]$ are composed by the sum of two integrals (integral 1 and integral 2) like following [2]:

$$\int_{x_i - l_{1i}}^{x_j + l_{2j}} dx' \int_{x_j - l_{1j}}^{x_j + l_{2j}} dx \int_{y_i - w_{1i}}^{y_i + w_{2i}} dy' \int_{y_j - w_{1i}}^{y_j + w_{2j}} F_i(x', y')\, F_j(x, y)\, G_\alpha(x - x', y - y') dy \tag{6}$$

where G_α are Sommerfeld Integrals with different expressions for each integral. The expression and computation of these integrals are collected in [4], where $F_i(x', y')$ and $F_j(x, y)$ are *basis* and *weight* functions. The table 1 shows the value of these functions for each integral (defined over rectangular shapes).

	Integral 1		Integral 2	
	$F_i(x',y')$	$F_j(x,y)$	$F_i(x',y')$	$F_j(x,y)$
Z_{xx}^{ij}	$T_i(x')Q_i(y')$	$T_j(x)Q_j(y)$	$\frac{\partial T_i(x')}{\partial x'}Q_i(y')$	$\frac{\partial T_j(x)}{\partial x}Q_j(y)$
Z_{xy}	$-$	$-$	$Q_i(x')\frac{\partial T_i(y')}{\partial y'}$	$\frac{\partial T_j(x)}{\partial x}Q_j(y)$
Z_{yx}^{ij}	$-$	$-$	$\frac{\partial T_i(x')}{\partial x'}Q_i(y')$	$Q_j(x)\frac{\partial T_j(y)}{\partial y}$
Z_{yy}^{ij}	$Q_i(x')T_i(y')$	$Q_j(x)T_j(y)$	$Q_i(x')\frac{\partial T_i(y')}{\partial y'}$	$Q_j(x)\frac{\partial T_j(y)}{\partial y}$

Table 1: $[Z]$ functions.

Since the functions F_i and F_j are defined within patches, the coupling between the patch i and j has to be computed by parts. This computation will be accomplished through the sum of the coupling of each one of the subdomains in which is divided the patch i, with each one of the subdomains of the patch j.

NUMERICAL INTEGRATION

The $[Z]$ terms are quadruple integrals that can be calculated directly through a Gauss-Laguerre quadrature but their cost would be large. In order to minimize this cost, a domain change transforming (6) to a double sum of double integrals is accomplished. Using the transformation of (7) and other similar for the domain (y, y'), the computation time reduces considerably.

$$\begin{aligned} u &= \frac{1}{\sqrt{2}}(x - x') \\ v &= \frac{1}{\sqrt{2}}(x + x') \end{aligned} \tag{7}$$

Each cuadruple integral defined by parts is converted in the sum of nine two fold integrals. The total integral has the following expression:

$$\begin{aligned} &\int_{x_i-l_{1i}}^{x_j+l_{2j}} dx' \int_{x_j-l_{1j}}^{x_j+l_{2j}} dx \int_{y_i-w_{1i}}^{y_i+w_{2i}} dy' \int_{y_j-w_{1j}}^{y_j+w_{2j}} F_i(x',y')\ F_j(x,y)\ G_\alpha(x-x',y-y')dy \\ &= \sum_{L=1}^{12}\sum_{K=1}^{12}\int_{ULI_L}^{ULS_L} du \int_{VLI_K}^{VLS_K} f_{TT_L}(u)\cdot f_{TT_K}(v)\cdot G_\alpha(\sqrt{2}u + x_j - x_i, \sqrt{2}v + y_j - y_i)dv \end{aligned} \tag{8}$$

When the expression (8) has been obtained and their limits calculated, the elements of the matrix $[Z]$ are calculated numerically. The functions $f_{TT}(u), f_{TT}(v)$ are the $basis$ and $weight$ functions in the transformed domain.

EXCITATION MODEL

The excitation model used in this article is the delta-gap model [3]. This model assumes that the voltage source V_t^m is within an infinitesimally small gap and across the extended ground plane. For this model is used a physical port within a $patch$ o several $patchs$. The impressed field originated is:

$$\overline{E}^i(\bar{\rho}) = V_m^t \delta(\bar{\rho} - \bar{\rho}_m)\hat{n}_m \tag{9}$$

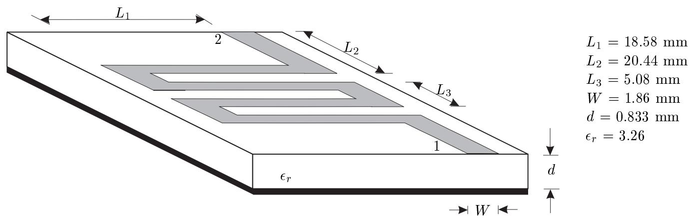
where $\bar{\rho}_m$ is the location of the port and $\hat{n}_m$ is a normal vector parallel to the feed-line. This voltage source originates induced currents which are modeled through half-basis function located at $\bar{\rho}_m$. Then, the $patchs$ that have the ports are composed of 2 physical rectangular subdomains and 2 of zero length. The choice of subdomains will depend of the position of the excitation.

The V_j coefficients are normalized to the unity value in these $patchs$ and zero on the rest. The total result is the linear superposition of each one of the excitations. It has supossed that a port is modeled through the discretation of a $patch$. However, the structure can be splitted in several $patches$ in function of the reability, the physical port can be discreted in various segments so several logic ports are conected in parallel to model a physical port.

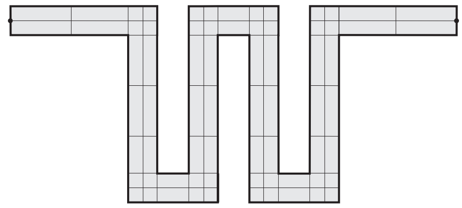
RESULTS

Several structures such as hybrids, meander lines and patch antennae have been simulated. For each one of these structures the full wave analysis is checked against measured results. In this article, the results of a meander with a 2U shape are showed.

The dimensions are showed in the figure 2(a) with a discretization of 32 *patches*. The simulated results are very closed to the measured ones though at high frecuencies there are little diferences. Measured and predicted results are in figures 3(a) and 3(b).

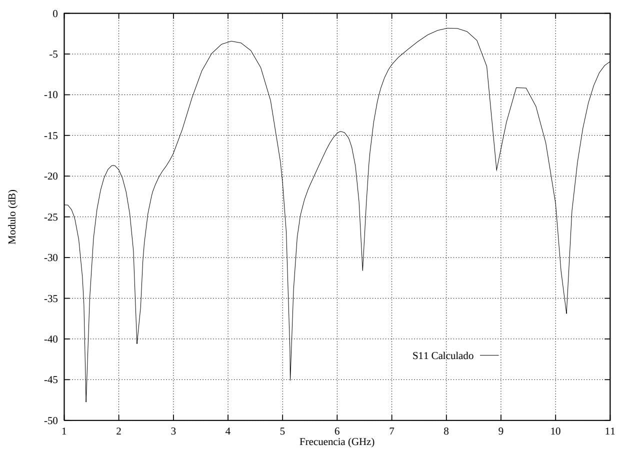


(a) meander dimensions.

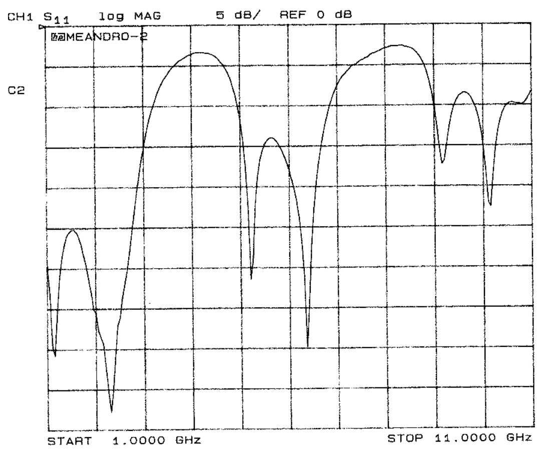


(b) meander layout.

Figure 2: Meander geometry.



(a) meander calculated.



(b) meander measurements.

Figure 3: Meander results.

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