SIANI
INSTITUTO UNIVERSITARIO
INGENIERIA COMPUTACIONAL

3-D Wind Field Simulation over Complex Terrain

University Institute for Intelligent Systems and Numerical Applications in Engineering

Congreso de la RSME 2015 – Soluciones Matemáticas e Innovación en la Industria

2-6 febrero 2015, Granada

http://www.siani.es/

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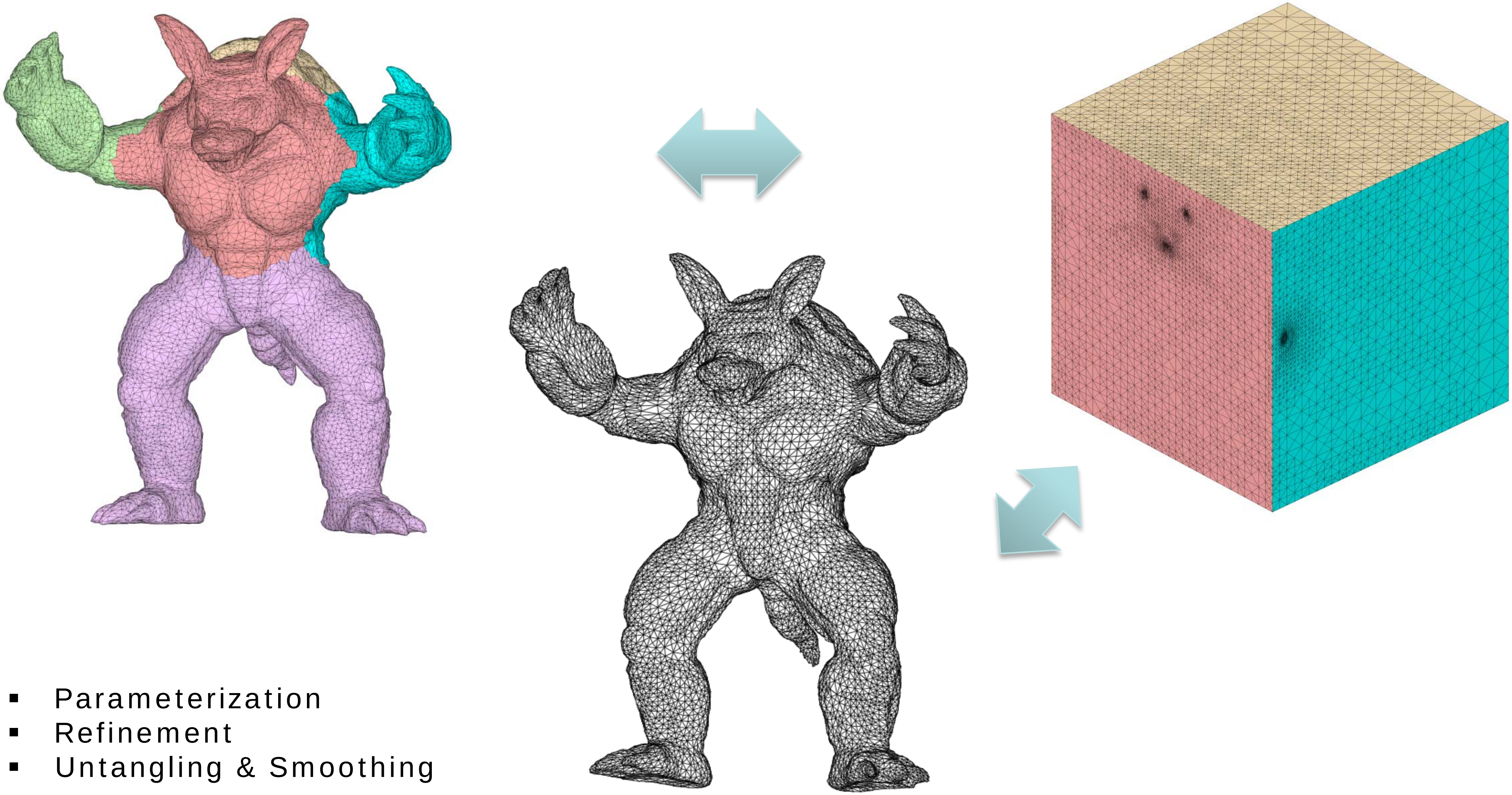
Meccano Method for Complex Solids

Algorithm steps

- Meccano construction
- Parameterization of the solid surface
  - Solid boundary partition
  - Floater’s parameterization
- Coarse tetrahedral mesh of the meccano
  - Partition into hexahedra
  - Hexahedron subdivision into six tetrahedra
- Solid boundary approximation
  - Kossaczky’s refinement
  - Distance evaluation
- Inner node relocation
  - Coons patches
- Simultaneous untangling and smoothing
  - SUS

Meccano Method

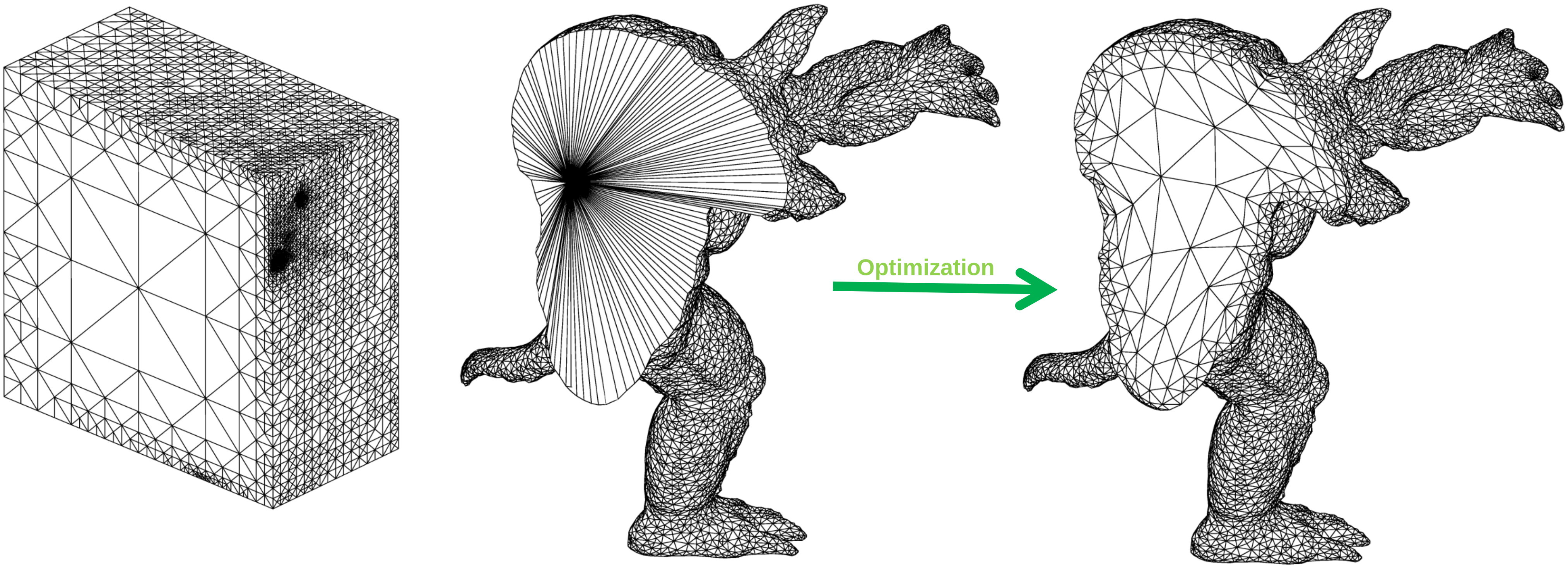
Simultaneous mesh generation and volumetric parameterization



- Parameterization
- Refinement
- Untangling & Smoothing

Meccano Method

Key of the method: SUS of tetrahedral meshes



Parameter space (meccano mesh)

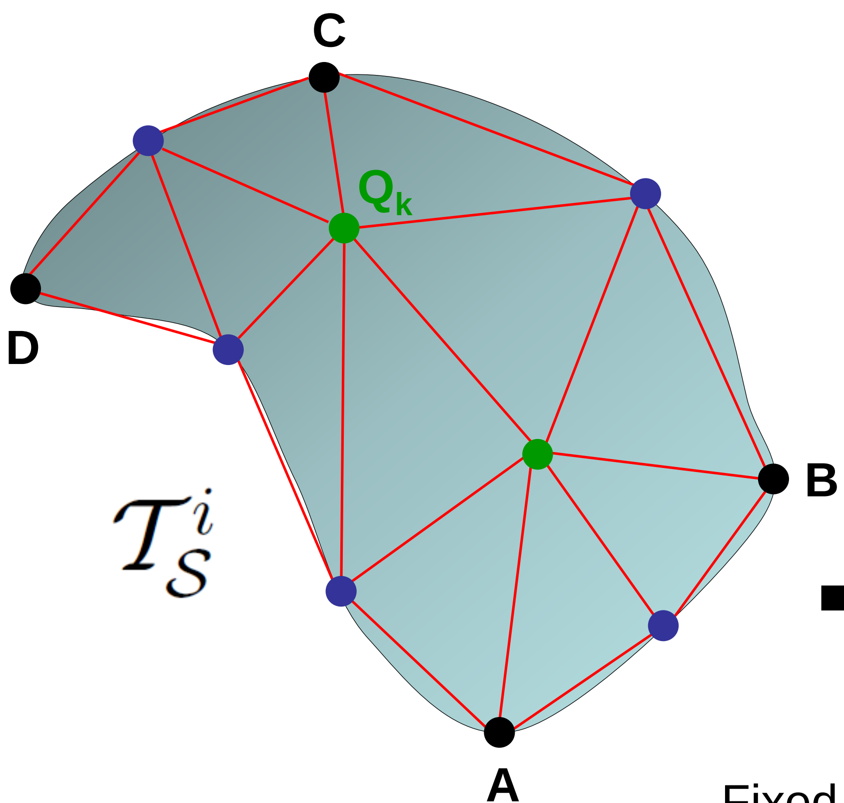
Physical space (tangled mesh)

Physical space (optimized mesh)

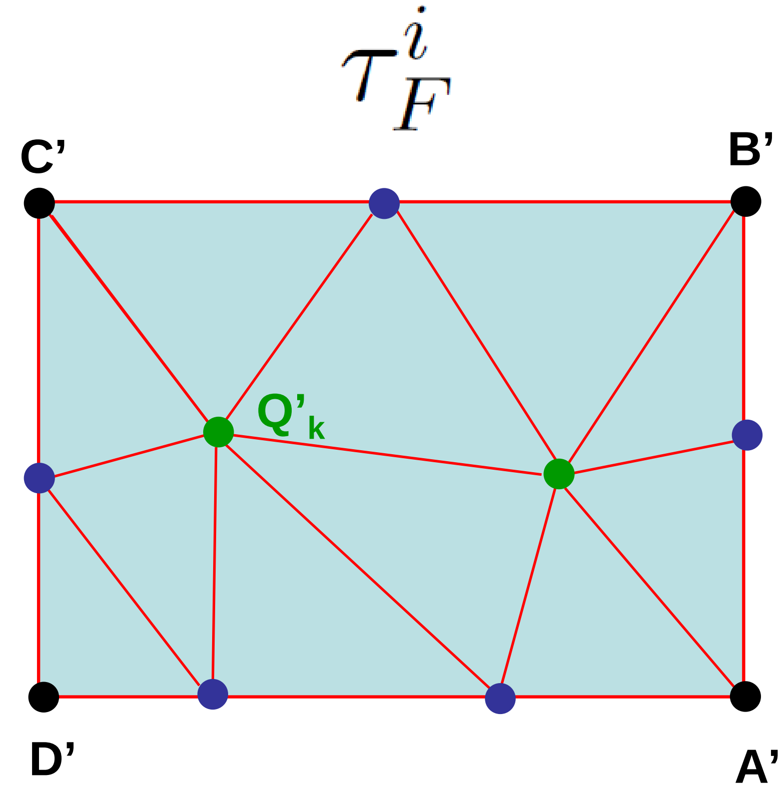
Meccano Method

Surface Parameterization of M.S. Floater (CAGD 1997)

From the *i-th* solid surface triangulation patch to the *i-th* meccano face



Physical Space



Parametric Space

Fixed boundary nodes,

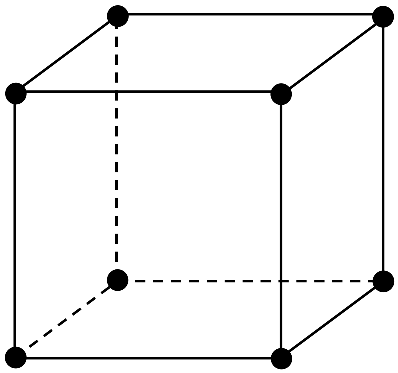
$Q'_k = \sum_j \lambda_{j,k} Q'_j$

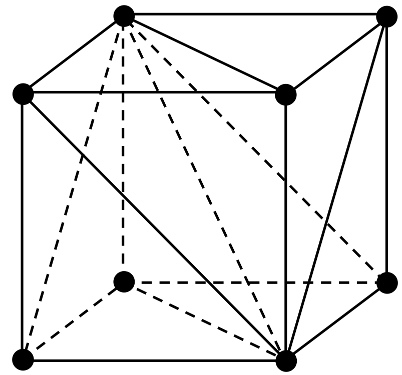
Compromise between area and shape

Meccano Method

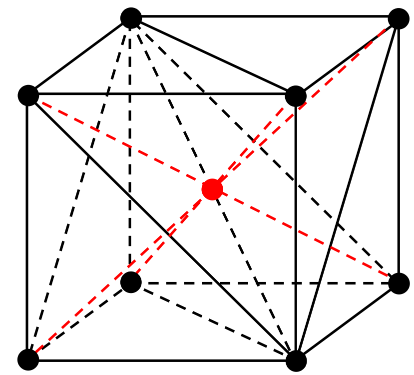
Local Refinement: Kossaczky’s Algorithm (JCAM 1994)

- Initial cube and its subdivision after three consecutive tetrahedron bisection

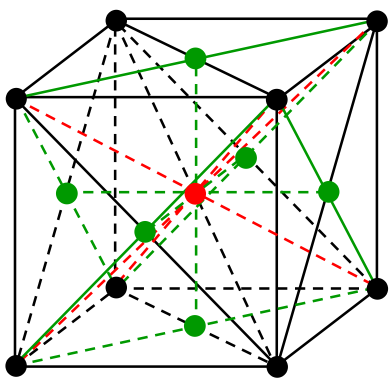




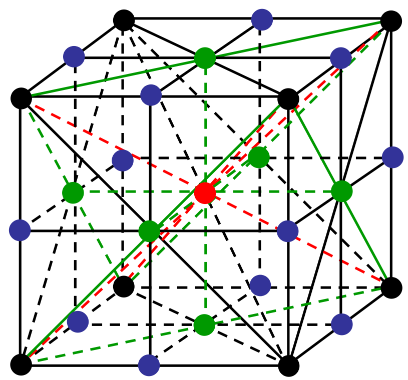
6 tetrahedra



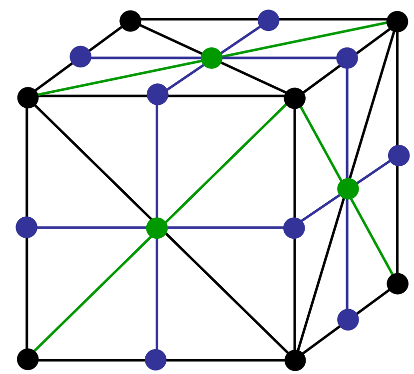
12 tetrahedra



24 tetrahedra



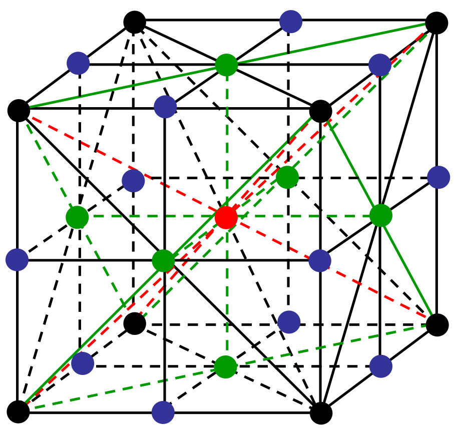
48 tetrahedra



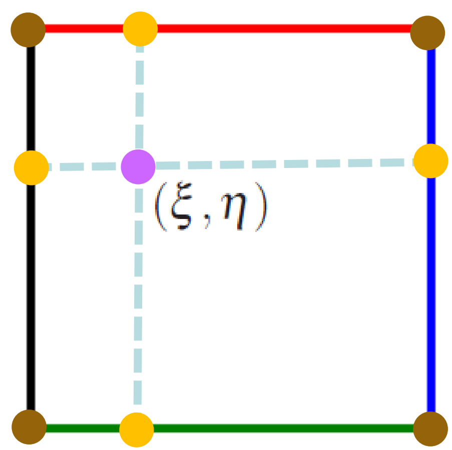
Coons patches provide a reasonable initial location in physical domain for inner nodes

Segment (1D), surface (2D) and cube (3D) interpolations

$$\mathbf{x}(\xi,\eta) = (1-\xi)\mathbf{x}(0,\eta) + \xi\mathbf{x}(1,\eta) + (1-\eta)\mathbf{x}(\xi,0) + \eta\mathbf{x}(\xi,1) - \begin{bmatrix} 1-\xi & \xi \end{bmatrix} \begin{bmatrix} \mathbf{x}(0,0) & \mathbf{x}(0,1) \\ \mathbf{x}(1,0) & \mathbf{x}(1,1) \end{bmatrix} \begin{bmatrix} 1-\eta \\ \eta \end{bmatrix}$$



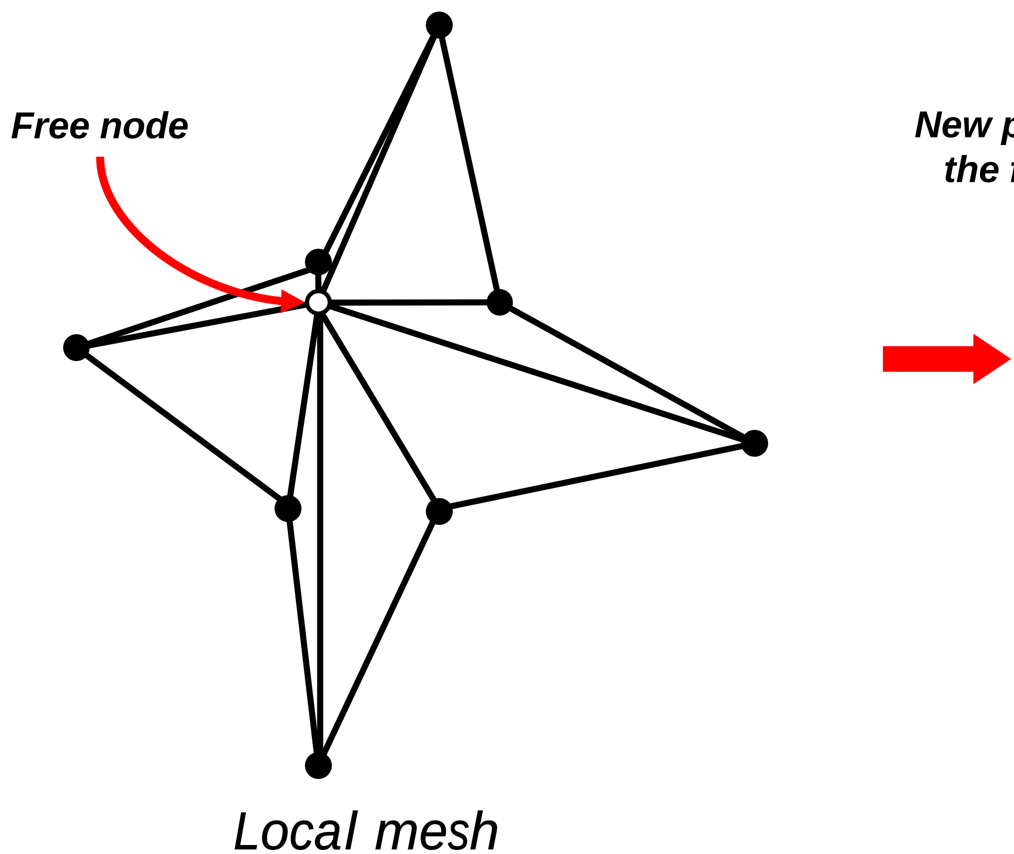
(a) parametric cube



(b) bilinear interpolation in a rectangle

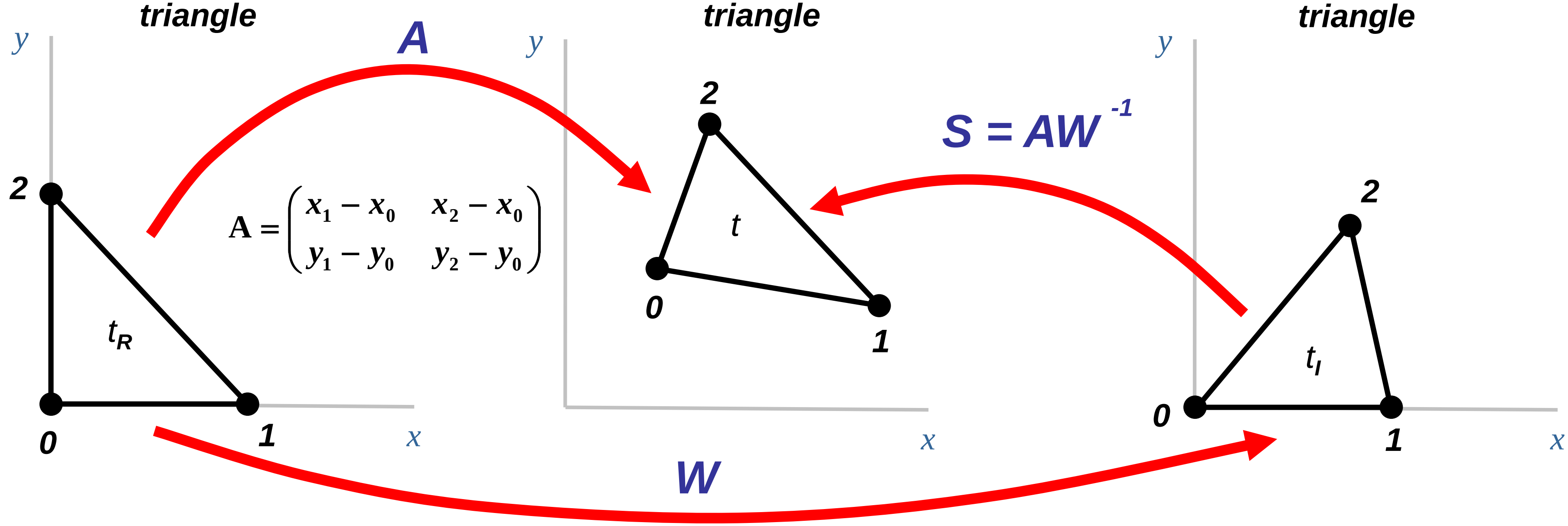
Local optimization

***Objective:* Improve the quality of the local mesh by minimizing an objective function**



Meccano Method

SUS Code: Weighted Jacobian Matrix on a Plane



$t_I \xrightarrow{\mathbf{S}} t$ $\quad S = AW^{-1}$: **Weighted Jacobian matrix**

An algebraic quality metric of t (mean ratio)

$$\boxed{\mathbf{q} = \frac{2\boldsymbol{\sigma}}{\|\mathbf{S}\|^2} = \frac{1}{\eta}}$$

where:

$$\|\mathbf{S}\| = \sqrt{\mathbf{tr}\left(\mathbf{S}^T\mathbf{S}\right)}$$

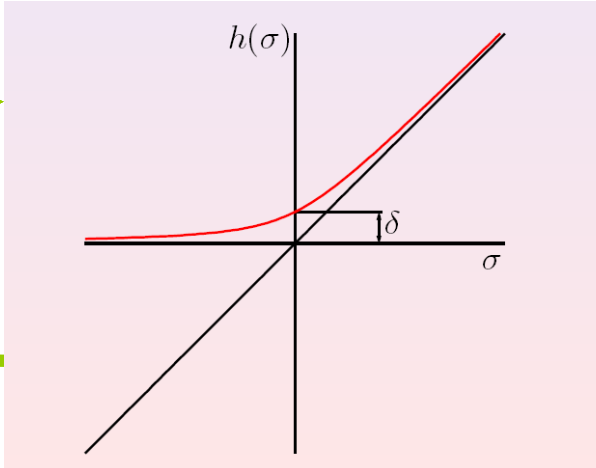
$$\boldsymbol{\sigma} = \det(\mathbf{S})$$

Meccano Method

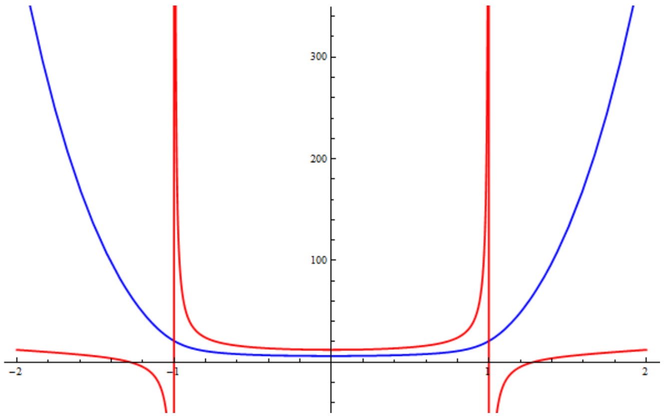
SUS Code: Local objective function for plane triangulations

Original function: $$K(\mathbf{x}) = \sum_{m=1}^{M} \frac{\|S_m\|^2}{2\,\sigma_m}$$

Modified function: $$K^*(\mathbf{x}) = \sum_{m=1}^{M} \frac{\|S_m\|^2}{2h(\sigma_m)}$$



$$h(\sigma) = \tfrac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\delta^2}\right)$$



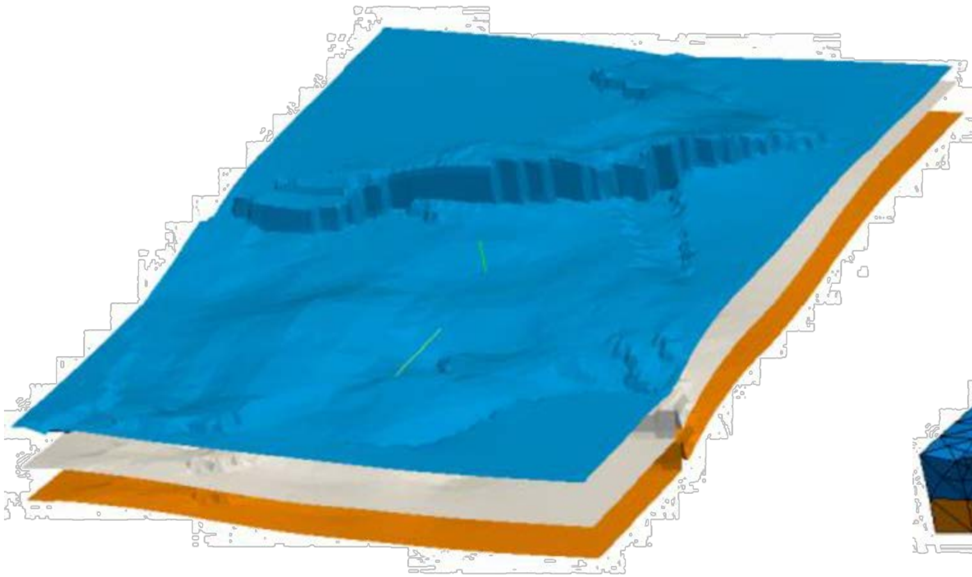
Modified function (blue) is regular in all R^2 and it approximates the same minimum that the original function (red). Moreover, it allows a simultaneous untangling and smoothing of triangular meshes

The extension to tetrahedral meshes is straightforward:

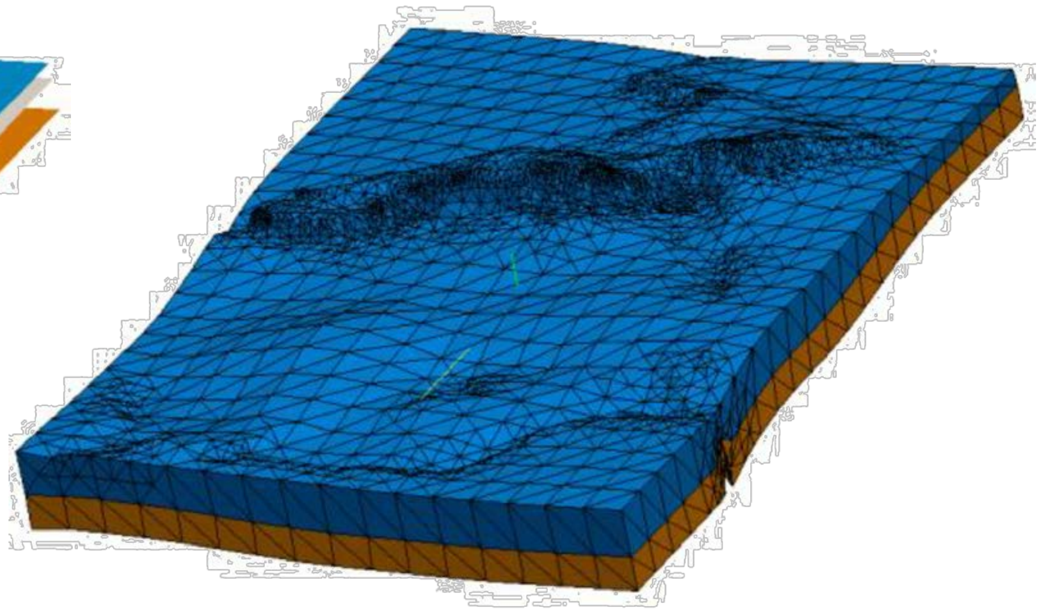
$$\eta_m = \frac{|S_m|^2}{3[h(\sigma_m)]^{\frac{2}{3}}}$$

Meccano Method

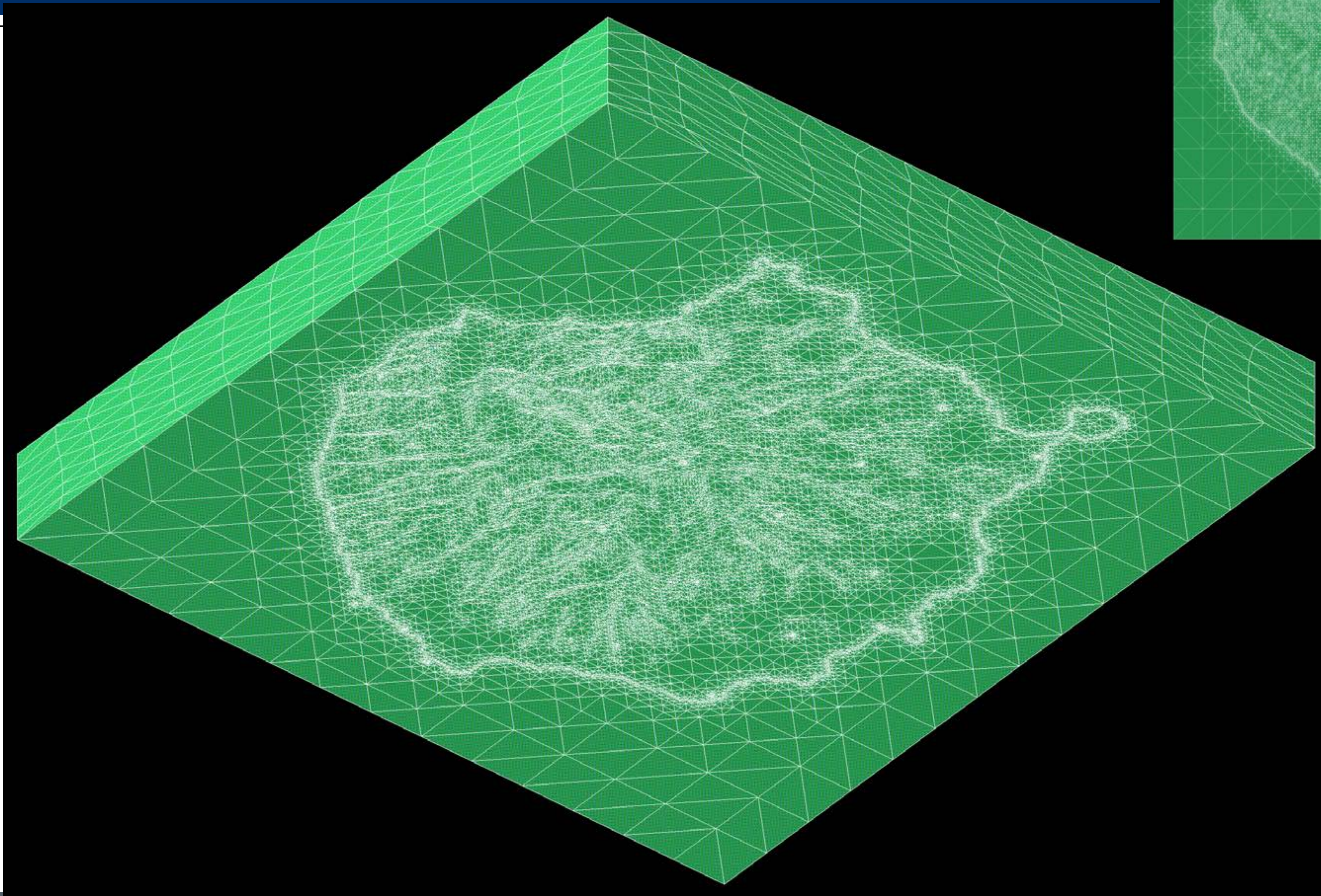
Oil field strata meshing

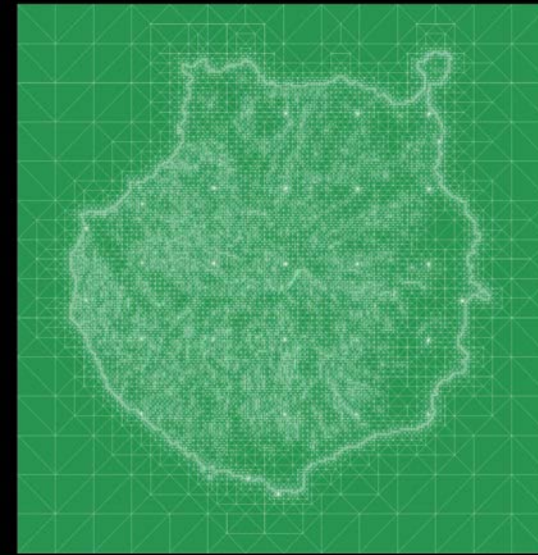


Boundary surfaces



Volume mesh






Wind Field Modeling

Wind Field Modeling

Conquered challenges

Local scale wind field model JWEIA(1998, 2001, 2010)

- Mass consistent model (MMC) provides a useful method for local scale wind studies.
- Local wind 3D field using data measurements.
- Automatic process. Few model parameters.
- Low computational cost due to use of adaptive tetrahedral meshes and efficient solvers.

Parameter estimation LNCS(2002), AES(2005)

- Use of parallel GAs for parameter estimation.

Integration with mesoscale models PAG (2015)

- Coupling with HARMONIE leads to predictive capabilities.
- Use of HARMONIE outputs as MMC inputs.

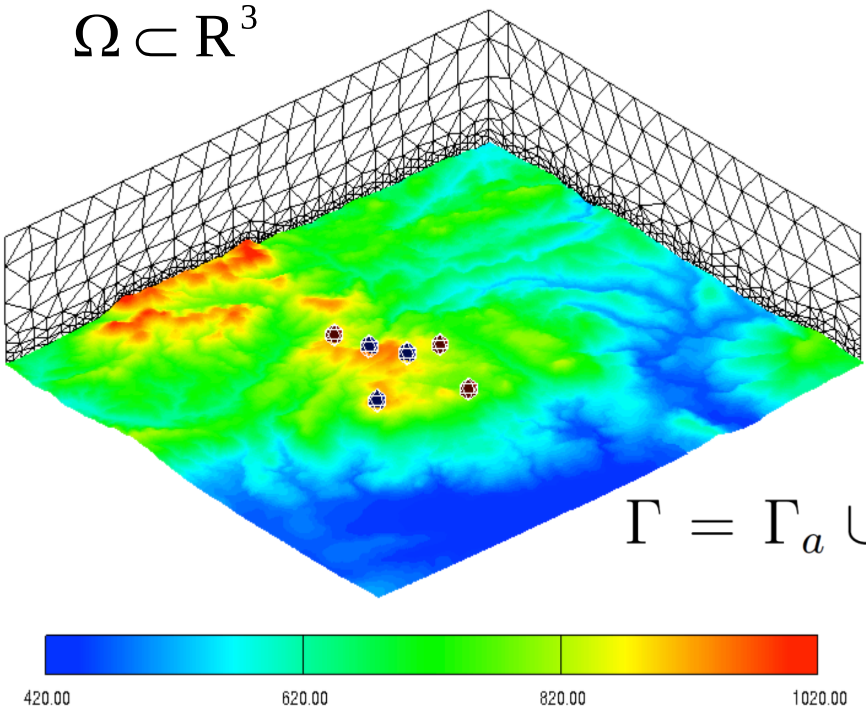
Ensemble methods PAG (2015)

- Use of ensemble methods to obtain local scale wind prediction.

Wind Field Modeling

Mass Consistent Wind Model

$\Omega \subset R^3$



$\Gamma = \Gamma_a \cup \Gamma_b$

$\mathbf{u}_0 = (u_0, v_0, w_0)$: observed wind, which is obtained with horizontal interpolation and vertical extrapolation of experimental or forecasted data.

Objective: find the velocity field $\mathbf{u}$ that it adjusts to $\mathbf{u}_0$ verifying

- Incompressibility condition in the domain: $\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega$
- Impermeability condition on the terrain: $\mathbf{n} \cdot \mathbf{u} = 0 \quad \text{on } \Gamma_a$

Then, $\mathbf{u}$ is the solution of the least-square problem: Find $\mathbf{u} \in \mathrm{K}$ verifying

$$\begin{cases} E(\mathbf{u}) = \min\limits_{\mathbf{v} \in \mathrm{K}} E(\mathbf{v}) \\ \mathrm{K} = \left\{ \mathbf{v};\ \nabla \cdot \mathbf{v} = 0,\ \ \mathbf{n} \cdot \mathbf{v}\big|_{\Gamma_a} = 0 \right\} \end{cases}$$

where $E(\mathbf{v}) = \dfrac{1}{2} \displaystyle\int_{\Omega} (\mathbf{v} - \mathbf{u}_0)^t \, \mathbf{P} \, (\mathbf{v} - \mathbf{u}_0) \, d\Omega$

We can write E as follows

$$E(\tilde{u}, \tilde{v}, \tilde{w}) = \int_{\Omega} \left[\alpha_1^2 \left((\tilde{u} - u_0)^2 + (\tilde{v} - v_0)^2 \right) + \alpha_2^2 (\tilde{w} - w_0)^2 \right] \, d\Omega$$

$$\boxed{\alpha = \frac{\alpha_1}{\alpha_2}}$$

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Governing equations

- Lagrange multiplier technique is used to solve this problem. So, if we introduce

$$L(\mathbf{v},\lambda) = E(\mathbf{v}) + \int_{\Omega} \lambda \nabla \cdot \mathbf{v} \, d\Omega$$

its saddle point $(\mathbf{u},\phi)$ verifies the Euler-Lagrange equations:

$$-\nabla \cdot \left(\mathbf{P}^{-1} \nabla \phi\right) = \nabla \cdot \mathbf{u}_0 \qquad \text{in } \Omega$$

$$-\mathbf{n} \cdot \mathbf{P}^{-1} \nabla \phi = \mathbf{n} \cdot \mathbf{u}_0 \qquad \text{on } \Gamma_a$$

$$\phi = 0 \qquad \text{on } \Gamma_b$$

and, finally, the adjusted velocity field is obtained by: $\mathbf{u} = \mathbf{u_0} + \mathbf{P}^{-1} \nabla \phi$

Construction of the observed wind

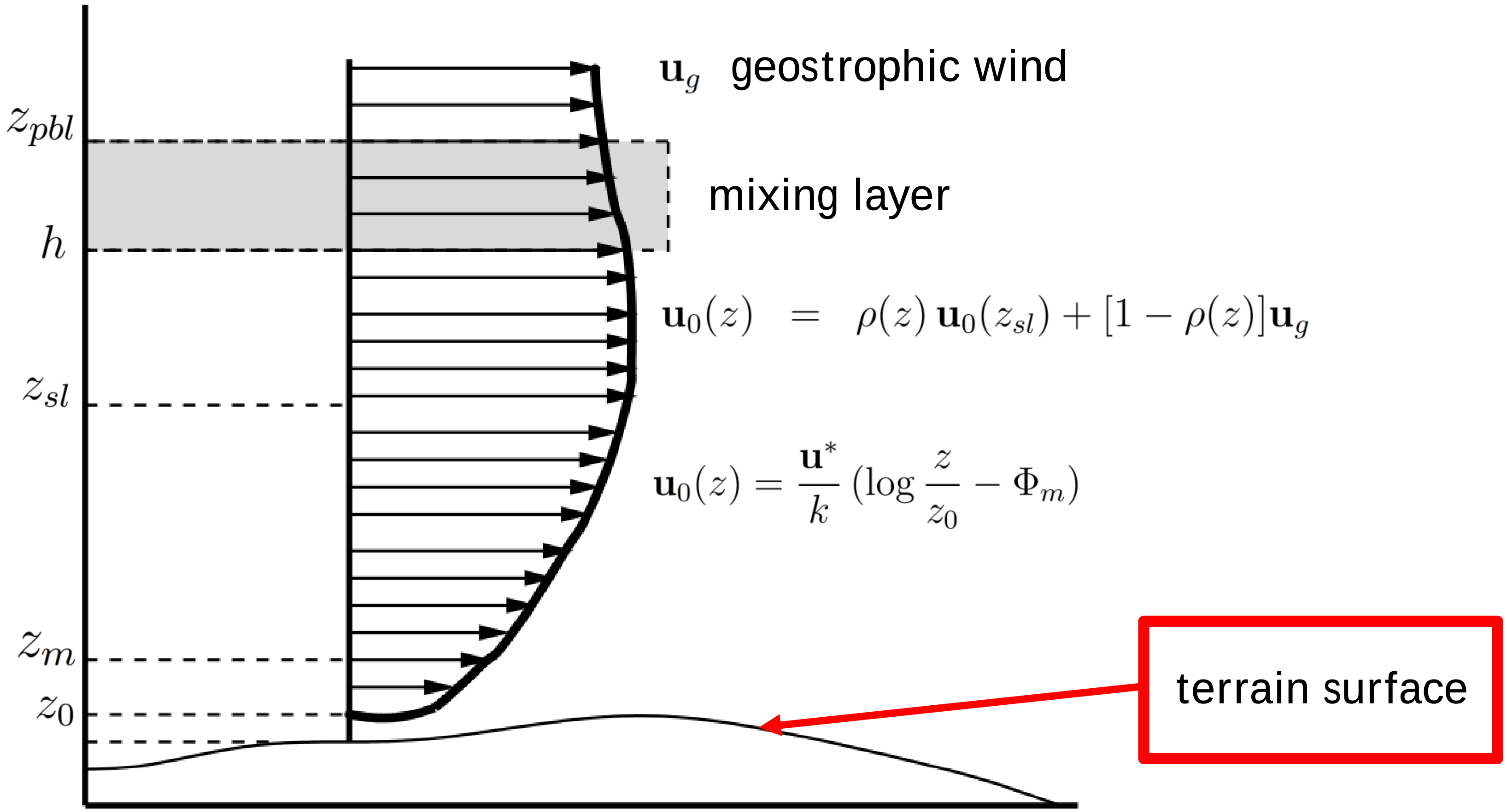
Horizontal interpolation

$$\mathbf{u}_0(z_m) = \xi \frac{\displaystyle\sum_{n=1}^{N} \frac{\mathbf{u}_n}{d_n^2}}{\displaystyle\sum_{n=1}^{N} \frac{1}{d_n^2}} + (1-\xi) \frac{\displaystyle\sum_{n=1}^{N} \frac{\mathbf{u}_n}{|\Delta h_n|}}{\displaystyle\sum_{n=1}^{N} \frac{1}{|\Delta h_n|}}$$

$$\boxed{0 \leq \xi \leq 1}$$

Construction of the observed wind

Vertical extrapolation (log-linear wind profile)



Wind Field Modeling

Mass Consistent Wind Model

- **Friction velocity:** $\mathbf{u}^* = \dfrac{k\ \mathbf{u}_0(z_m)}{\log \dfrac{z_m}{z_0} - \Phi_m}$

- **Height of the planetary boundary layer:** $z_{pbl} = \dfrac{\gamma\ |\mathbf{u}^*|}{f}$

  $f = 2\Omega \sin\phi$ **is the Coriolis parameter, being** Ω **the Earth rotation and** ϕ **is the latitude**

  γ **is a parameter depending on the atmospheric stability**

- **Mixing height:**

  $h = z_{pbl}$ **in neutral and unstable conditions**

  $h = \gamma' \sqrt{\dfrac{|\mathbf{u}^*|\ L}{f}}$ **in stable conditions**

- **Height of the surface layer:** $z_{sl} = \dfrac{h}{10}$

Wind Field Modeling

Estimation of Model Parameters

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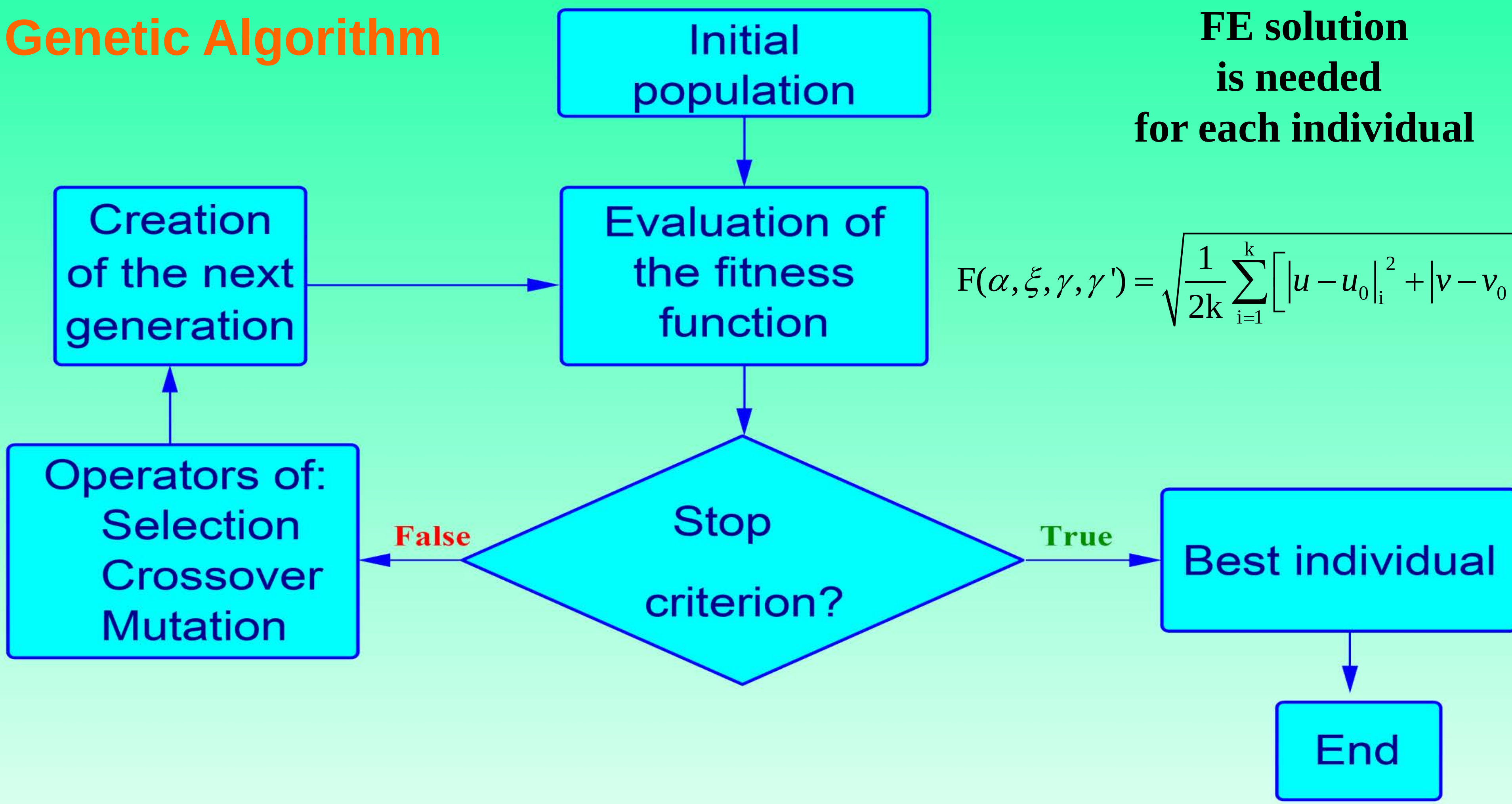
Genetic Algorithm



FE solution is needed for each individual

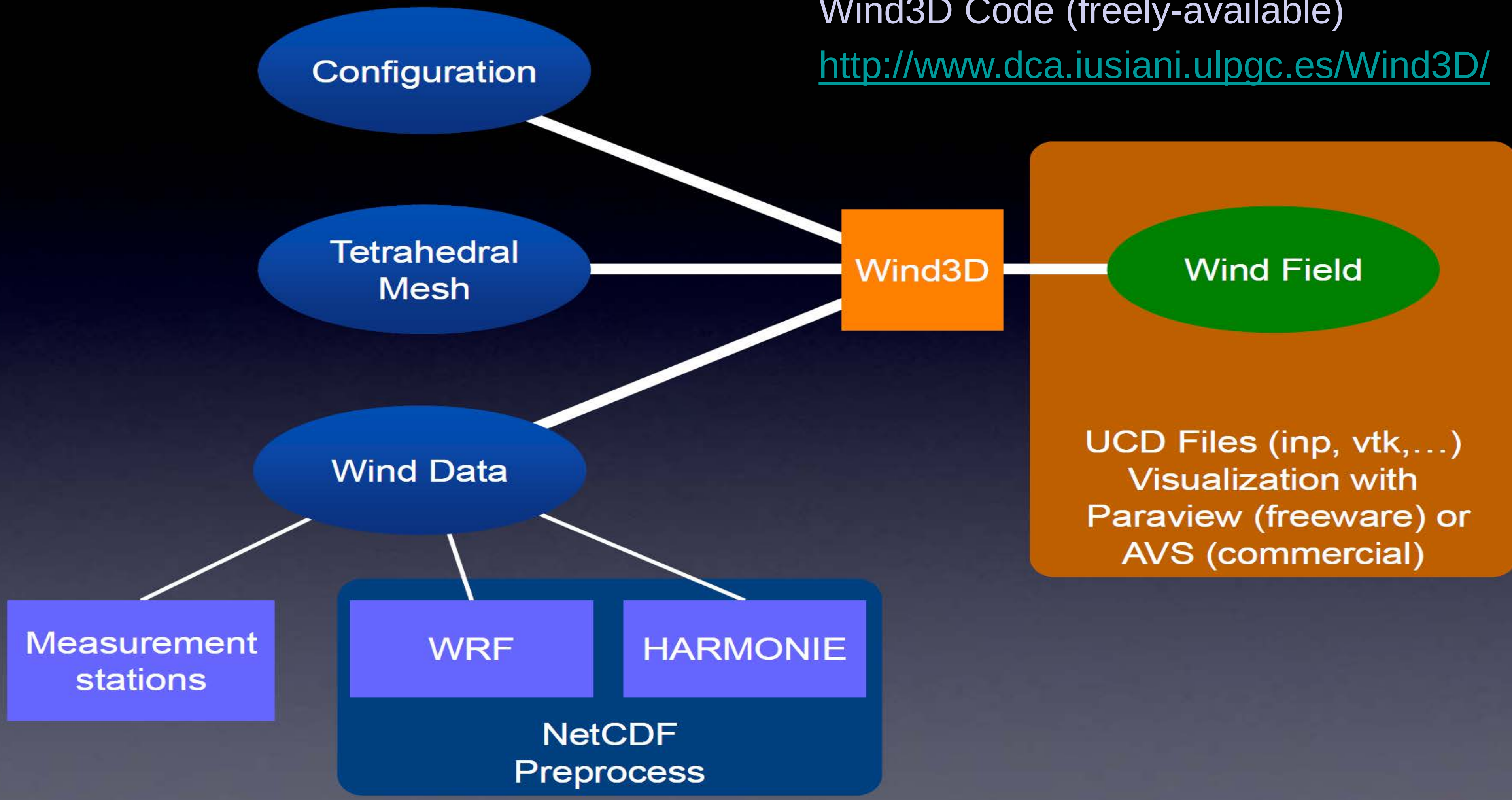
$$F(\alpha,\xi,\gamma,\gamma') = \sqrt{\frac{1}{2k}\sum_{i=1}^{k}\left[\left|u-u_0\right|_i^2+\left|v-v_0\right|_i^2\right]}$$

Wind Field Modeling

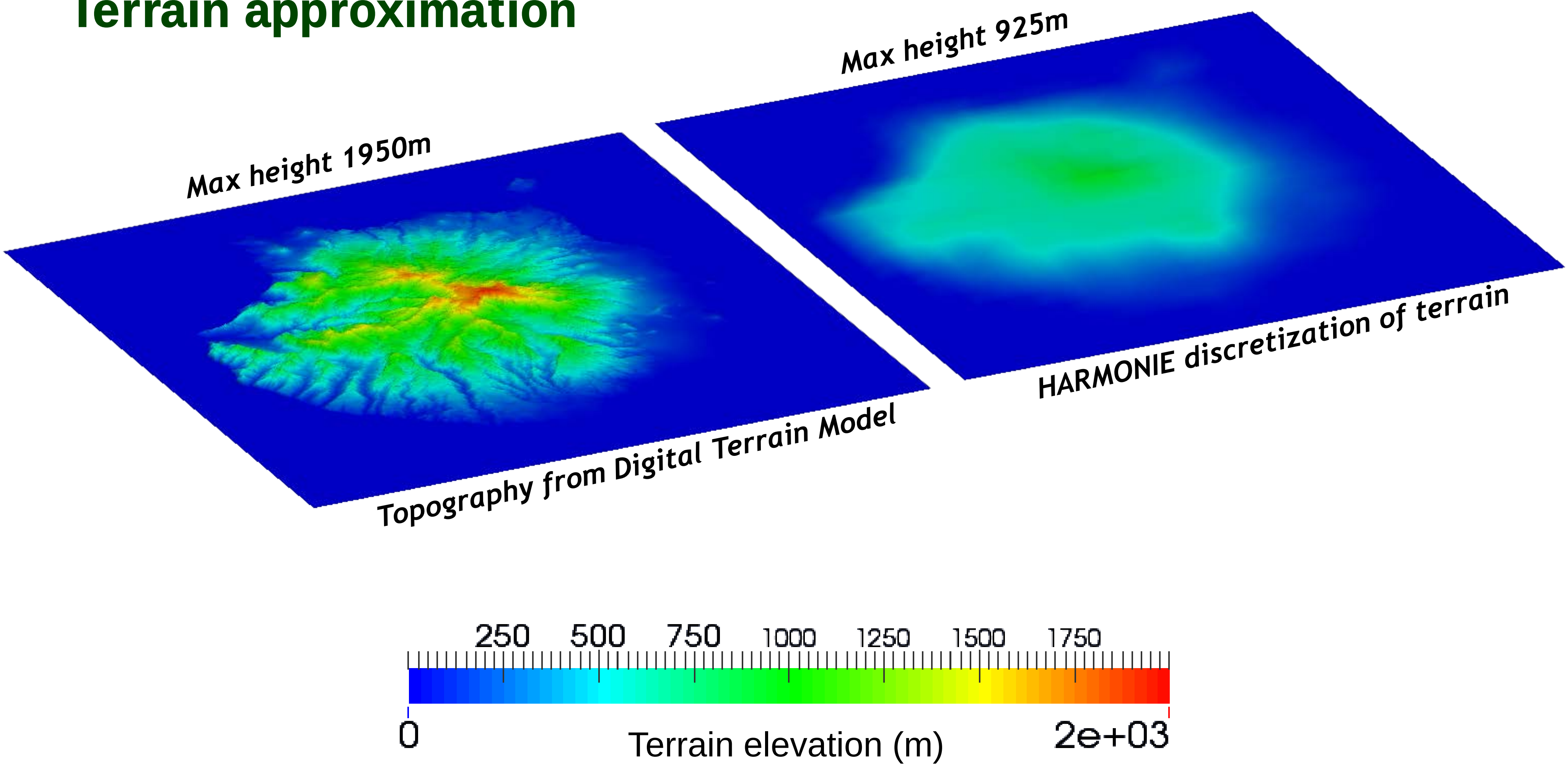
Forecasting over Complex Terrain

Wind3D Code (freely-available)

http://www.dca.iusiani.ulpgc.es/Wind3D/



Terrain approximation



Wind Field Modeling

HARMONIE-FEM wind forecast

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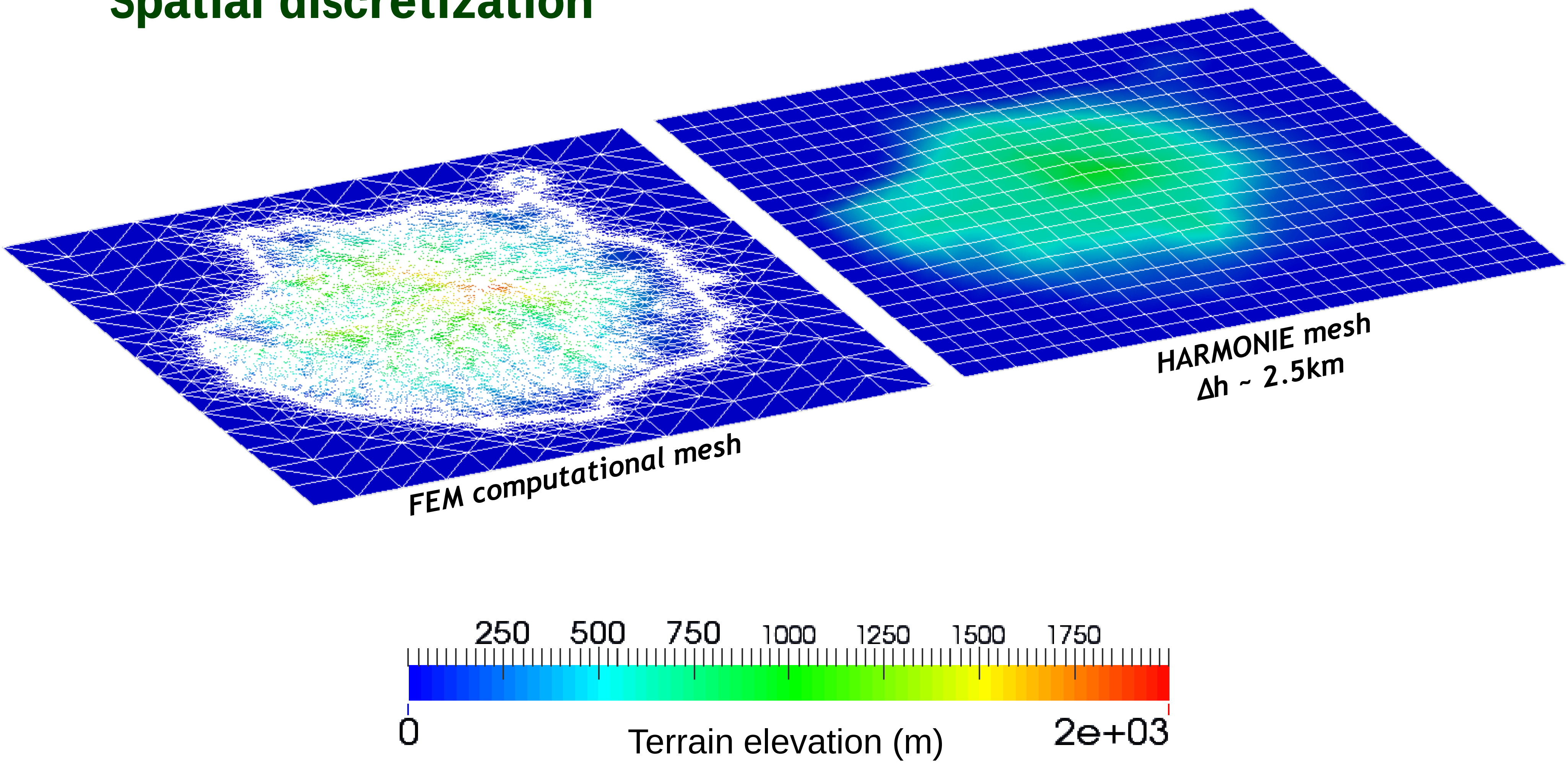
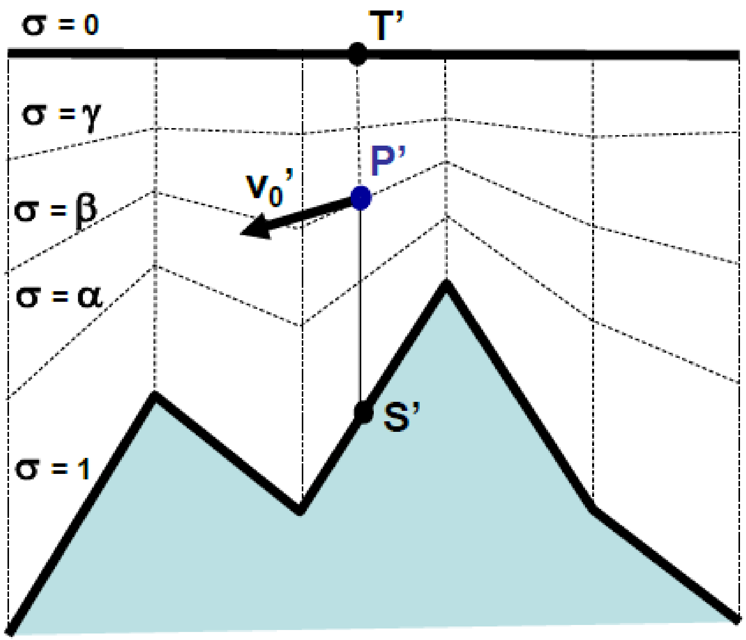
Spatial discretization

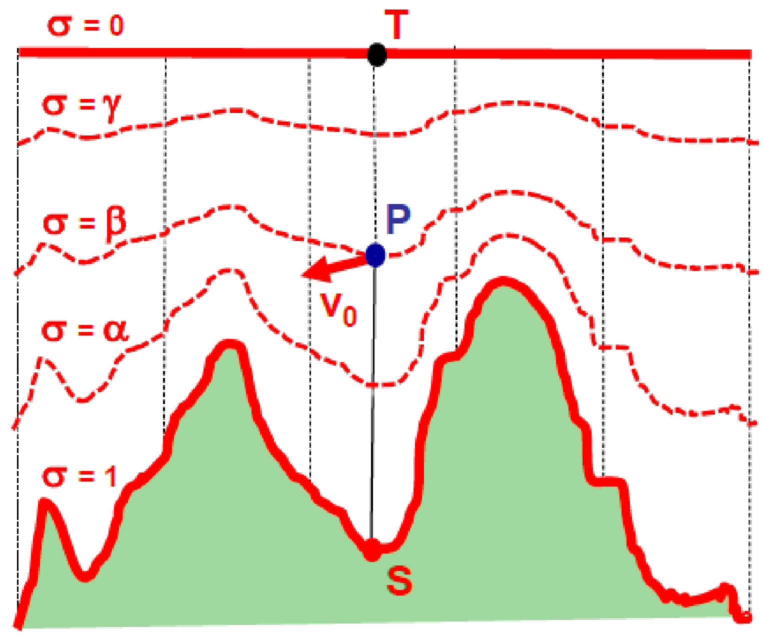


Wind Field Modeling

Data interpolation in FE domain: Problems with terrain discretization



HARMONIE FD domain

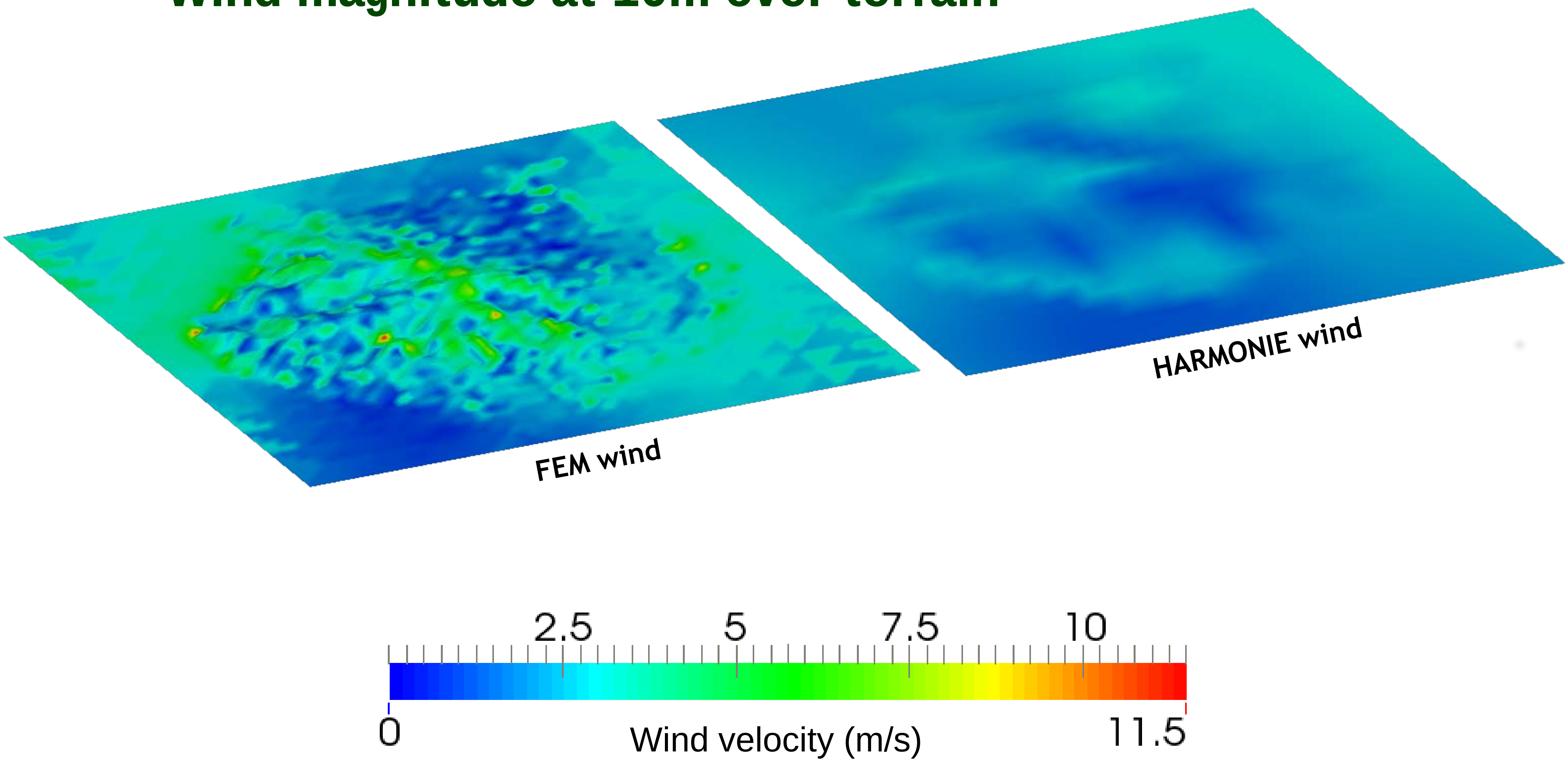


FE domain

Possible solutions for a suitable data interpolation in the FE domain:

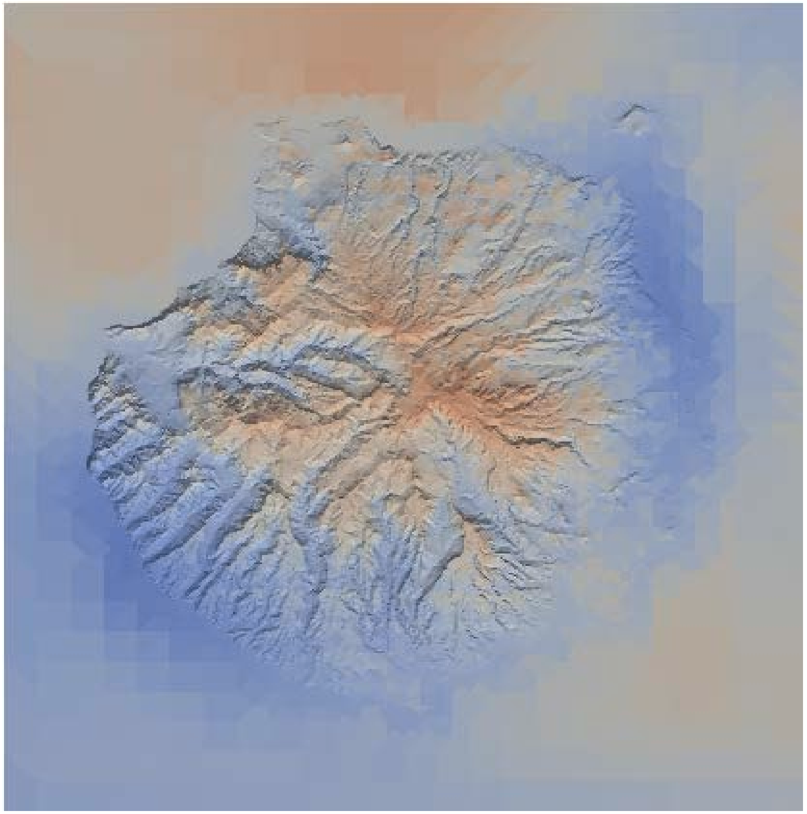
- Use U10 and V10 supposing it is 10m over the FE terrain (used in this talk)
- Given a point of FE domain, find the closest one in HARMONIE domain grid
- Other possibilities can be considered

Wind magnitude at 10m over terrain

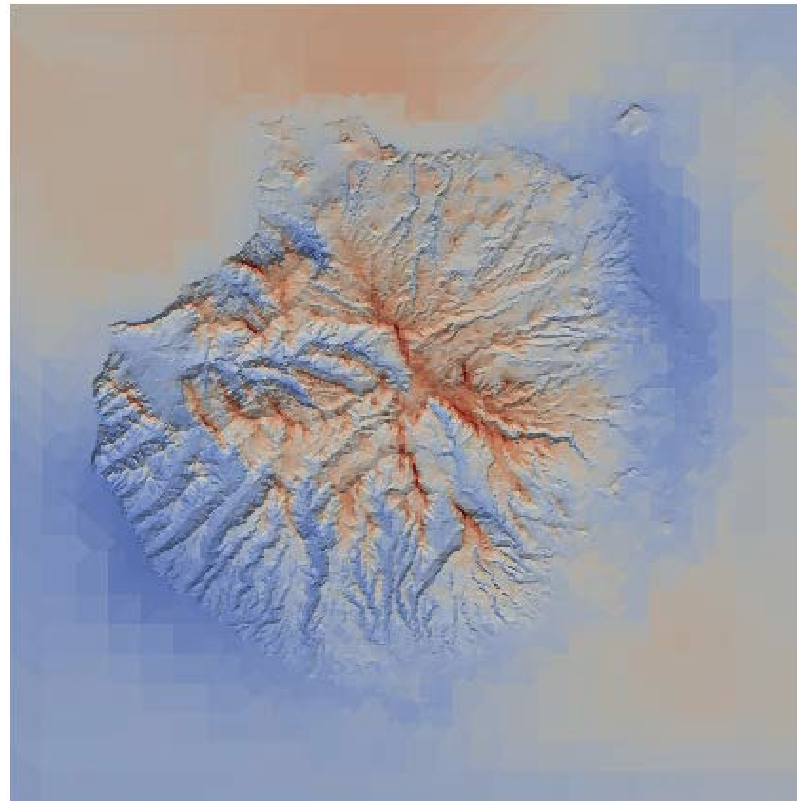


Velocity Module - 23/12/2009 - 18:00 h

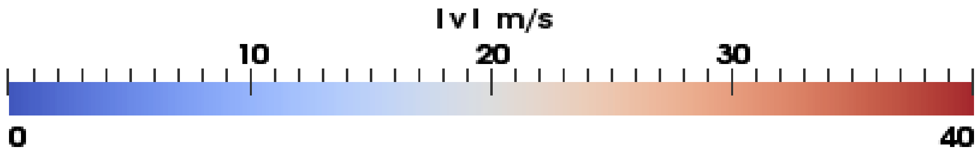
Wind on the terrain surface



Interpolated field from HARMONIE



Resulting field with the mass consistent model



Wind Field Modeling

HARMONIE-FEM wind forecast

1. Multiple numerical predictions using slightly different conditions

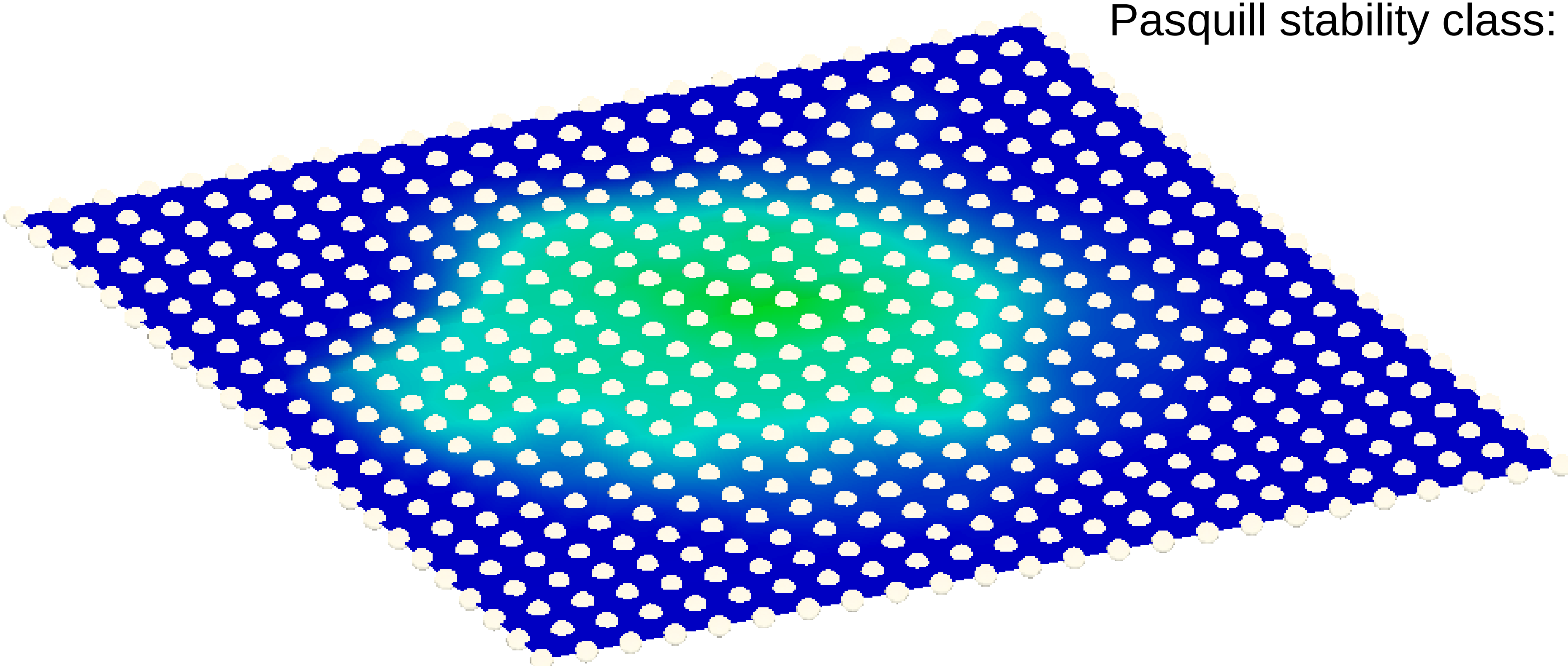
1. HARMONIE data + MMC model → local wind forecast

1. The main idea
    a. Use HARMONIE data as reliable points, as stations or as control points for GAs
    b. Run different sets of experiments → perturbation

Wind Field Modeling

HARMONIE-FEM wind forecast

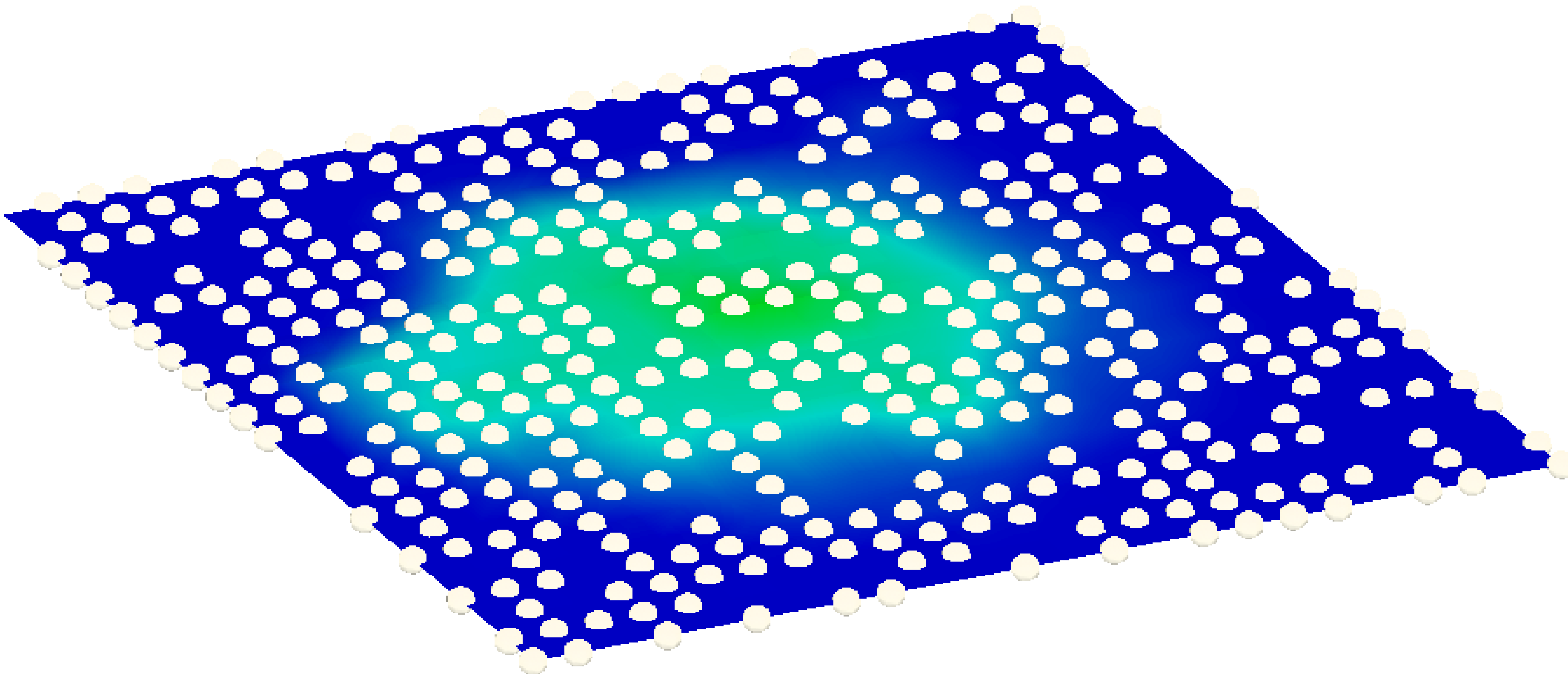
U_{10} V_{10} horizontal velocities
Geostrophic wind = (27.3, -3.9)
Pasquill stability class: Stable



HARMONIE U_{10} V_{10} data points

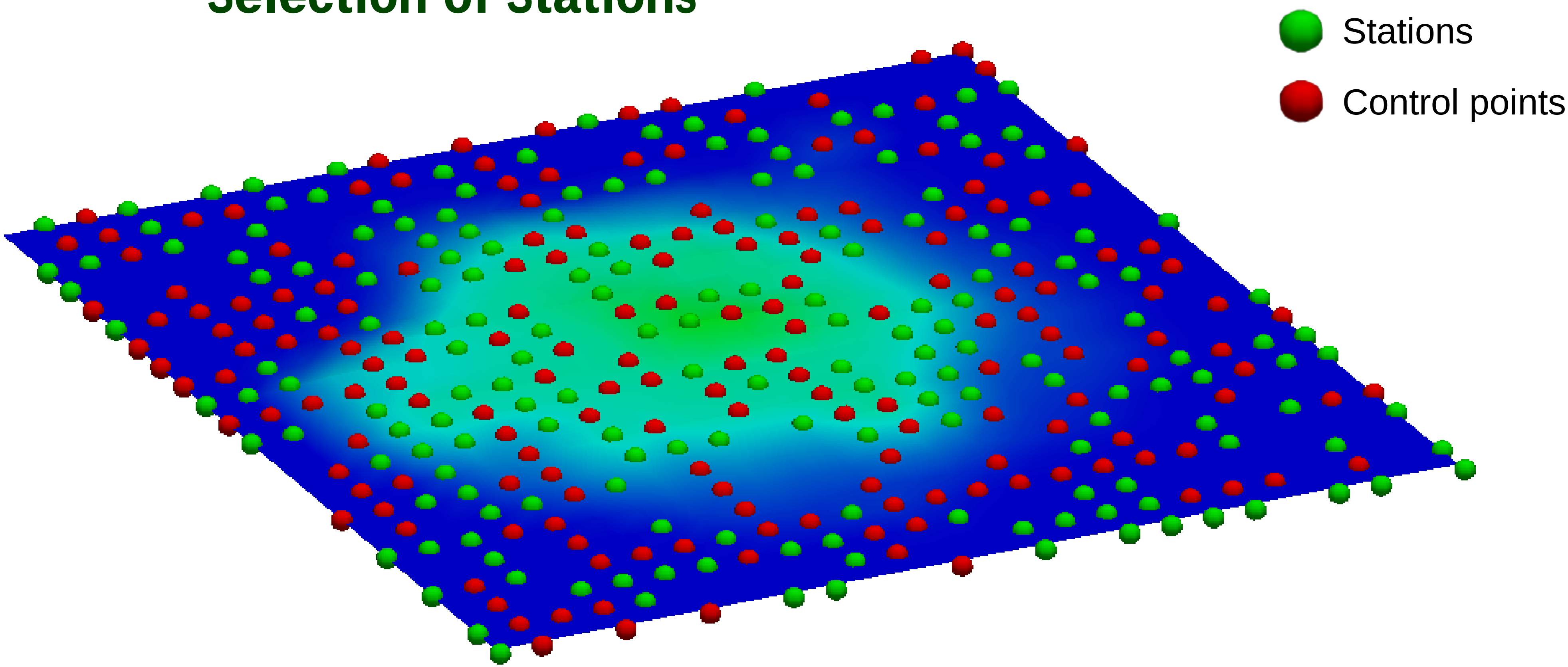
Wind Field Modeling

HARMONIE-FEM wind forecast



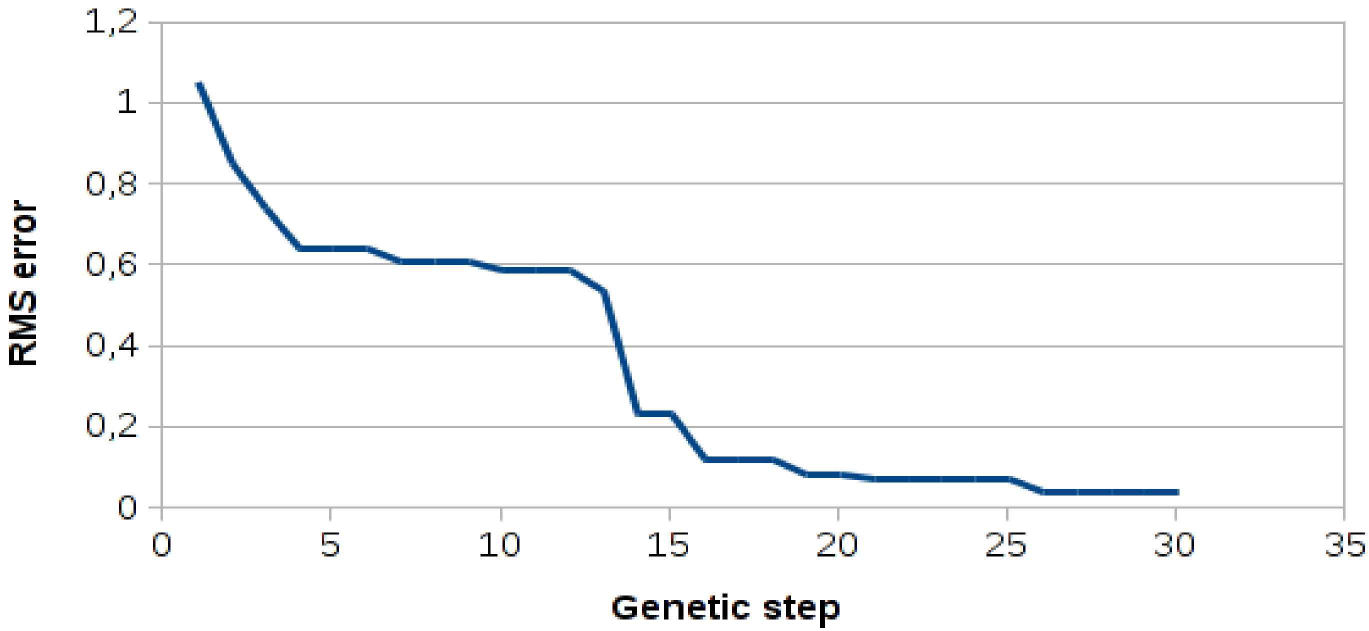
Used data (2/3 of total)

Selection of Stations



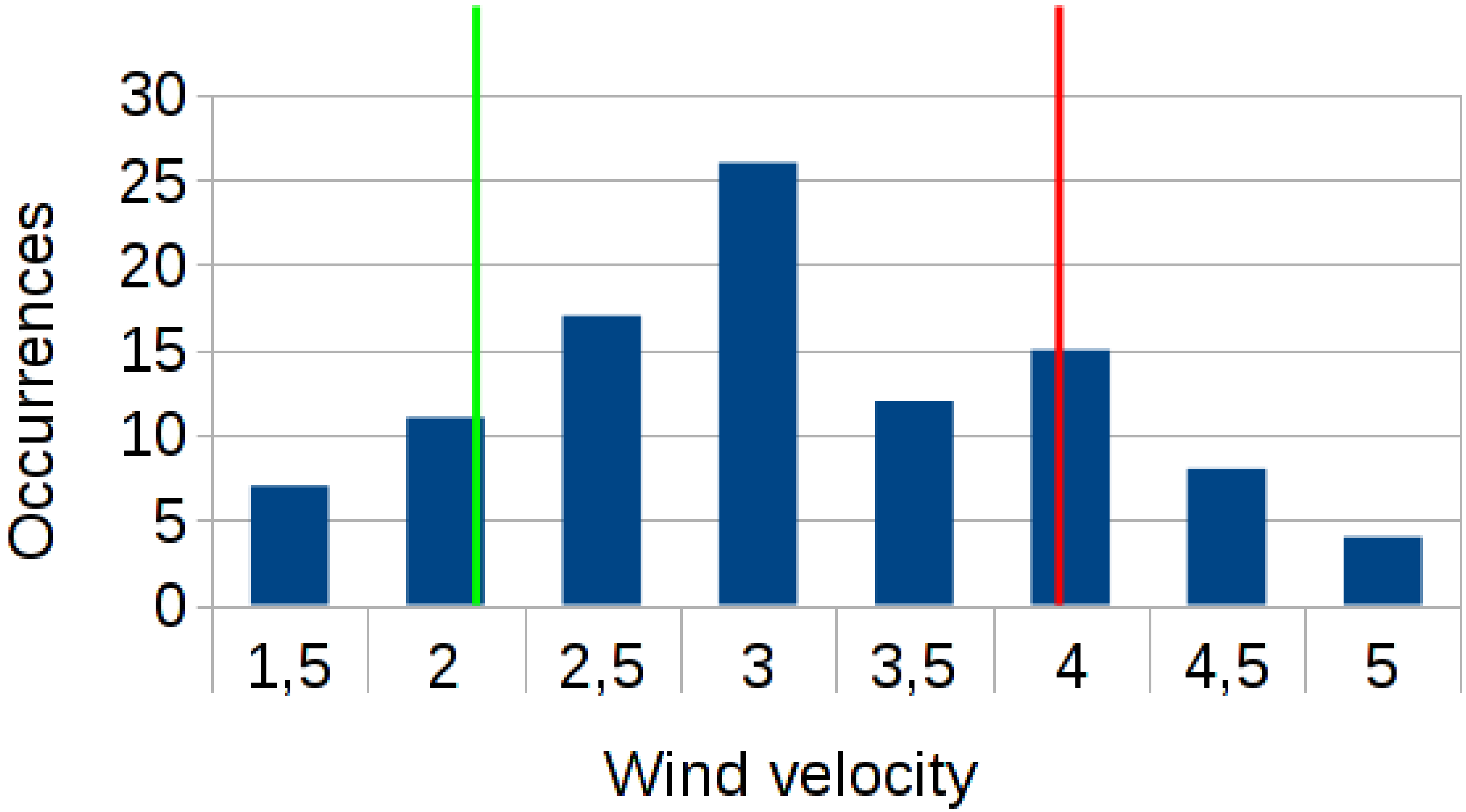
Stations (1/3) and control points (1/3)

Ensemble methods



New		Small changes	**Previous**	
Alpha	= 2.057920		Alpha	= 2.302731
Epsilon	= 0.950898		Epsilon	= 0.938761
Gamma	= 0.224911		Gamma	= 0.279533
Gamma'	= 0.311286		Gamma'	= 0.432957

Ensemble methods



Convection-diffusion-reaction equation

$$\frac{\partial \mathbf{c}}{\partial t} + \overbrace{\mathbf{u} \cdot \nabla \mathbf{c}}^{\text{Convective term}} = \overbrace{\nabla \cdot (\mathbf{K} \nabla \mathbf{c})}^{\text{Diffusive term}} + \underbrace{\mathbf{e}}_{\text{Emissión term}} + \overbrace{\mathbf{s}(\mathbf{c})}^{\text{Reactive term}} \qquad \text{in } \Omega$$

Boundary Conditions

$$\begin{cases} \mathbf{c}(\mathbf{x}, t) = \mathbf{c}^{emi}(\mathbf{x}) & \text{in } \Gamma_D\text{: Stack source point} \\ \mathbf{n} \cdot \mathbf{K} \nabla \mathbf{c} = -\mathbf{V}^d \mathbf{c} & \text{in } \Gamma_R\text{: Terrain} \\ \mathbf{n} \cdot \nabla \mathbf{c} = 0 & \text{in } \Gamma_N = \partial\Omega \backslash (\Gamma_D \cup \Gamma_R) \end{cases}$$

Initial Condition $\mathbf{c}(\mathbf{x}, 0) = \mathbf{c}^{ini}(\mathbf{x})$

c = concentration
$\mathbf{c}^{emi}$ = stack concentration
$\mathbf{c}^{ini}$ = initial concentration
u = wind velocity

K = diffusion matrix
e = emissión
s(c) = reactive term
$\mathbf{V}^d$ = deposition velocity