T-spline Parameterization of 2D Geometries Based on the Meccano Method with a New T-mesh Optimization Algorithm

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http://www.dca.iusiani.ulpgc.es/proyecto2012-2014

16th IMR (2007)

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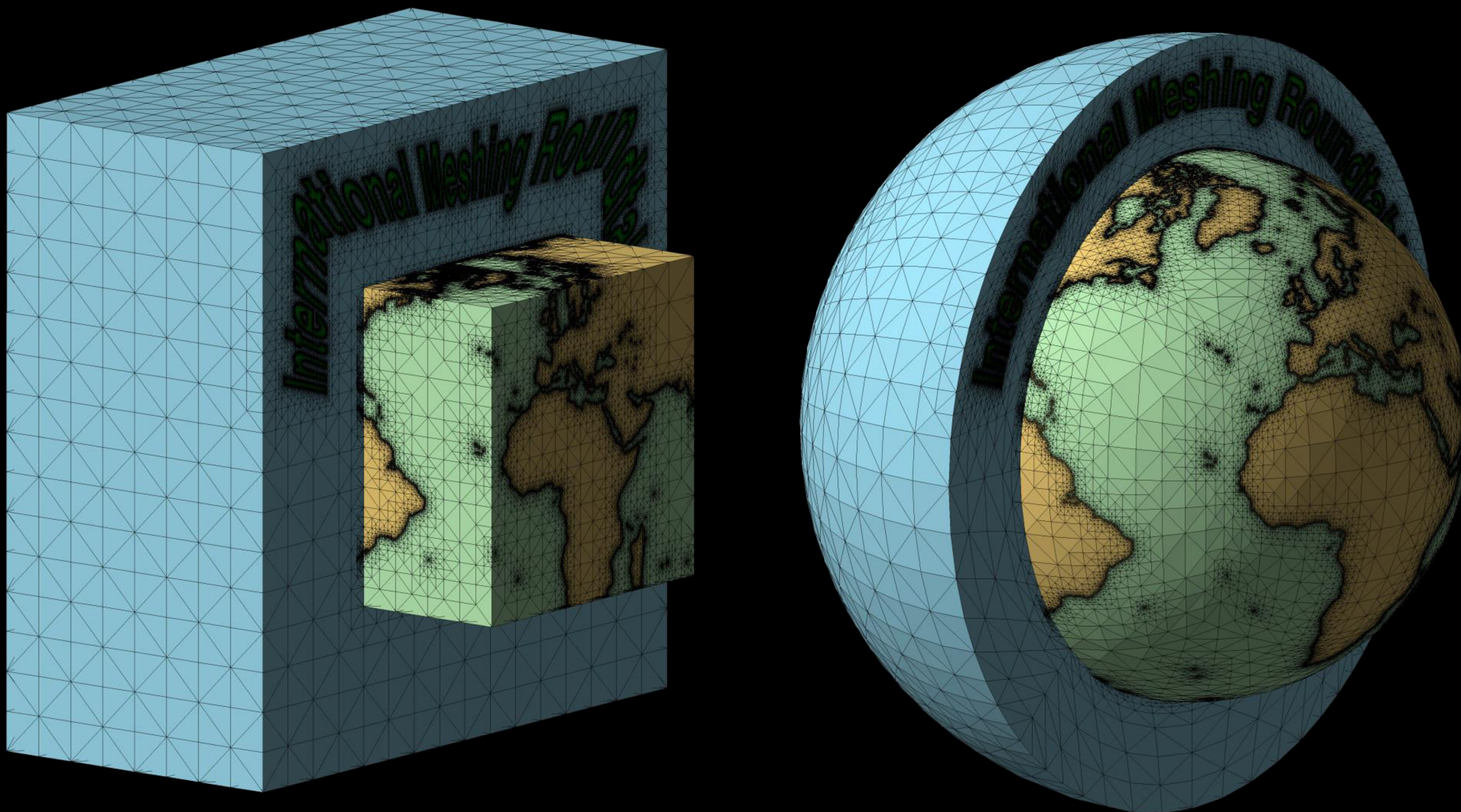

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  - Application to isogeometric modeling and analysis
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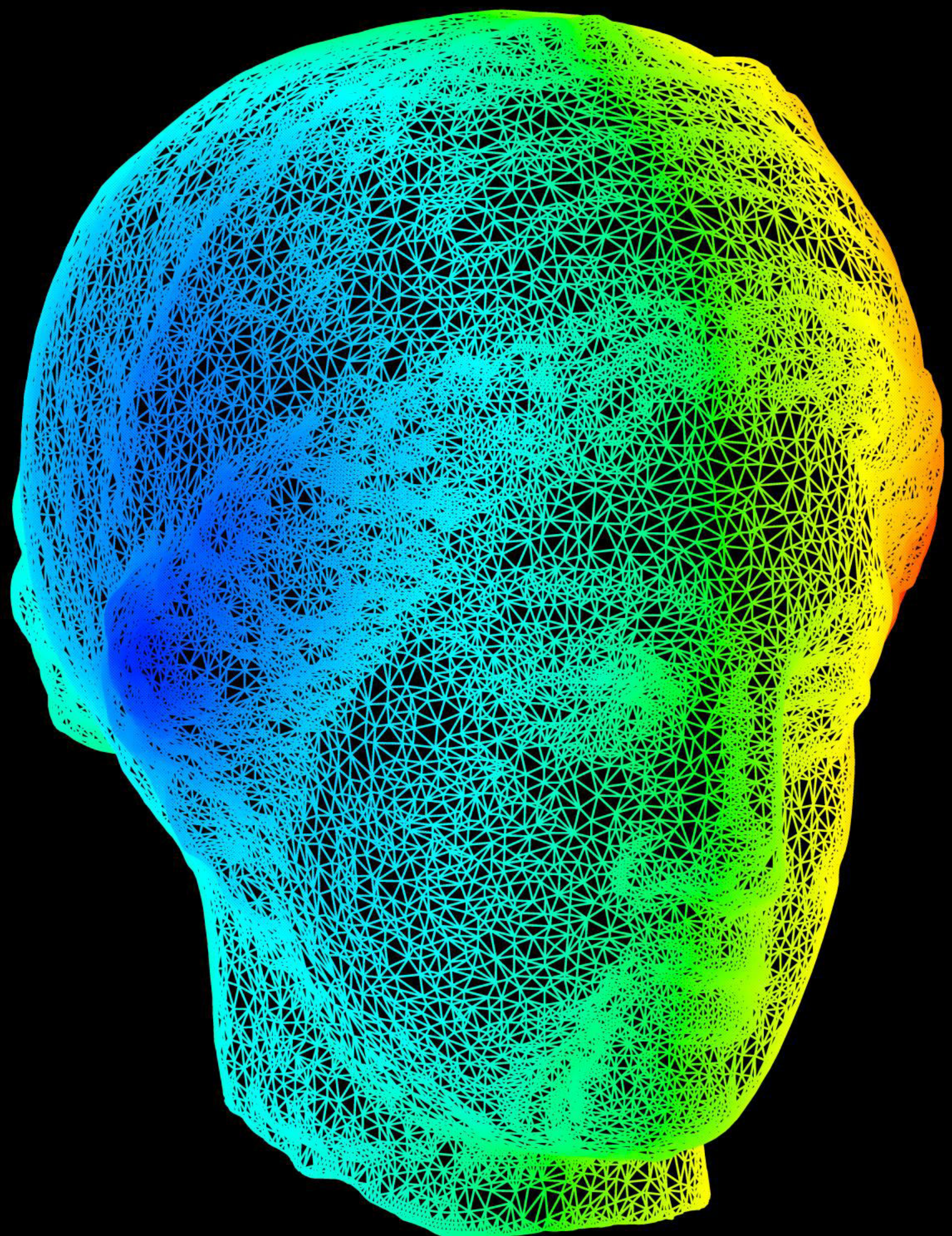
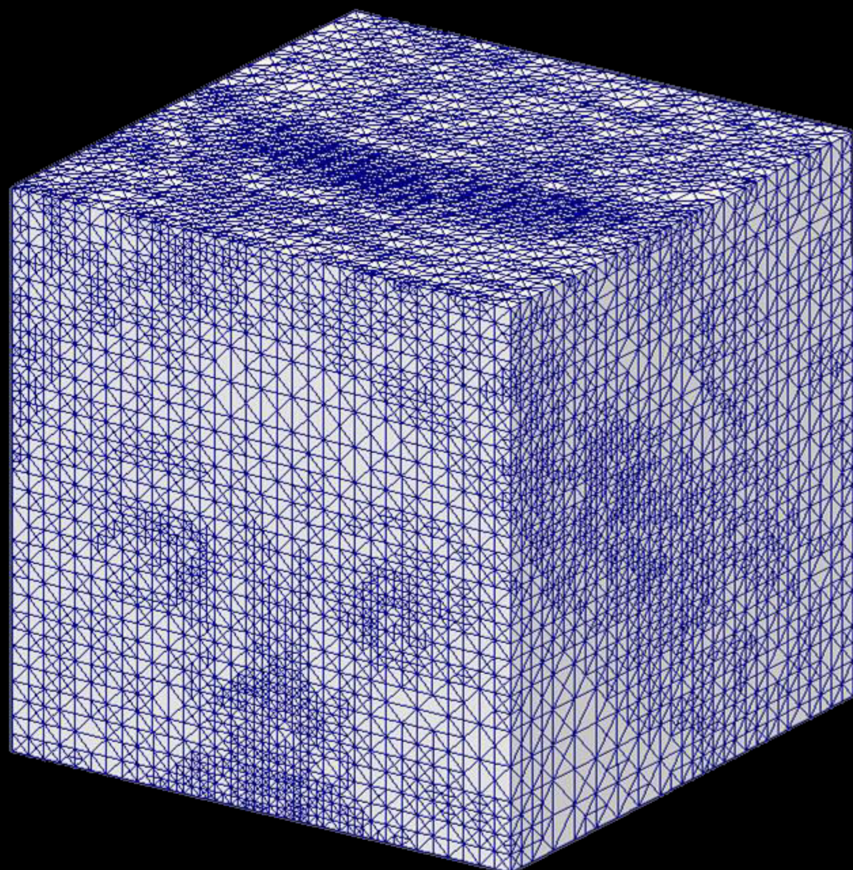
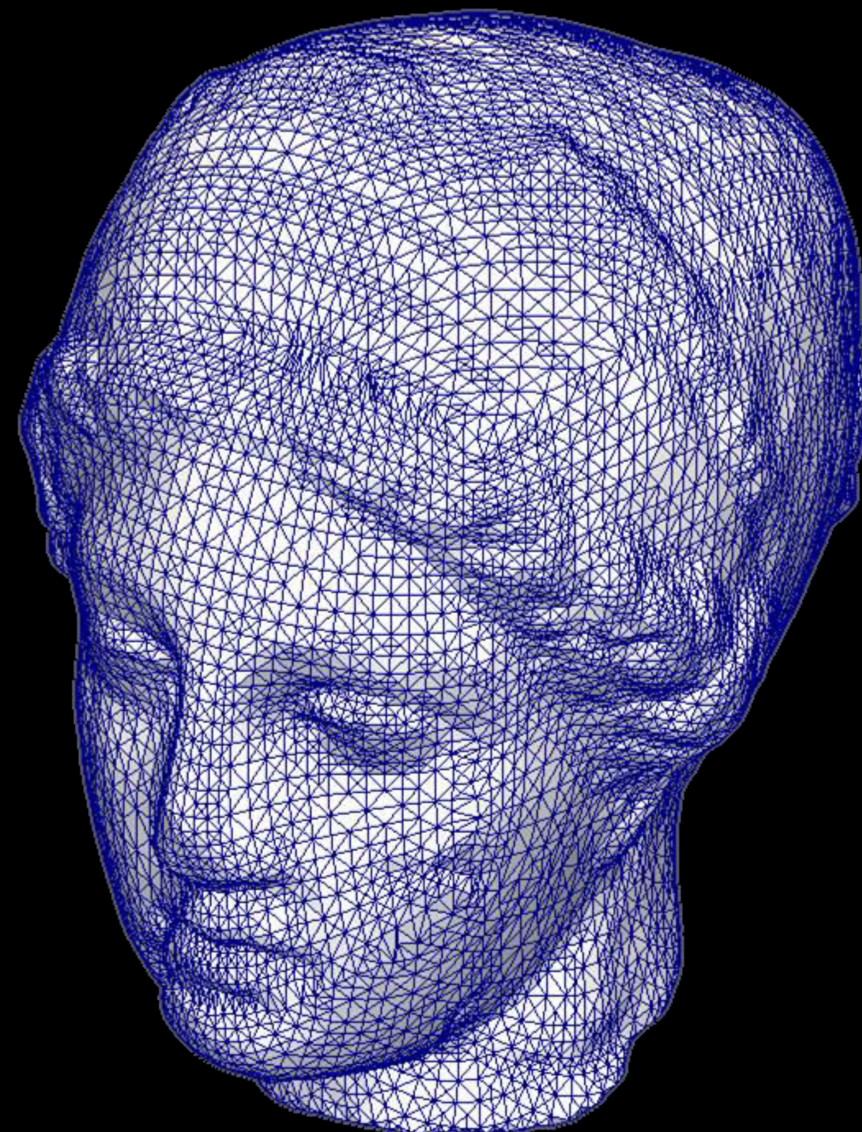


INPUT DATA: Surface Triangulation

http://www.cyberware.com/

Adaptive Isogeometric Refinement

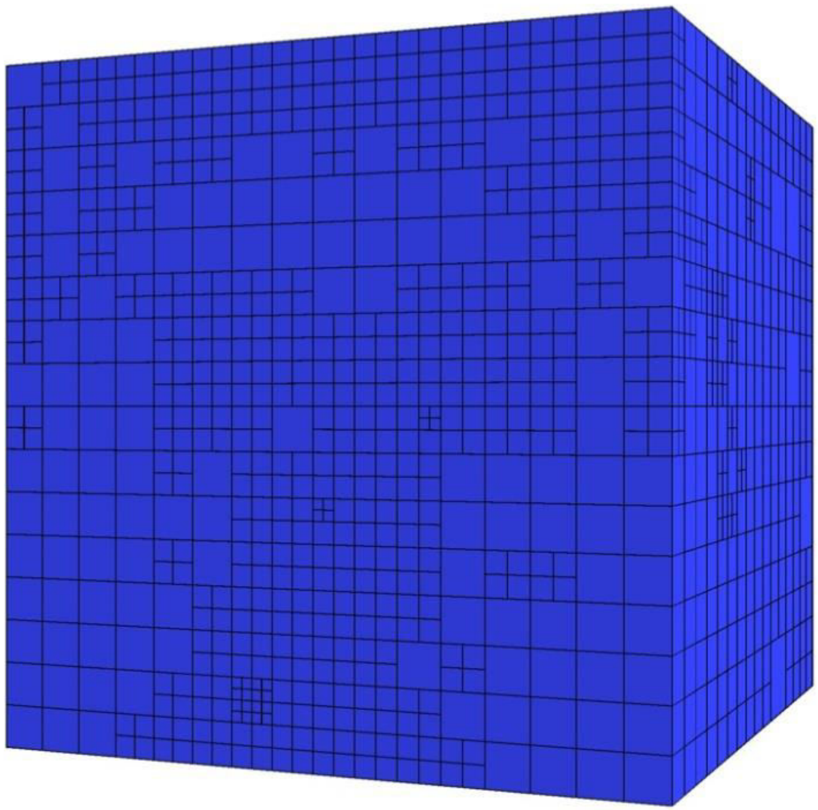
Application in Igea: Poisson problem with a central source

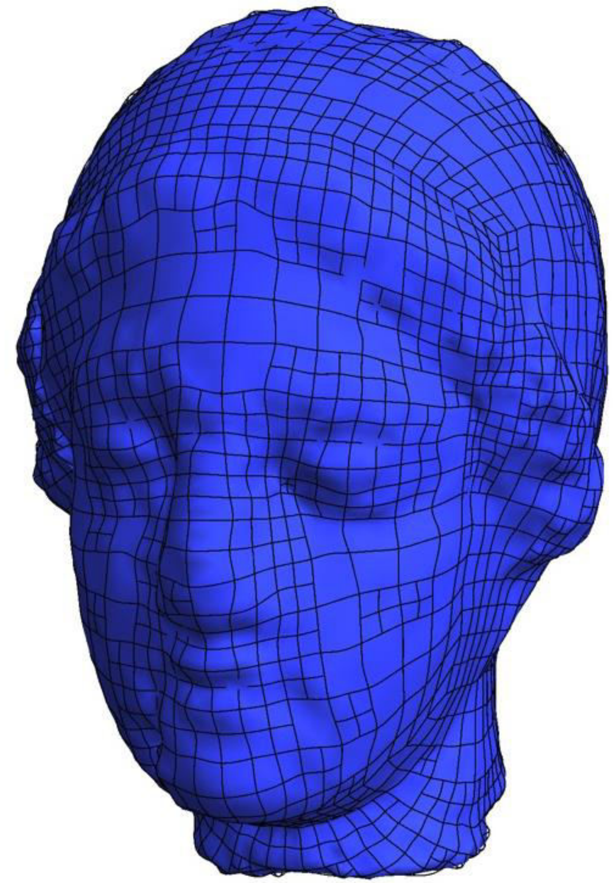
$$\Delta u = \frac{1}{25} e^{-\frac{(x^2+y^2+z^2)}{10}} \left(-15 + x^2 + y^2 + z^2\right) \quad \text{in } \Omega$$

$$u\big|_{\partial\Omega} = 0$$

Exact solution:

$$u \approx e^{-\frac{(x^2+y^2+z^2)}{10}}$$





T-mesh $\xrightarrow{\mathbf{S}(\xi)}$ T-spline

Adaptive Isogeometric Refinement

Igea: T-spline of Numerical Solution

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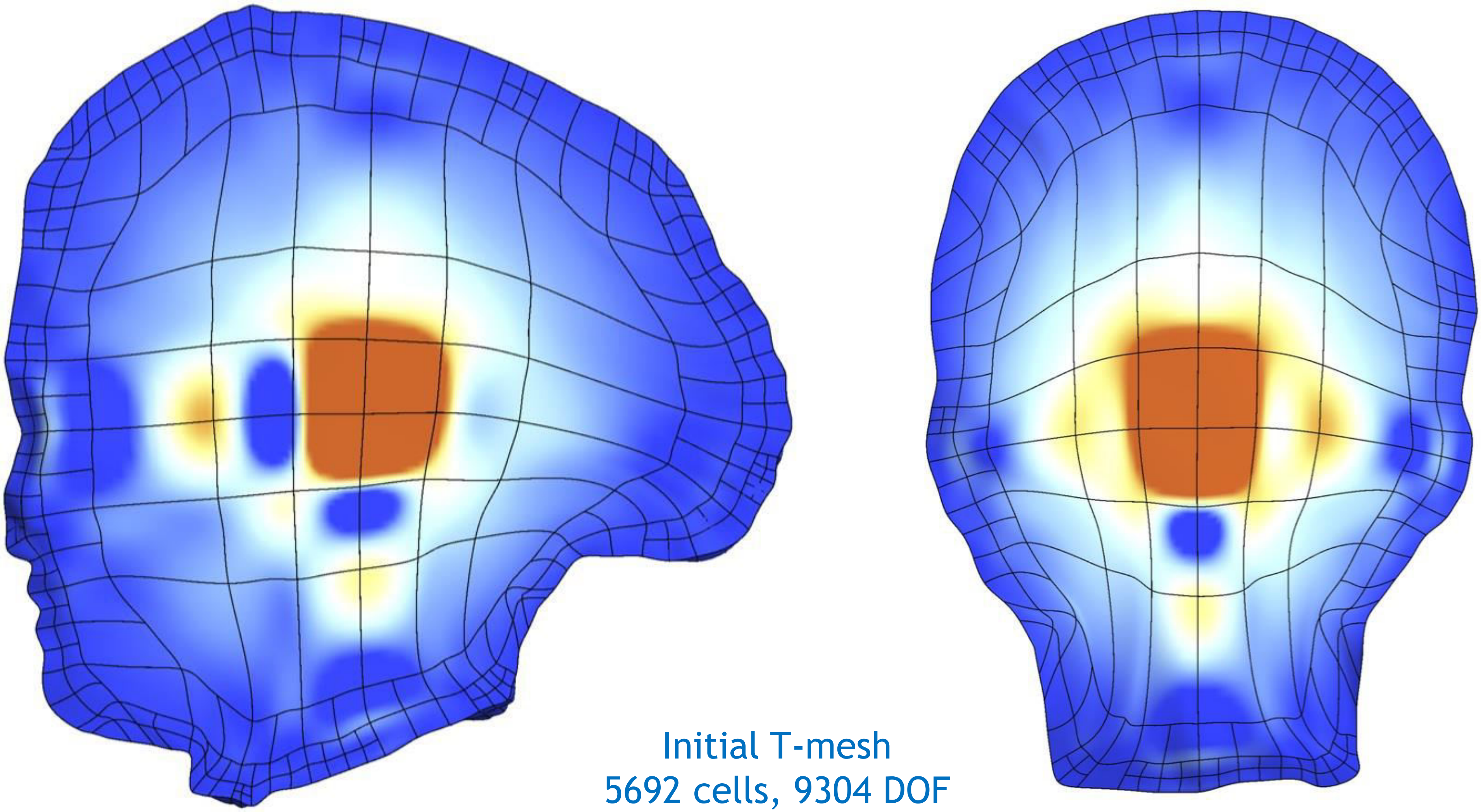
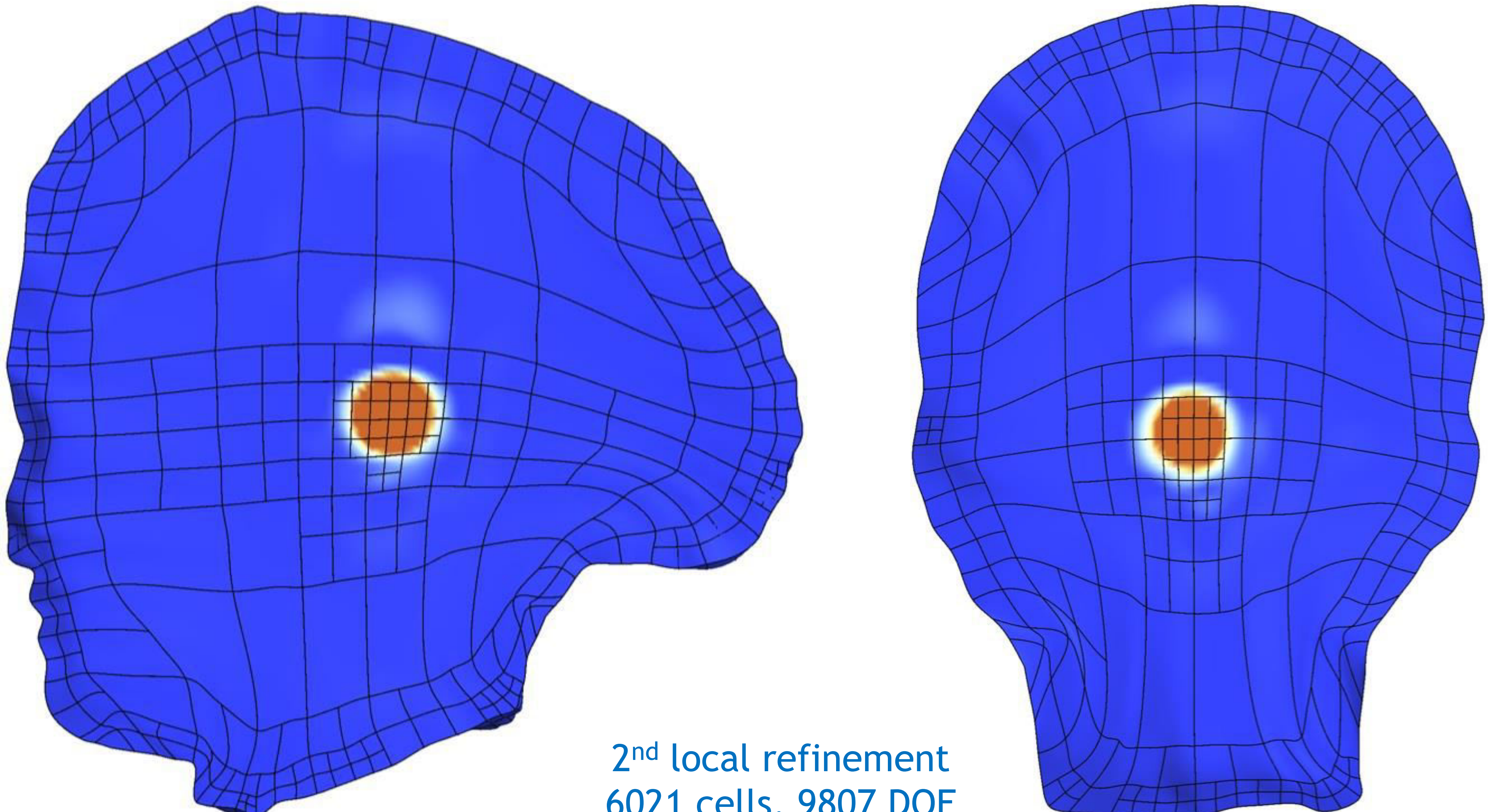


Initial T-mesh
5692 cells, 9304 DOF

Adaptive Isogeometric Refinement

Igea: T-spline of Numerical Solution

Error indicator : $\eta\left(\Omega_e\right)^2 = \int_{\Omega_e} h^2 \left(f + \Delta u_h\right)^2 \, d\Omega$



2nd local refinement
6021 cells, 9807 DOF

Adaptive Isogeometric Refinement

Igea: T-spline of Numerical Solution

Error indicator : $\eta(\Omega_e)^2 = \int_{\Omega_e} h^2 (f + \Delta u_h)^2 \, d\Omega$

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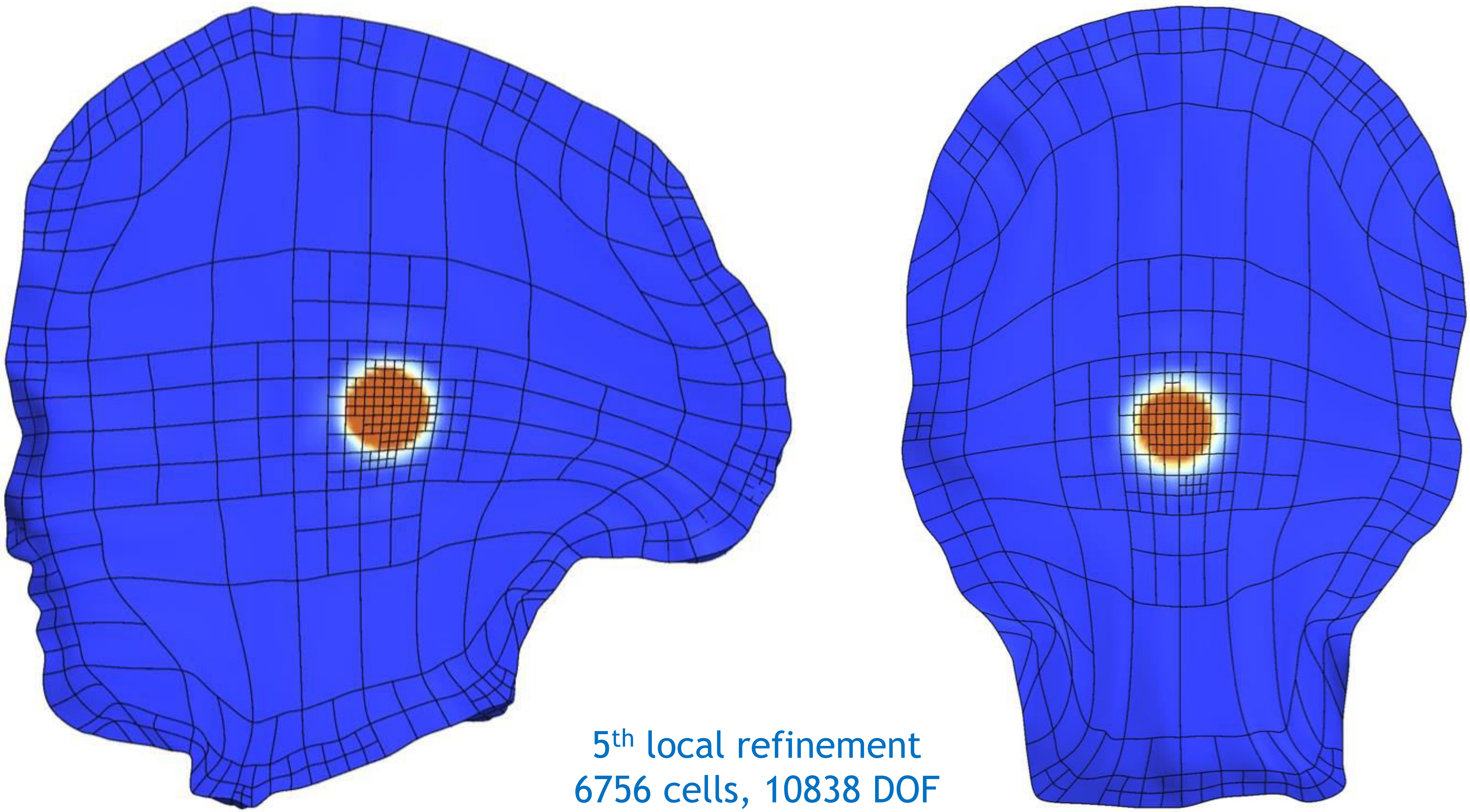
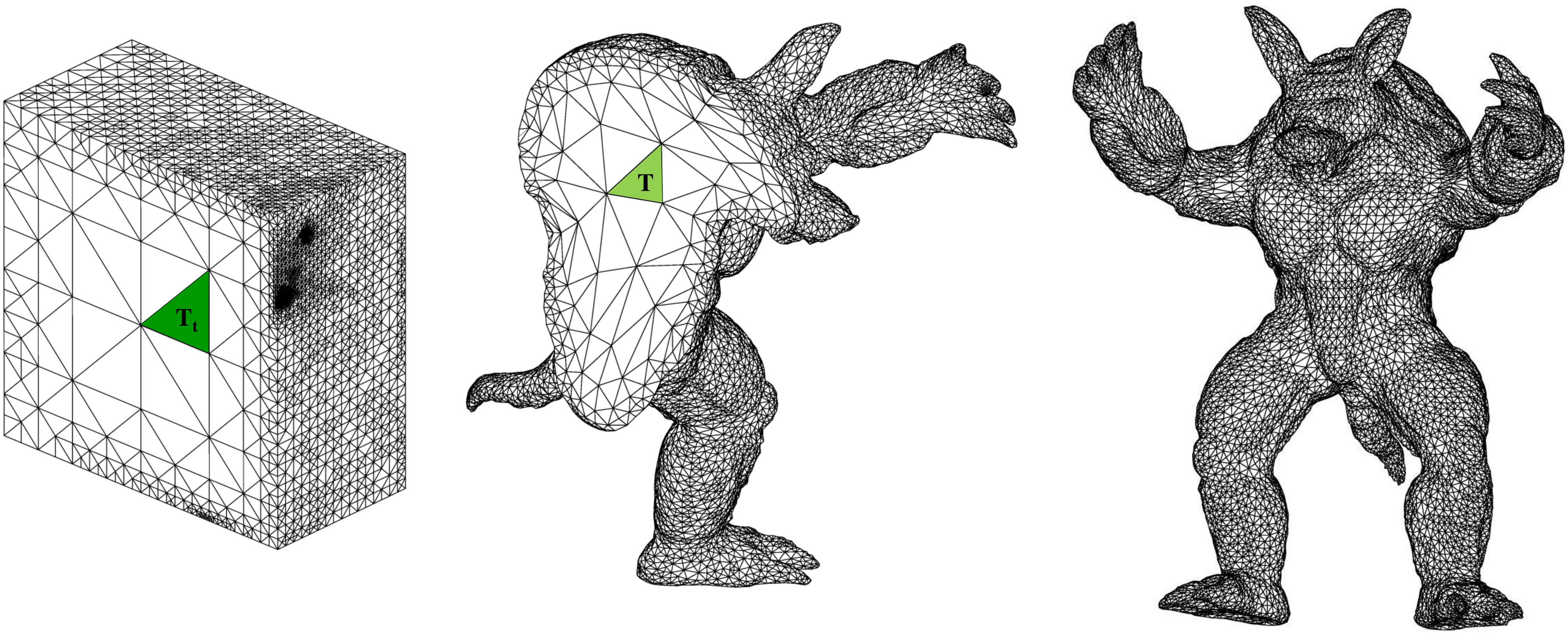


5th local refinement
6756 cells, 10838 DOF

Meccano Method for Complex Solids

Volume parameterization based on SUS of tetrahedral meshes



Optimization

Physical Element $\mathbf{T}$ → **Target Element** $\mathbf{T_t}$ (to get less distortion in the parameterization)

1. The key of the meccano method is the simultaneous untangling and smoothing (SUS) procedure.

2. The quality of the T-spline mapping (i.e., positive Jacobian, good uniformity and orthogonality of the isoparametric curves) depends on the quality of the T-mesh in the physical space. We have to fix a quality metric for this mapping.

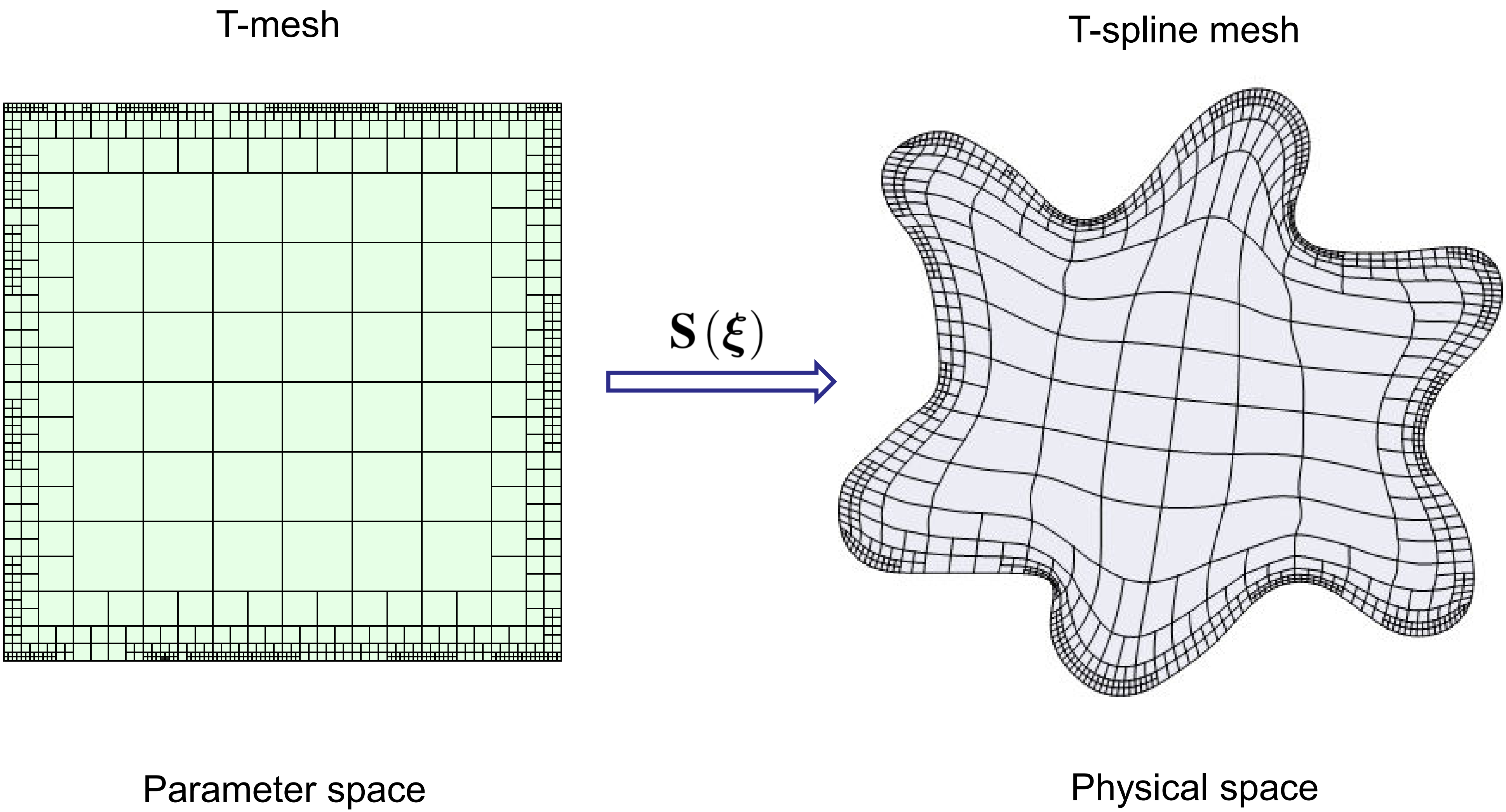
3. In order to simplify the procedure and to get less distortion in the volume parameterization, it should be interesting to **directly apply the meccano method on T-meshes instead of tetrahedral meshes**.

4. We have started analysing the problem in 2-D.

The Meccano Method on T-meshes in 2-D

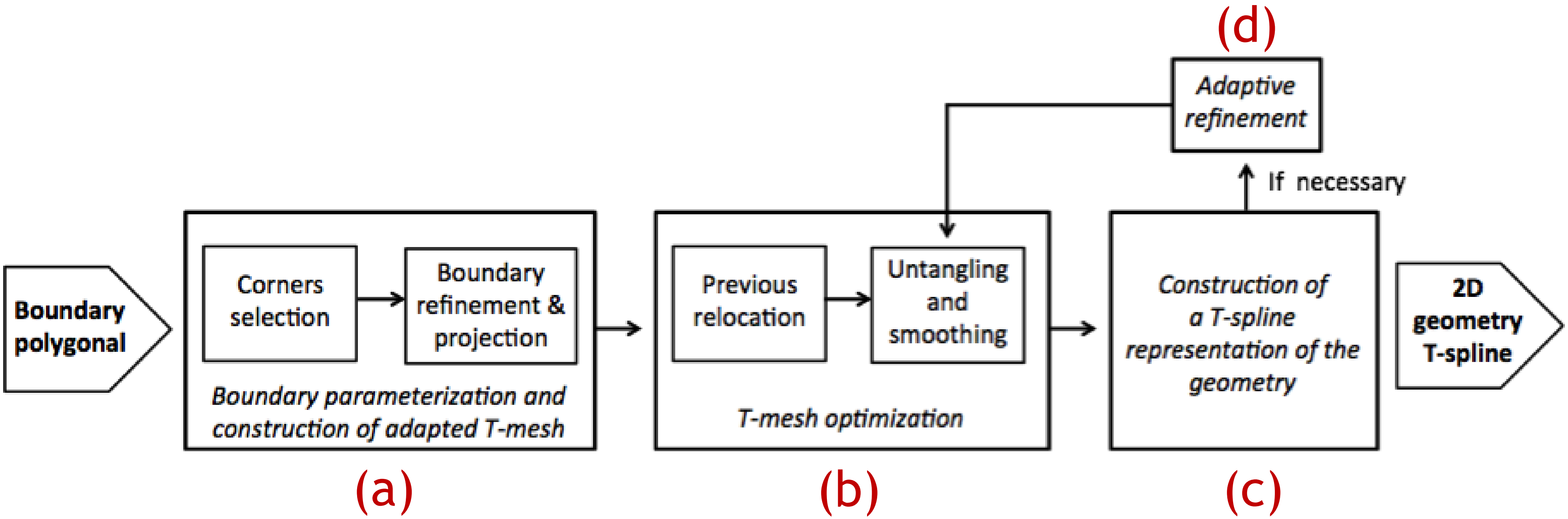
Input data: Boundary representation of the object

Objective: Construction of a high quality T-spline parameterization



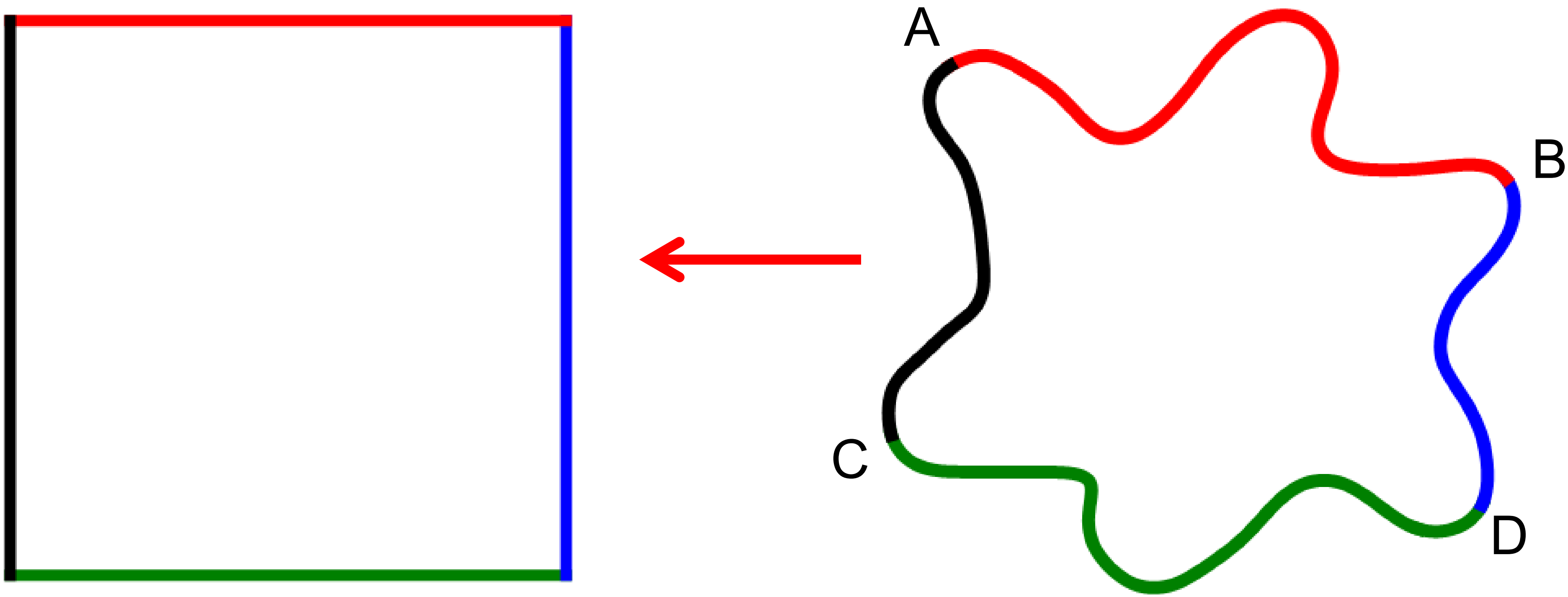
General scheme of the method

a) Boundary parameterization and adapted T-mesh construction.
b) T-mesh optimization.
c) Construction of a T-spline representation of the geometry.
d) Adaptive refinement to remove negative Jacobians and to improve the parameterization.



The Meccano Method on T-meshes in 2-D

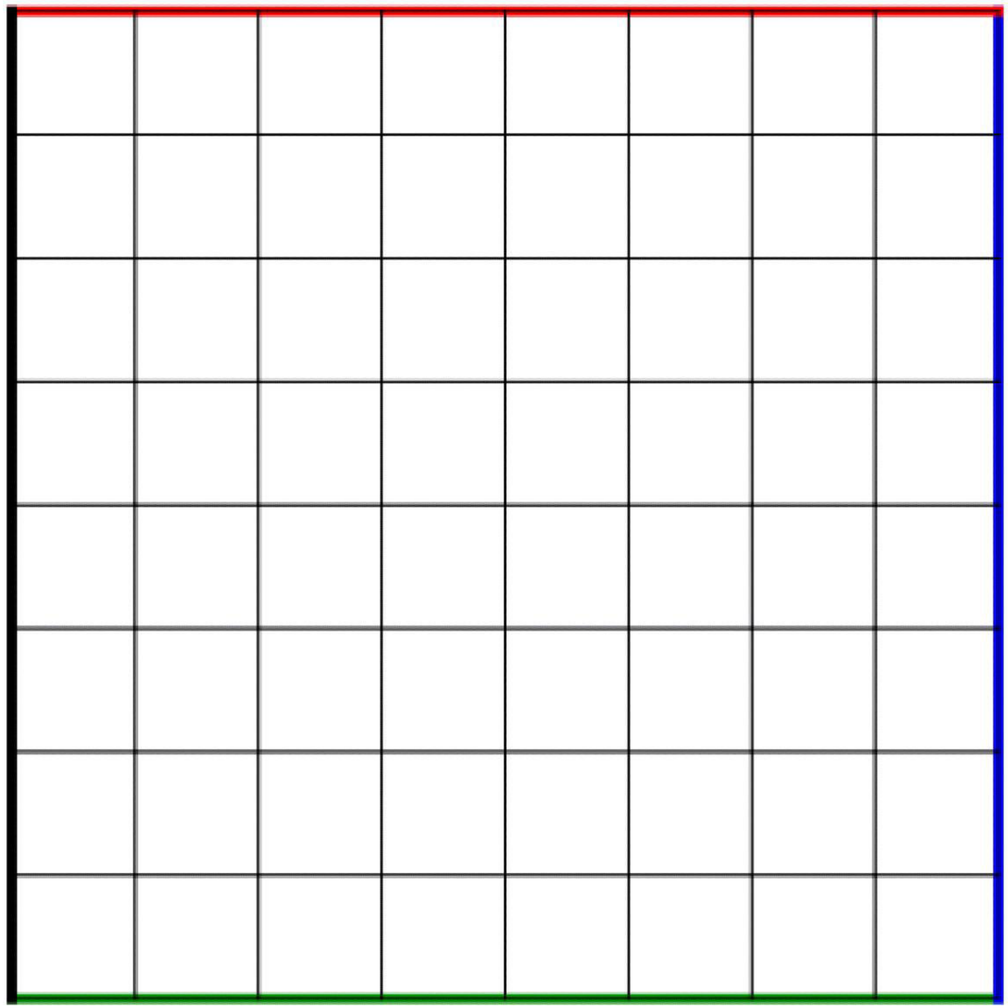
Step 1: Input boundary (image, polyline, curve, etc.) and boundary mapping



- Select four points (A, B, C, D) of the input boundary
- Boundary parameterization via chord-length

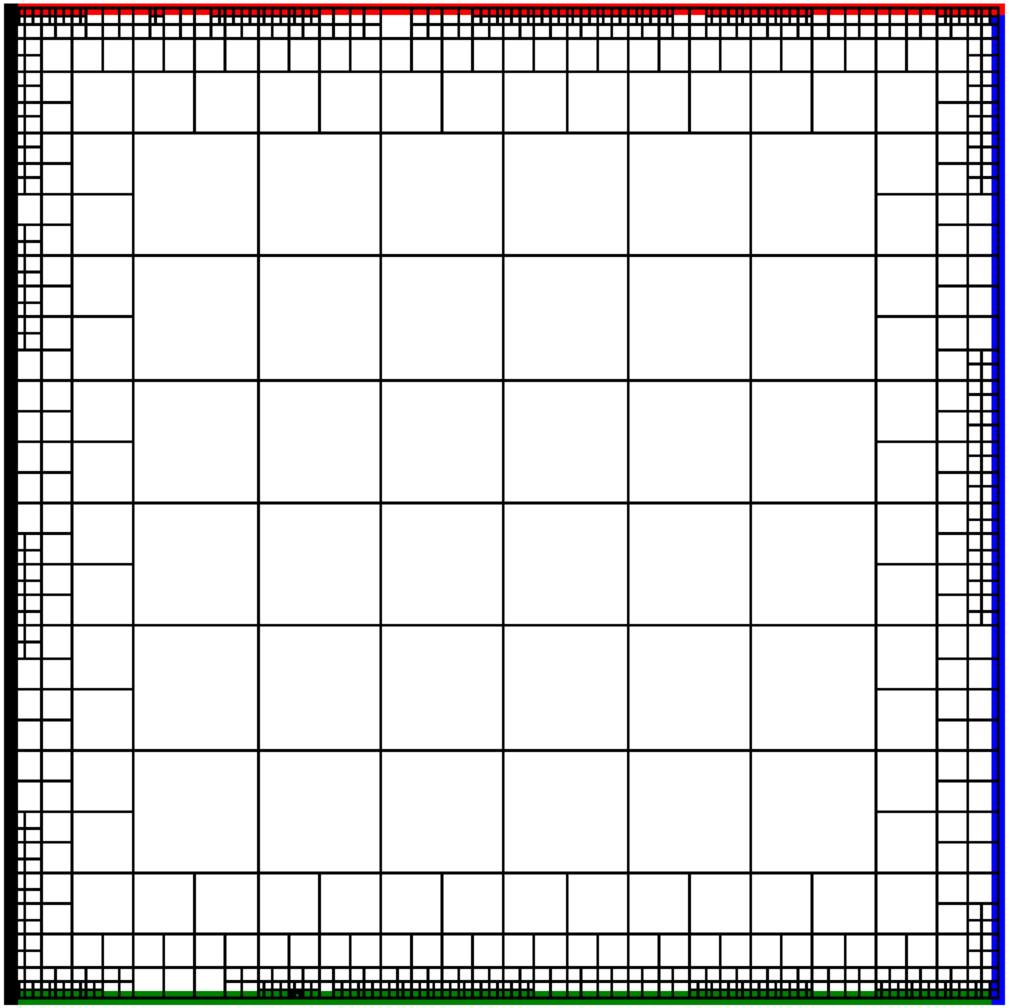
The Meccano Method on T-meshes in 2-D

Step 2: Coarse quadrilateral mesh of the meccano (parameter space)



The Meccano Method on T-meshes in 2-D

Step 3: Refine mesh with quadtree subdivisions to approach the boundary



The Meccano Method on T-meshes in 2-D

Step 4: Move the meccano boundary nodes to the object boundary

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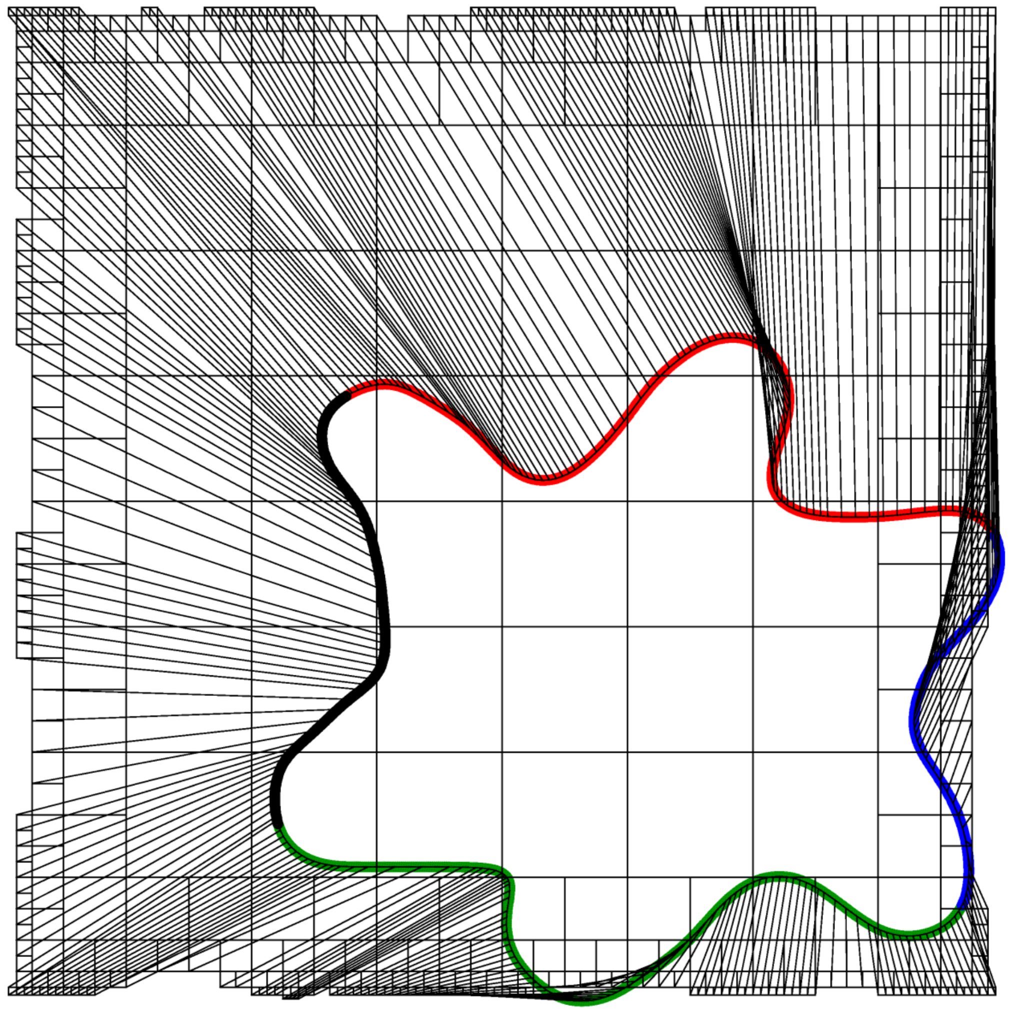
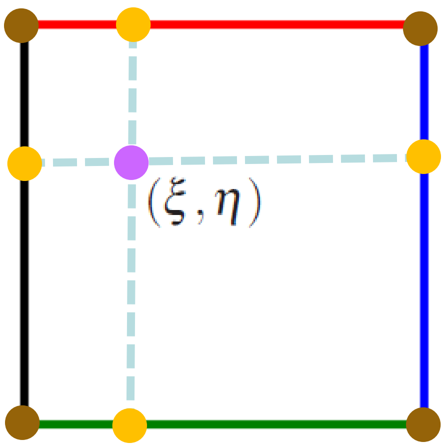
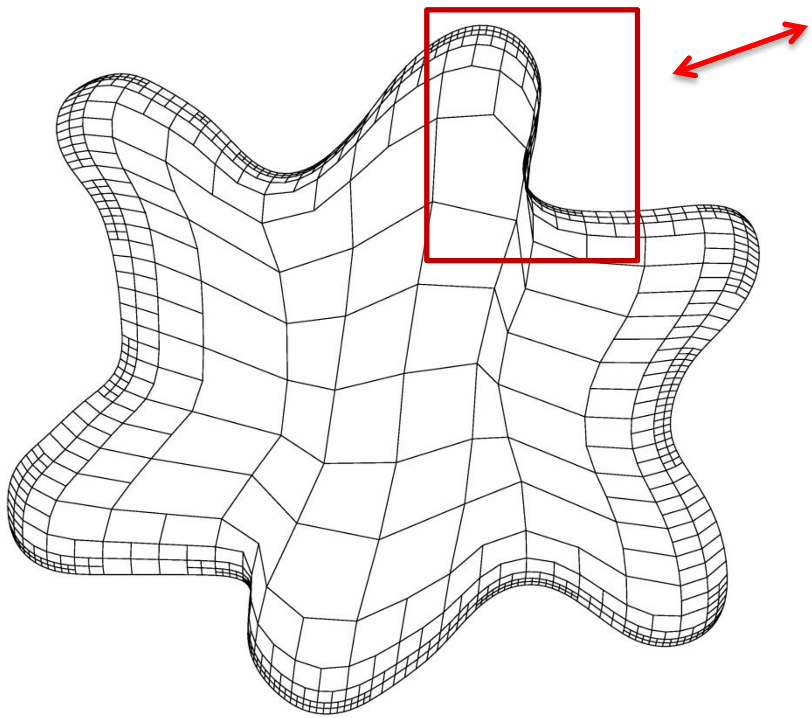


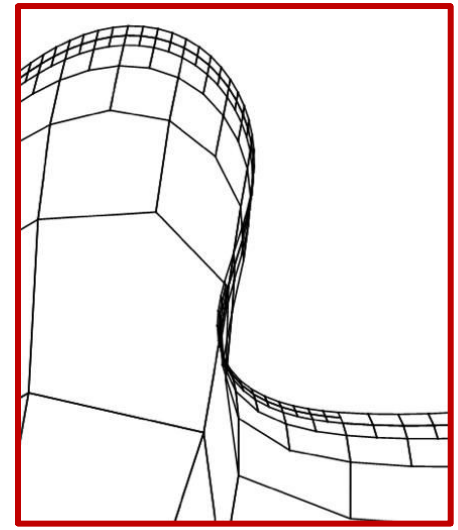
The Meccano Method on T-meshes in 2-D

Step 5: Inner node relocation with Coons patch to facilitate the optimization

$$
\begin{aligned}
\mathbf{x}(\xi,\eta) = &(1-\xi)\mathbf{x}(0,\eta)+\xi\mathbf{x}(1,\eta)\\
&+(1-\eta)\mathbf{x}(\xi,0)+\eta\mathbf{x}(\xi,1)\\
&-\begin{bmatrix}1-\xi & \xi\end{bmatrix}\begin{bmatrix}\mathbf{x}(0,0) & \mathbf{x}(0,1)\\ \mathbf{x}(1,0) & \mathbf{x}(1,1)\end{bmatrix}\begin{bmatrix}1-\eta\\ \eta\end{bmatrix}
\end{aligned}
$$



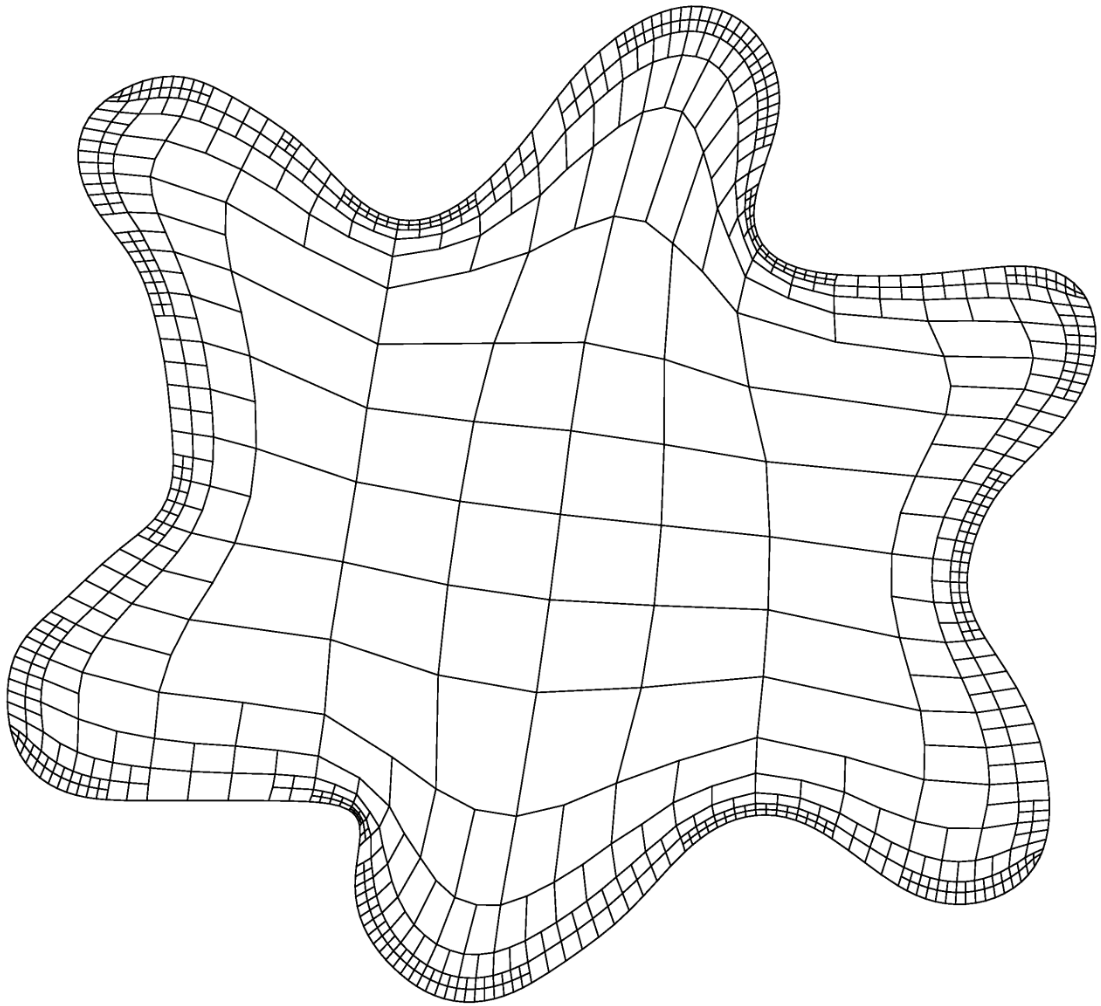




Mesh folder

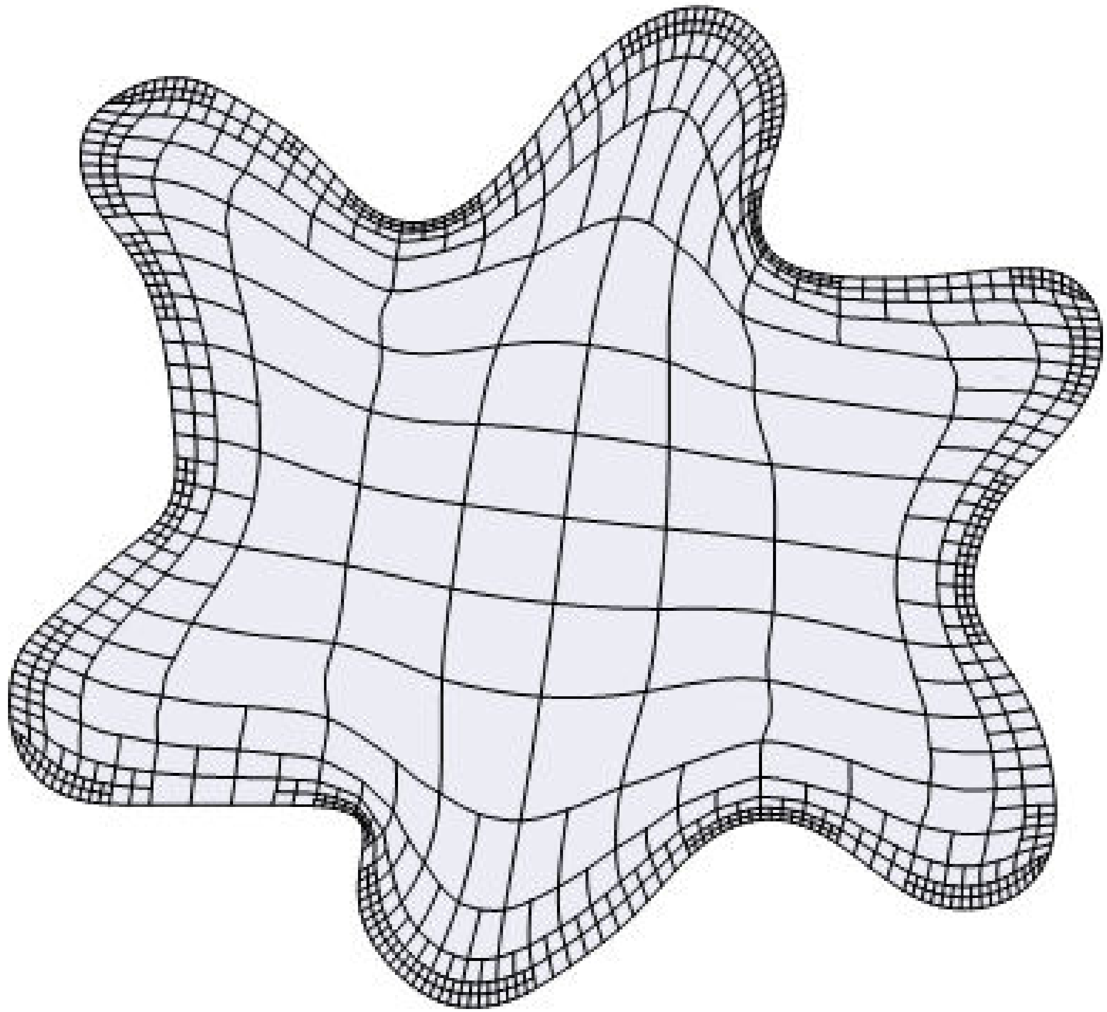
The Meccano Method on T-meshes in 2-D

Step 6: Simultaneous Untangling and Smoothing (SUS) of the T-mesh



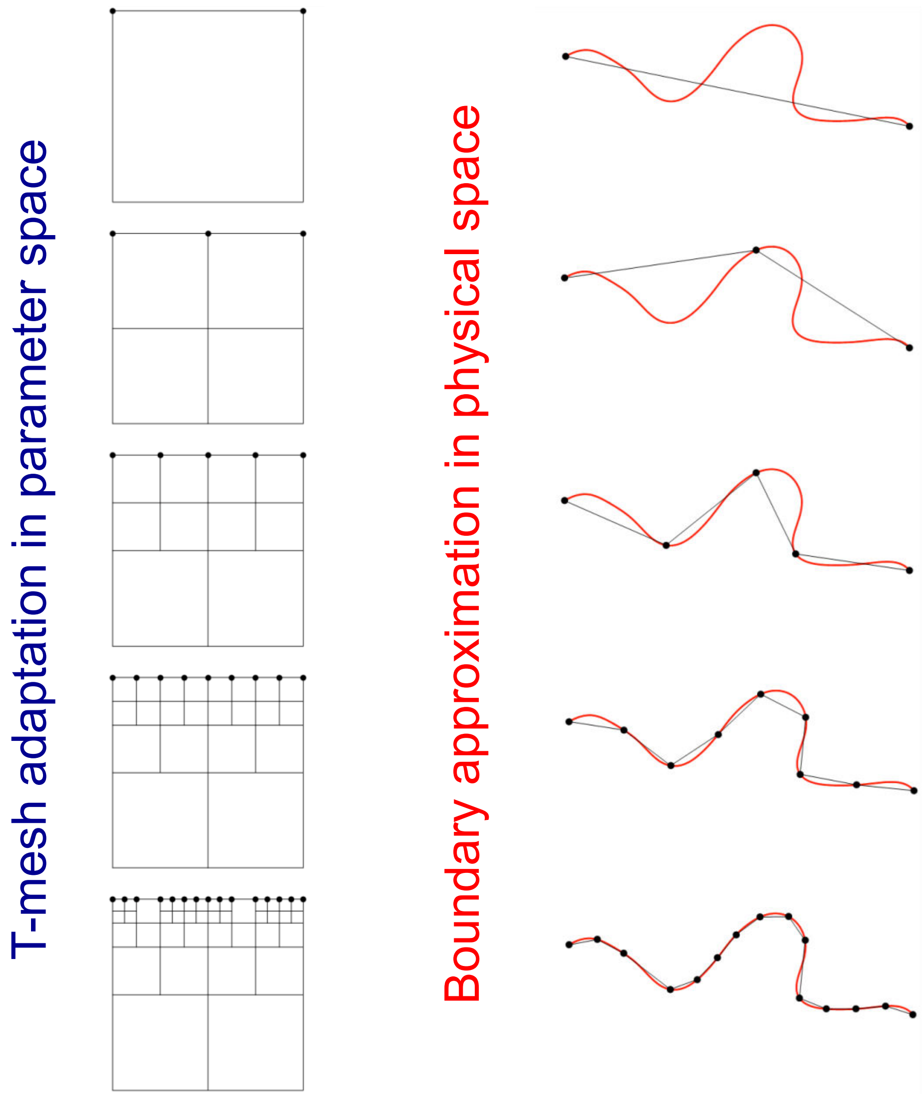
The Meccano Method on T-meshes in 2-D

Step 7: T-spline representation of the spot

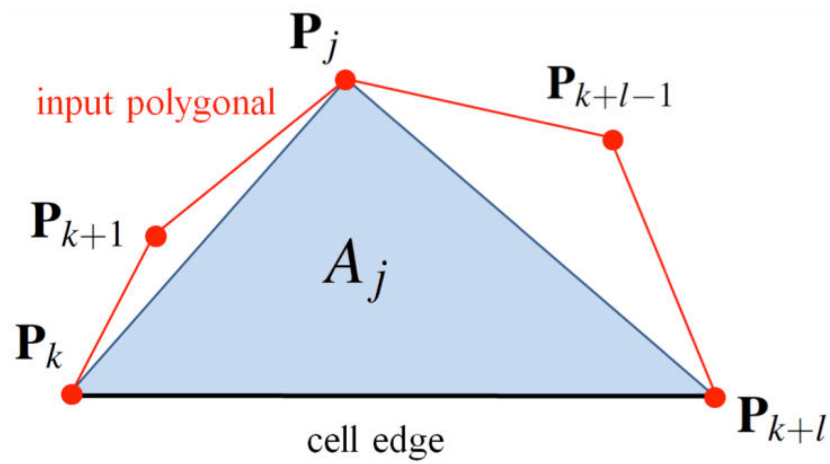


Boundary Approach in 2-D

Input data: Boundary polyline approximation (red color line)



Boundary edge refinement criterion:



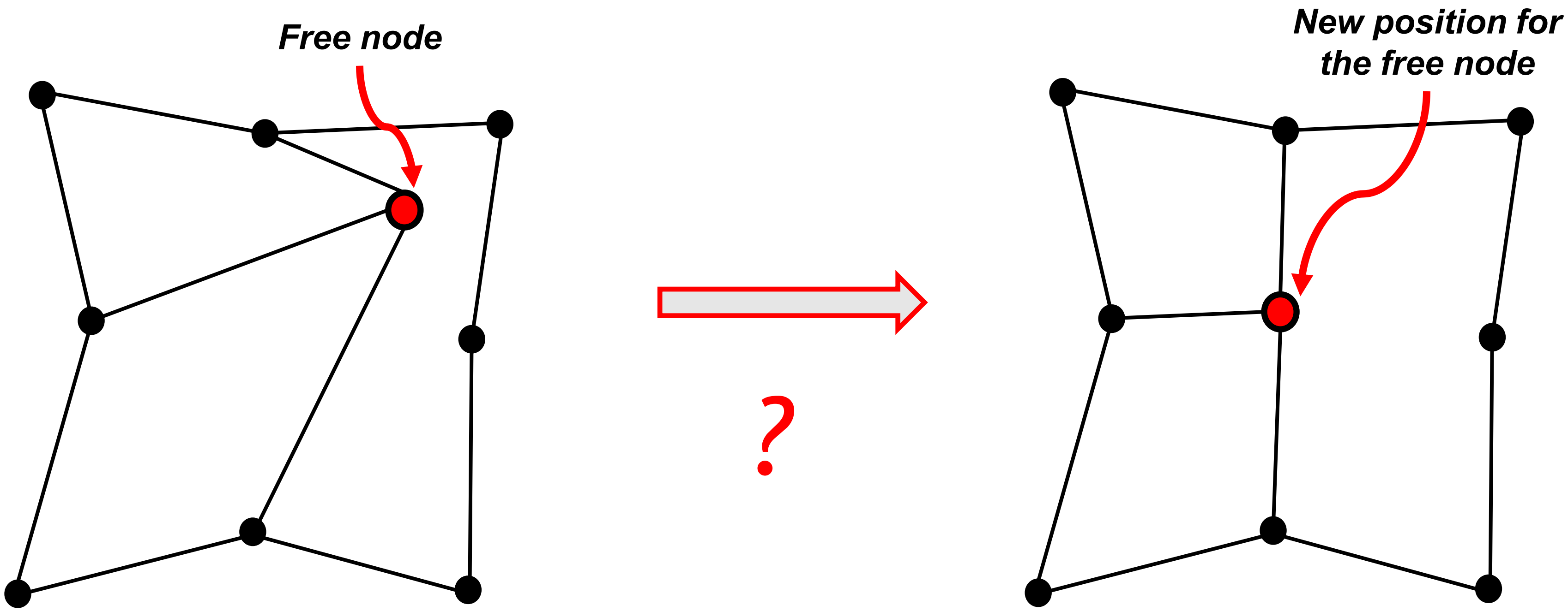
$$\exists j : \; A_j > \varepsilon \Rightarrow \text{refine}$$

$$A_j = \text{Area}(\mathbf{P}_k, \mathbf{P}_j, \mathbf{P}_{k+l})$$

$$k+1 \leqslant j \leqslant k+l-1$$

Local optimization

***Objective:* Improve the quality of the local mesh by minimizing an objective function**



Local mesh

Optimized local mesh

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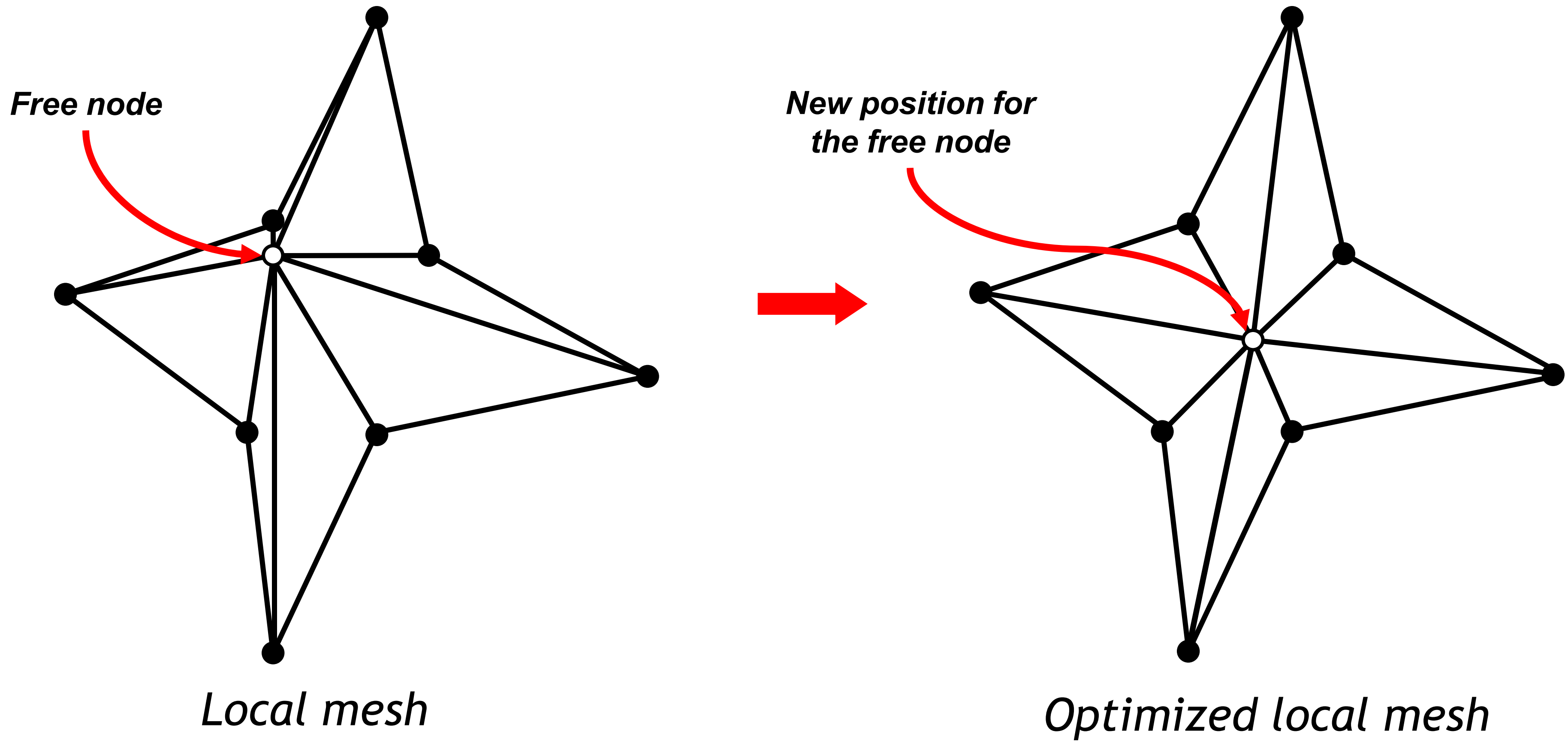
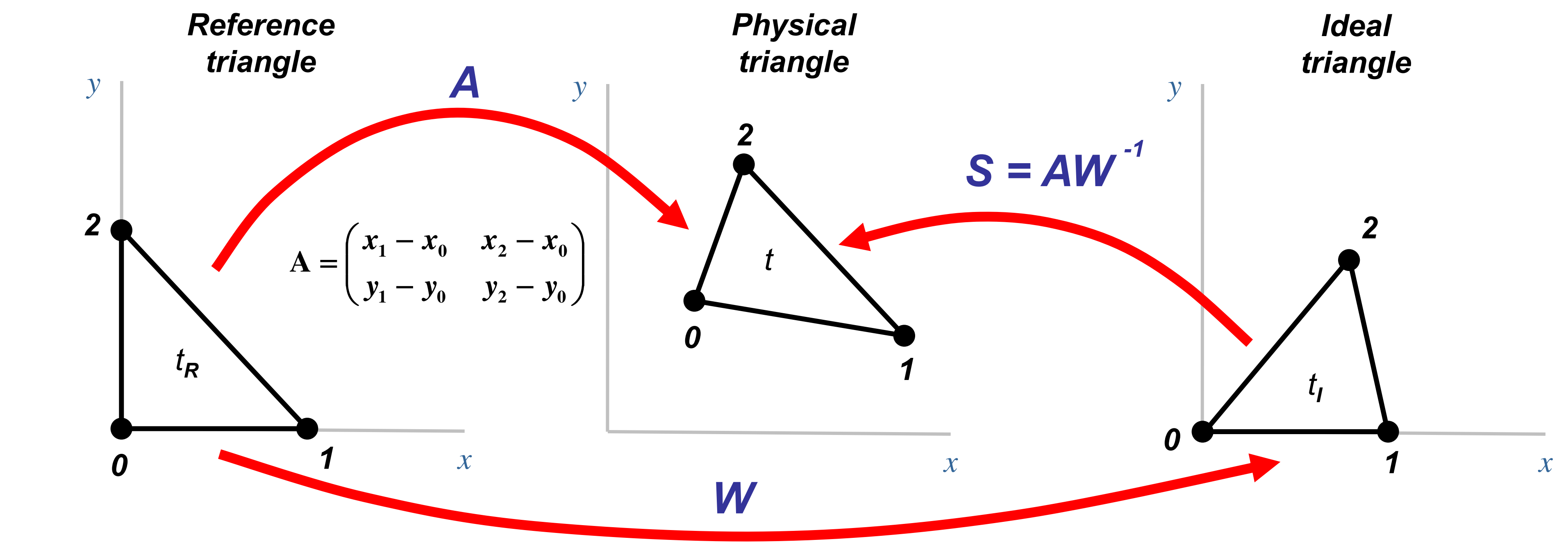
Local optimization

***Objective:* Improve the quality of the local mesh by minimizing an objective function**



Simultaneous Untangling and Smoothing (CMAME 2003)

Weighted Jacobian Matrix on a Plane



$t_I \xrightarrow{\mathbf{S}} t$ $\quad S = AW^{-1}$: **Weighted Jacobian matrix**

An algebraic quality metric of t (mean ratio)

$$\mathbf{q} = \frac{2\sigma}{\|\mathbf{S}\|^2} = \frac{1}{\eta}$$

where:

$$\|\mathbf{S}\| = \sqrt{\mathrm{tr}\left(\mathbf{S}^T\mathbf{S}\right)}$$

$$\sigma = \det(\mathbf{S})$$

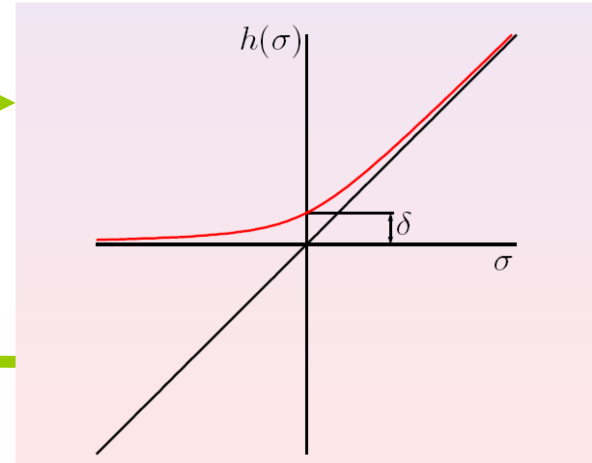
Simultaneous Untangling and Smoothing (CMAME 2003)

Local objective function for valid plane triangulations

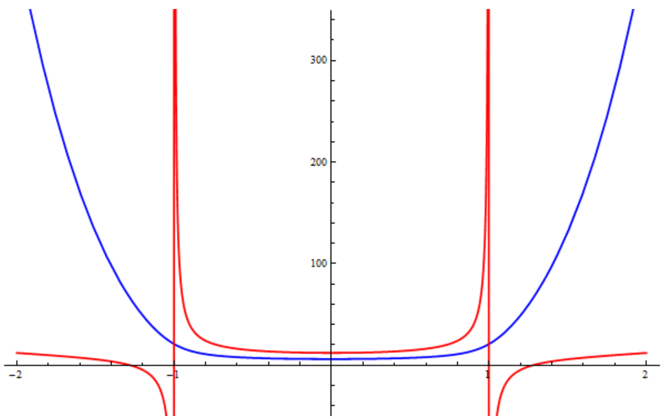
SUS Code: Freely-available in http://www.dca.iusiani.ulpgc.es/proyecto2012-2014

Original function: $$K(\mathbf{x}) = \sum_{m=1}^{M} \frac{\|S_m\|^2}{2\,\sigma_m}$$

Modified function: $$K^*(\mathbf{x}) = \sum_{m=1}^{M} \frac{\|S_m\|^2}{2h(\sigma_m)}$$



$$h(\sigma) = \tfrac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\delta^2}\right)$$

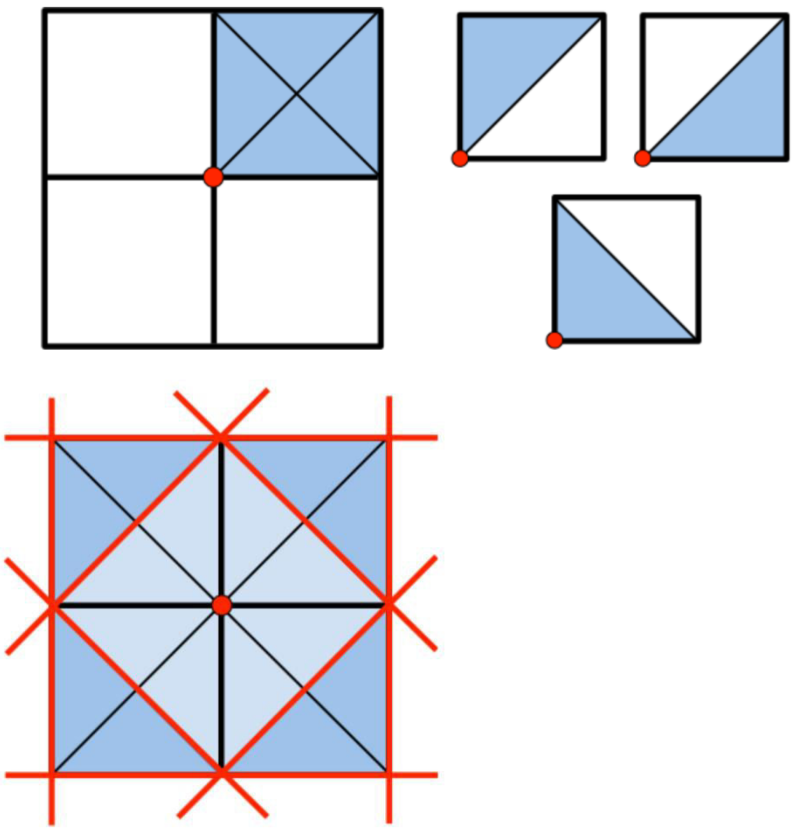


Modified function (blue) is regular in all R^2 and it approximates the same minimum that the original function (red). Moreover, it allows a simultaneous untangling and smoothing of triangular meshes

Simultaneous Untangling and Smoothing of T-meshes

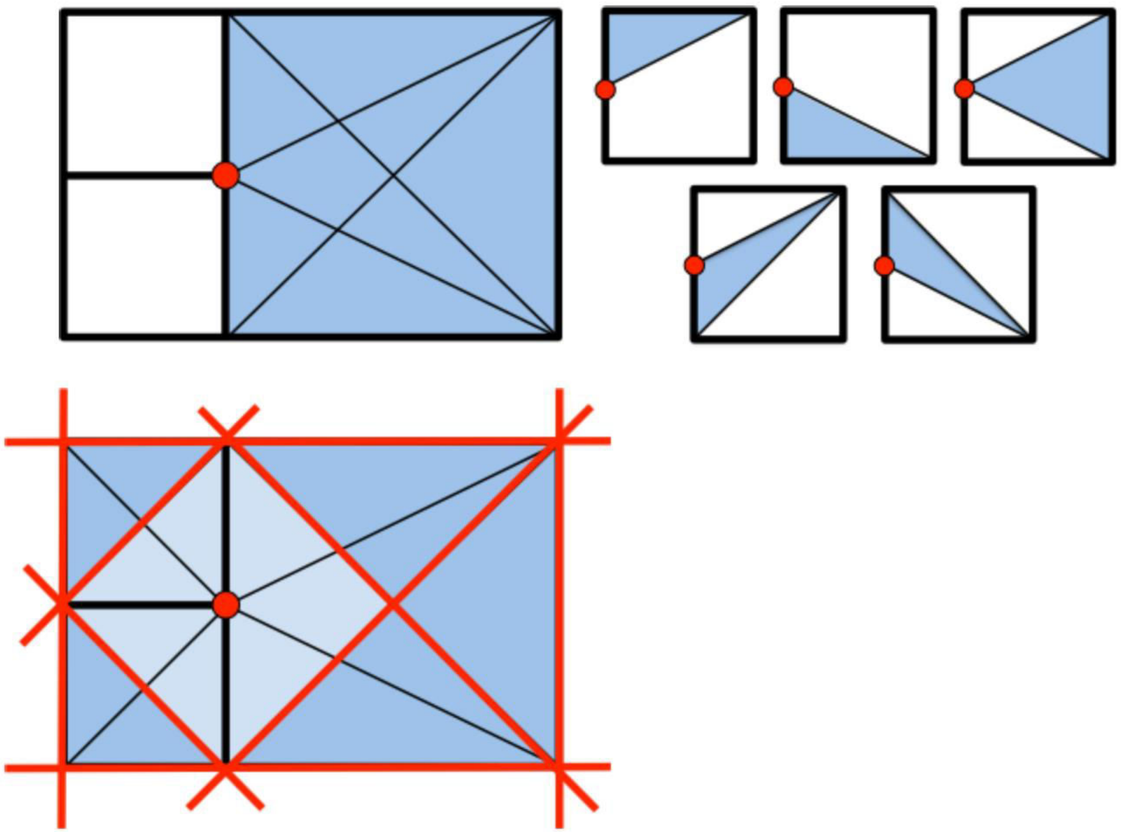
Triangle decomposition of the T-mesh cells

Case 1: Free node is a regular node



Barriers and feasible region for a regular node

Case 2: Free node is a hanging node

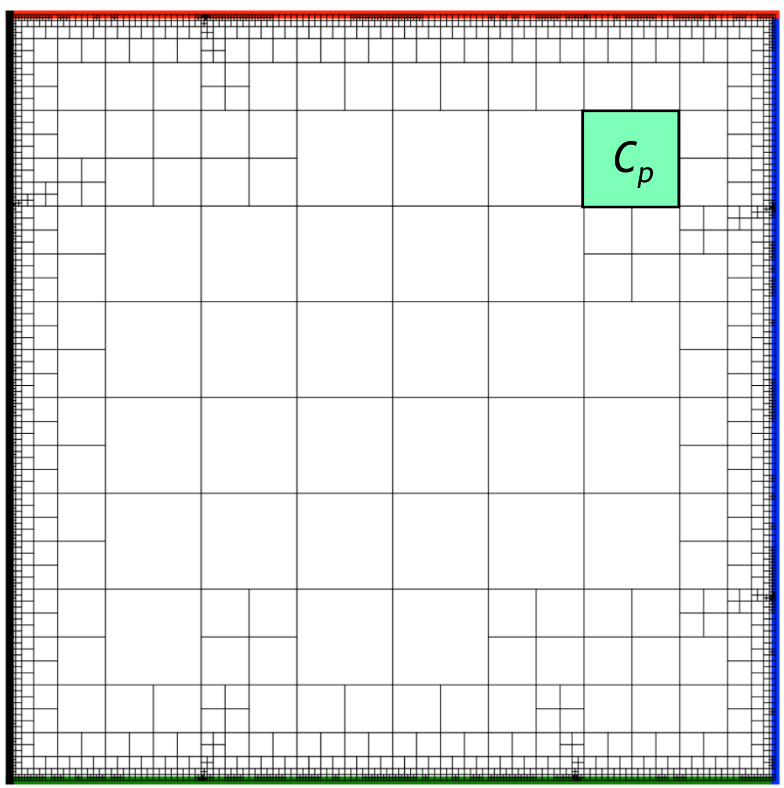


Barriers and feasible region for a hanging node

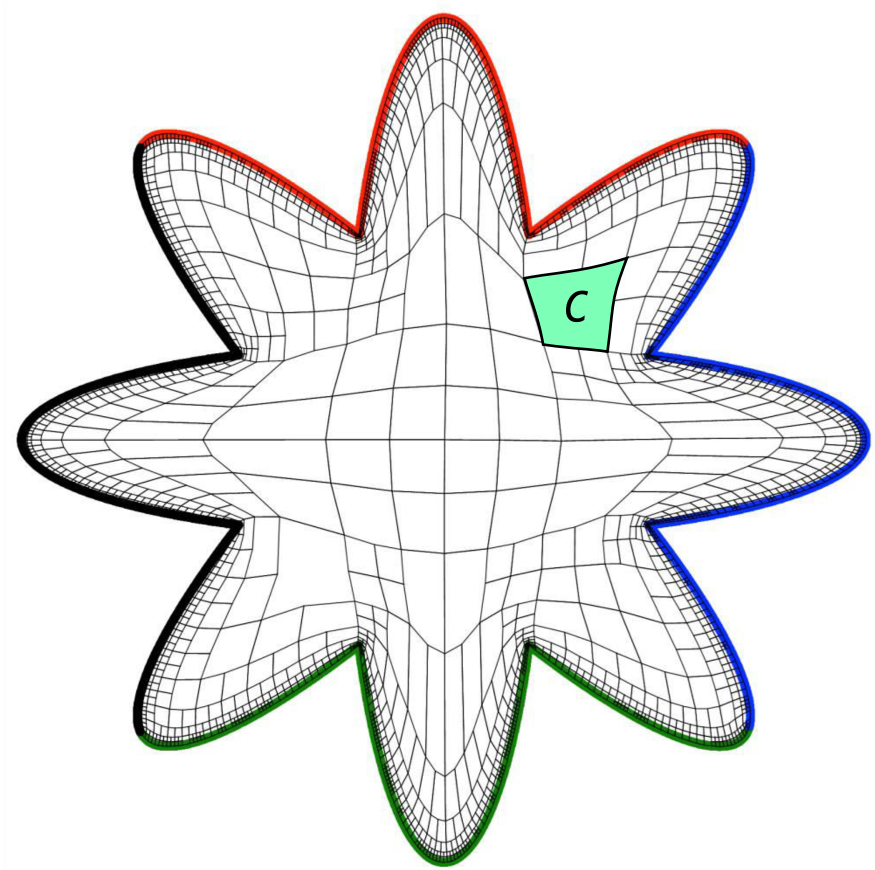
Simultaneous Untangling and Smoothing of T-meshes

Optimization is guided by the parametric T-mesh

Physical cell C must be as similar as possible to the counterpart in the parametric space C_p



Parametric T-mesh

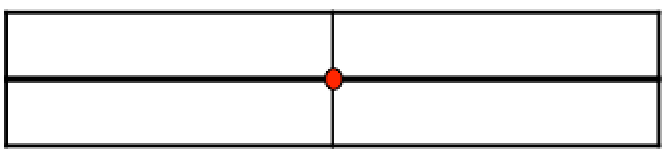


Optimized physical T-mesh

Simultaneous Untangling and Smoothing of T-meshes

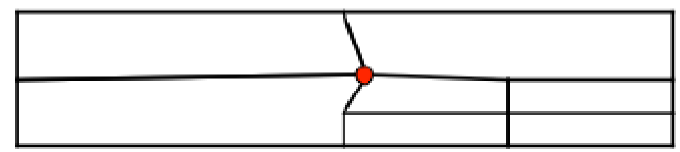
Problems appears with the non-weighted objective function K*

*Satisfactory result for a conformal submesh using K**



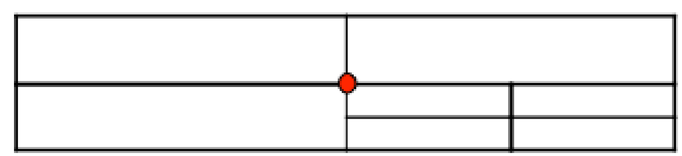
(a)

*Not satisfactory result for a non-conformal submesh using K**



(b)

Desirable result: Orthogonal mesh for a non-conformal case ***(weighted objective function K_τ^*)***



(c)

Simultaneous Untangling and Smoothing of T-meshes

Solution by using weighted objective functions (regular node)

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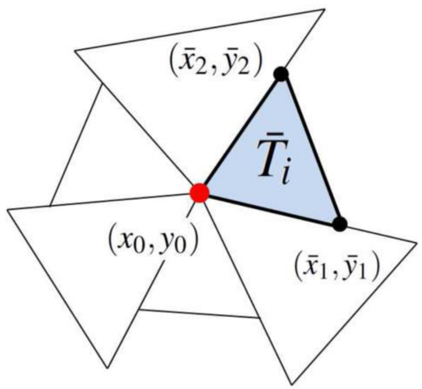
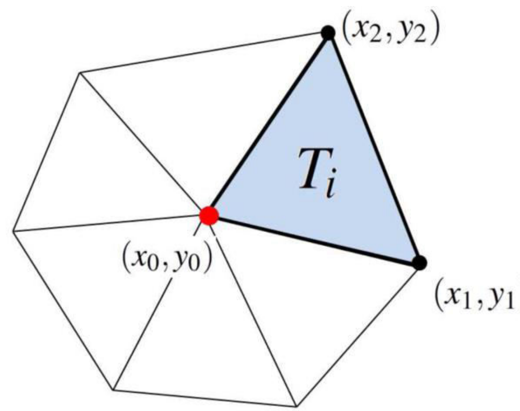
Objective: The optimal position for non-conformal mesh is the same as for the conformal case



Non-conformal mesh



Conformal mesh

$$\begin{cases} \partial_x \bar{K}(x_0, y_0) = \sum_{i=1}^{M} \partial_x \bar{\eta}_i(x_0, y_0) = \sum_{i=1}^{M} \frac{1}{\tau_i} \partial_x \eta_i(x_0, y_0) \\ \partial_y \bar{K}(x_0, y_0) = \sum_{i=1}^{M} \partial_y \bar{\eta}_i(x_0, y_0) = \sum_{i=1}^{M} \frac{1}{\tau_i} \partial_y \eta_i(x_0, y_0), \end{cases}$$

$\Longrightarrow$

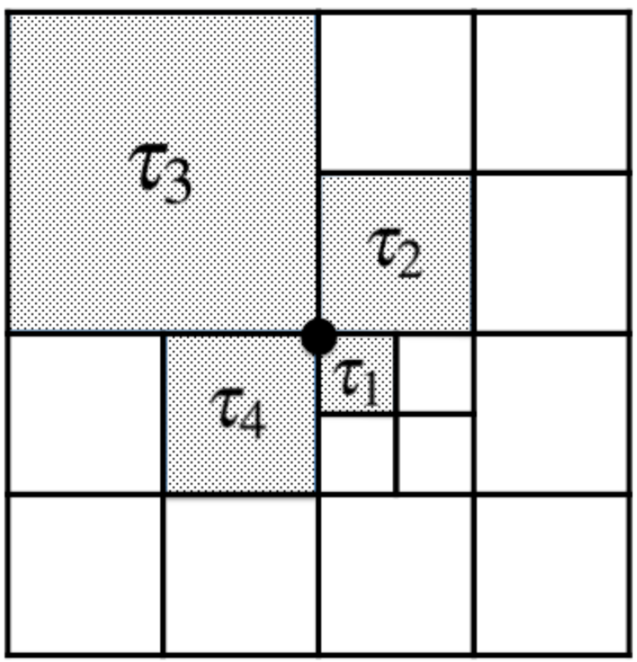
$$\bar{K}_\tau(x, y) = \sum_{i=1}^{M} \tau_i \, \bar{\eta}_i(x, y)$$

τ_i is scale factor of the element

Simultaneous Untangling and Smoothing of T-meshes

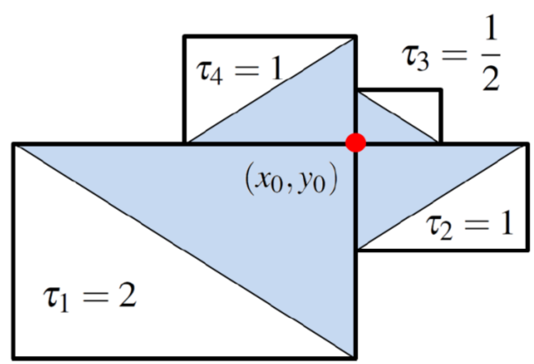
Solution by using weighted objective functions (regular node)

$$K_{\tau}^{*}(\mathbf{x}) = \tau_1 \sum_{m=1}^{3} \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_2 \sum_{m=4}^{6} \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_3 \sum_{m=7}^{9} \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_4 \sum_{m=10}^{12} \frac{\|S_m\|^2}{2h(\sigma_m)}$$

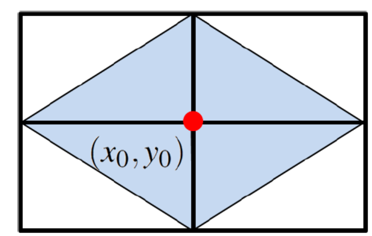


All possible weights for balanced quadtrees:

$$\tau_1 = 1,\ \tau_2 = \tau_4 = 2 \text{ y } \tau_3 = 4$$



Non-conformal mesh

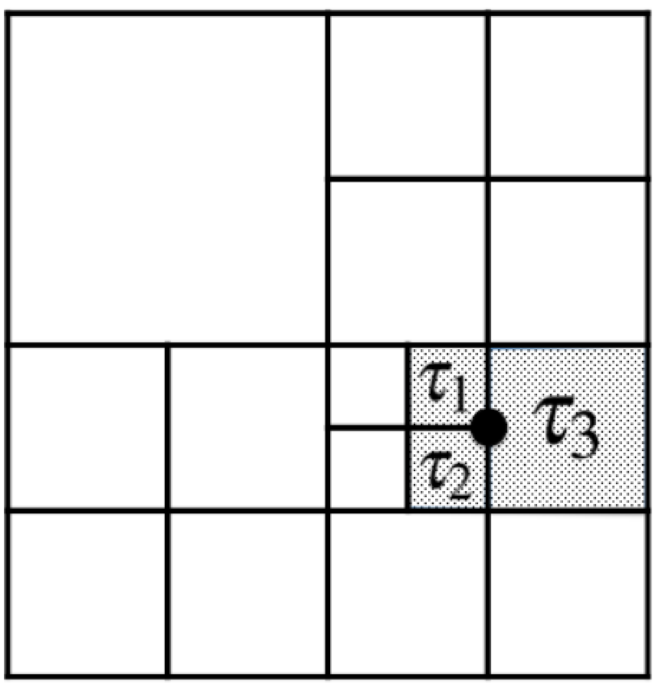


Conformal mesh

Simultaneous Untangling and Smoothing of T-meshes

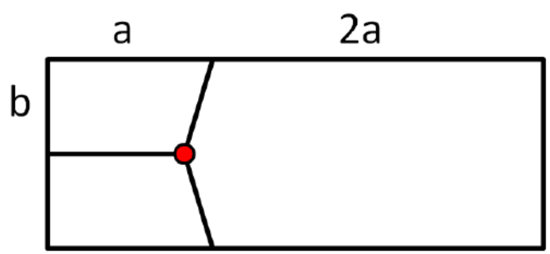
Solution by using weighted objective functions (hanging node)

$$K_{\tau}^{*}(\mathbf{x}) = \tau_1 \sum_{m=1}^{3} \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_2 \sum_{m=4}^{6} \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_3 \sum_{m=7}^{11} \frac{\|S_m\|^2}{2h(\sigma_m)}$$

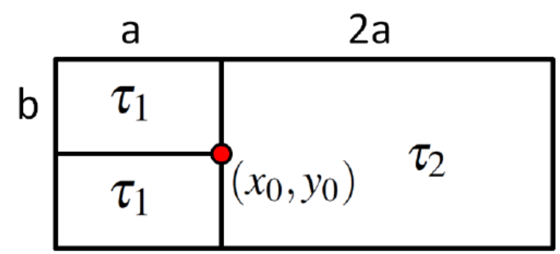


All possible weights for balanced quadtrees:

$$\tau_1 = \tau_2 = 1 \text{ y } \tau_3 = \frac{8}{5}$$



Optimized mesh without weights



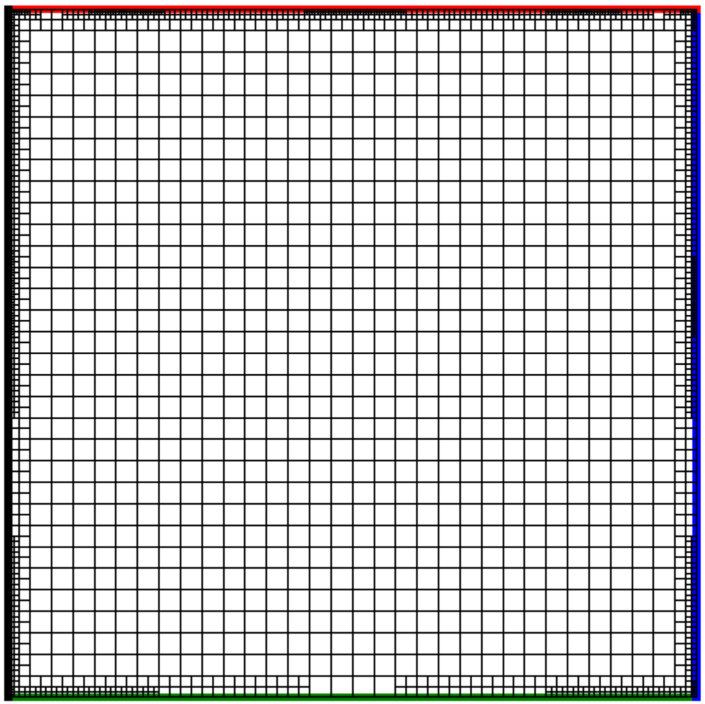
Optimized mesh with weights

Simultaneous Untangling and Smoothing of T-meshes

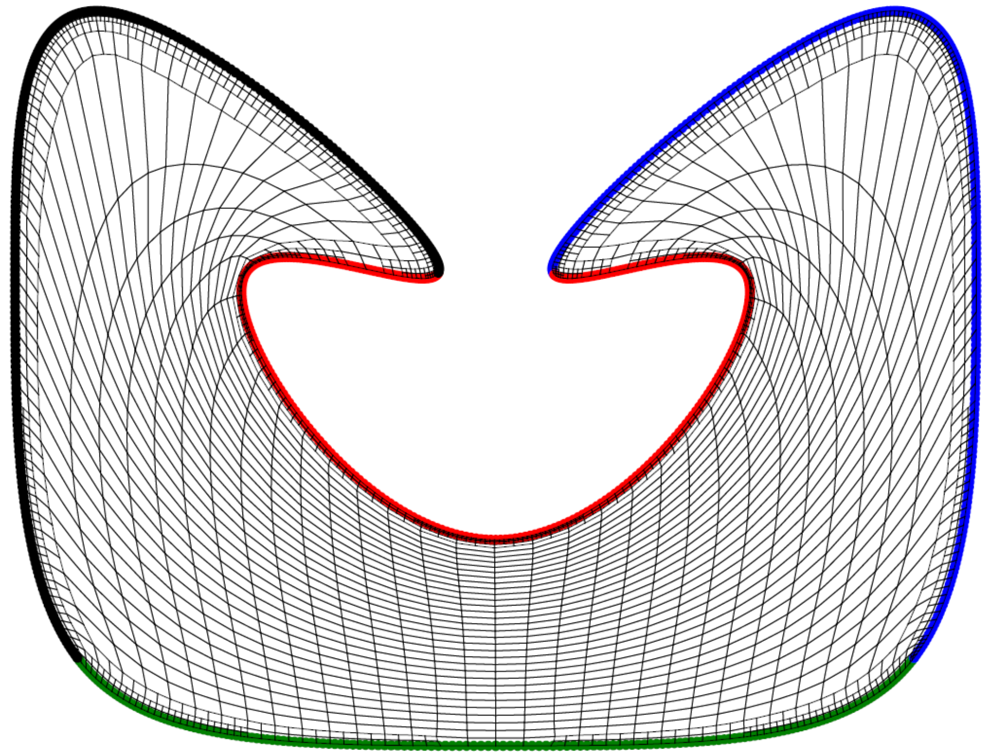
T-mesh transformation along the SUS process: Example

T-mesh

T-spline



$\mathbf{S}(\xi)$



Parameter space

Physical space

Simultaneous Untangling and Smoothing of T-meshes

T-mesh transformation along the SUS process: Video

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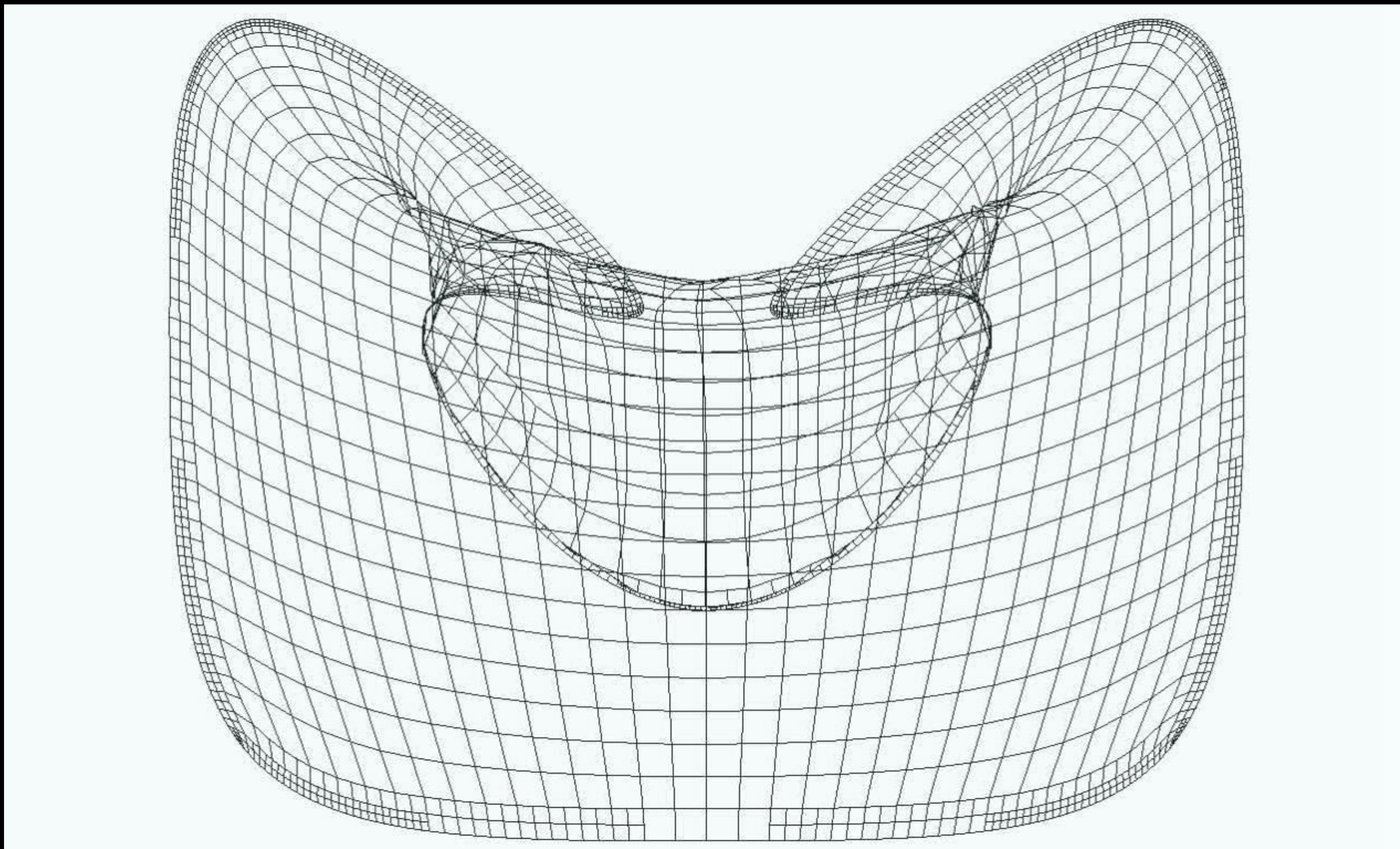
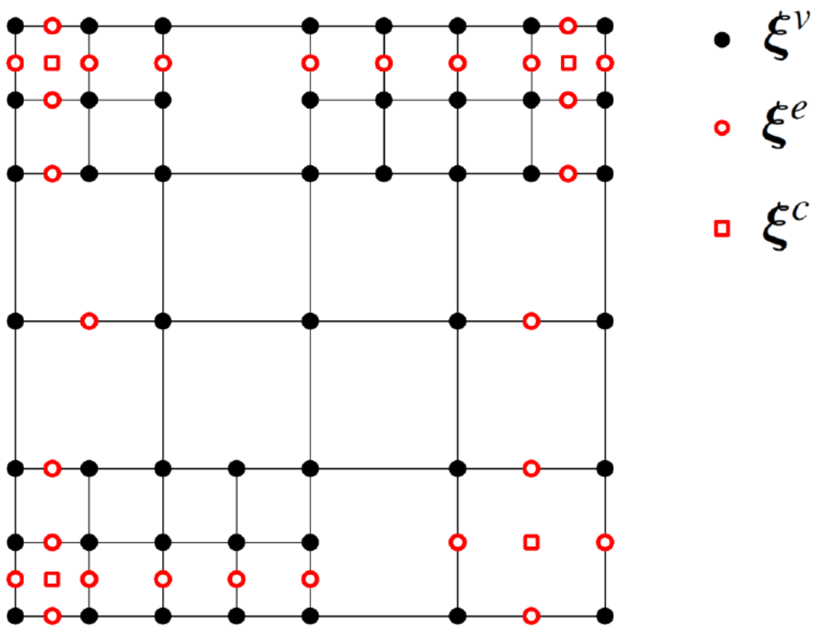
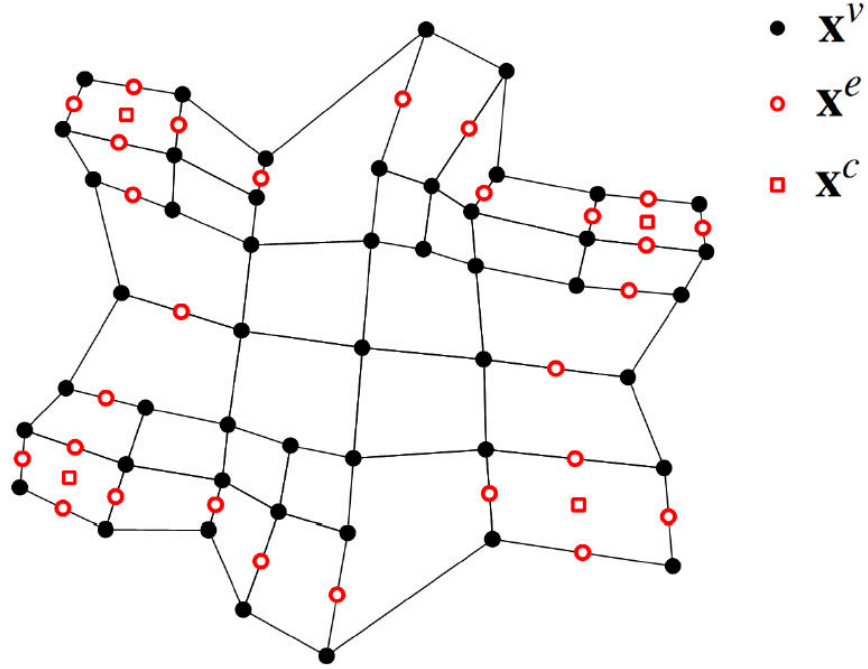


T-spline Parameterization

Determination of control points by imposing interpolation conditions

$$\mathbf{S}(\boldsymbol{\xi}) = \sum_{\alpha \in A} \mathbf{P}_\alpha \, R_\alpha(\boldsymbol{\xi})$$



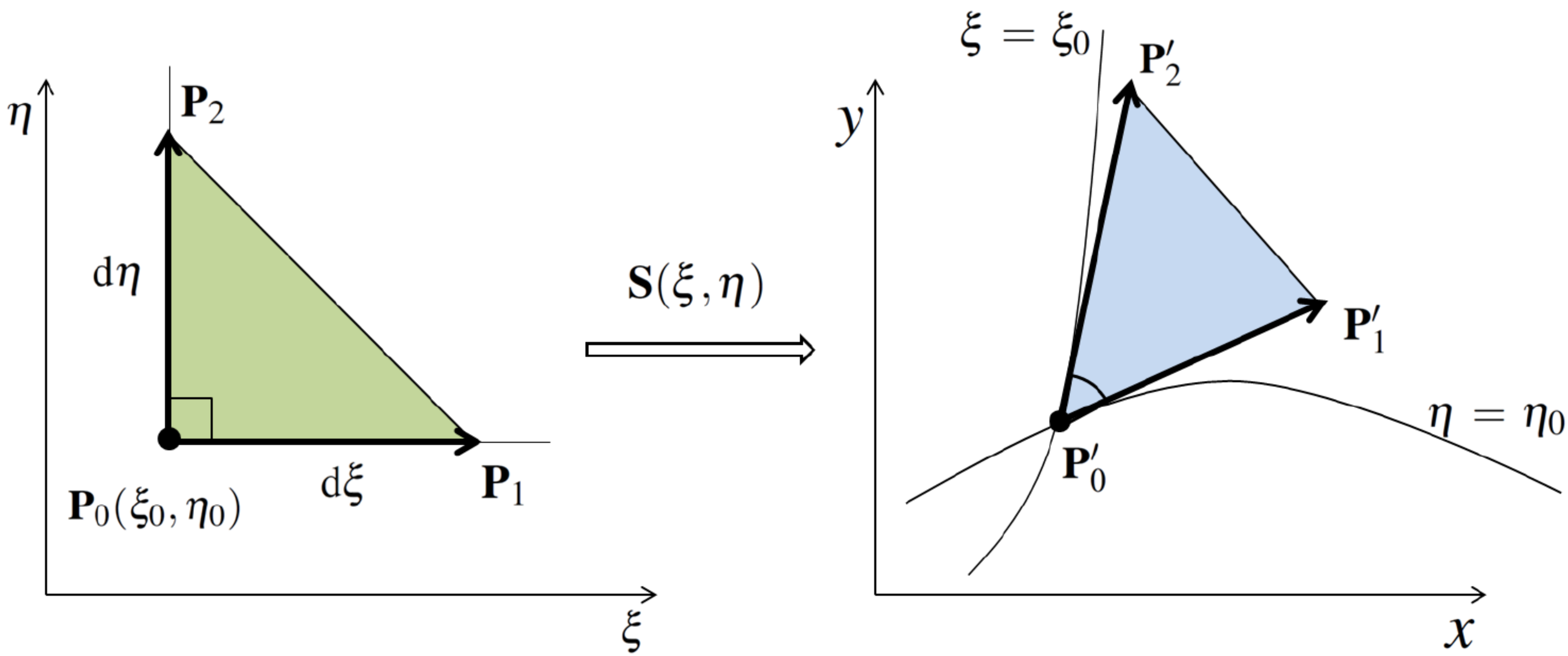


Mean Ratio Jacobian of Parametric Transformation

A quality metric of the T-spline mapping at any point P_0

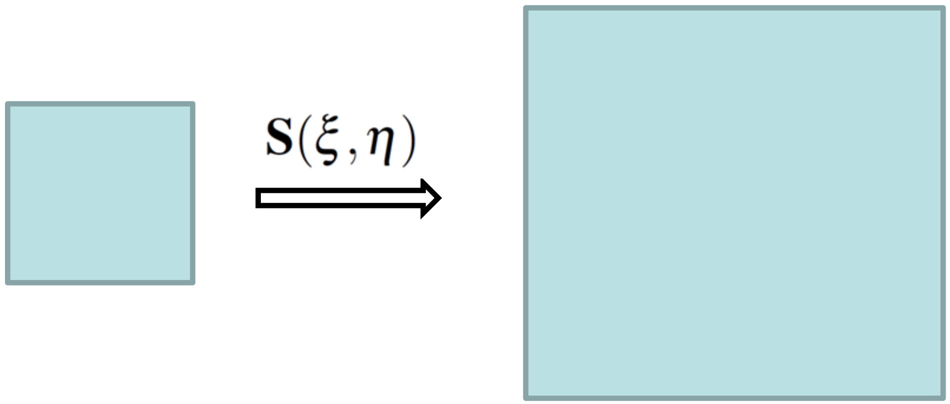
$$-1 \le J_r(\boldsymbol{\xi}) = \frac{2\det(J)}{\|J\|^2} \le 1$$

where $\boldsymbol{J}$ is the jacobian matrix of the T-spline mapping $\mathbf{S}$



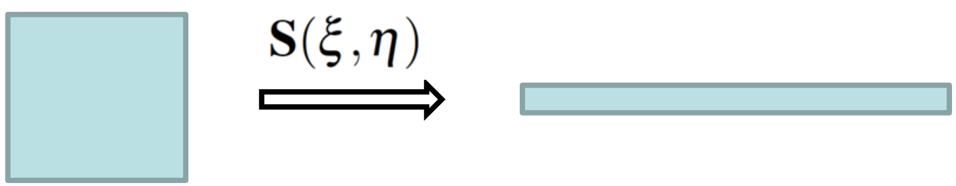
Mean Ratio Jacobian of Parametric Transformation

Comparison between mean ratio Jacobian and scaled Jacobian



Square Square

- Scaled Jacobian = 1
- Mean ratio Jacobian = 1

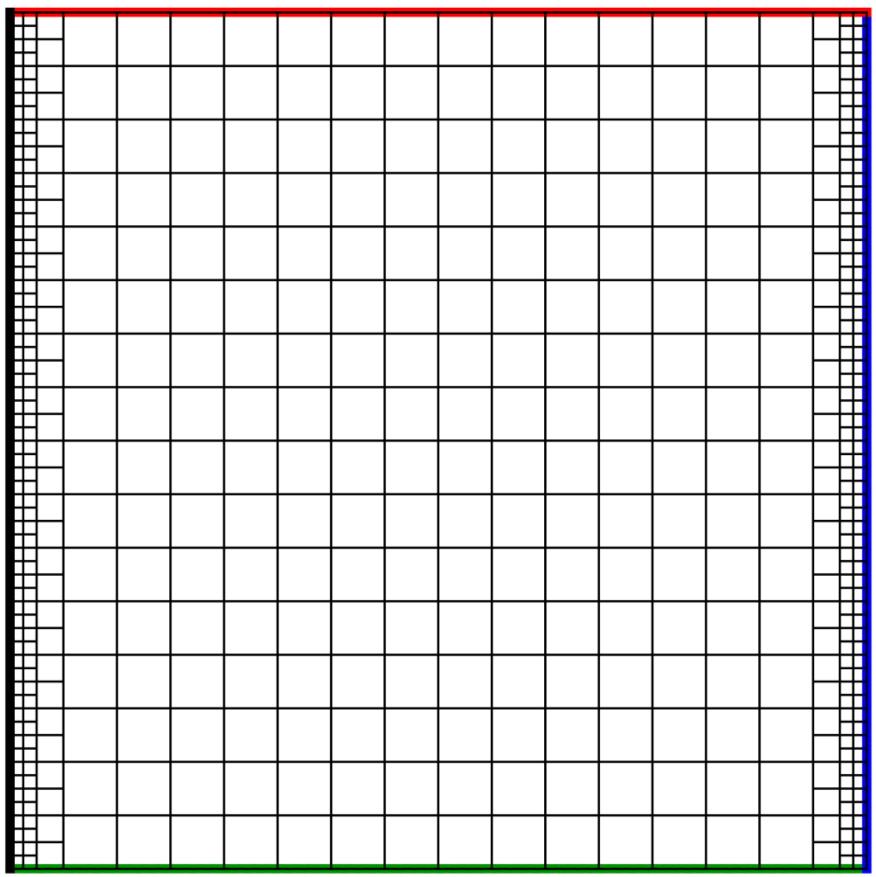


Square Flattened Rectangle

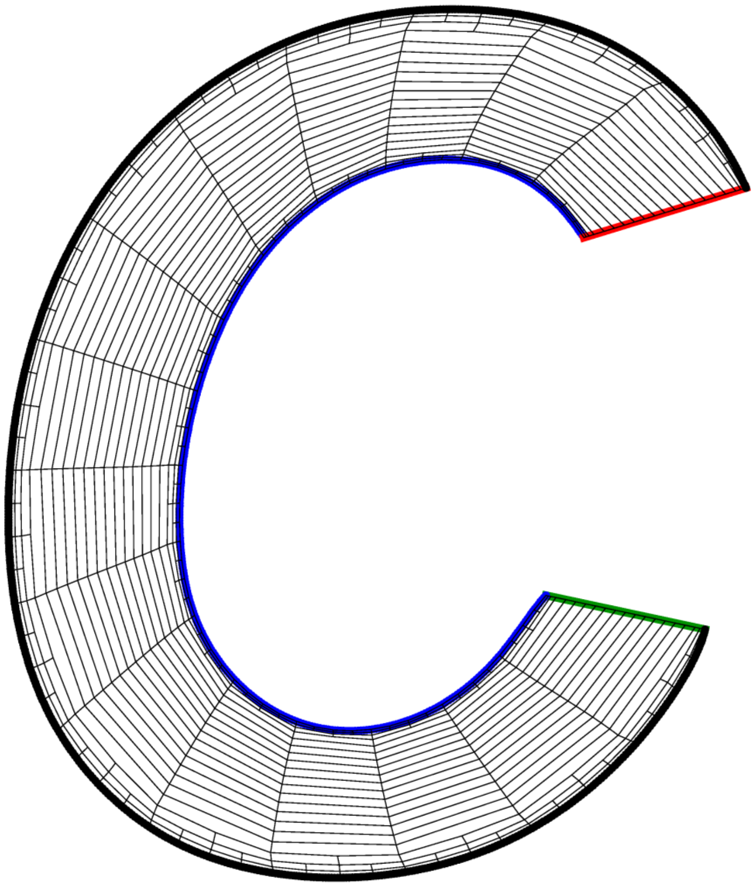
- Scaled Jacobian = 1
- Mean ratio Jacobian = close to 0

Mean Ratio Jacobian of Parametric Transformation

Comparison between mean ratio and scaled Jacobian



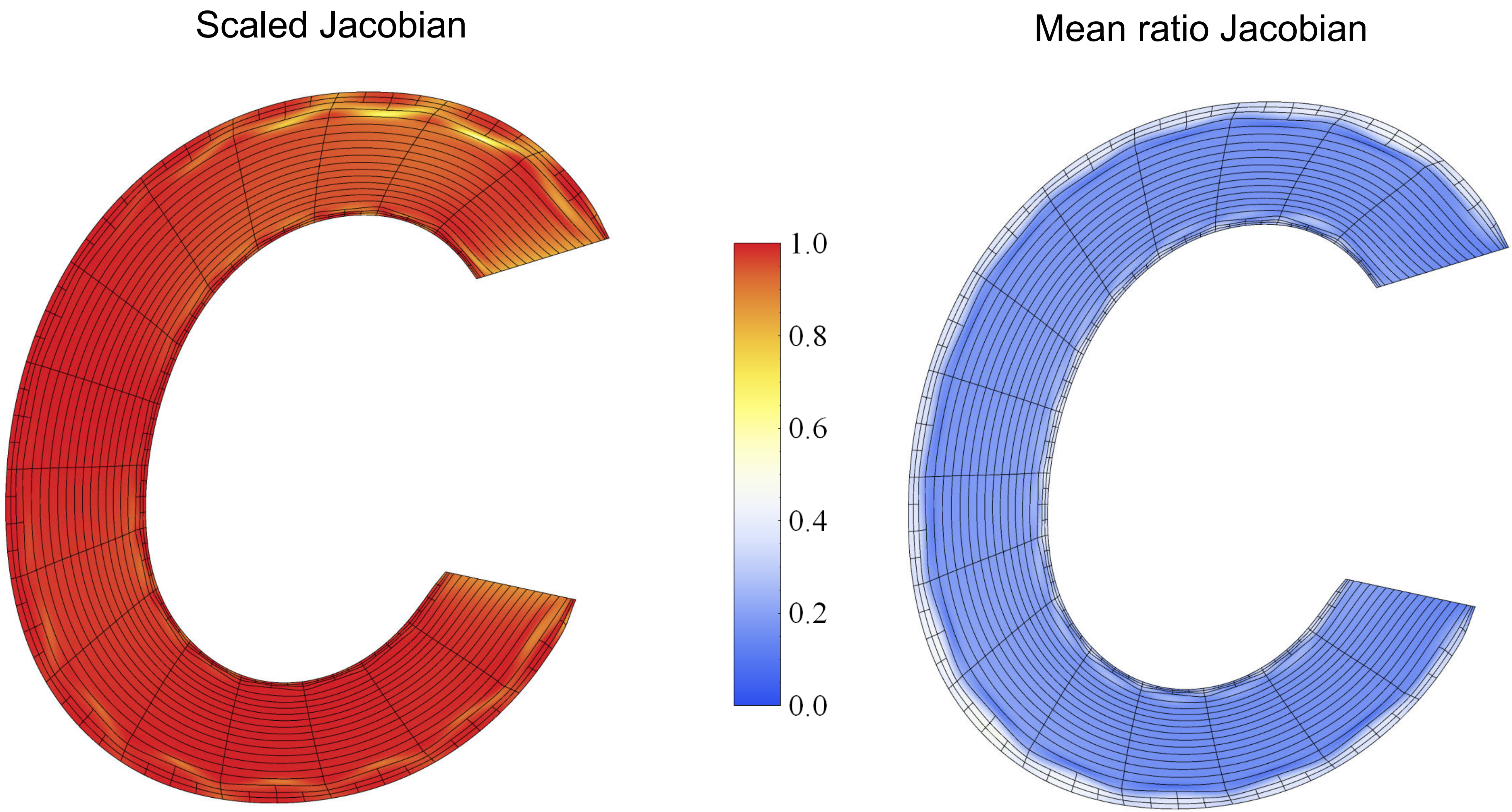
Parameter space



Physical space

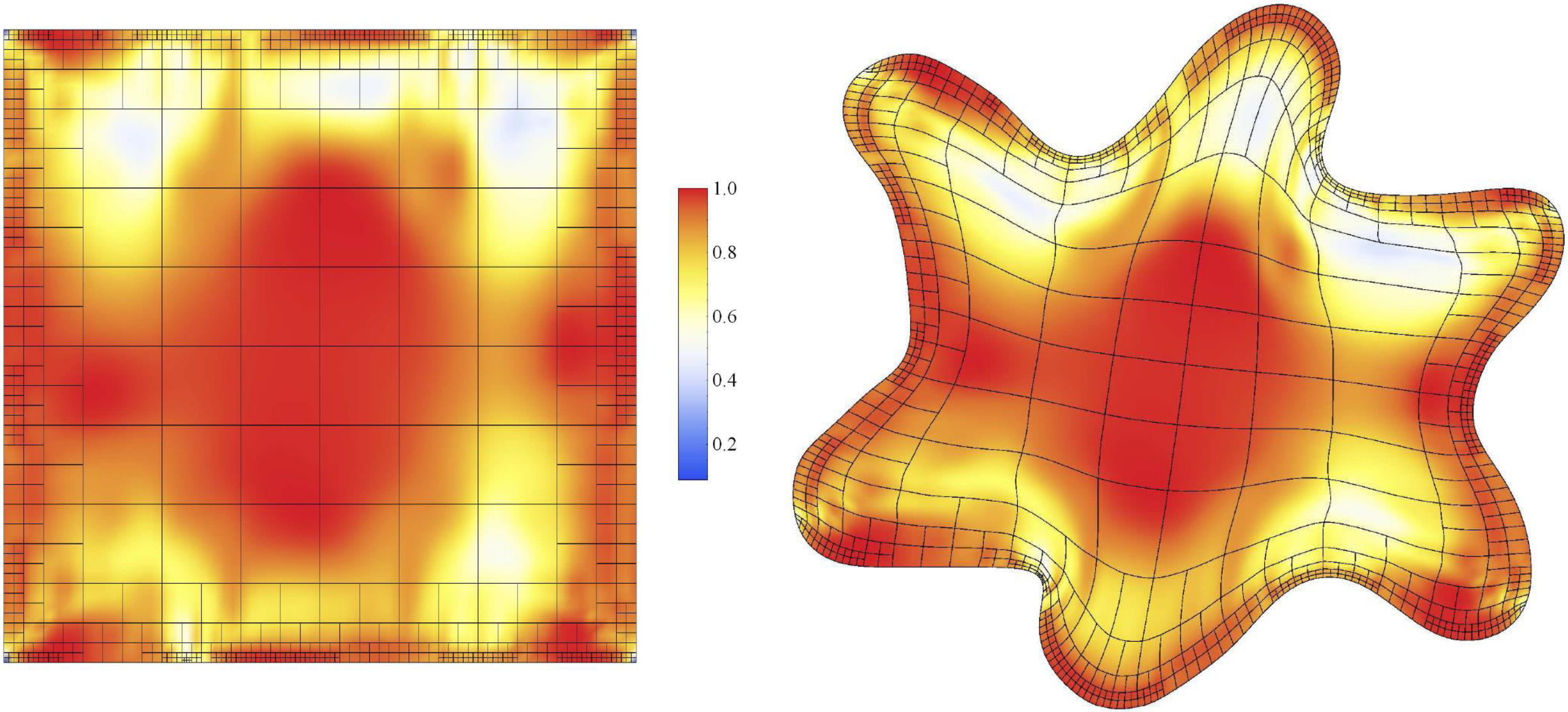
Mean Ratio Jacobian of Parametric Transformation

Comparison between mean ratio and scaled Jacobian



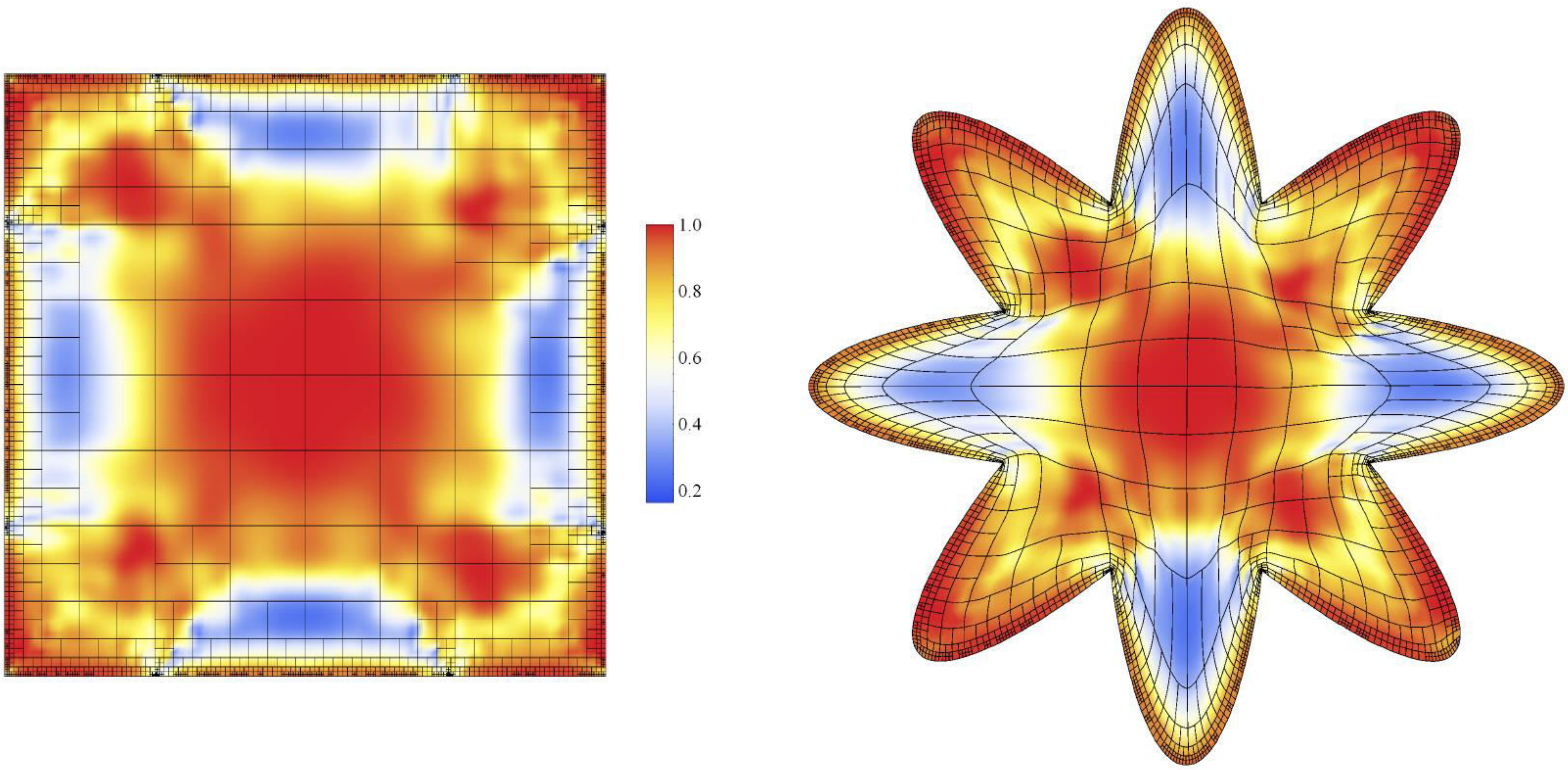
Applications: Isogeometric Modeling

The Spot (Mean Ratio Jacobian)



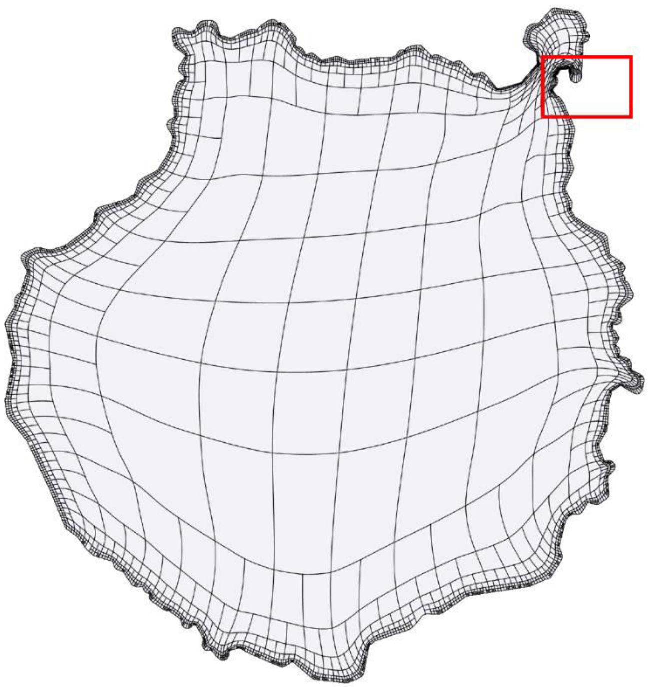
Applications: Isogeometric Modeling

The Flower (Mean Ratio Jacobian)

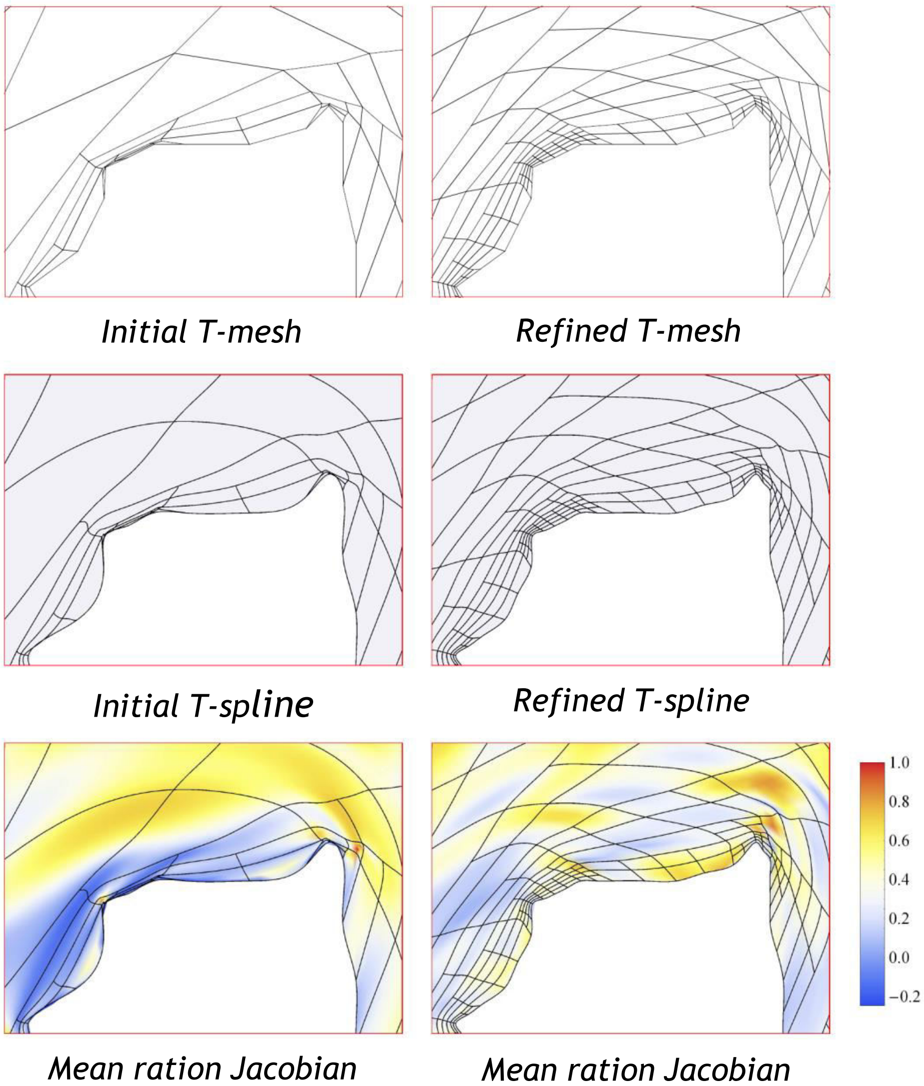


Applications: Isogeometric Modeling

Gran Canaria Island (Adaptive refinement to improve the Mean Ratio Jacobian)



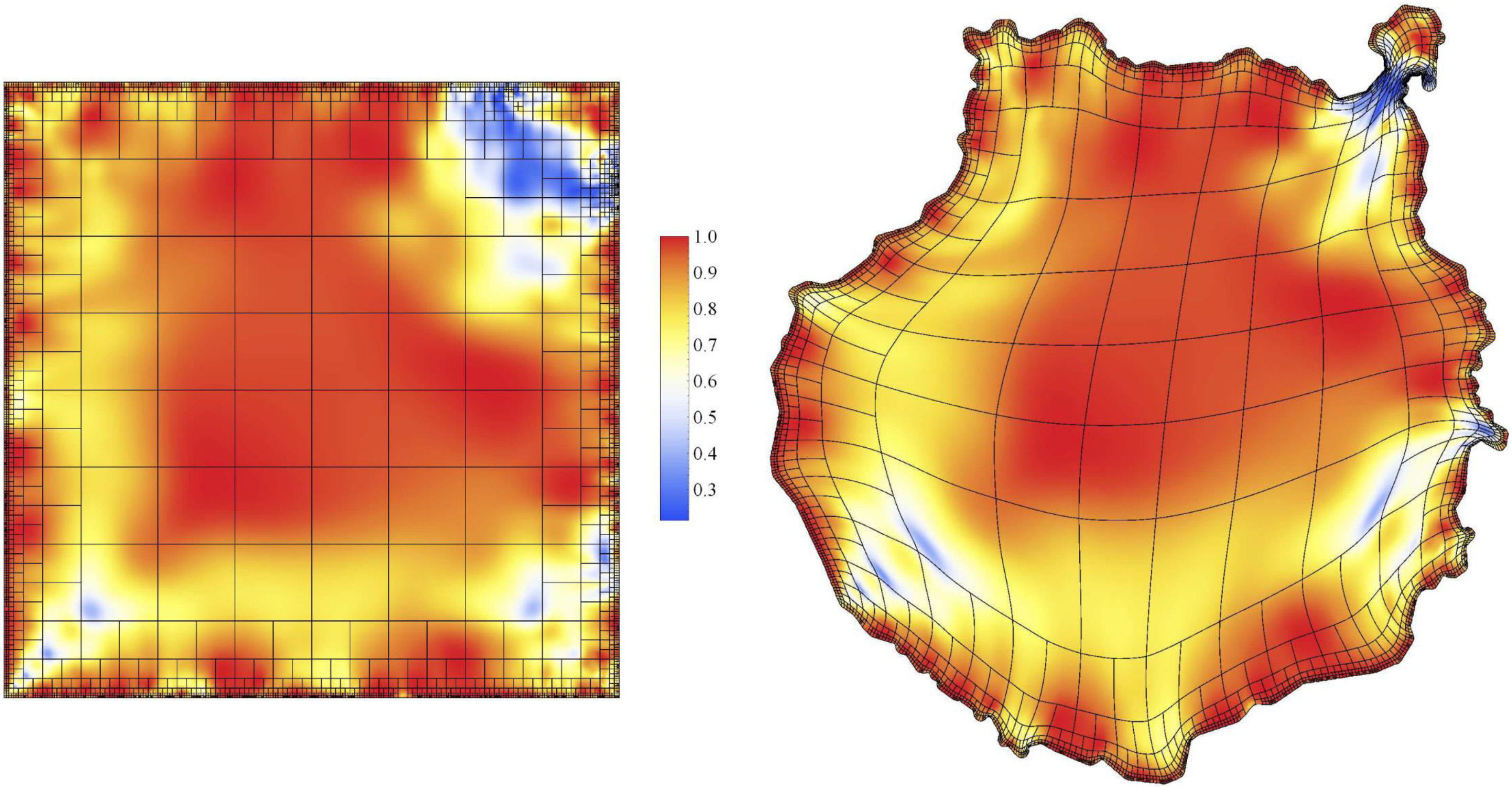
(a)



No negative Jacobian after refinement!

Applications: Isogeometric Modeling

Gran Canaria Island (Mean Ratio Jacobian)

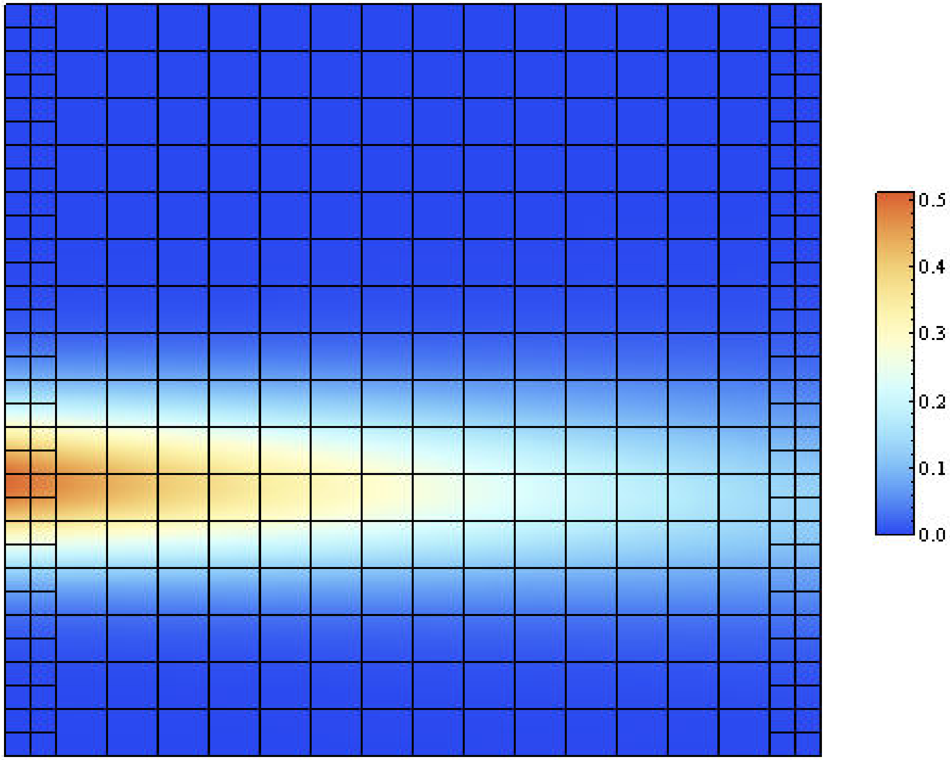


Applications: Isogeometric Analysis

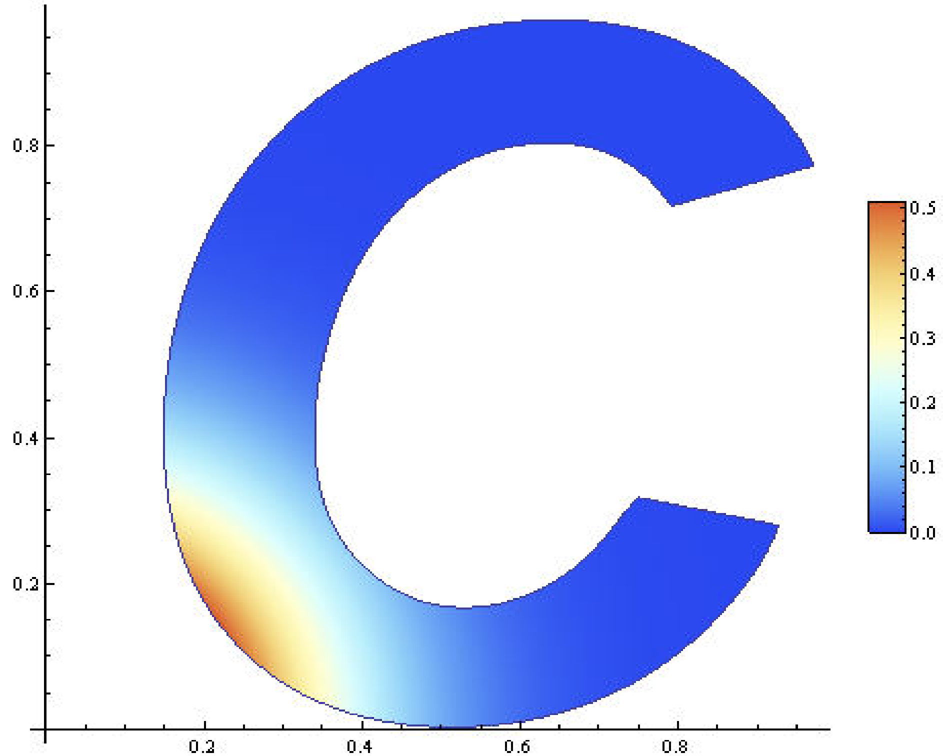
Numerical solution of a Poisson problem

$$-\triangle u = f \quad \text{in } \Omega,$$
$$u = g \quad \text{on } \partial\Omega.$$

Exact solution of elliptic problem: $u(x) = e^{-10(x^2+y^2)}$



Solution in parameter space



Solution in physical space

Local Nested Adaptive Refinement

Numerical solution of a Poisson problem

Concentrate source in relation to the initial mesh size

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$$
\begin{aligned}
-\triangle u &= f && \text{in } \Omega, \\
u &= g && \text{on } \partial\Omega.
\end{aligned}
$$

Exact solution:

$$u(x,y) = \exp\left[-10^3\left((x-0.6)^2 + (y-0.35)^2\right)\right]$$

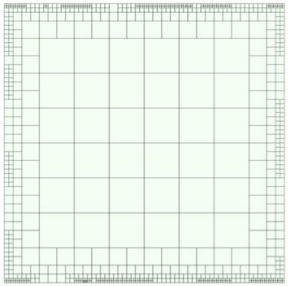
Residual-type error indicator:

$$\eta(\Omega_e)^2 = \|h(f+\Delta u_h)\|^2_{0,\Omega_e} = \int_{\Omega_e} h^2 (f+\Delta u_h)^2 \, \mathrm{d}\Omega$$

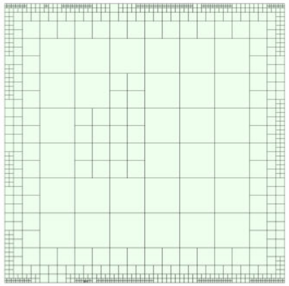
Local Nested Adaptive Refinement

Numerical solution of a Poisson problem

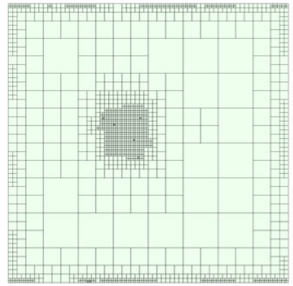
Error indicator : $\eta\left(\Omega_e\right)^2=\int_{\Omega_e} h^2\left(f+\Delta u_h\right)^2 \, d\Omega$



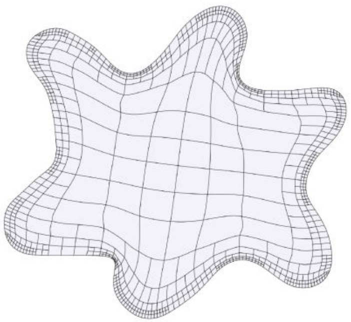
(a) 844 cells,1456 DOF



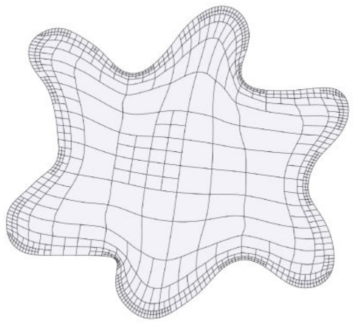
(b) 859 cells, 1476 DOF



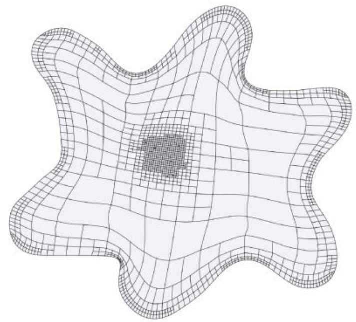
(c) 1552 cells, 2233 DOF



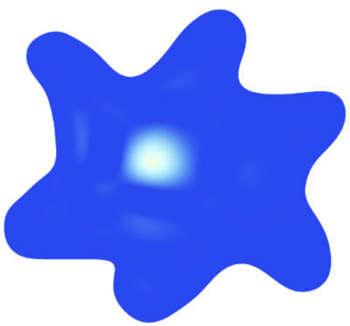
(d) Initial mesh



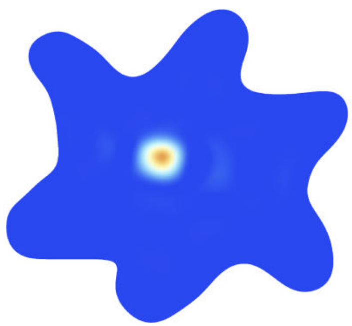
(e) 1-st refinement



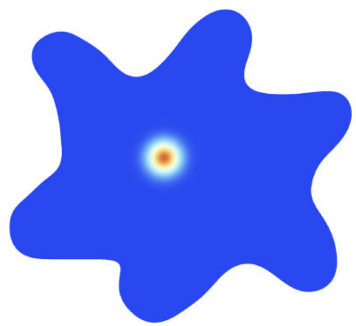
(f) 14-th refinement



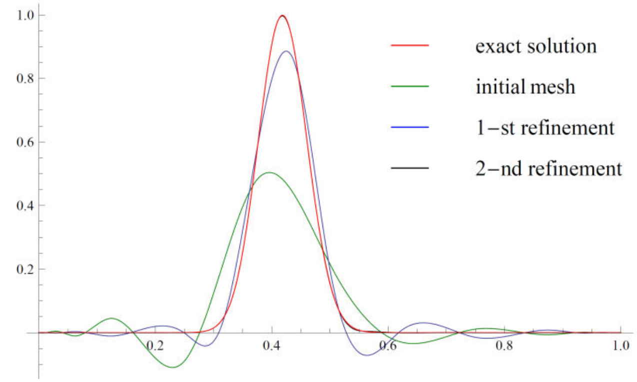
(g) Initial solution



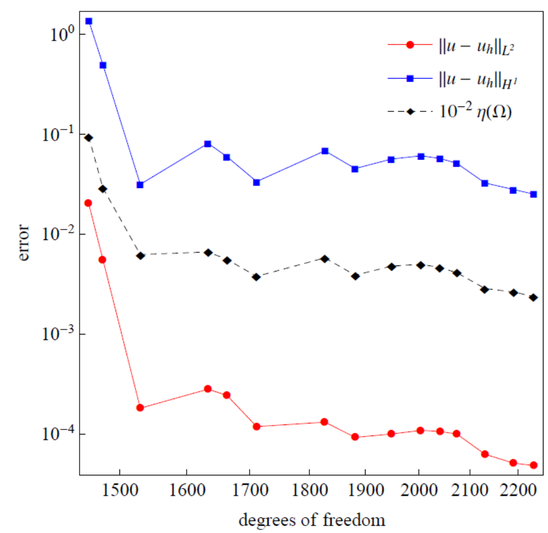
(h) 1-st refinement



(i) 14-th refinement



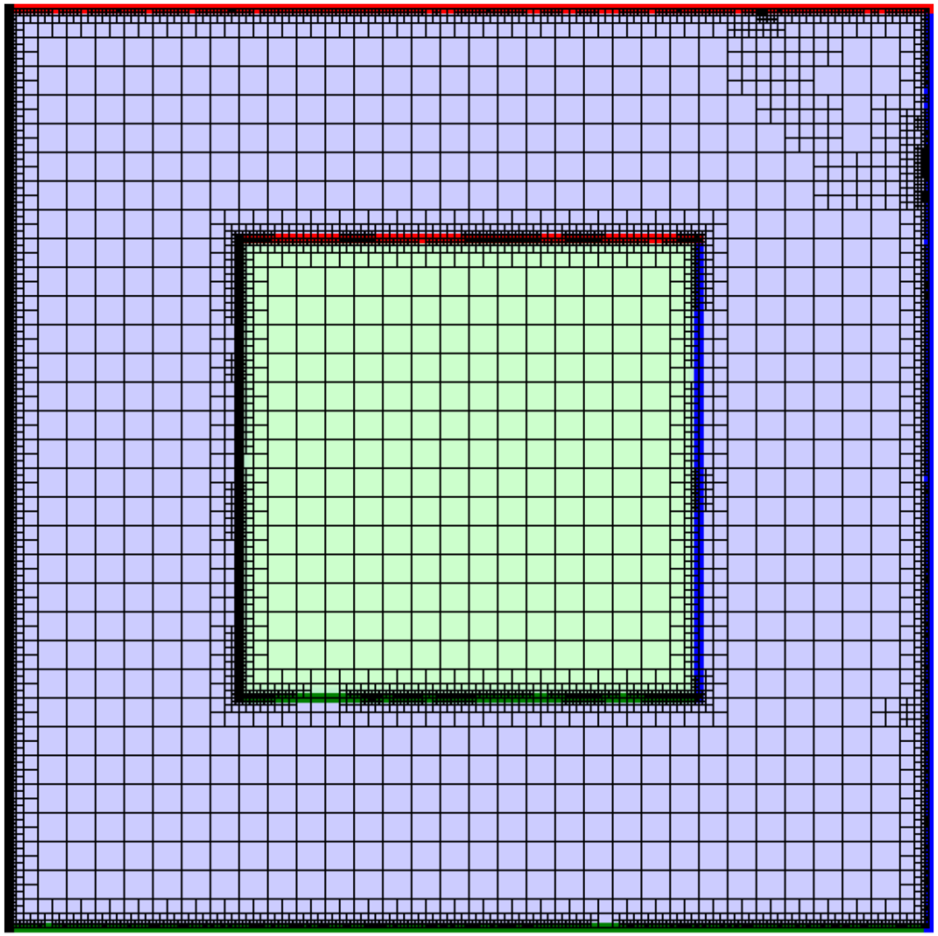
Numerical solution across a section

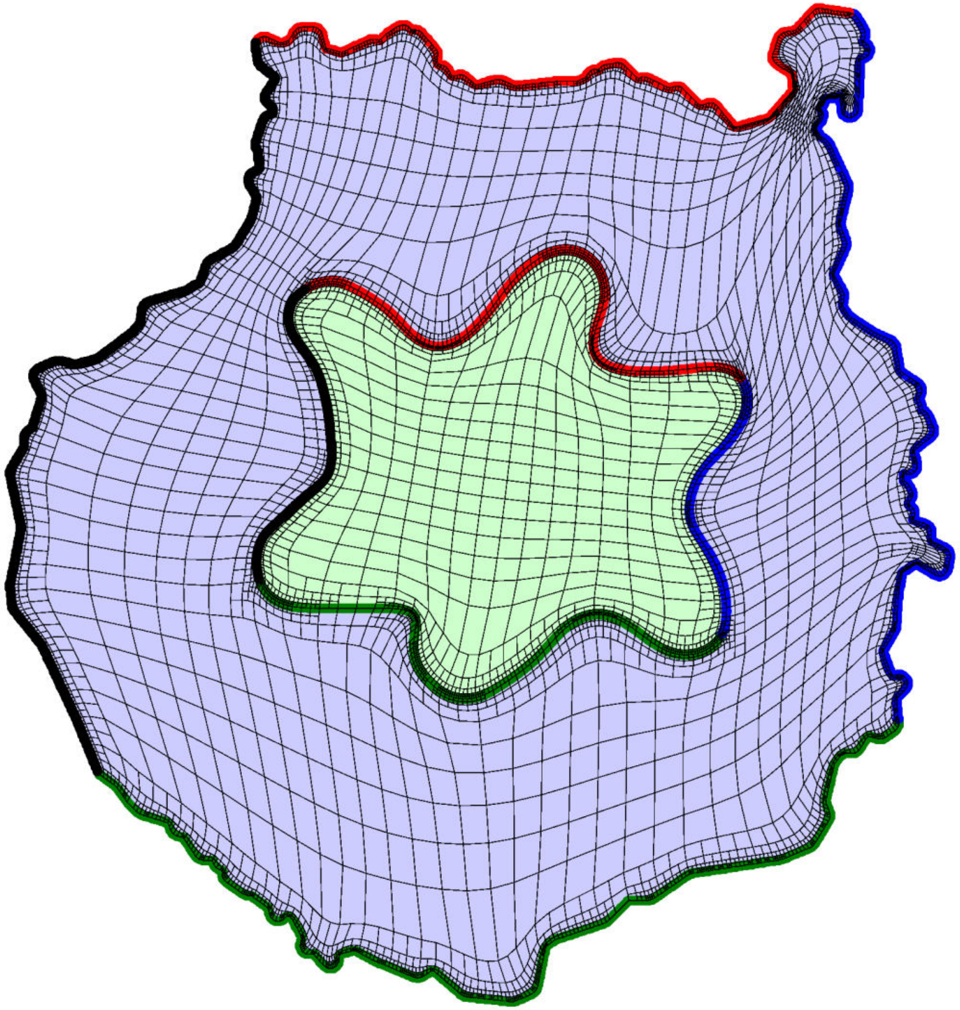


Convergence behavior

Final Comments and Future Works

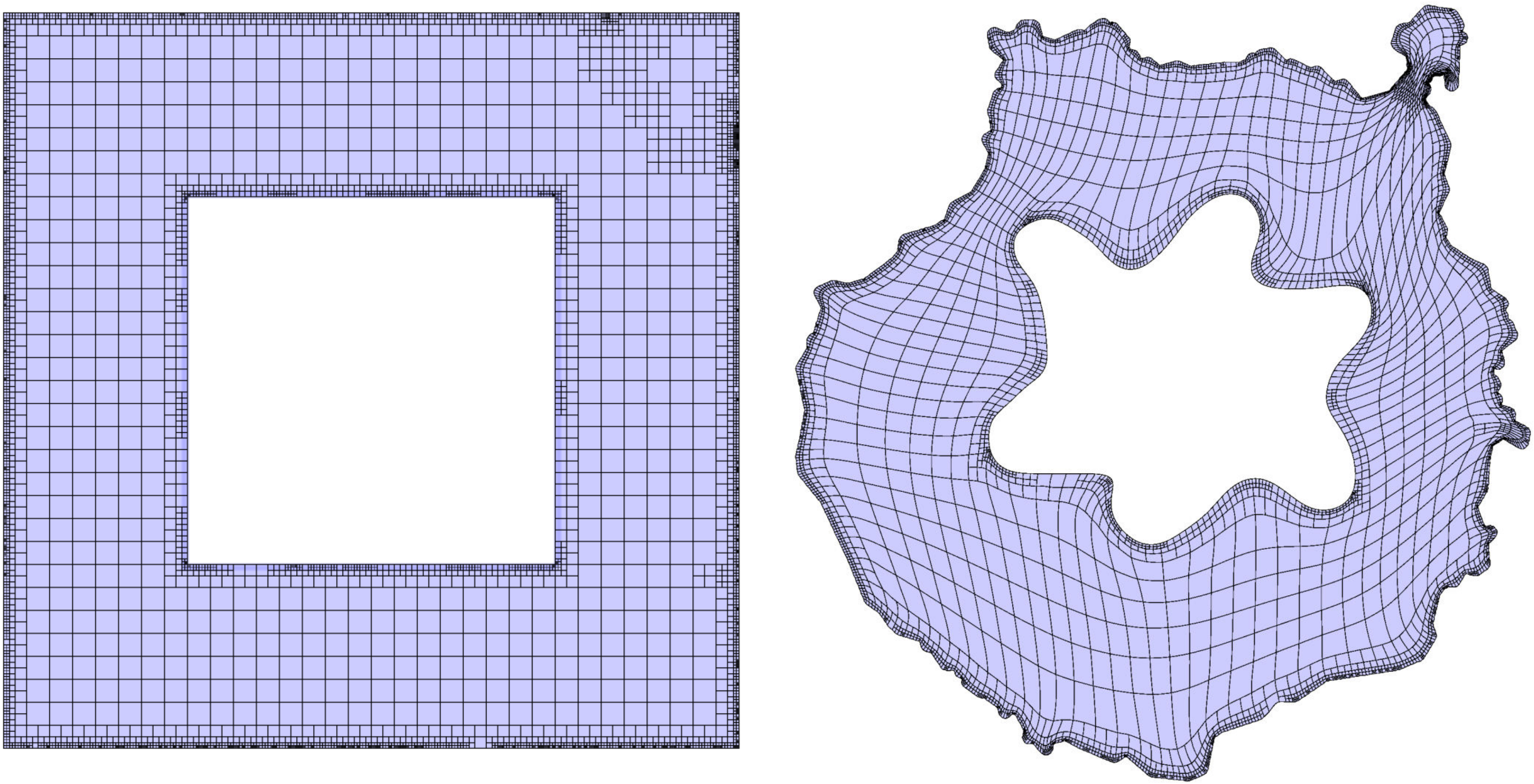
Isogeometric modeling of geometries with several materials





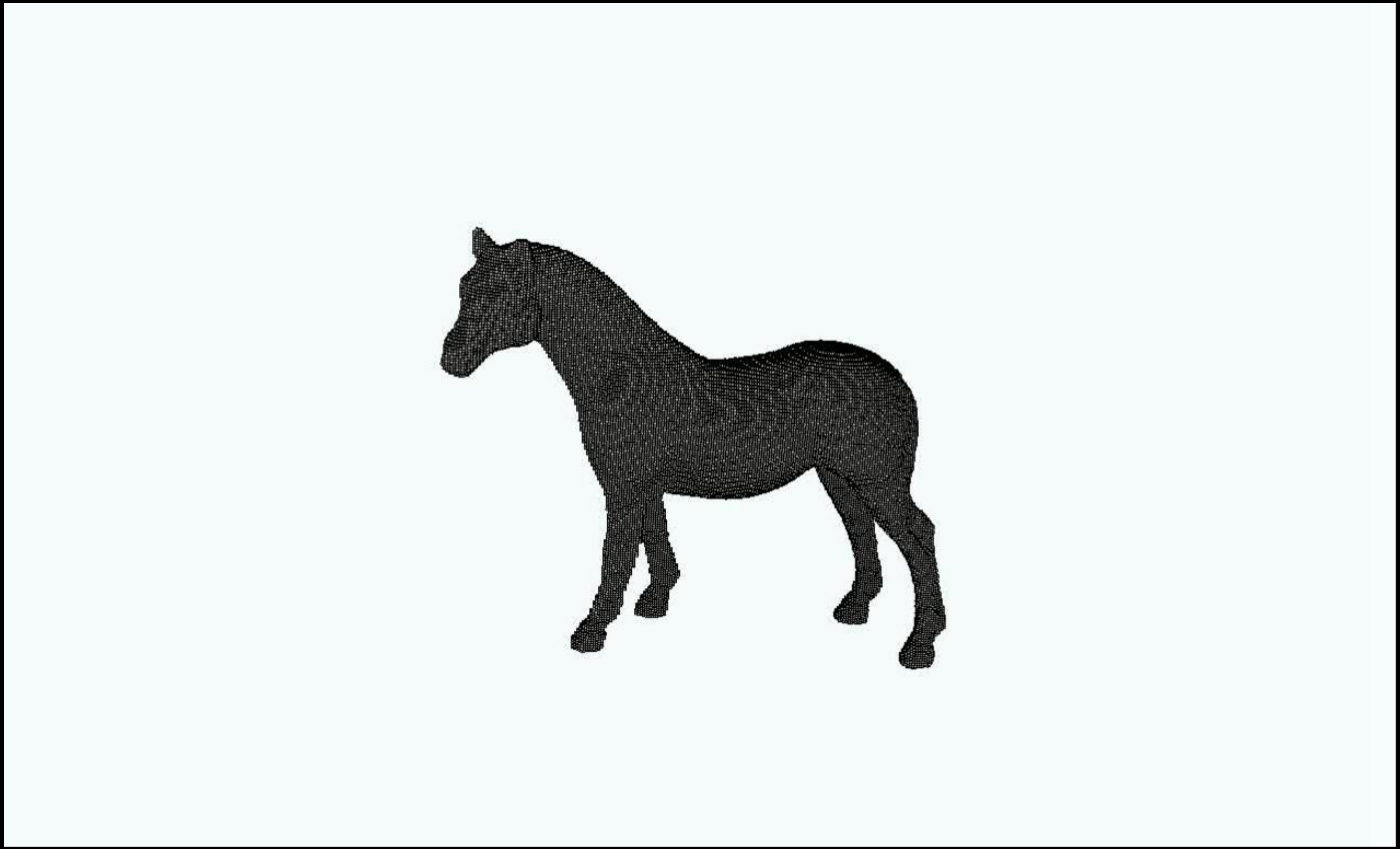
Final Comments and Future Works

Isogeometric modeling of geometries with holes



Final Comments and Future Works

Automatic Construction of the Meccano



Final Comments and Future Works

Automatic Construction of the Meccano

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T-spline Parameterization of 2D Geometries Based on the Meccano Method with a New T-mesh Optimization Algorithm

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22nd International Meshing Roundtable, October 13-16, 2013, Orlando, USA

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