3-D Wind Field Simulation over Complex Terrain

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http://www.siani.es/
Meccano Method for Complex Solids
Algorithm steps

- Meccano construction
- Parameterization of the solid surface
  - Solid boundary partition
  - Floater’s parameterization
- Coarse tetrahedral mesh of the meccano
  - Partition into hexahedra
  - Hexahedron subdivision into six tetrahedra
- Solid boundary approximation
  - Kossaczky’s refinement
  - Distance evaluation
- Inner node relocation
  - Coons patches
- Simultaneous untangling and smoothing
  - SUS
Meccano Method
Simultaneous mesh generation and volumetric parameterization

- Parameterization
- Refinement
- Untangling & Smoothing
Meccano Method
Key of the method: SUS of tetrahedral meshes

Parameter space (meccano mesh)

Physical space (tangled mesh)

Optimization

Physical space (optimized mesh)
From the $i$-th solid surface triangulation patch to the $i$-th meccano face

Fixed boundary nodes,

$$Q'_k = \sum_j \lambda_{j,k} Q'_j$$

Compromise between area and shape

Physical Space  Parametric Space
Initial cube and its subdivision after three consecutive tetrahedron bisection

- 6 tetrahedra
- 12 tetrahedra
- 24 tetrahedra
- 48 tetrahedra

Local Refinement: Kossaczky’s Algorithm (JCAM 1994)

Meccano Method
Coons patches provide a reasonable initial location in physical domain for inner nodes

Segment (1D), surface (2D) and cube (3D) interpolations

\[
x(\xi, \eta) = (1 - \xi)x(0, \eta) + \xi x(1, \eta) + (1 - \eta)x(\xi, 0) + \eta x(\xi, 1) - [1 - \xi \xi] \begin{bmatrix} x(0, 0) & x(0, 1) \\ x(1, 0) & x(1, 1) \end{bmatrix} \begin{bmatrix} 1 - \eta \\ \eta \end{bmatrix}
\]

(a) parametric cube  
(b) bilinear interpolation in a rectangle
Local optimization

Objective: Improve the quality of the local mesh by minimizing an objective function

Local mesh

Optimized local mesh
Meccano Method
SUS Code: Weighted Jacobian Matrix on a Plane

Reference triangle

Physical triangle

Ideal triangle

\[ A = \begin{pmatrix} x_1 - x_0 & x_2 - x_0 \\ y_1 - y_0 & y_2 - y_0 \end{pmatrix} \]

\[ S = AW^{-1} \]

An algebraic quality metric of \( t \) (mean ratio)

\[ q = \frac{2\sigma}{||S||^2} = \frac{1}{\eta} \]

where:

\[ ||S|| = \sqrt{\text{tr}(S^T S)} \]

\[ \sigma = \det(S) \]
Original function:  \[ K(x) = \sum_{m=1}^{M} \frac{||S_m||^2}{2\sigma_m} \]

Modified function:  \[ K^*(x) = \sum_{m=1}^{M} \frac{||S_m||^2}{2h(\sigma_m)} \]

Modified function (blue) is regular in all \( \mathbb{R}^2 \) and it approximates the same minimum that the original function (red). Moreover, it allows a simultaneous untangling and smoothing of triangular meshes.

The extension to tetrahedral meshes is straightforward:

\[ \eta_m = \frac{||S_m||^2}{3[h(\sigma_m)]^{\frac{2}{3}}} \]

Meccano Method
SUS Code: Local objective function for plane triangulations
Meccano Method
Oil field strata meshing

Boundary surfaces

Volume mesh
Wind Field Modeling
Conquered challenges

- Mass consistent model (MMC) provides a useful method for local scale wind studies.
- Local wind 3D field using data measurements.
- Automatic process. Few model parameters.
- Low computational cost due to use of adaptive tetrahedral meshes and efficient solvers.

- Use of parallel GAs for parameter estimation.

Integration with mesoscale models PAG (2015)
- Coupling with HARMONIE leads to predictive capabilities.
- Use of HARMONIE outputs as MMC inputs.

Ensemble methods PAG (2015)
- Use of ensemble methods to obtain local scale wind prediction.
**Objective:** find the velocity field $\mathbf{u}$ that it adjusts to $\mathbf{u}_0$ verifying

- Incompressibility condition in the domain: $\nabla \cdot \mathbf{u} = 0$ in $\Omega$
- Impermeability condition on the terrain: $\mathbf{n} \cdot \mathbf{u} = 0$ on $\Gamma_a$
Then, $\mathbf{u}$ is the solution of the least-square problem: Find $\mathbf{u} \in \mathbb{K}$ verifying

$$E(\mathbf{u}) = \min_{\mathbf{v} \in \mathbb{K}} E(\mathbf{v})$$

$$\mathbb{K} = \{ \mathbf{v}; \nabla \cdot \mathbf{v} = 0, \mathbf{n} \cdot \mathbf{v} |_{\Gamma_a} = 0 \}$$

where $E(\mathbf{v}) = \frac{1}{2} \int_{\Omega} (\mathbf{v} - \mathbf{u}_0)^t \mathbf{P} (\mathbf{v} - \mathbf{u}_0) \, d\Omega$

We can write $E$ as follows

$$E(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \int_{\Omega} \left[ \alpha_1^2 \left( (\mathbf{u} - \mathbf{u}_0)^2 + (\mathbf{v} - \mathbf{v}_0)^2 \right) + \alpha_2^2 (\mathbf{w} - \mathbf{w}_0)^2 \right] \, d\Omega$$

$$\alpha = \frac{\alpha_1}{\alpha_2}$$
Lagrangian multiplier technique is used to solve this problem. So, if we introduce

\[ L(v, \lambda) = E(v) + \int_{\Omega} \lambda \nabla \cdot v \, d\Omega \]

its saddle point \((u, \phi)\) verifies the Euler-Lagrange equations:

\[-\nabla \cdot \left(P^{-1} \nabla \phi \right) = \nabla \cdot u_0 \quad \text{in } \Omega \]

\[-n \cdot P^{-1} \nabla \phi = n \cdot u_0 \quad \text{on } \Gamma_a \]

\[ \phi = 0 \quad \text{on } \Gamma_b \]

and, finally, the adjusted velocity field is obtained by:

\[ u = u_0 + P^{-1} \nabla \phi \]
Construction of the observed wind

Horizontal interpolation

\[
\mathbf{u}_0(z_m) = \xi \frac{\sum_{n=1}^{N} \frac{u_n}{d_n^2}}{\frac{1}{N} \sum_{n=1}^{N} \frac{1}{d_n^2}} + (1 - \xi) \frac{\sum_{n=1}^{N} \frac{u_n}{|\Delta h_n|}}{\frac{1}{N} \sum_{n=1}^{N} \frac{1}{|\Delta h_n|}}
\]

\[0 \leq \xi \leq 1\]
Construction of the observed wind

Vertical extrapolation (log-linear wind profile)

\[ u_0(z) = \rho(z) u_0(z_{sl}) + [1 - \rho(z)] u_g \]

\[ u_0(z) = \frac{u^*}{k'} \left( \log \frac{z}{z_0} - \Phi_m \right) \]
Friction velocity:  
\[ u^* = \frac{k \ u_0(z_m)}{\log \frac{z_m}{z_0} - \Phi_m} \]

Height of the planetary boundary layer:  
\[ z_{pbl} = \frac{\gamma |u^*|}{f} \]

\( f = 2\Omega \sin \phi \) is the Coriolis parameter, being \( \Omega \) the Earth rotation and \( \phi \) is the latitude.

\( \gamma \) is a parameter depending on the atmospheric stability.

Mixing height:

- in neutral and unstable conditions:  
  \[ h = z_{pbl} \]

- in stable conditions:  
  \[ h = \gamma' \sqrt{\frac{|u^*| L}{f}} \]

Height of the surface layer:  
\[ z_{sl} = \frac{h}{10} \]
FE solution is needed for each individual.

\[ F(\alpha, \xi, \gamma, \gamma') = \sqrt{\frac{1}{2k} \sum_{i=1}^{k} \left[ u - u_0 \right]_i^2 + \left[ v - v_0 \right]_i^2} \]
Wind Field Modeling
Forecasting over Complex Terrain

Wind3D Code (freely-available)
http://www.dca.iusiani.ulpgc.es/Wind3D/

- Configuration
- Tetrahedral Mesh
- Wind Data
  - Measurement stations
  - WRF
  - HARMONIE

Wind3D
UCD Files (inp, vtk,...)
Visualization with Paraview (freeware) or AVS (commercial)
Terrain approximation

Max height 1950m

Max height 925m

Topography from Digital Terrain Model

HARMONIE discretization of terrain

Terrain elevation (m)
Spatial discretization

FEM computational mesh

HARMONIE mesh
\( \Delta h \sim 2.5 \text{km} \)

Terrain elevation (m)
Possible solutions for a suitable data interpolation in the FE domain:

- Use U10 and V10 supposing it is 10m over the FE terrain (used in this talk)
- Given a point of FE domain, find the closest one in HARMONIE domain grid
- Other possibilities can be considered
Wind Field Modeling
HARMONIE-FEM wind forecast

Wind magnitude at 10m over terrain

FEM wind

HARMONIE wind

Wind velocity (m/s)

0 2.5 5 7.5 10 11.5
Wind on the terrain surface

Interpolated field from HARMONIE

Resulting field with the mass consistent model
1. Multiple numerical predictions using slightly different conditions

1. HARMONIE data + MMC model → local wind forecast

1. The main idea
   a. Use HARMONIE data as reliable points, as stations or as control points for GAs
   b. Run different sets of experiments → perturbation
HARMONIE U10 V10 data points

U_{10} V_{10} horizontal velocities
Geostrophic wind = (27.3, -3.9)
Pasquill stability class: Stable
Used data (2/3 of total)
Selection of Stations

Stations (1/3) and control points (1/3)
Ensemble methods

New
Alpha = 2.057920
Epsilon = 0.950898
Gamma = 0.224911
Gamma' = 0.311286

Previous
Alpha = 2.302731
Epsilon = 0.938761
Gamma = 0.279533
Gamma' = 0.432957

Small changes
Ensemble methods
Convection-diffusion-reaction equation

\[ \frac{\partial c}{\partial t} + u \cdot \nabla c = \nabla \cdot (K \nabla c) + e + s(c) \quad \text{in } \Omega \]

**Emissión term**

**Boundary Conditions**

\[
\begin{align*}
    c(x, t) &= c^{emi}(x) \quad \text{in } \Gamma_D: \text{ Stack source point} \\
    n \cdot K \nabla c &= -V^d c \quad \text{in } \Gamma_R: \text{ Terrain} \\
    n \cdot \nabla c &= 0 \quad \text{in } \Gamma_N = \partial \Omega \setminus (\Gamma_D \cup \Gamma_R)
\end{align*}
\]

**Initial Condition**

\[ c(x, 0) = c^{ini}(x) \]

- \( c = \text{ concentration} \)
- \( c^{emi} = \text{ stack concentration} \)
- \( c^{ini} = \text{ initial concentration} \)
- \( u = \text{ wind velocity} \)
- \( K = \text{ diffusion matrix} \)
- \( e = \text{ emissión} \)
- \( s(c) = \text{ reactive term} \)
- \( V^d = \text{ deposition velocity} \)