Surface insertion in a tetrahedral mesh using the Meccano method

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CONACYT-SENER Project, Fondo Sectorial, contract: 163723

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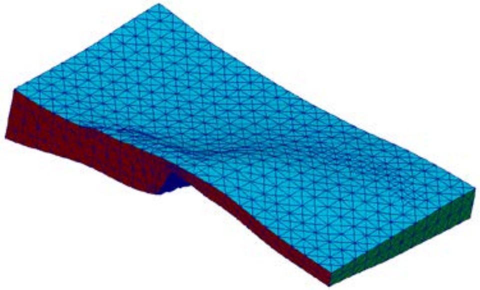
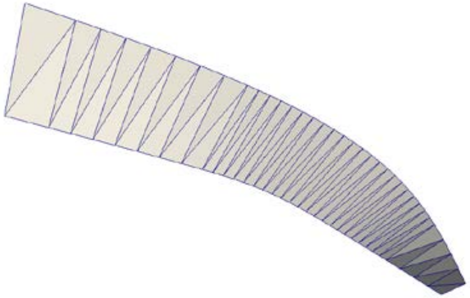
Motivation

Immerse surface insertion in tetrahedral mesh

- **Input data**
  - Tetrahedral mesh of a solid generated using the Meccano method
  - immerse surface description

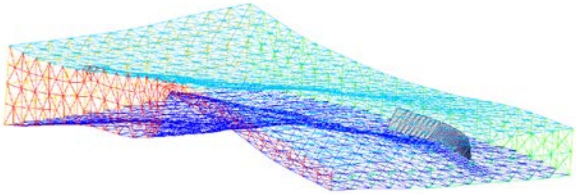


Volume tetrahedral mesh



Surface triangulation

- **Requirements**
  - Smooth approximation
  - The triangles of the approximation are mesh faces.



immersed surface

Meccano Method for complex solids

Simultaneous mesh generation and volumetric parameterization

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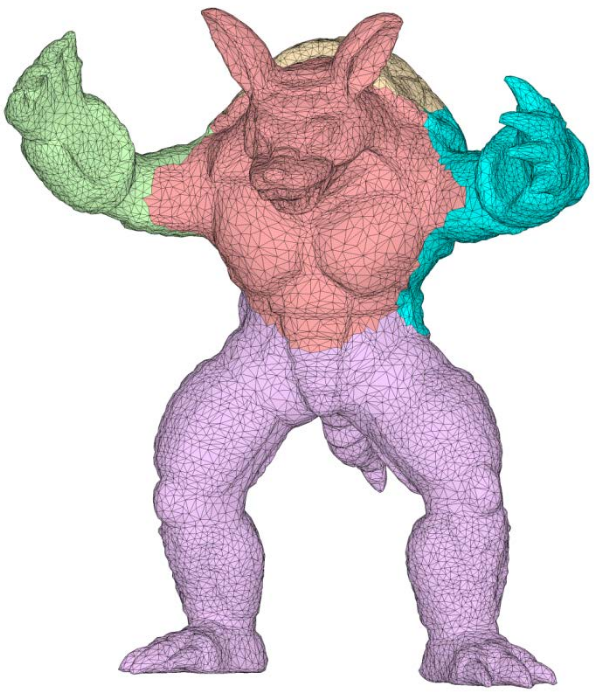
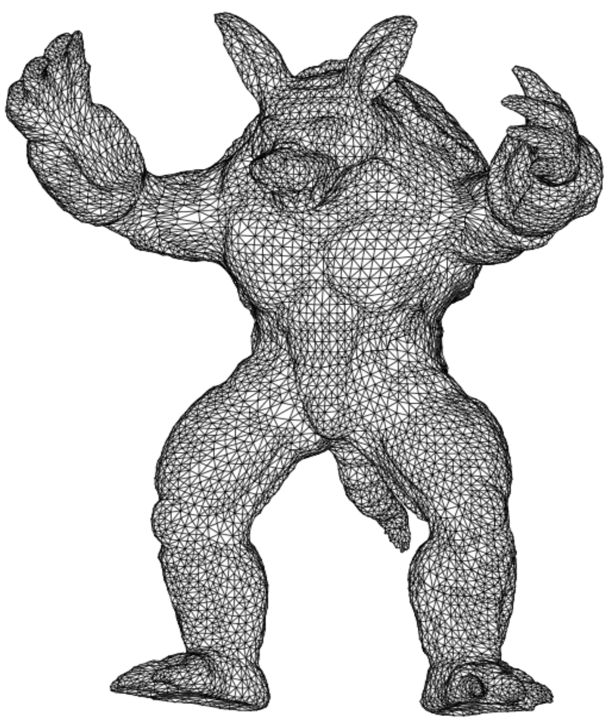
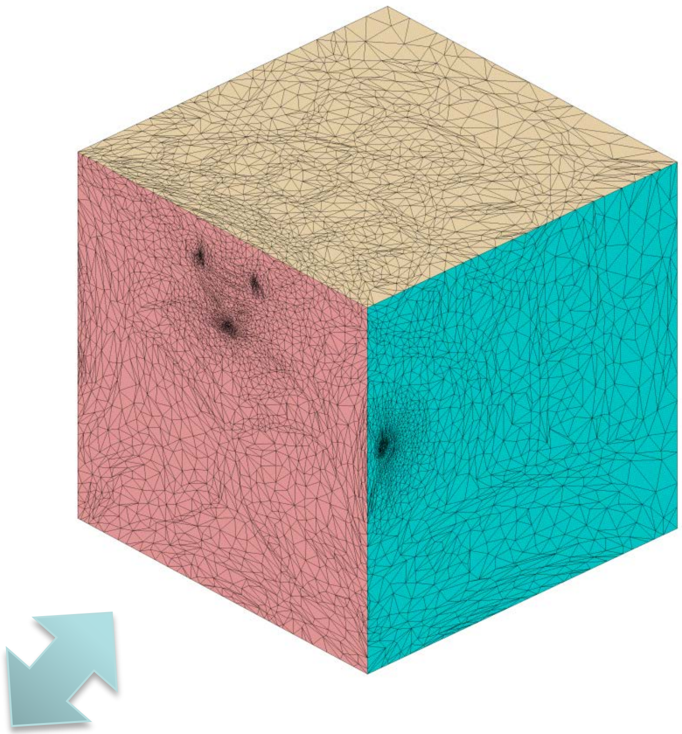


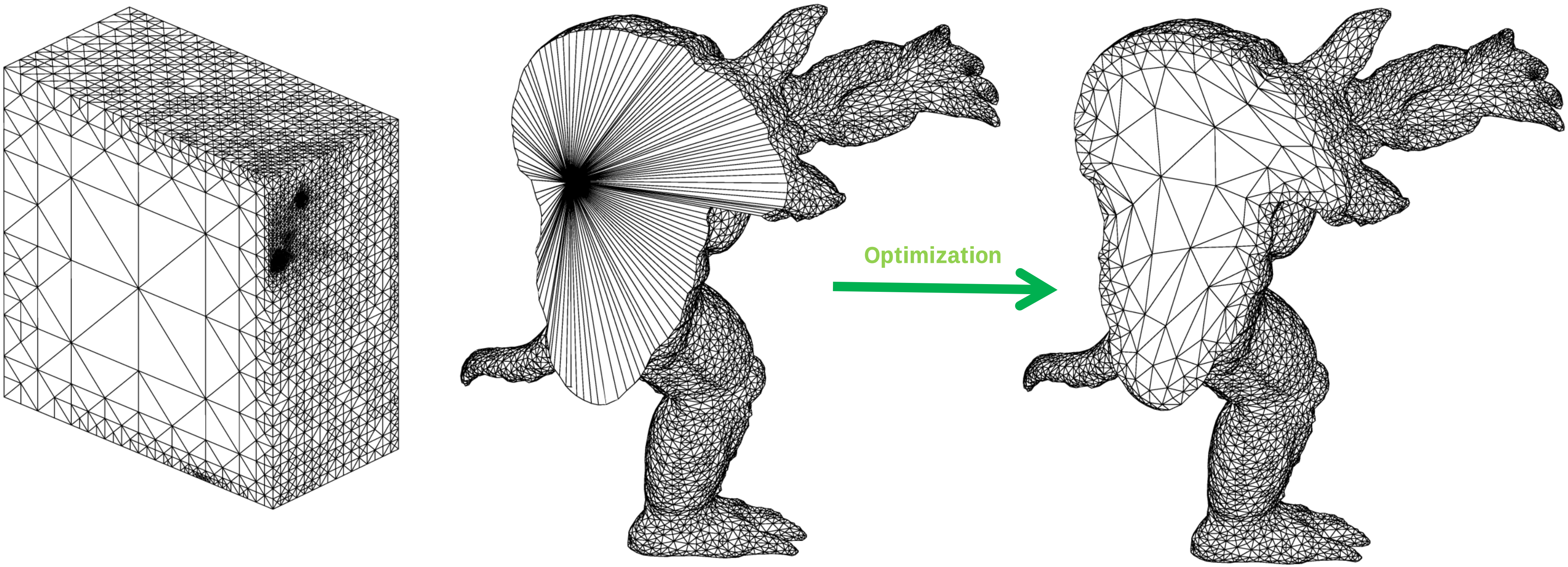




- Parameterization
- Refinement
- Untangling & Smoothing

Meccano Method for complex solids

Mesh generation. SUS of tetrahedral meshes



Parametric space
(meccano mesh)

Physical space
(tangled mesh)

Physical space
(optimized mesh)

Meccano Method for complex solids

Simultaneous mesh generation and volumetric parameterization

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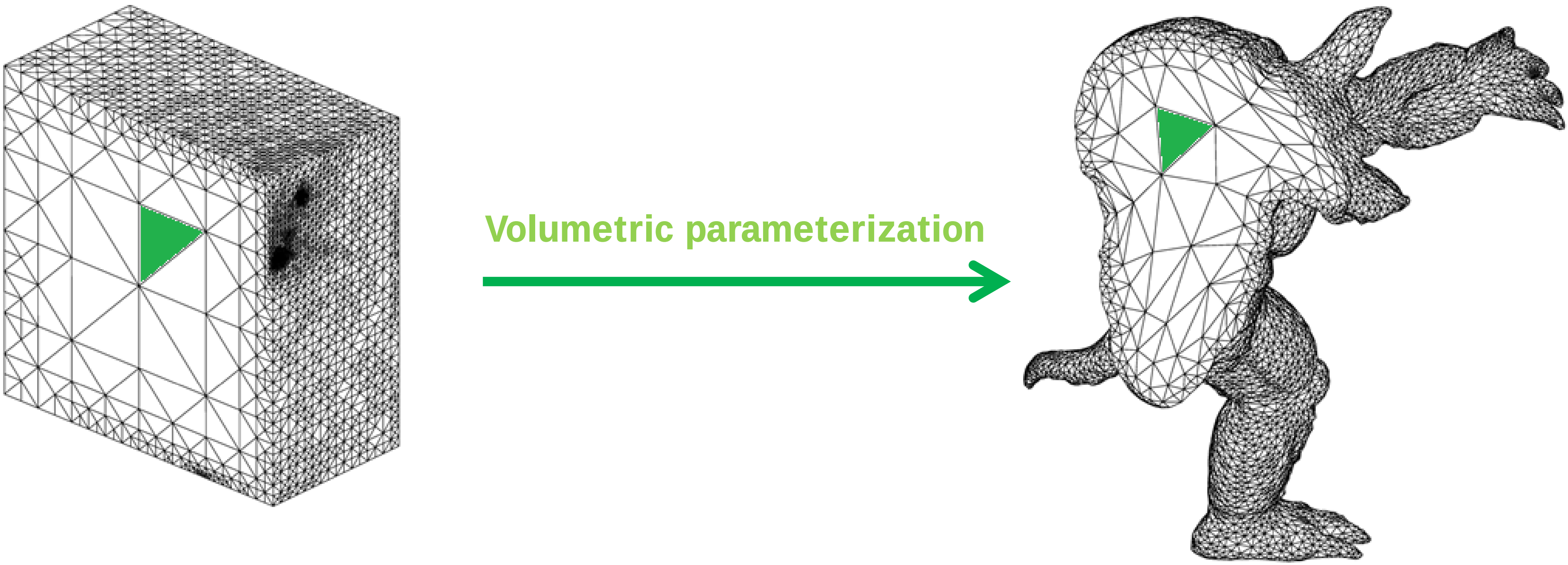


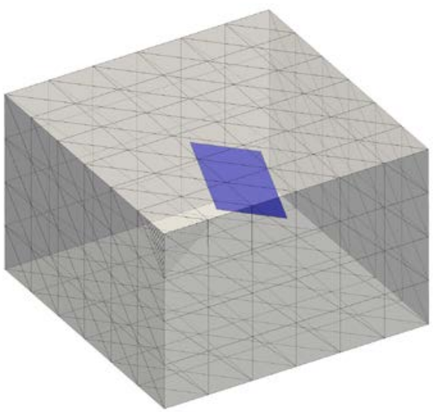
Parametric space
(meccano mesh)

Physical space
(final mesh)

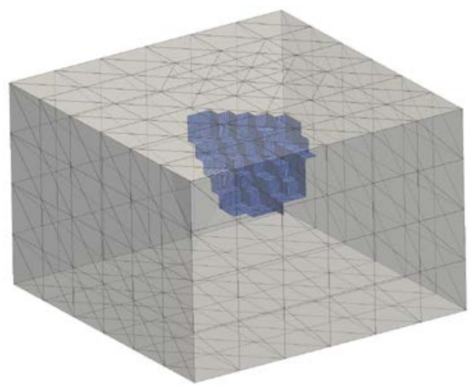
Algorithm steps

Three stages: approximation, parameterization and smoothing

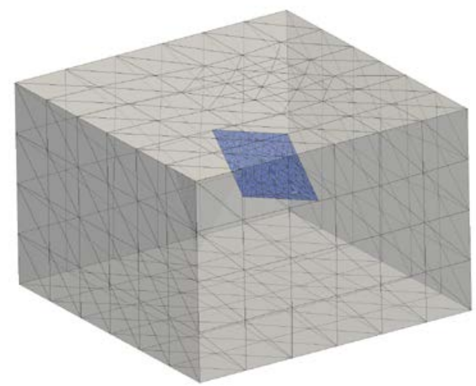
- **Initial approximation**: Mesh-surface intersection → Volumetric Approximation → Surface approximation
- **Parameterization**: Surface parameterization → Approximation parameterization
- **Smoothing**: Projection → SUS



Real surface
(input data)



Initial rough
approximation



Surface projection
and smoothing

Surface initial approximation

Volumetric approximation

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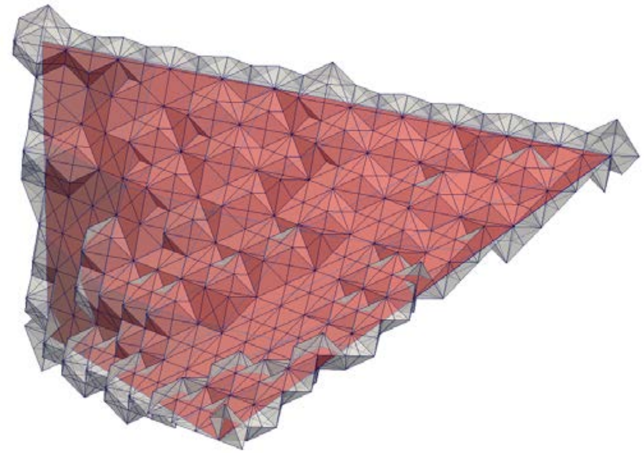
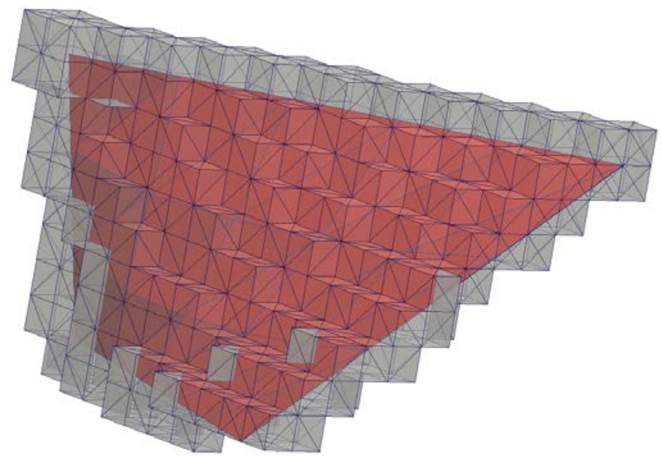
- **Calculate mesh-surface intersection**
  - **Encapsule the immerse surface**
  - **Adaptive Kossaczký refinement with a prescribed max size for cells intersecting the surface**



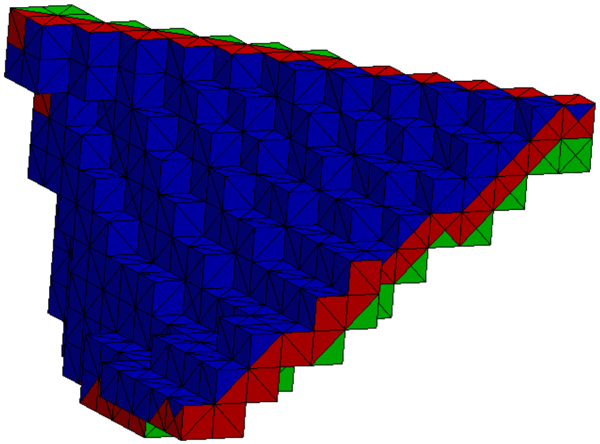
- **Repair volumetric approximation**
  - **Volume topologically equivalent to 3D ball (spherical volume)**
  - **Boundary faces parallel to coordinate surfaces in parametric domain**



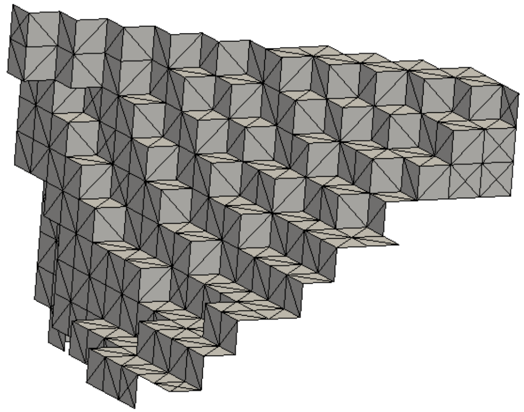
Surface initial approximation

Surface approximation

- **Mark boundary faces of volumetric aproximation**
  - **Local expansion of immerse surface defines two regions**
  - **Consider triangles with nodes in the same (arbitrary) region**



- **Repair surface approximation**
  - **Surface topologically equivalent to a 2D ball (disc).**
  - **Boundary edges parallel to coordinate axis in parametric domain**



Mandatory properties to get a valid surface approximation

- **Property 0: Surface approximation is an orientable, conex mesh topologically equivalent to a disc**

- **Property 1: If all the nodes of an element (tetrahedron, triangle or edge) lie in the surface approximation, then the element belongs to the surface approximation**

- **Property 2: If all the nodes of an element (triangle or edge) lie in the surface approximation boundary, then the element belongs to the surface approximation boundary**

- **Property 3: If any of the nodes of a triangle, but not all of them, lie on the surface approximation, then at least one node of the triangle must be an inner node of the tetrahedral mesh**

Topological and Geometric Properties

Repairing the initial surface approximation.

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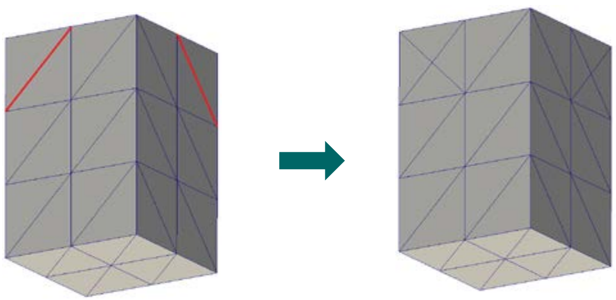
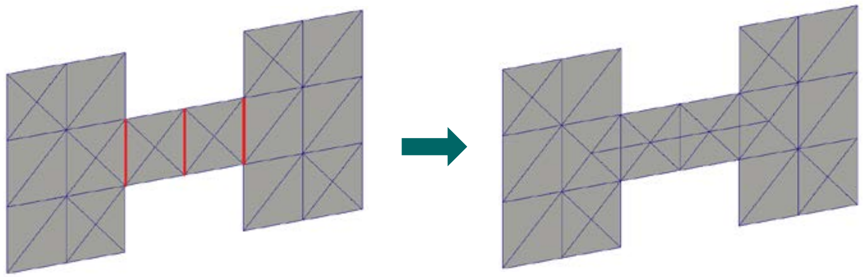
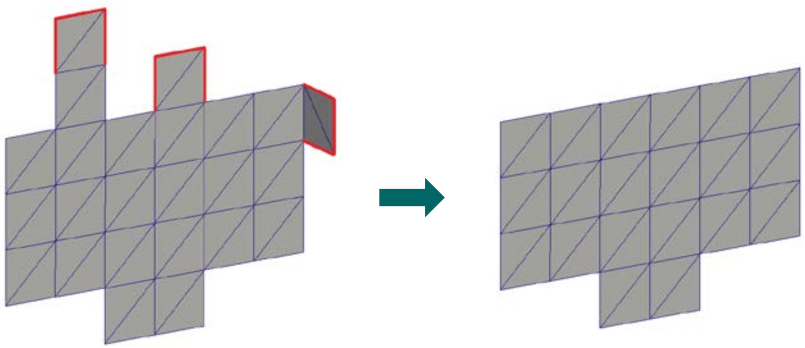
- **Avoid unsolvable cell / face collapse**
  - At most one face in approximation per cell
  - At most one edge at boundary per face in the approximation



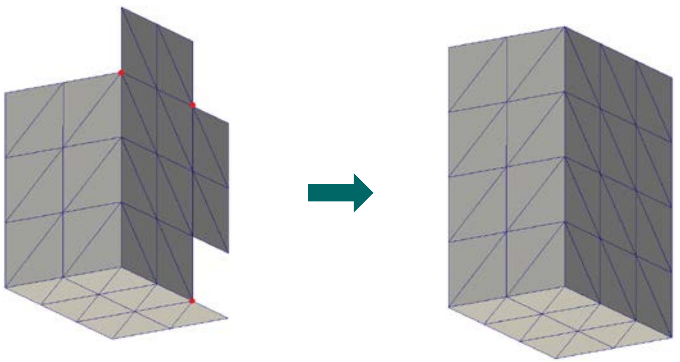
- **Erase dividing edges**



- **Erode corbels**

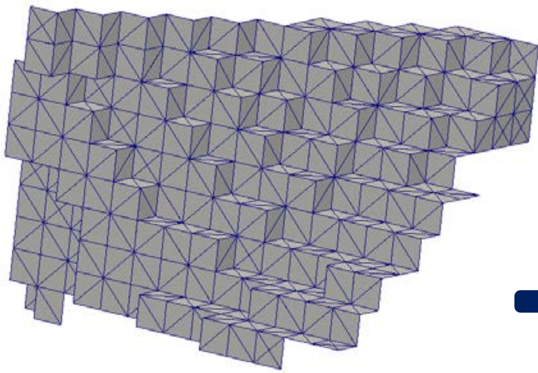


- **Control connectivity of boundary nodes**

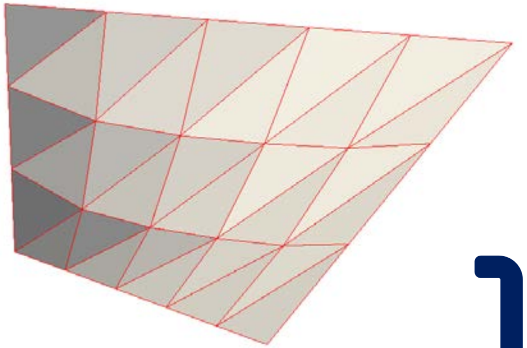


Surface parameterizations

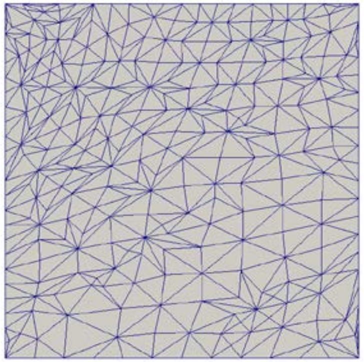
Parameterization of actual surface and its approximation



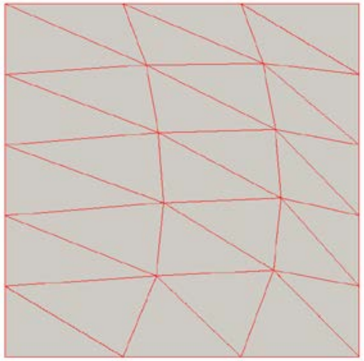
approximation (physical domain)



surface (physical domain)



approximation (parametric domain)

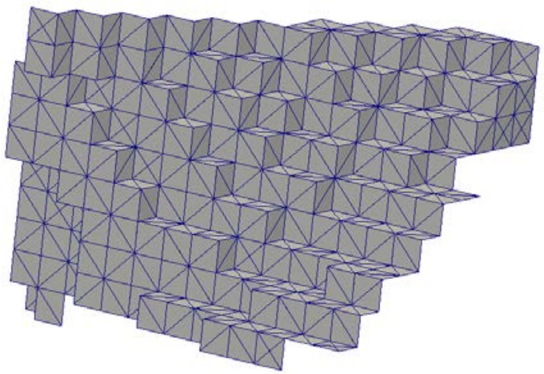


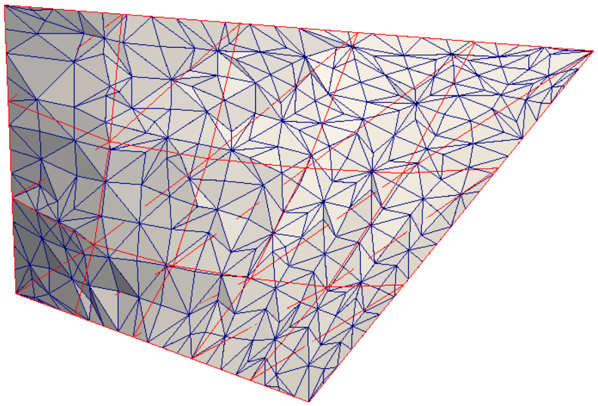
surface (parametric domain)

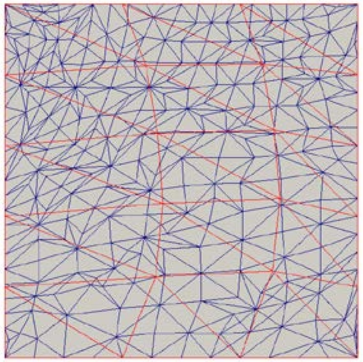
Projection

Simultaneous parameterization to project nodes

Simultaneous parameterization of the immerse surface and its initial approximation to the same parametric domain provides an initial location for nodes in the actual surface.





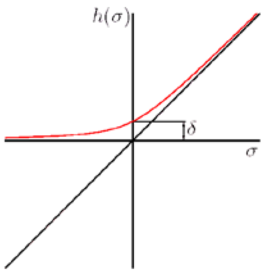


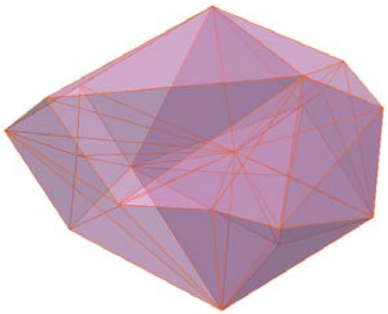
▪ Inner nodes

- 3D optimization in the physical domain
- A term per adjacent tetrahedron, that penalties poor quality respect to the cell counterpart in the parametric domain

$$\left|K_\eta\right|_p^*(\mathbf{x}) = \left[\sum_{m=1}^{M} \eta_m^{*\,p}(\mathbf{x})\right]^{1/p}$$

$$\eta_m^{*\,p} = \frac{|S|^2}{3\left[h(\sigma_m)\right]^{\frac{2}{3}}}$$





star in the physical domain

Smoothing

SUS: Simultaneous Untangling and Smoothing

- **Nodes attached to surface parameterization**

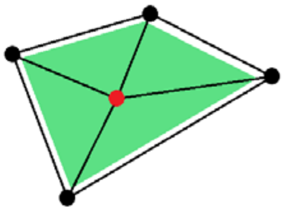
  - 2D optimization in the parametric domain

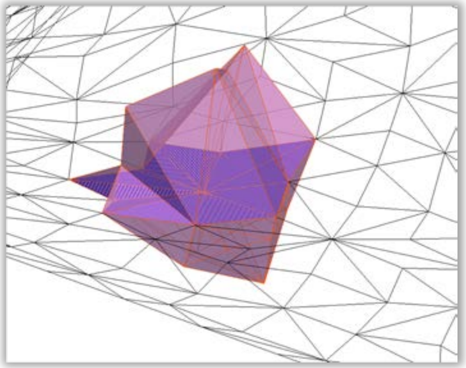
$$\left|K_\eta\right|_p^*(\mathrm{u},\ \mathrm{v}) = \left[\sum_{m=1}^{M} \eta_m^{*\,p}\,(\mathbf{x}(\mathrm{u},\ \mathrm{v}))\right]^{1/p}$$

  - An extra term in the objective function per triangle parametrized to the surface, that penalties collapse of the counterpart triangle in the parametric domain

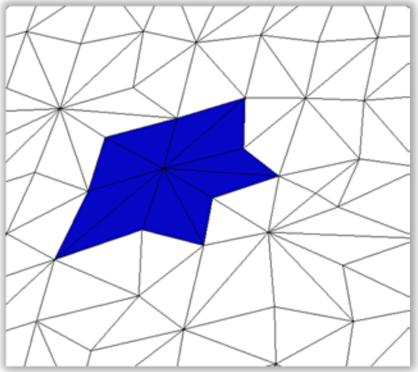
$$V(\mathrm{u},\mathrm{v}) = \sum_{m=1}^{M} \frac{1}{h(\mathit{mín}\{a_m,\tau\})}$$

a_m: area ratio over star área
τ: thickness of forbidden region





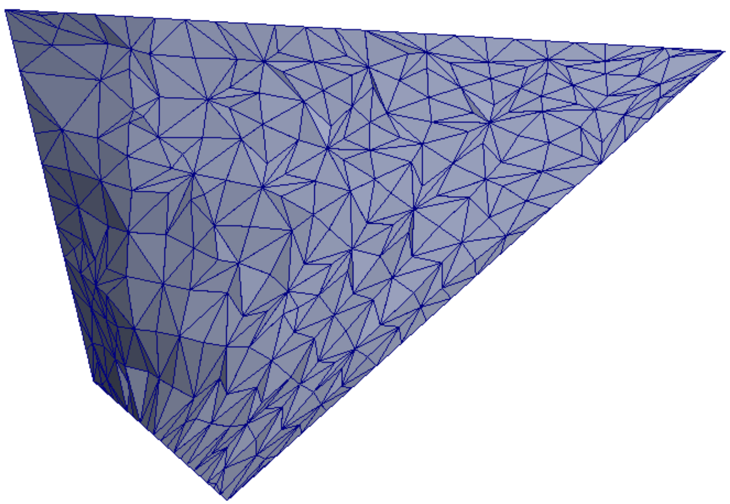
star in the physical domain

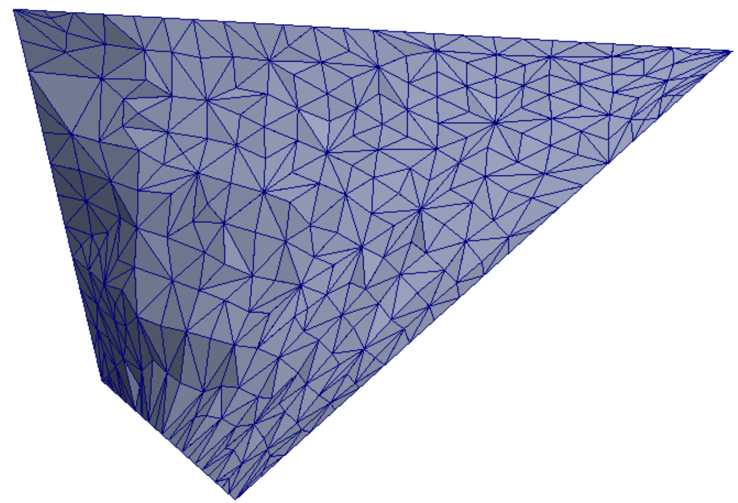


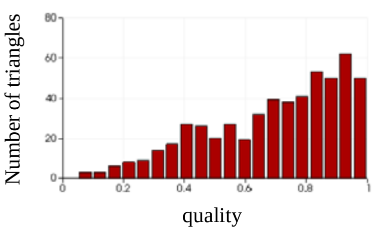
star in the parametric domain

Smoothing

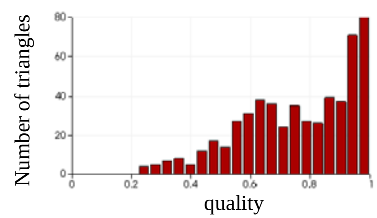
Surface approximation. Triangle Quality.





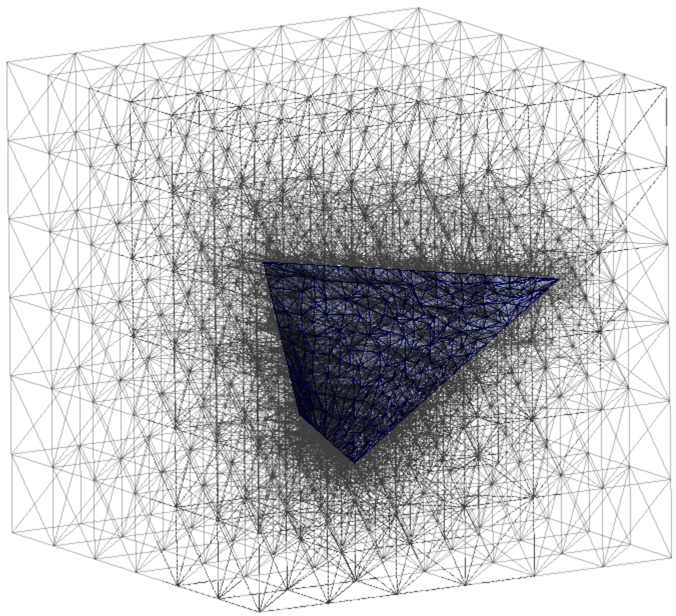


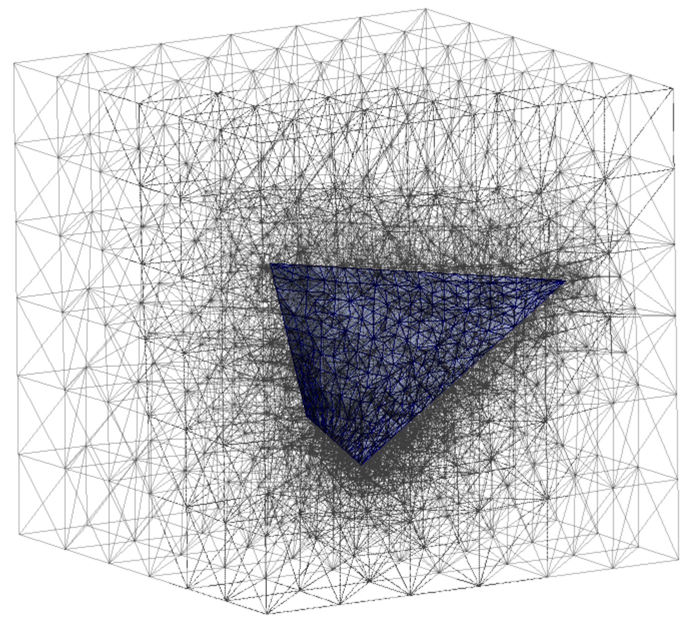
Mesh	Triangle quality		
	Min	Mean	Max
original	0.050	0.694	0.999
smoothed	0.221	0.755	0.999

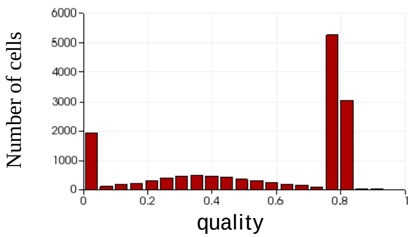


Smoothing

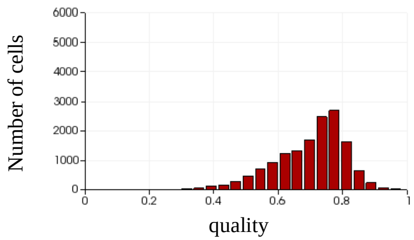
Volume mesh. Cell quality.







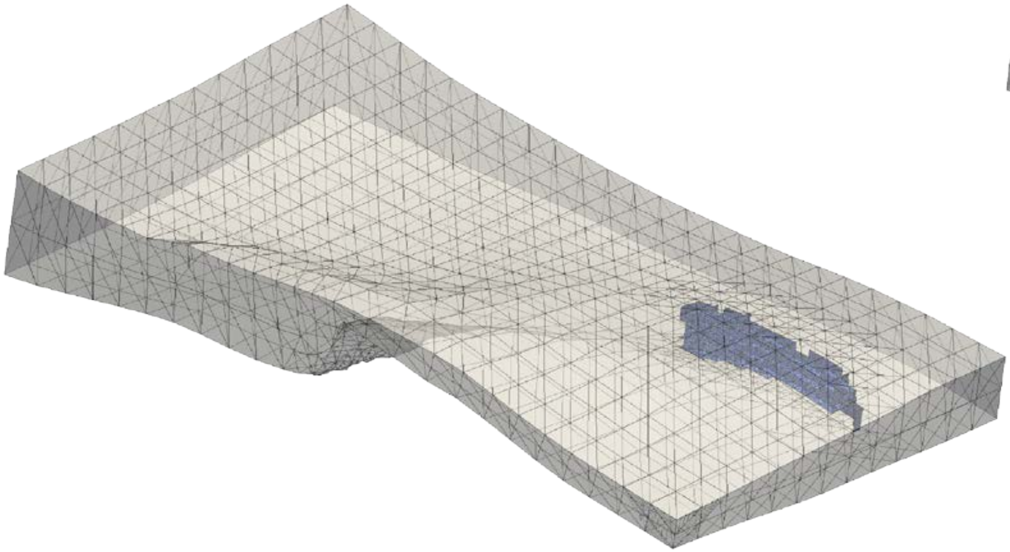
Mesh	Cell quality		
	Min	Mean	Max
original	0.000	0.555	0.935
smoothed	0.218	0.697	0.983



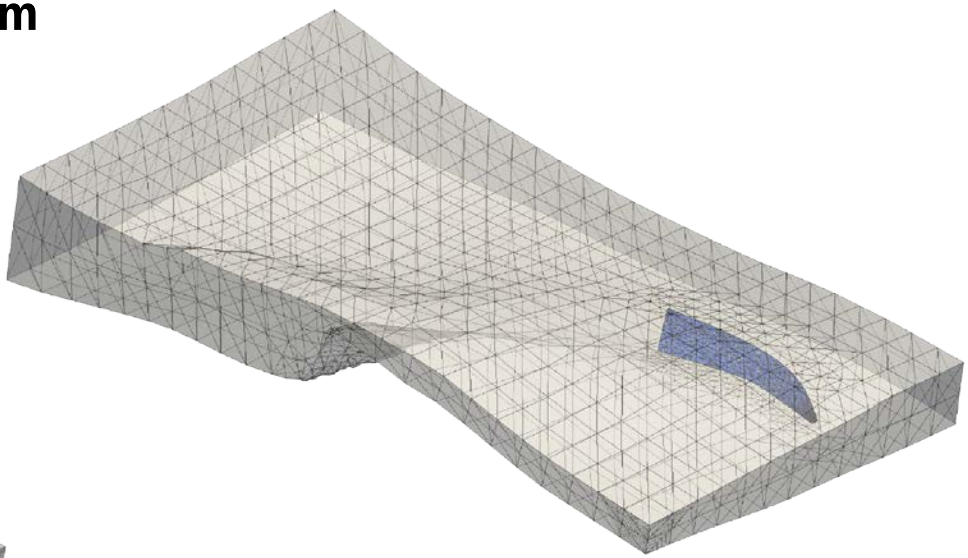
Applications (I)

Fault approximation in oil field exploitation

- **Goal: Capture a fault in a subsoil stratum**



Rough approximation



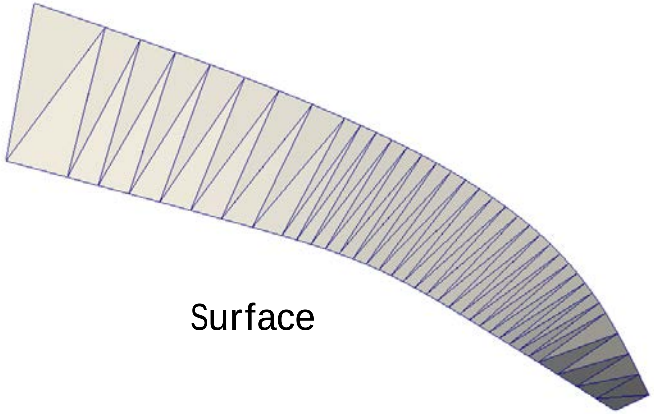
Smooth approximation

Mesh size
5850 nodes
25559 cells

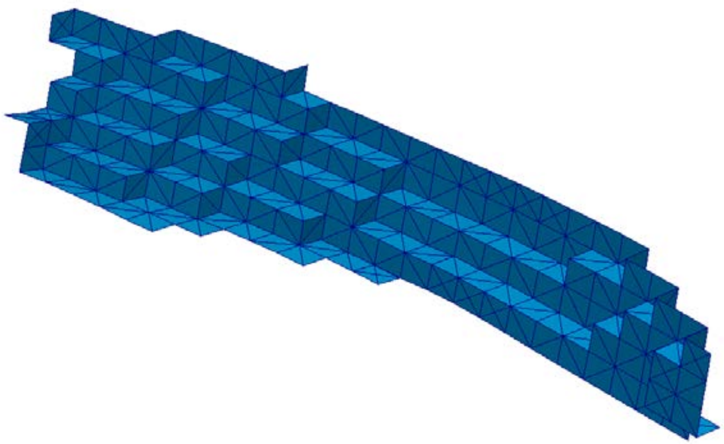
Cell quality		
Min	Mean	Max
0,209	0,725	0,983

Applications (I)

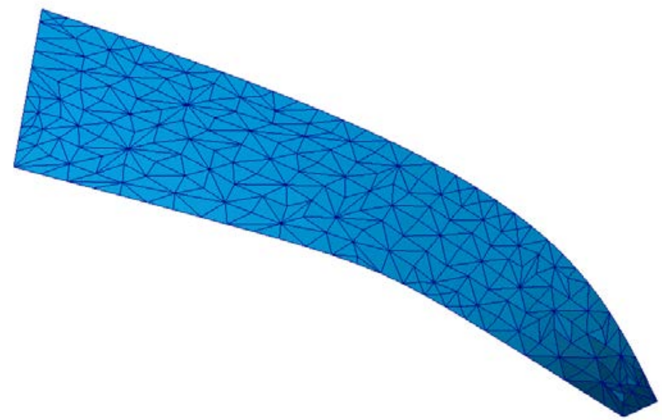
Fault approximation in oil field exploitation



Surface



Rough approximation



Smooth approximation

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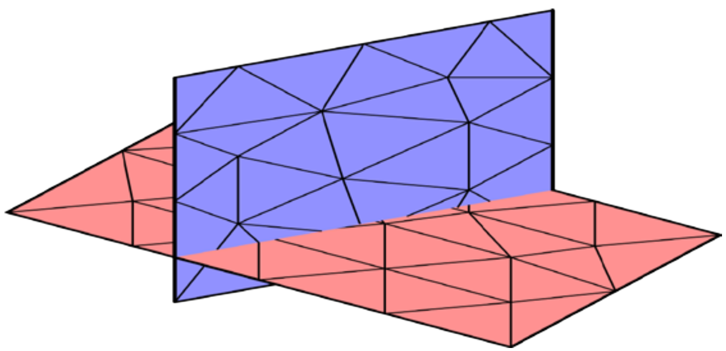
- **Conclusions**

  - **The proposed algorithm provides a smooth aproximation of a surface immersed in a tetrahedral mesh generated by the Meccano method**
  - **The triangles of the approximation are faces of cells of the mesh**
  - **Simple operations are involved in the construction of the initial approximation: refinement and mesh element (cell, face, edge) selection**
  - **The adaptive refinement, combined with the optimization of nodes constrained to the surface parameterization, allows to increase the resolution, i. e., the number of nodes in the surface approximation, while preserving quality of cells in the tetrahedral mesh**

Conclusions and Future research

Future research

- **Future research**
  - **Extend the algorithm to manage the intersection of several immersed surfaces**
  - **Consider not immersed surfaces, which intersect the mesh boundary**



  - **Generalize the algorithm to arbitrary surfaces (closed surfaces, complex topologies, etc.)**

Questions ?

End of the presentation

Thank you for your attention!

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