# Tetrahedral Mesh Optimization Combining Boundary and Inner Node Relocation and Adaptive Local Refinement

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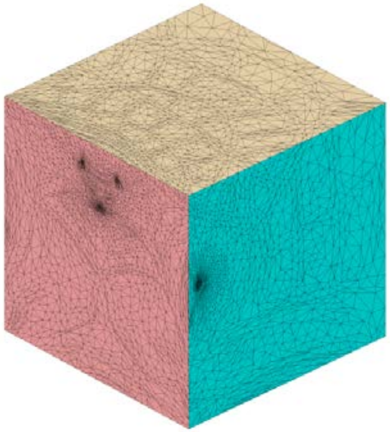
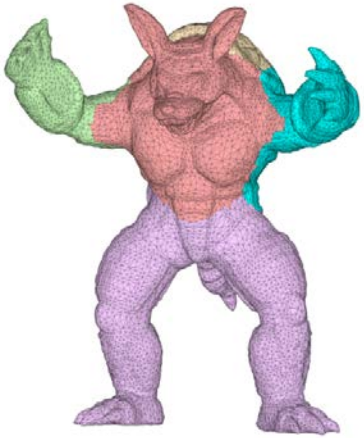
# Motivation

## Mesh Genus-Zero Solids

- **Mesh genus-zero solids and get a valid volumetric parameterization**



(a) boundary surfaces



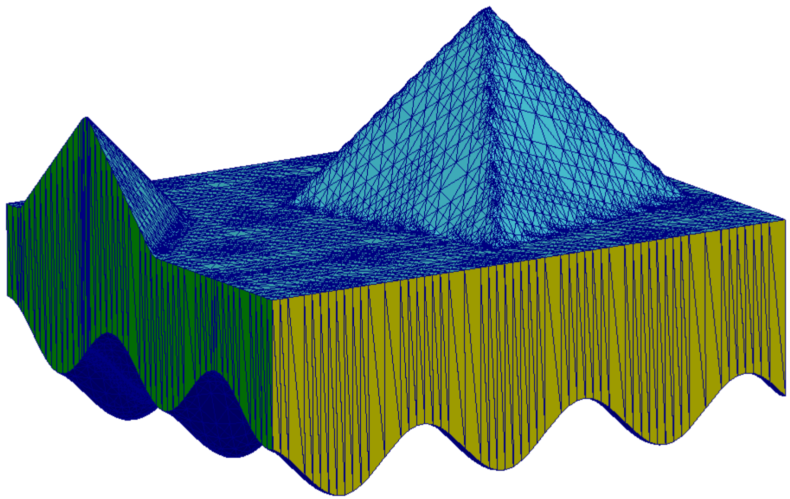
(b) parametric domain

- **Improve the quality of meshes generated ussing the Meccano method**

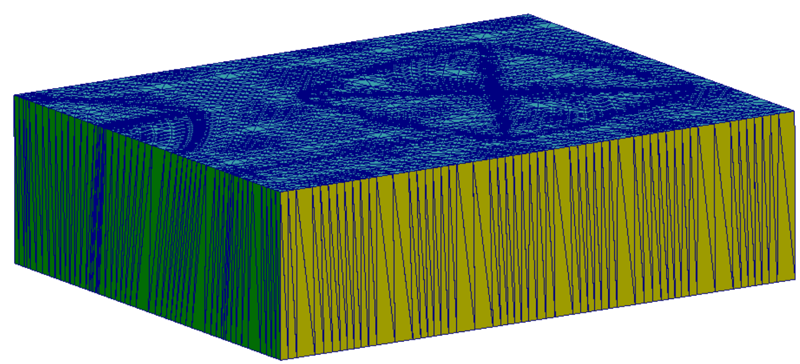
- **Main drawback: boundary nodes attached at their location**

  **Smooth of bondary nodes**

## 1. Map physical boundary surface to meccano boundary

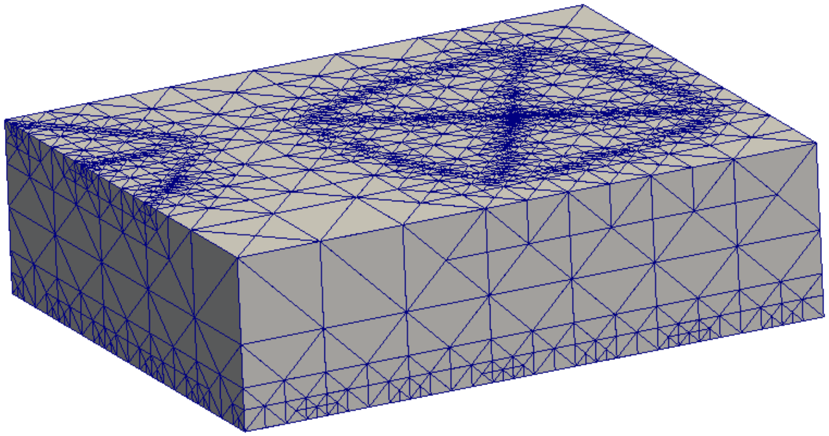


(a) solid boundary surfaces

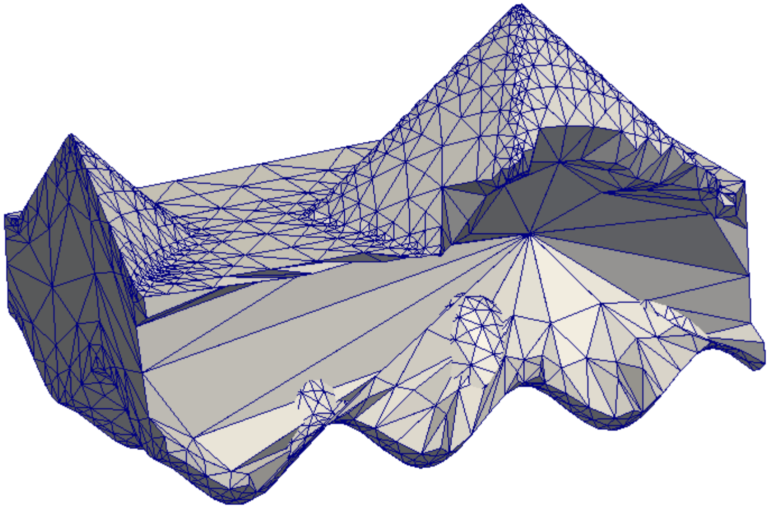


(b) parametric domain

## 2. Mesh and refine the meccano to capture solid surfaces

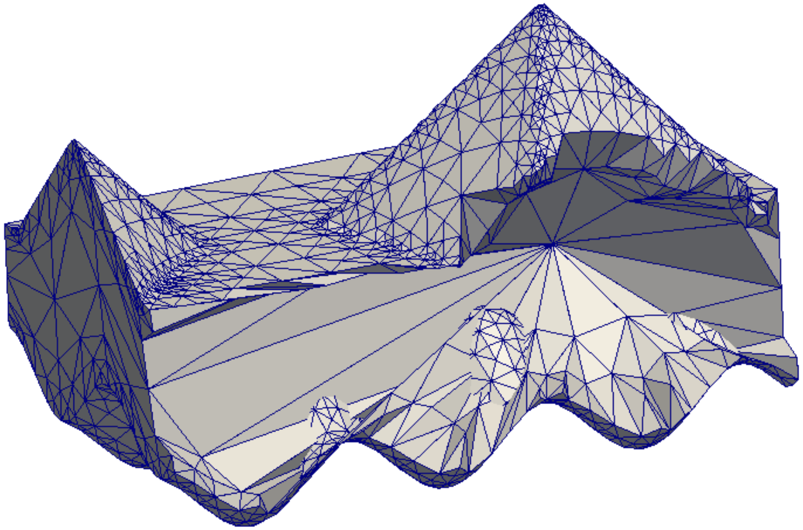


(a) refined meccano mesh

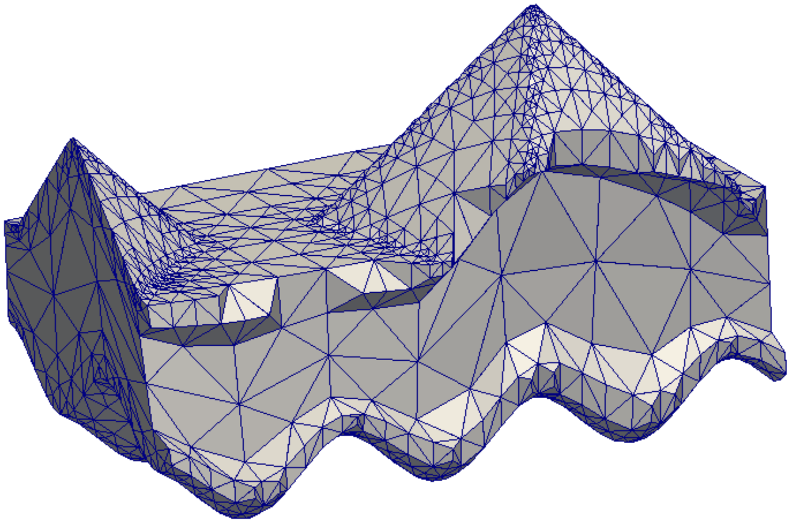


(b) solid mesh

## 3. Apply Coons patches to get a reasonable location for inner nodes

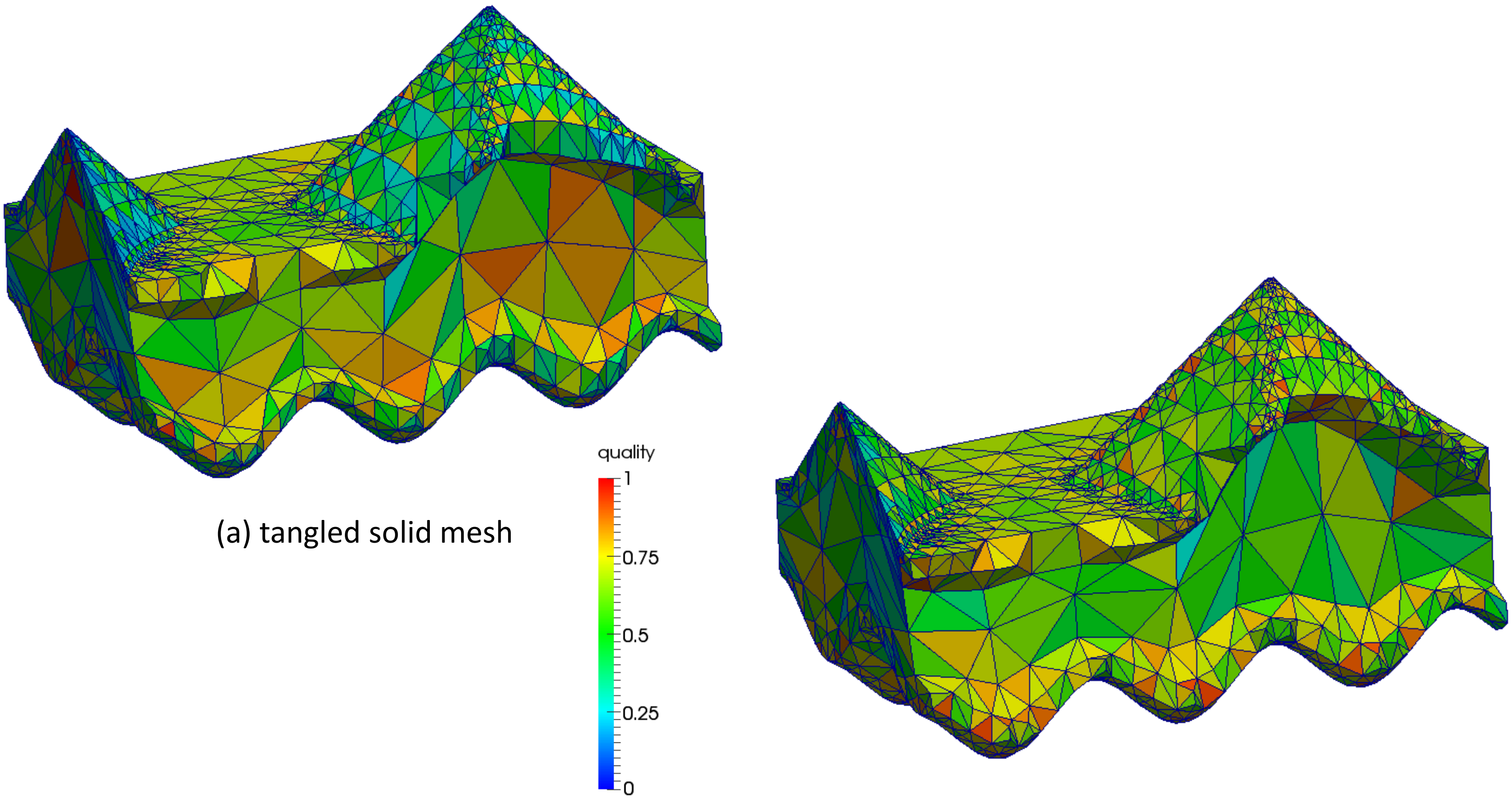


(a) solid mesh before Coons patches



(b) solid mesh after Coons patches

## 4. Simultaneous untangle and smoothing to get a valid mesh



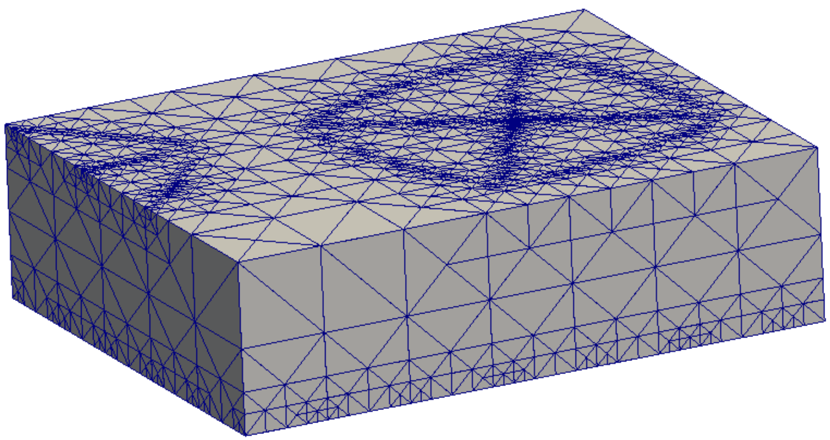

(a) tangled solid mesh

(b) untangled and smoothed solid mesh

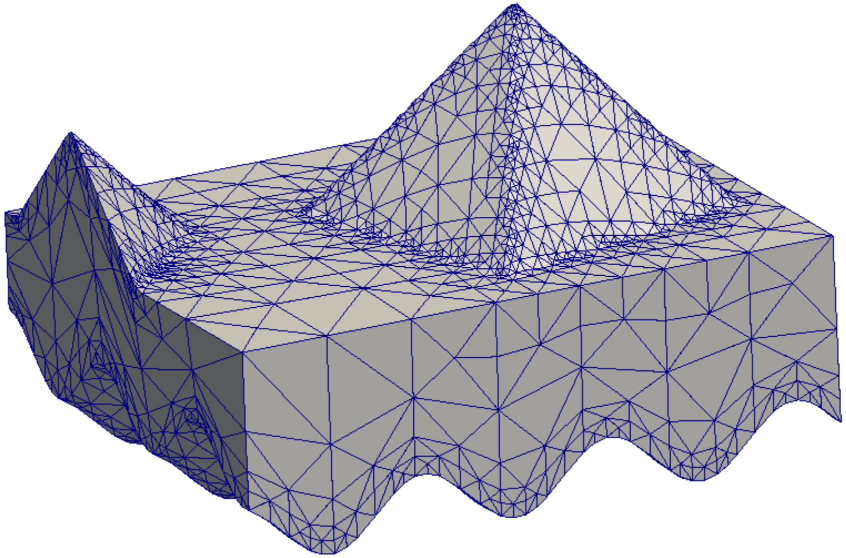
# The Meccano Method

## Resulting Volume Parameterization

- **Volumetric parameterization from uniform structure Kossascky mesh to solid in physical domain**



(a) uniform Kossascky meccano mesh



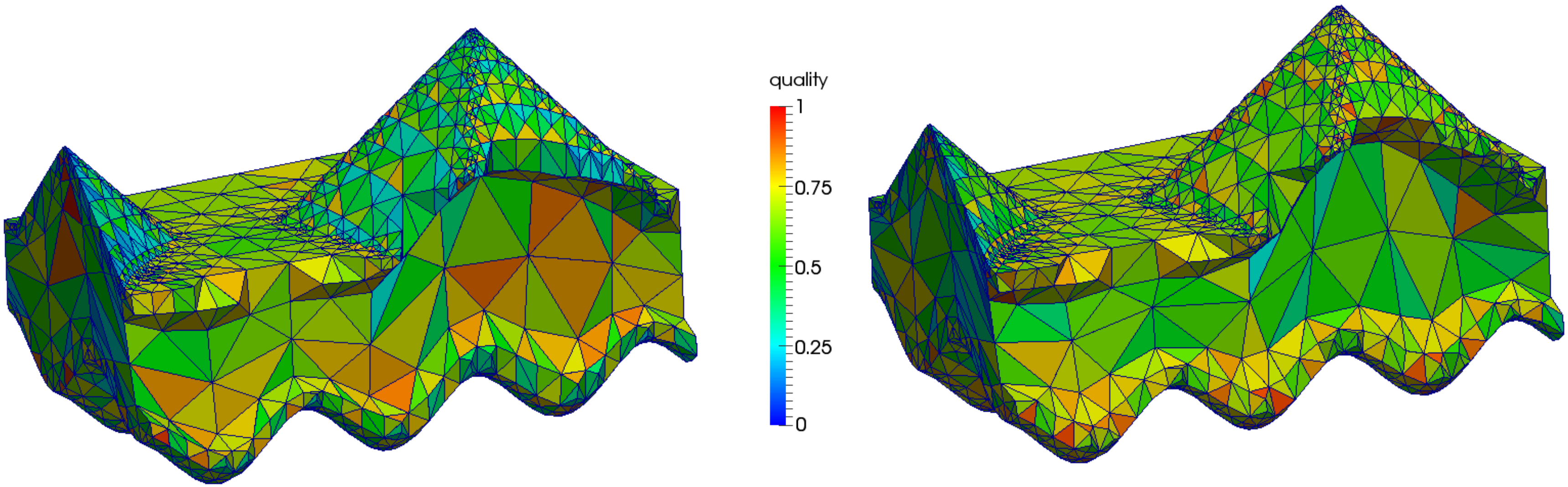
(b) solid mesh in physical domain

# Tetrahedral Mesh Smoothing

## Inner Node Smoothing

- **Unconstrained optimization problem in 3D physical domain**
  - Physical coordinates as optimization variables
  - Quality measurement in physical domain

- **Parametric *counterpart* as ideal → reduce the distortion**




(a) before optimization

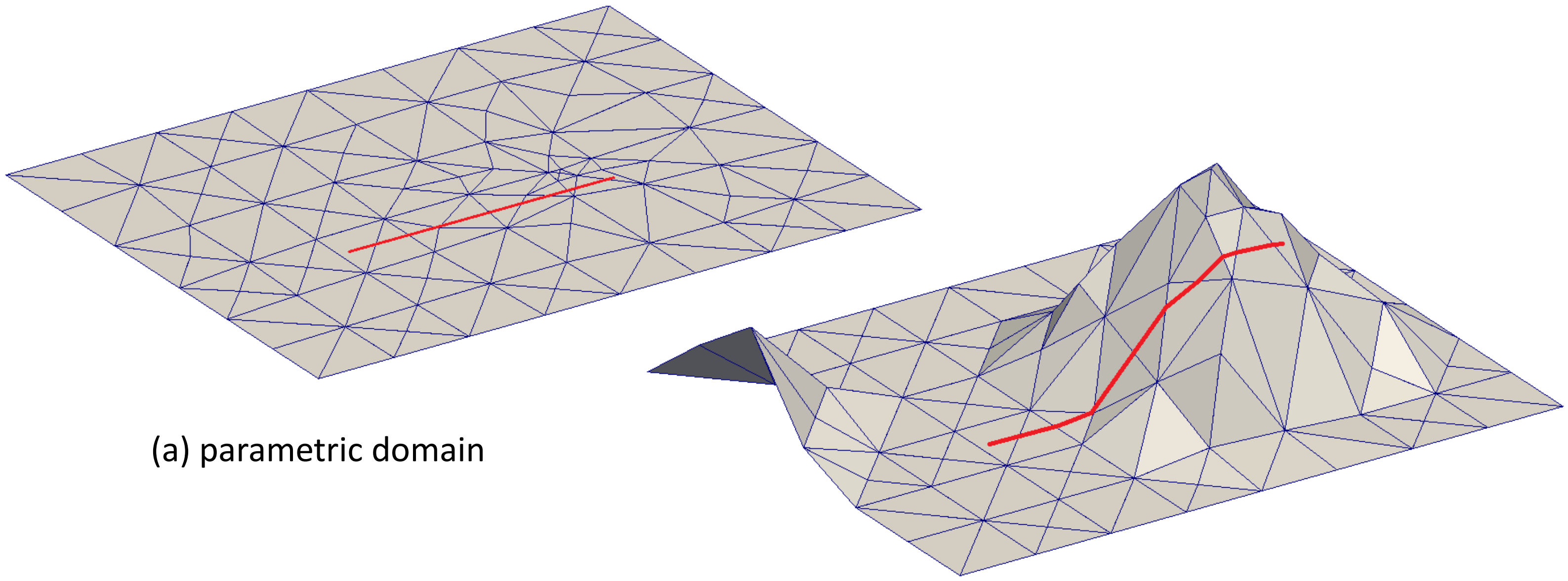
(b) after optimization

# Tetrahedral Mesh Smoothing

## Boundary Node Smoothing. Parameterizations

- **Exploit mesh parametrization to simplify the optimization problem**

**3D (physical) surface constrained** → **mesh parameterization** → **2D (parametric) rectangle constrained**



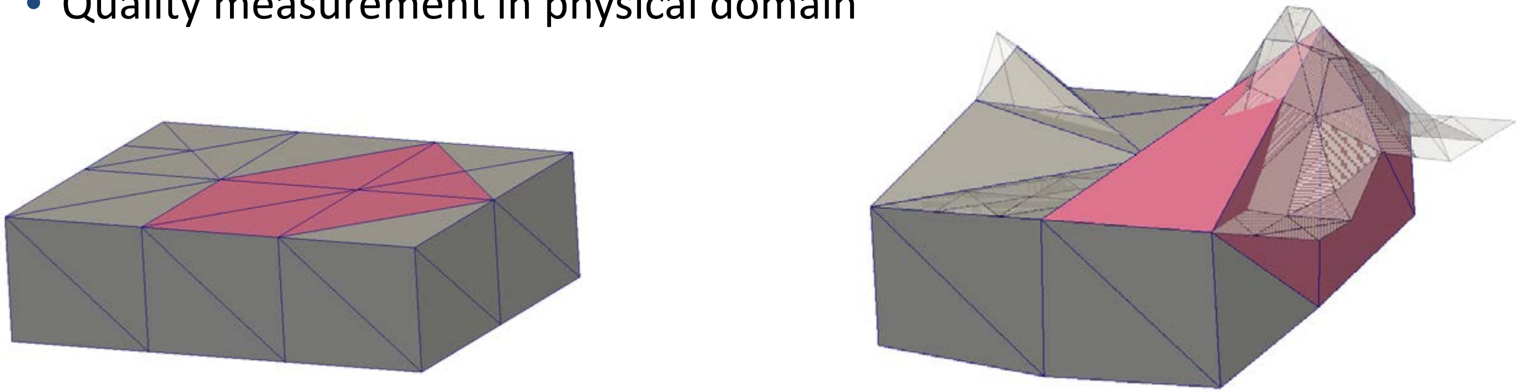
(a) parametric domain

(b) physical domain

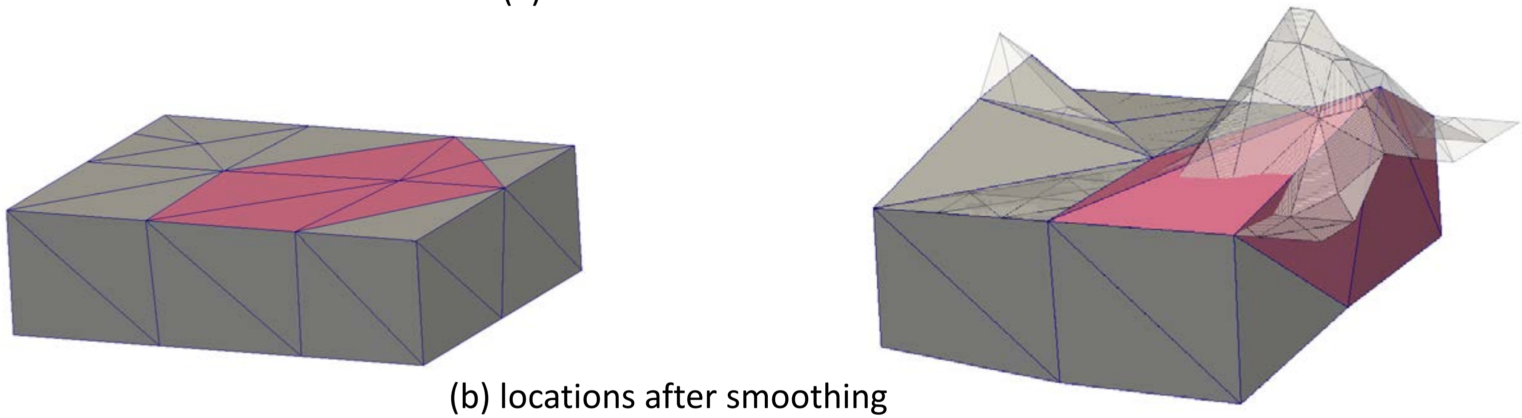
# Tetrahedral Mesh Smoothing

## Boundary Node Smoothing. Optimization

- **Optimization**
  - Parametric coordinates as optimization variables
  - Quality measurement in physical domain



(a) Initial locations

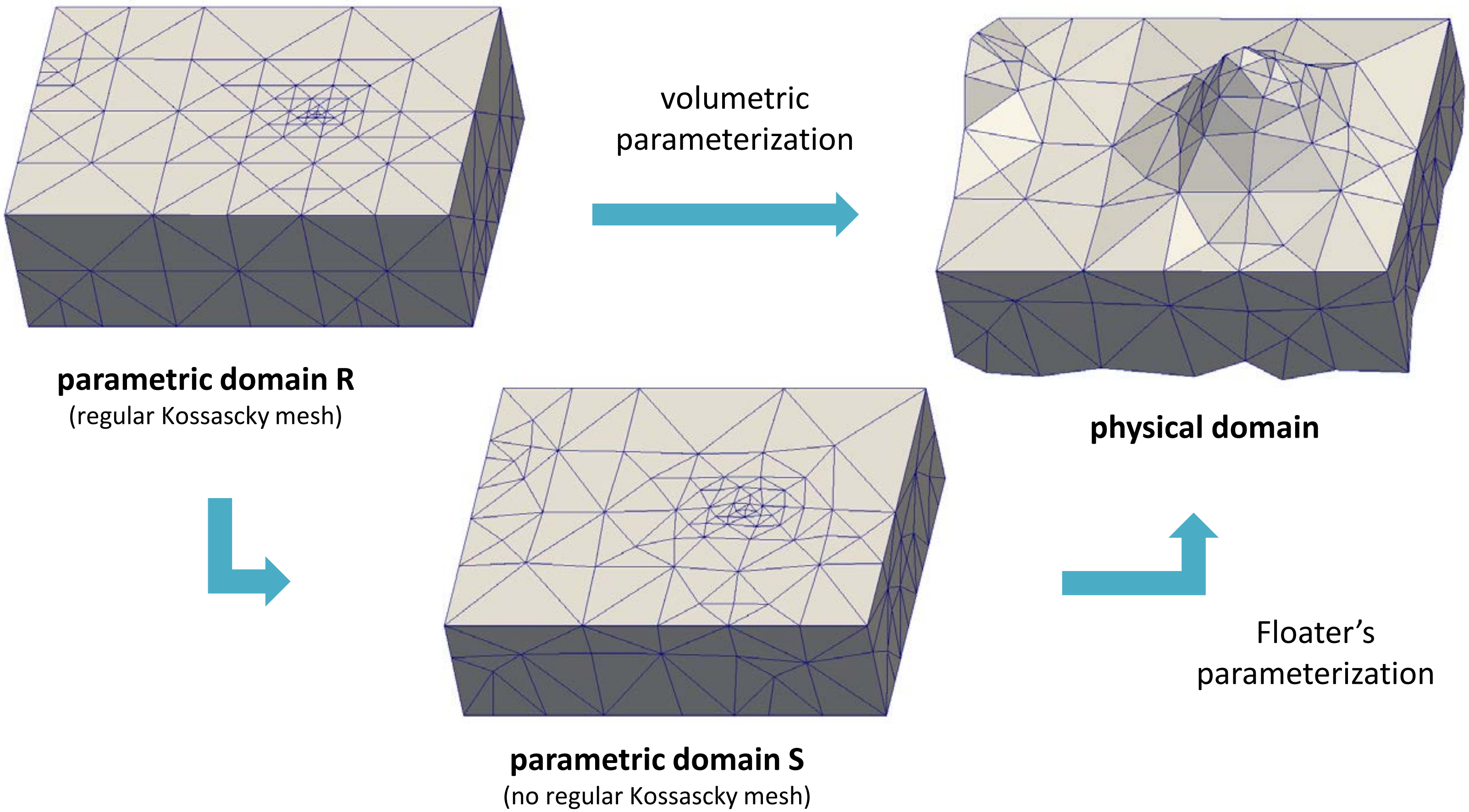


(b) locations after smoothing

# Tetrahedral Mesh Smoothing

## Boundary Node Smoothing. Parametric Domains

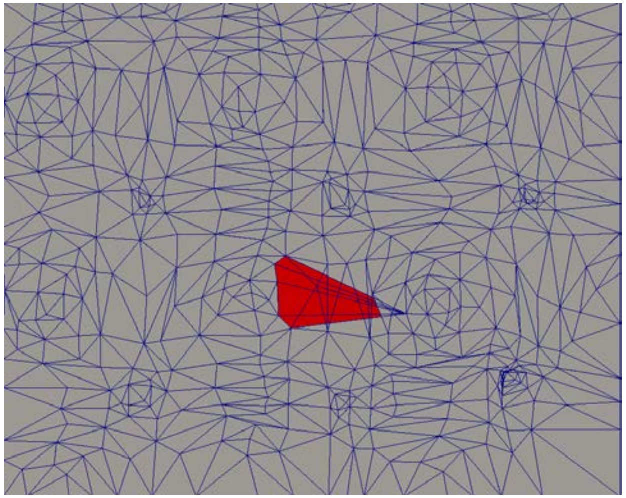
- **Creating an additional parametric domain for smoothing porpouse only**



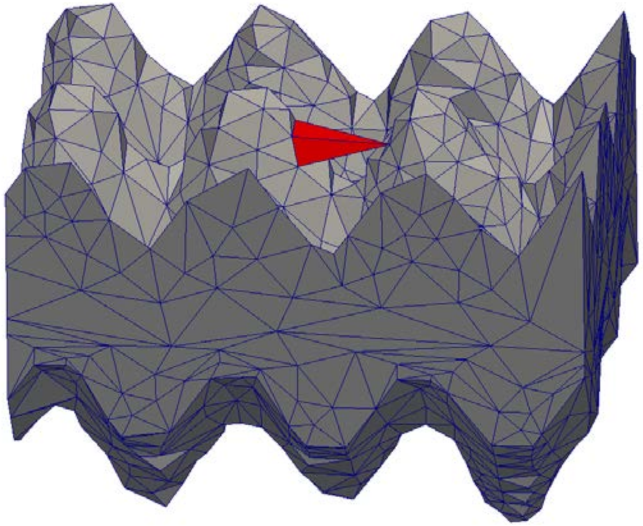
# Tetrahedral Mesh Smoothing

## Validity of Boundary Mesh

- **Geometry preservation**



(a) parametric domain



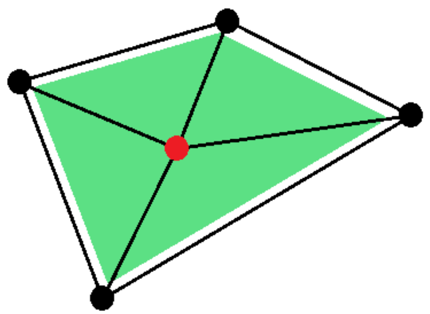
(b) physical domain

- **The resulting mesh is tangled. There are no inverted tetrahedral but some of them are overlapped ⟶ no valid mesh**

- **Imposing validity for boundary faces of the meccano**

$$\left|K_\eta\right|_p^*(\mathrm{u},\mathrm{v}) = \left[\sum_{m=1}^{M} \eta_m^{*\,p}\left(\mathbf{x}(\mathrm{u},\mathrm{v})\right)\right]^{1/p}$$

Extra term in the objective function that penalties tetrahedral collapse in the parametric domain.



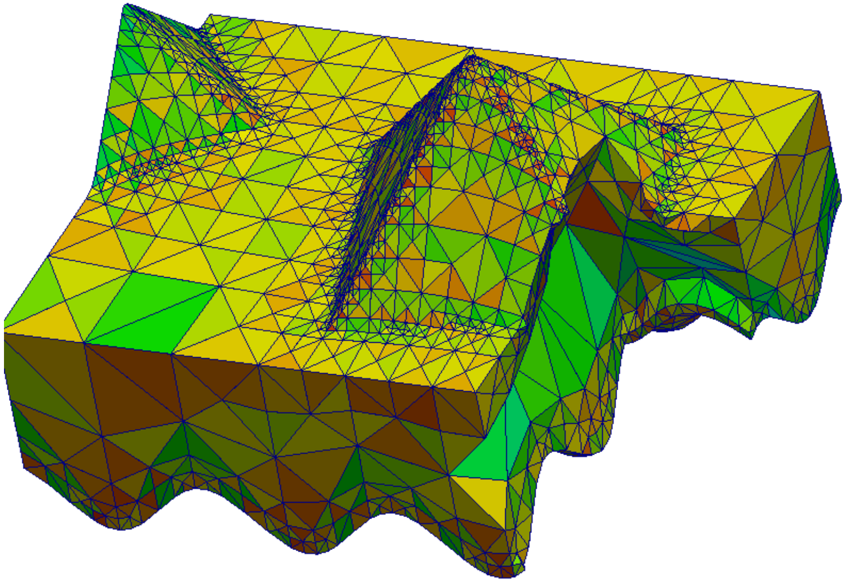
$$V(\mathrm{u},\mathrm{v}) = \sum_{m=1}^{M} \frac{1}{h(mín\{a_m,\tau\})}$$

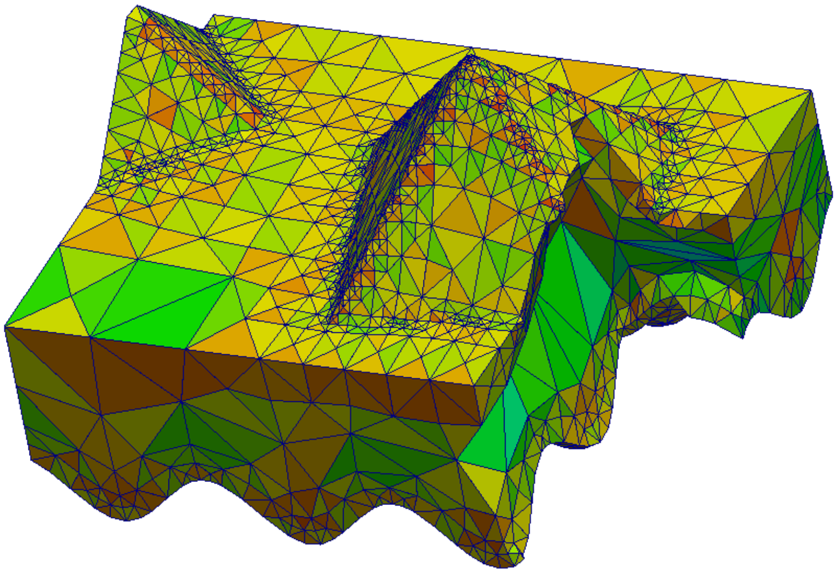
$a_m$: area ratio over star area  $\tau$: thickness of forbidden region

# Tetrahedral Mesh Smoothing

## Boundary Node Smoothing

- **Analogous procedure for edge (1D) optimization**





- **Boundary nodes smoothing could cause loose of both volume and geometry**

  **Iterative Refinement, untangle and smoothing stages**

# Tetrahedral Mesh Smoothing

## Bound Smoothing Area (Buffering algorithm)

- **Mesh must be smoothed after refinement**

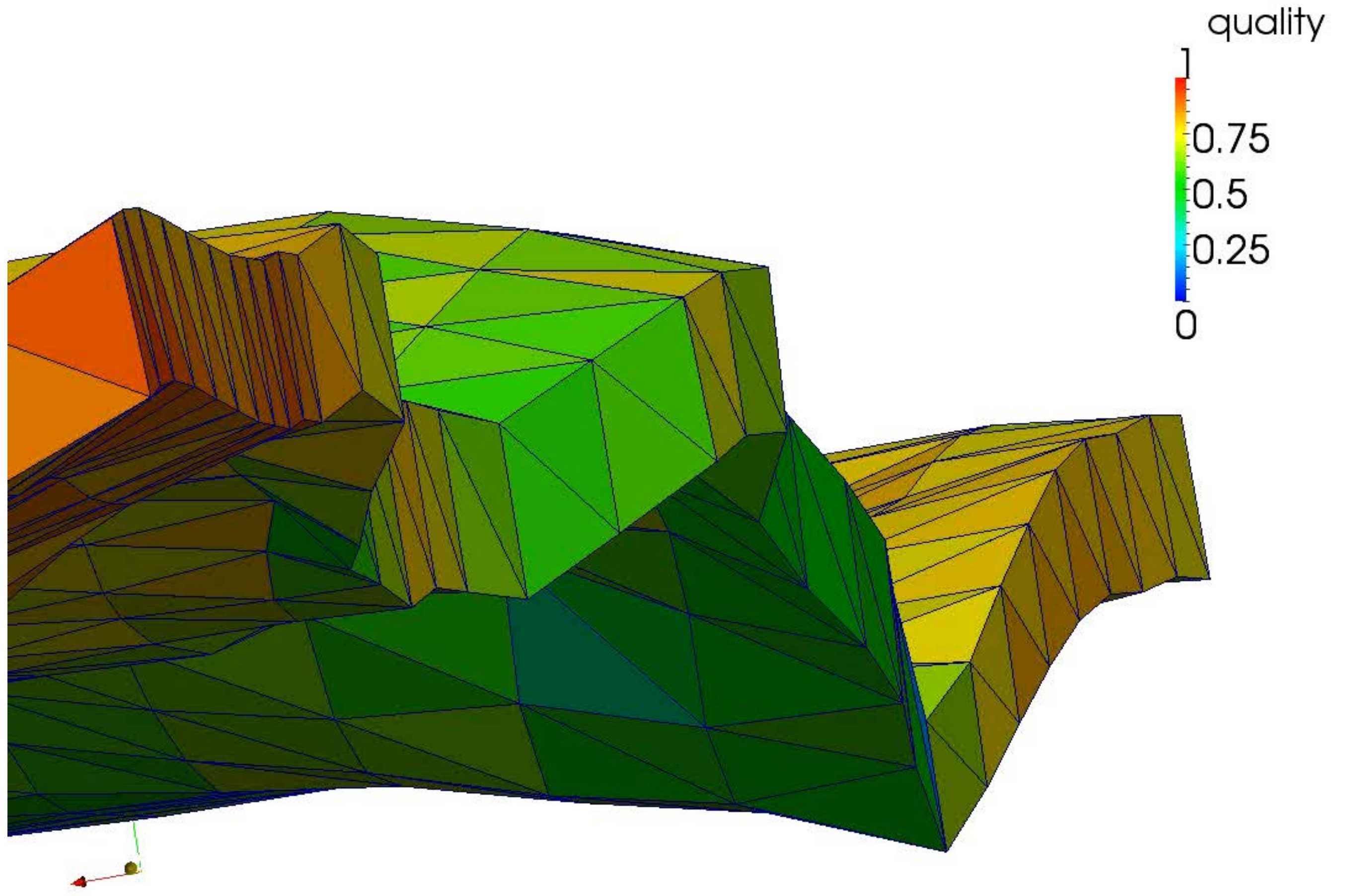
- **Buffering algorithm focus smoothing in areas where refinement toke place**

  1. **Initialize with new nodes a buffer of pending nodes to smooth**
  2. **Smooth first pending node**
  3. **If displacement ≥ prescribed threshold ⇒ enqueue neighbor nodes**
  4. **If any pending node ⇒ return to 2**

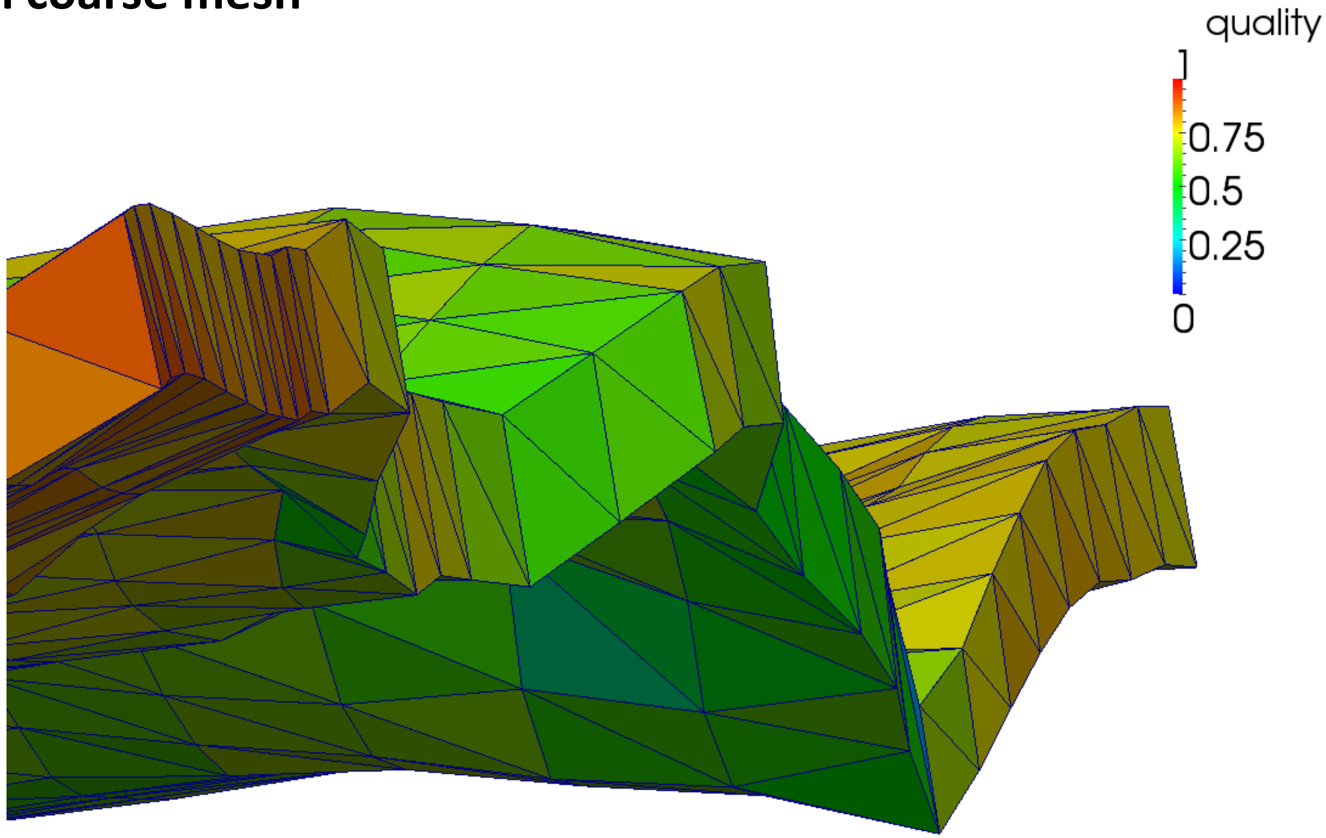
- **Drastic reduction in the number of star to smooth**

# Aplications

## Oil Field Meshing



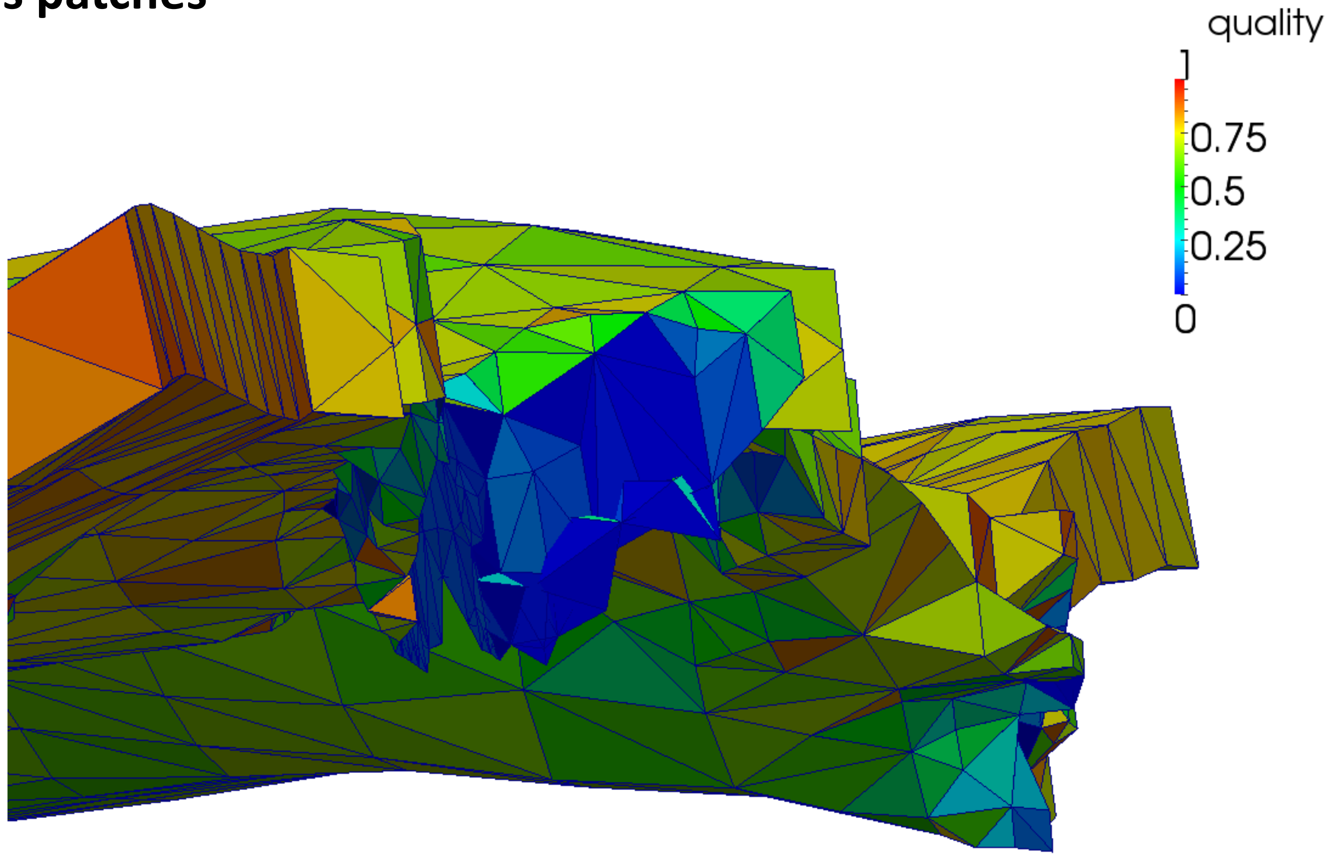
- **Initial coarse mesh**



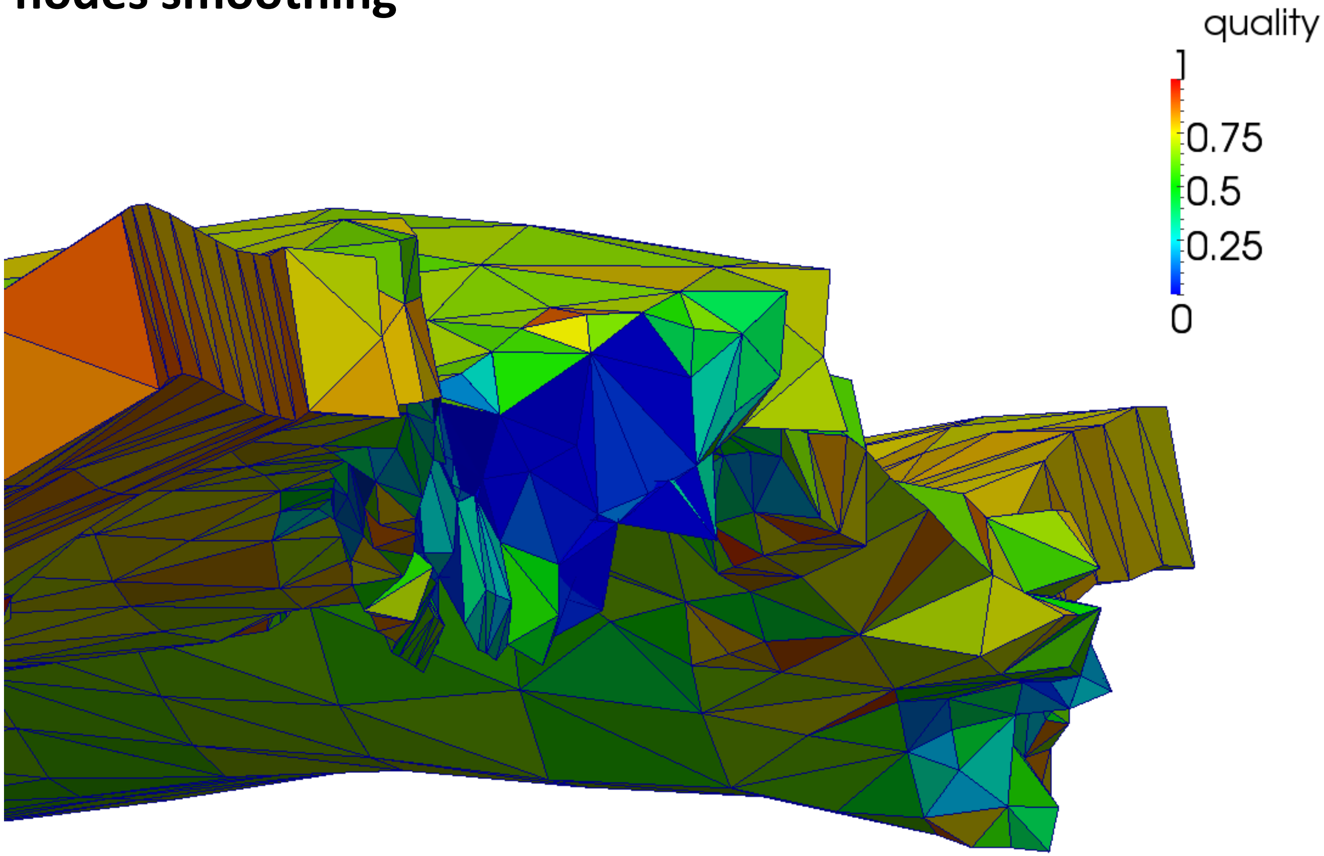
# Aplications

## Oil Field Meshing

- **Coons patches**



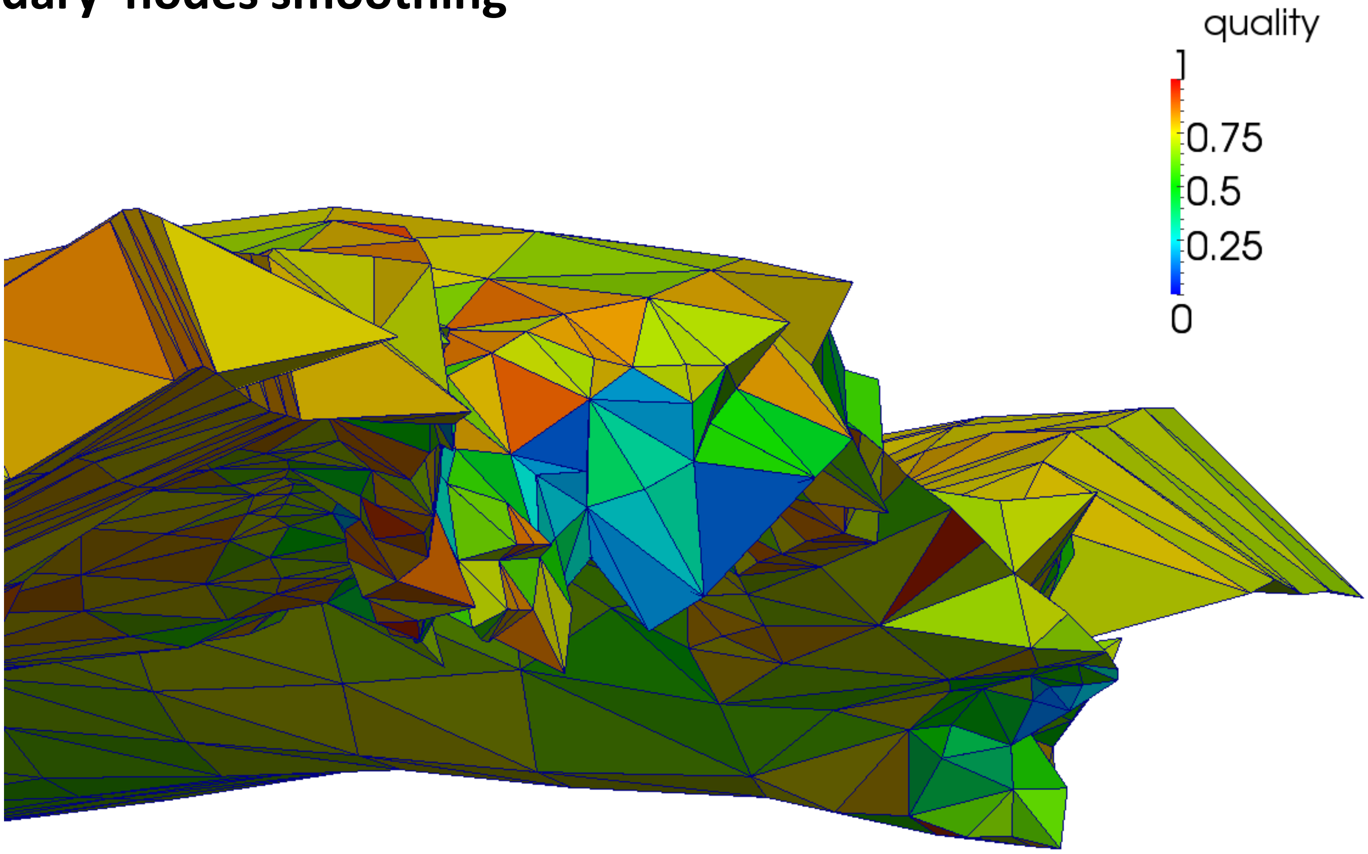
- **Inner nodes smoothing**



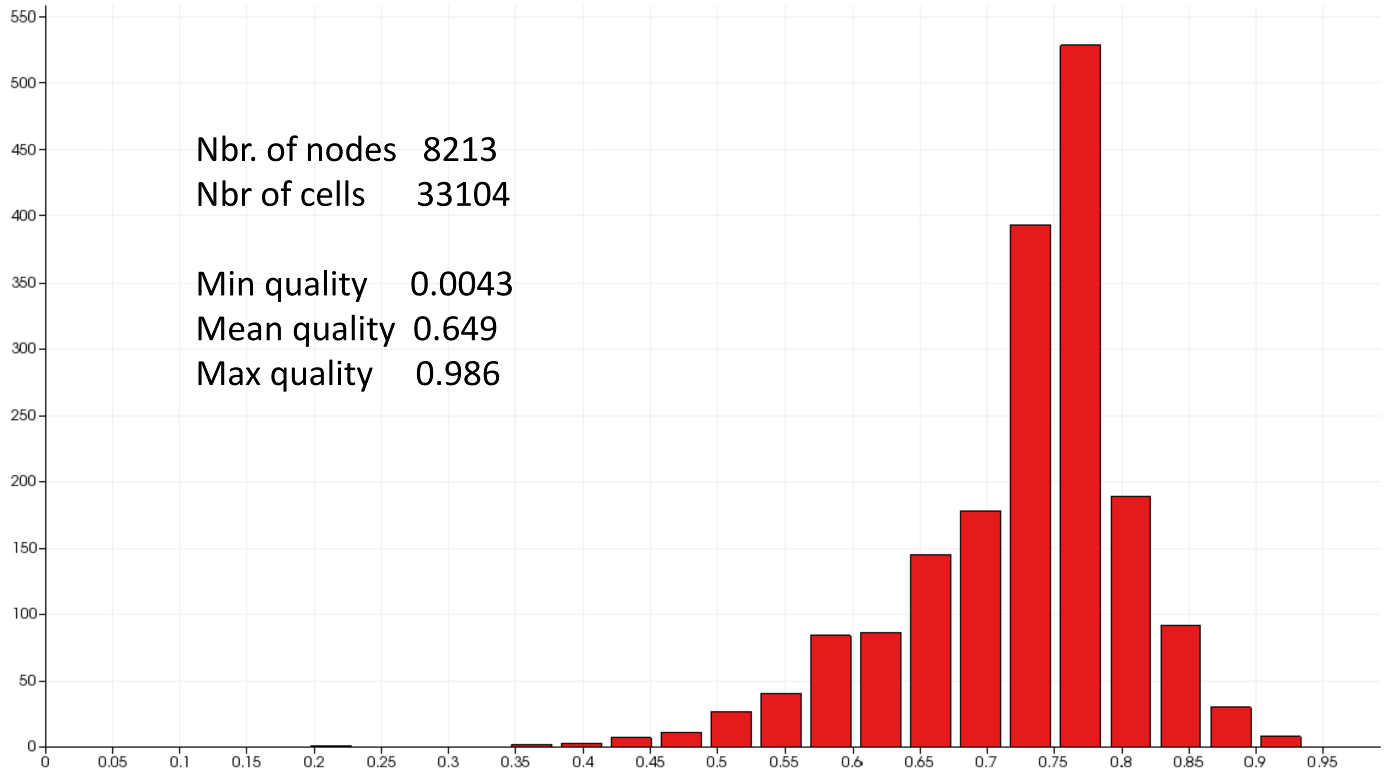
# Aplications

## Oil Field Meshing

- **Boundary nodes smoothing**

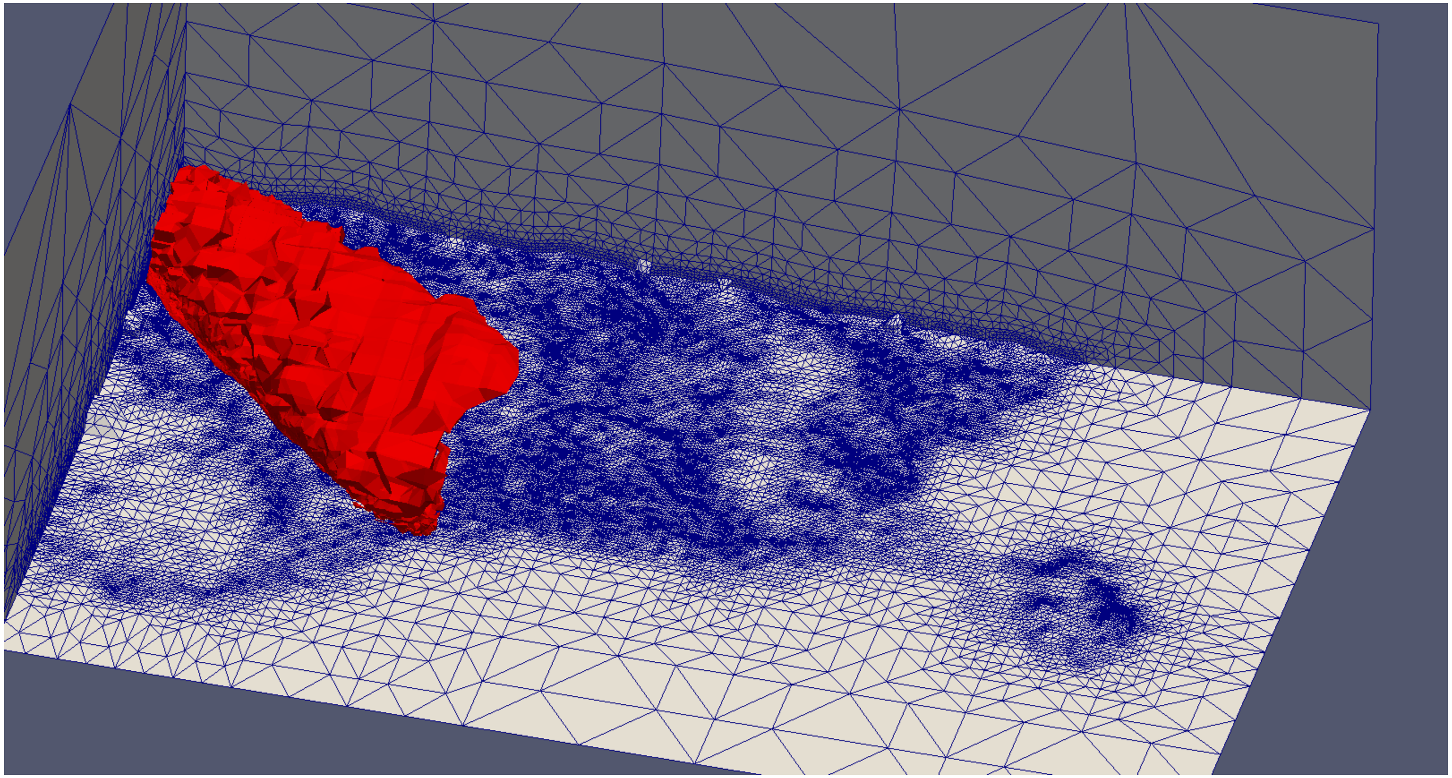


- **Tetrahedral quality**



# Aplicatio

## Troposphere Meshing for Wind Forec



# Conclusio

- **Conclusions**
  - An innovative method has been developed for bot boundary smoothing and volumetric parametrization genus-zero solid
  - This work upgrades The Meccano Method by addi boundary nodes relocation
  - The procedure obtains good quality meshes
  - The idea can be applied to any mesh generation metho that uses surface parameterization
  - The procedure requires minimum user intervention
  - The algorithm has a low computational cost and admi paralellization

# Thanks for your attention!