



Mesh Generation and Isogeometric Analysis

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<http://www.dca.iusiani.ulpgc.es/proyecto2012-2014>

Avances en Simulación de Campos de Viento y Radiación Solar



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Proyecto
Descripción
Objetivos
Investigadores
Resultados
Links de interés

Publicaciones
Artículos
Congresos
Libros y capítulos
Otras publicaciones

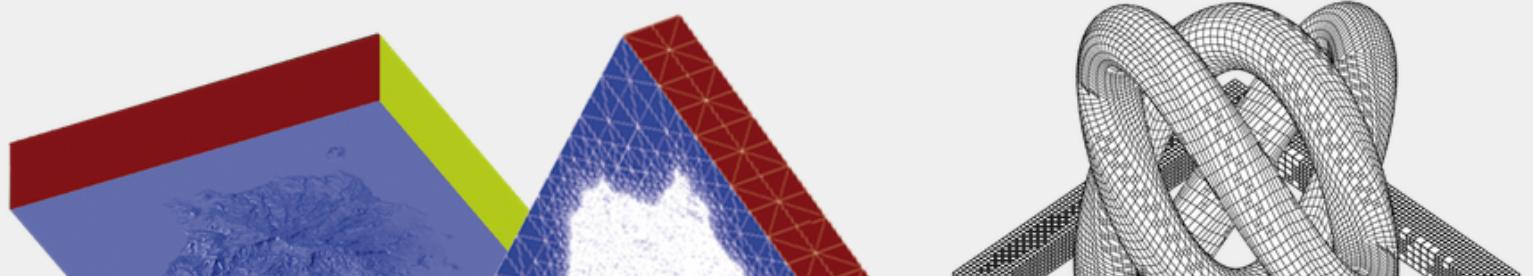
Otras actividades
Vall de Nuria

Ministerio de Economía y Competitividad y FEDER
Referencia: CGL2011-29396-C03-01
Plazo de ejecución: 01/01/2012 - 31/12/2014

Descripción del proyecto



Este Subproyecto de Investigación de la Universidad de Las Palmas de Gran Canaria, titulado “Avances en Simulación de Campos de Viento y Radiación Solar”, con referencia CGL2011-29396-C03-01, se enmarca dentro del Proyecto Coordinado “Métodos Numéricos Avanzados para Gestión Medioambiental”, con referencia CGL2011-29396-C03-00, financiado por el Ministerio de Economía y Competitividad y FEDER a través de la convocatoria de proyectos de investigación, subprograma de Proyectos de Investigación Fundamental no Orientada, dentro del Programa Nacional de Proyectos de Investigación Fundamental, VI Plan Nacional de Investigación Científica, Desarrollo e Innovación Tecnológica 2008-2011.

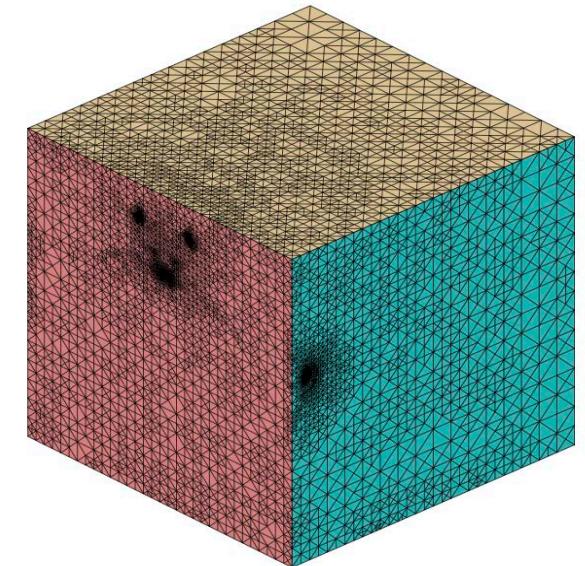
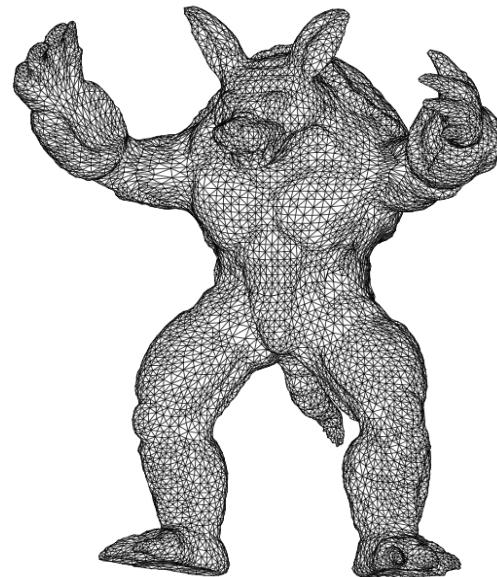
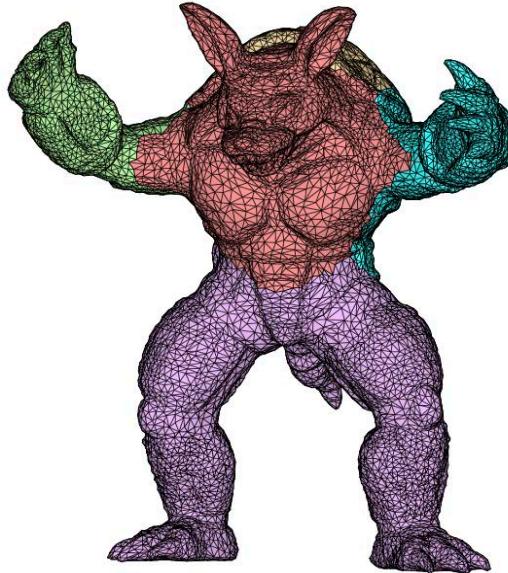


<http://www.dca.iusiani.ulpgc.es/proyecto2012-2014>

- **The Meccano Method (based on tetrahedral mesh optimization)**
 - The initial algorithm for tetrahedral mesh generation
 - Volumetric parameterization of tetrahedral meshes
 - Application to finite element and isogeometric solid modeling and analysis
- **The Meccano Method (based on a new 2D T-mesh optimization)**
 - The new algorithm for two-dimensional T-mesh generation
 - T-spline parameterization of 2D geometries
 - Geometries with several materials or holes
 - Application to isogeometric modeling and analysis
- **Comments and Future Research**

Meccano Method for Complex Solids

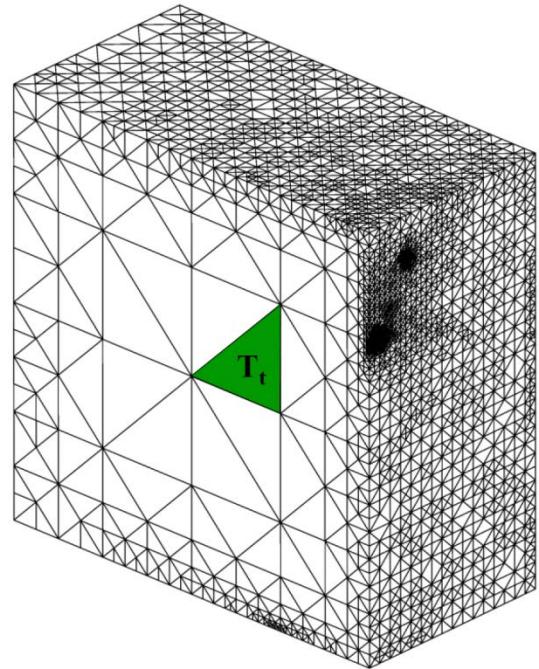
Volume parameterization based on SUS of tetrahedral meshes



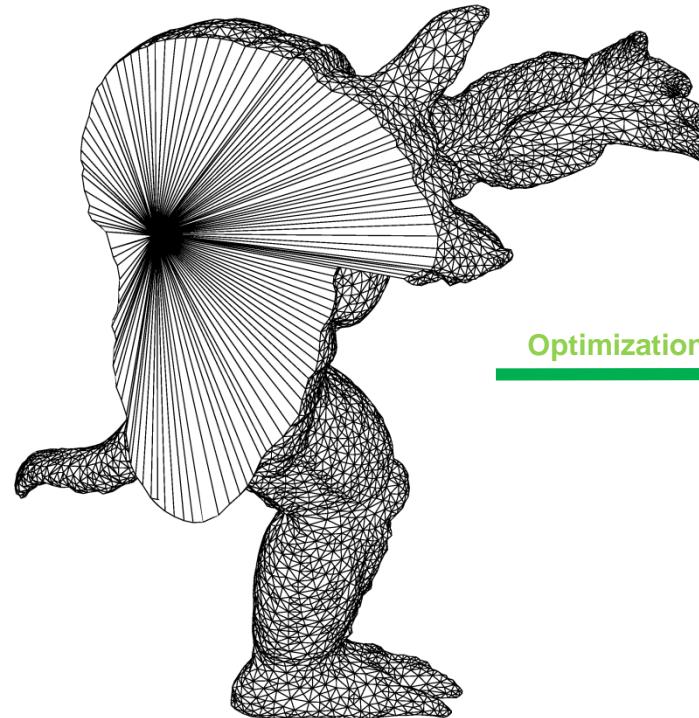
- Parameterization
- Refinement
- Untangling/Smoothing

Meccano Method for Complex Solids

Volume parameterization based on SUS of tetrahedral meshes

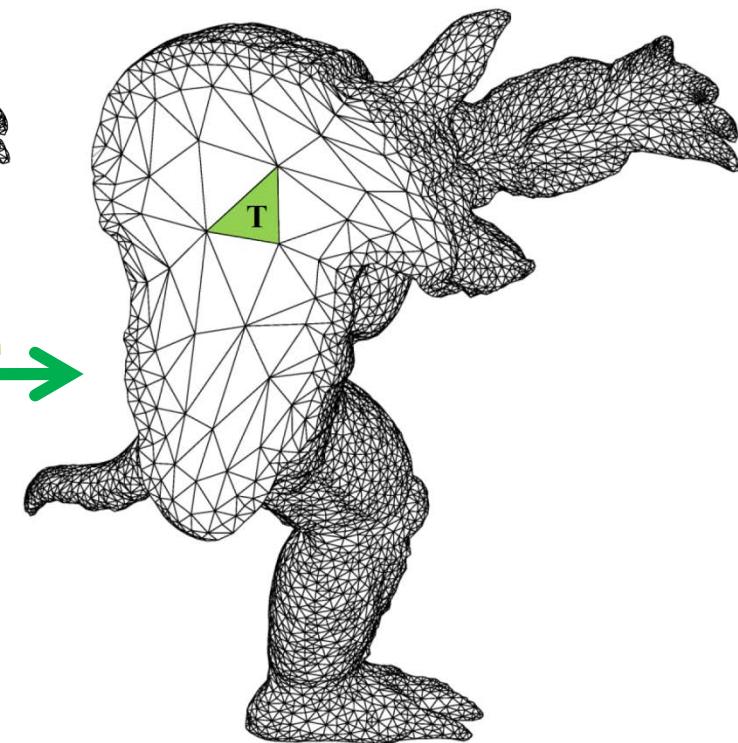


Parameter space
(meccano mesh)



Physical space
(tangled mesh)

Optimization



Physical space
(optimized mesh)

Target Element T_t

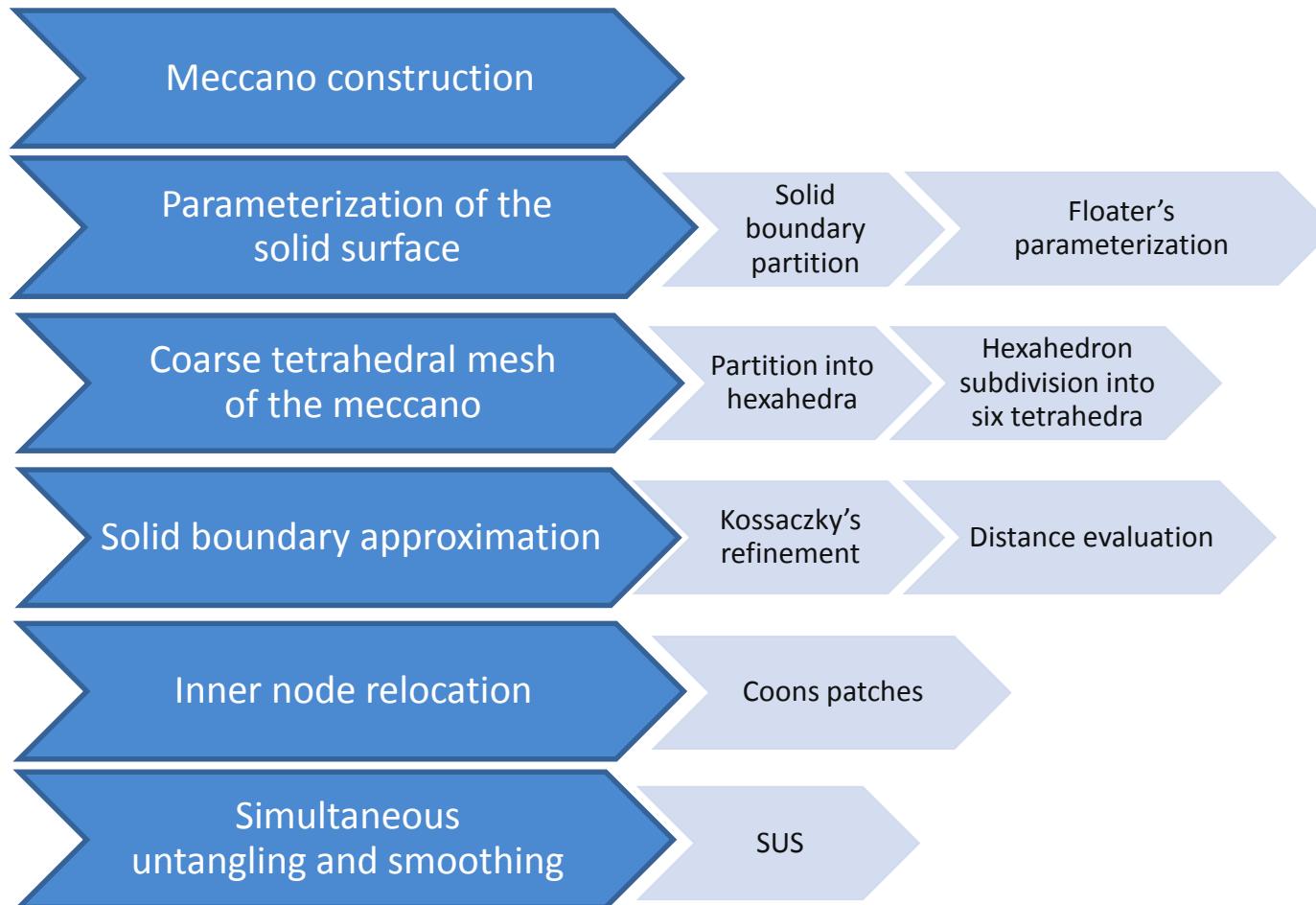
Optimization

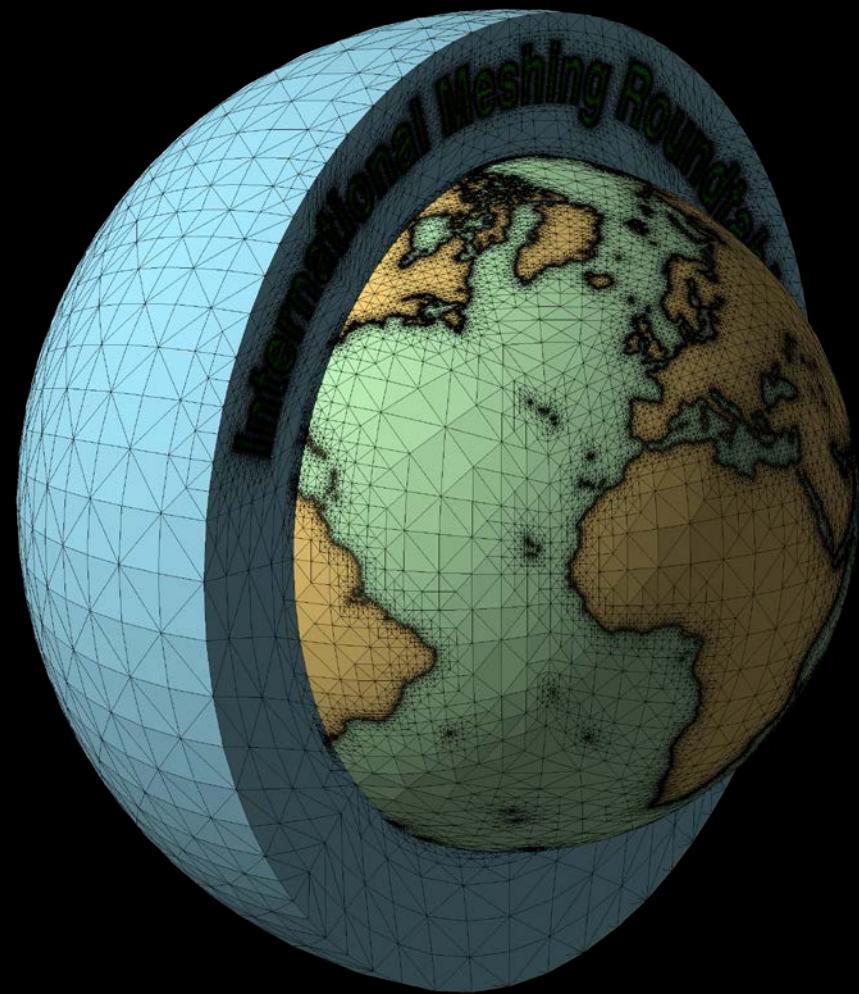
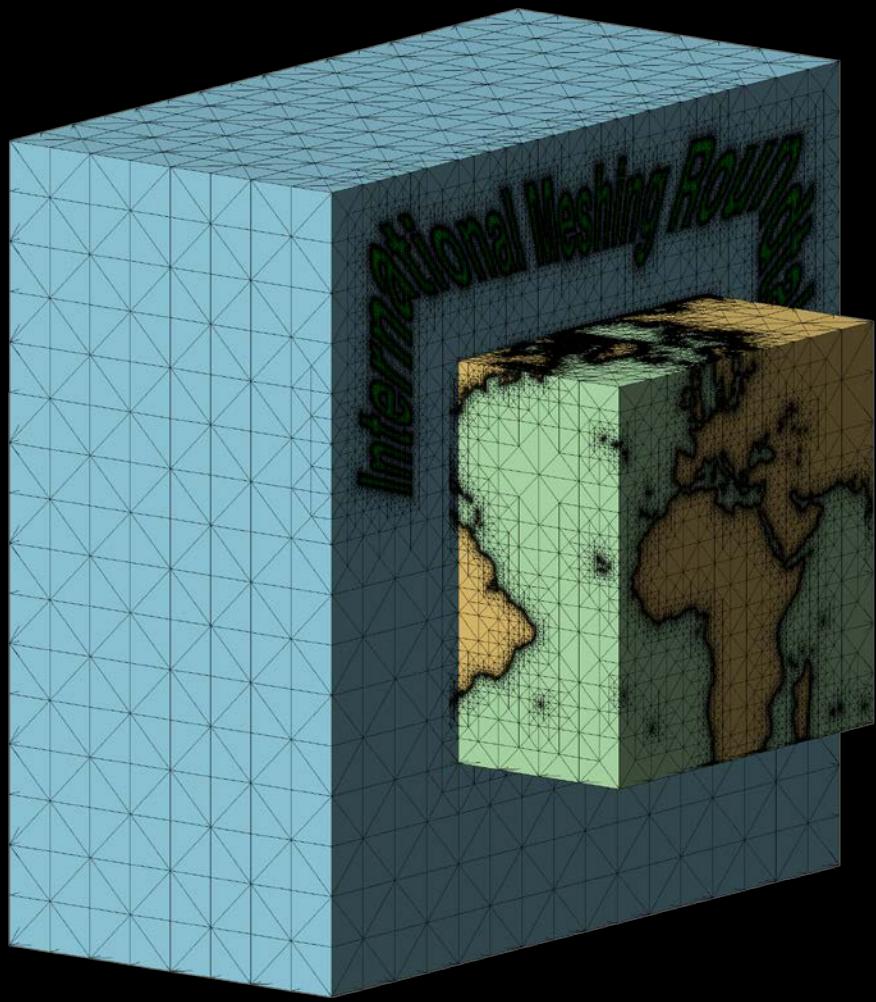
Physical Element T

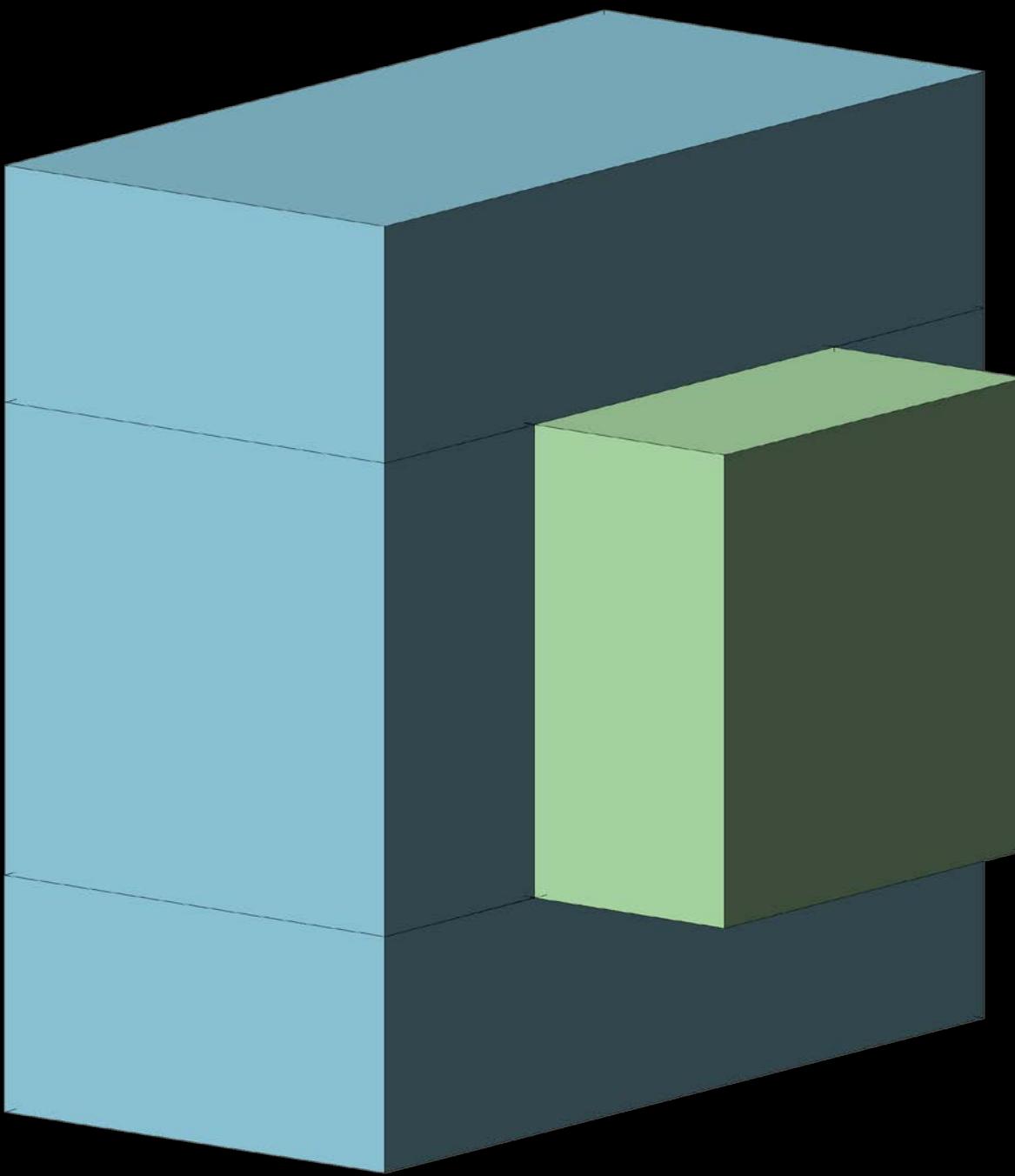
(to get less distortion in the parameterization)

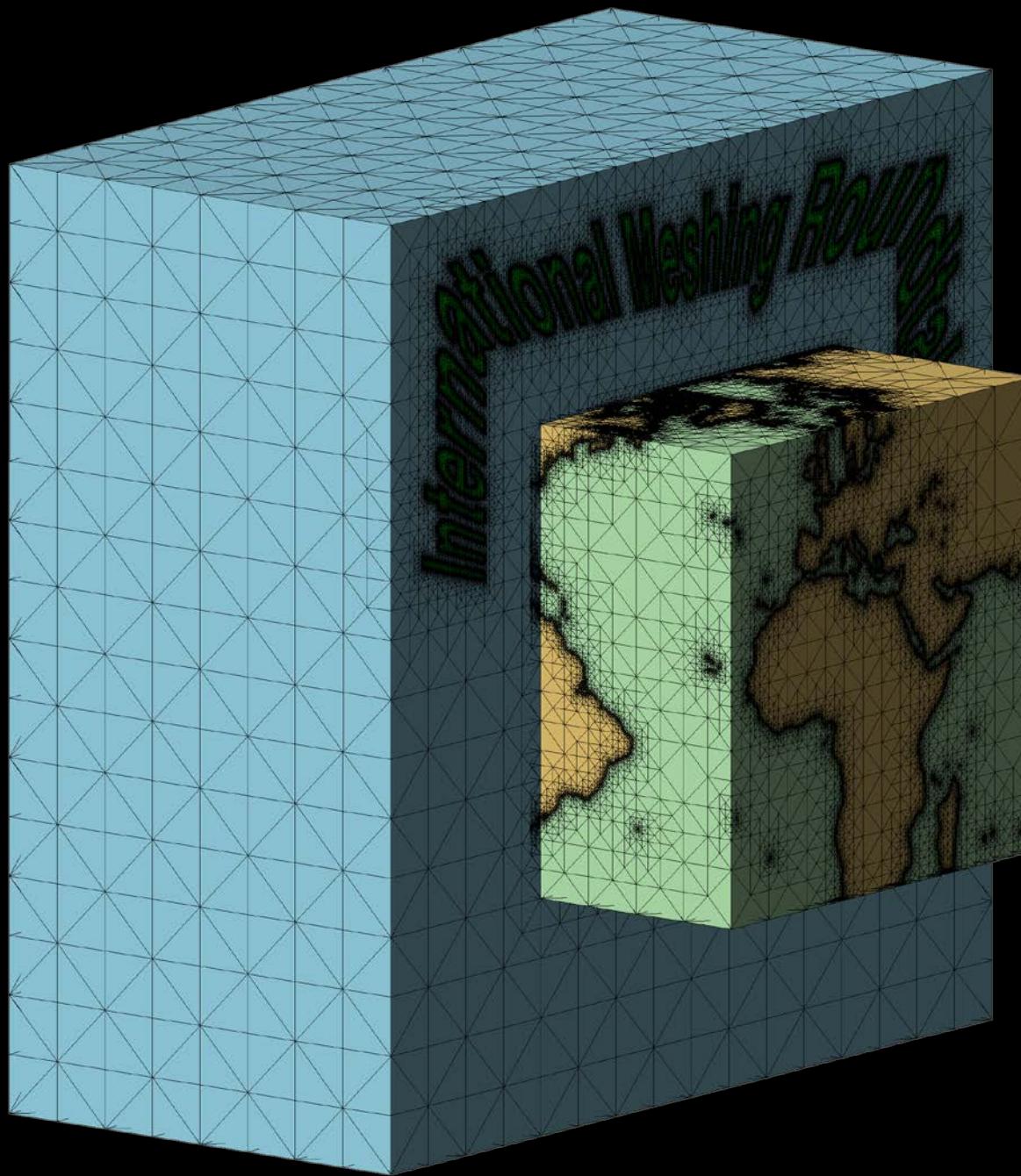
Meccano Method for Complex Solids

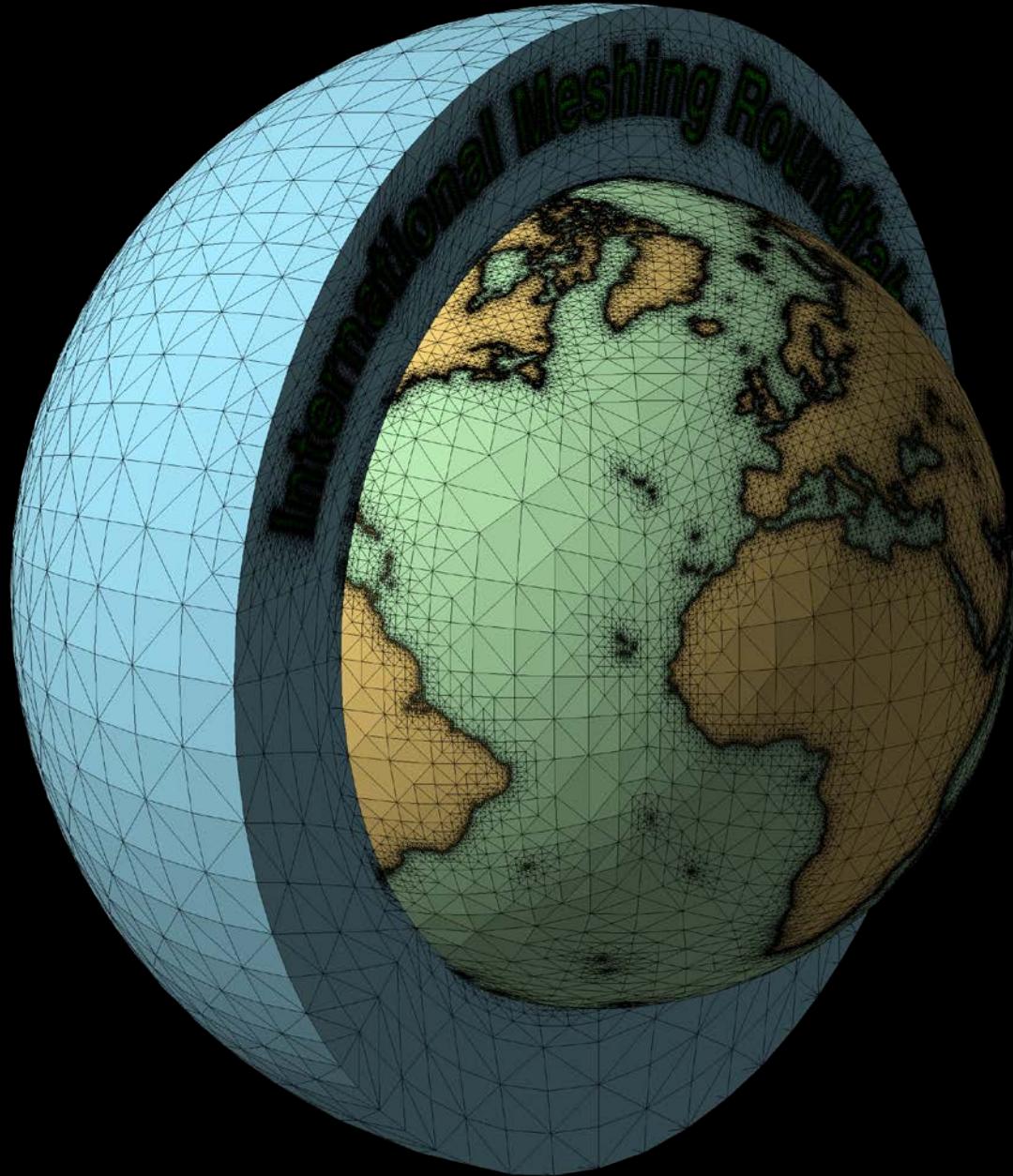
Algorithm steps

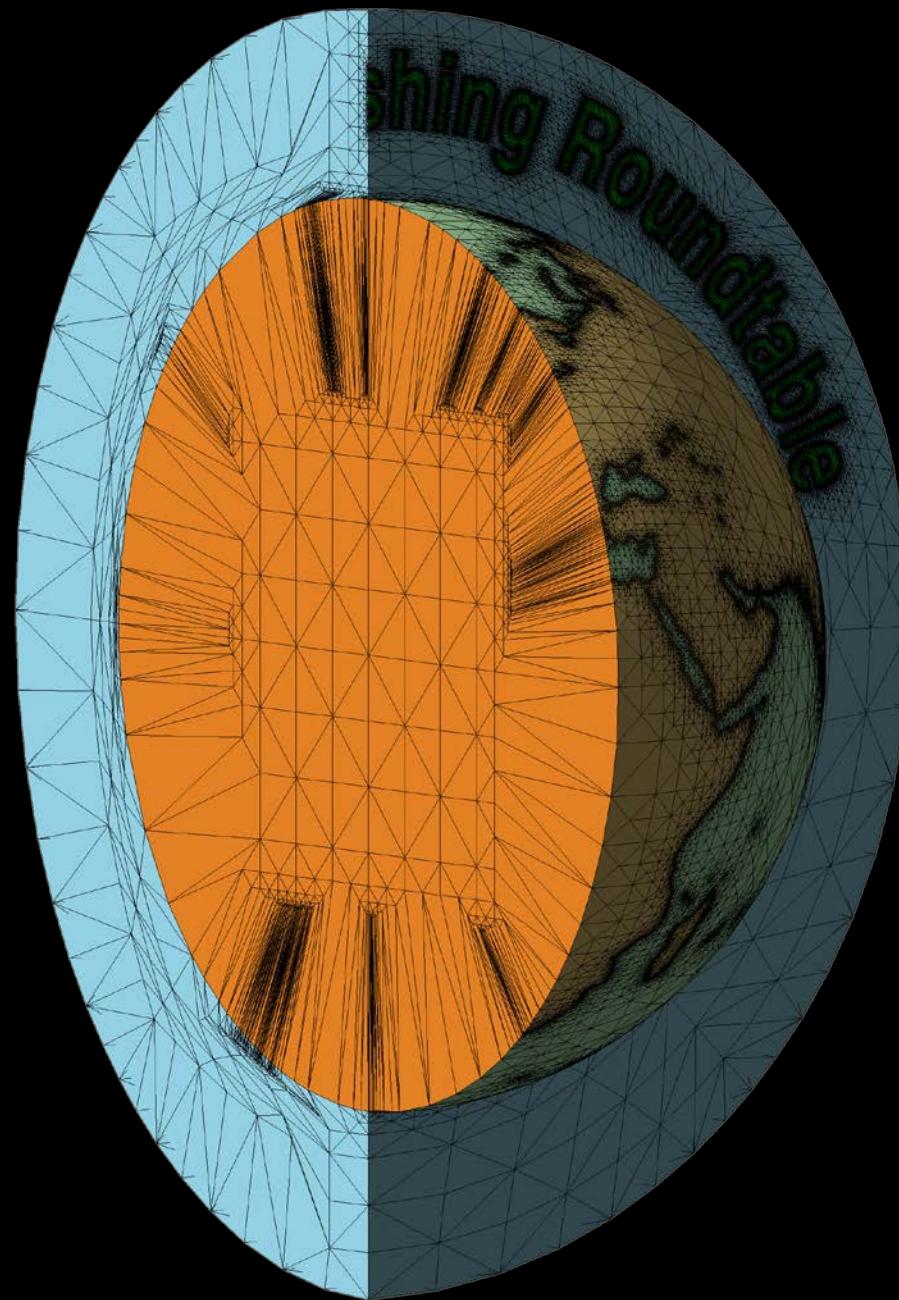


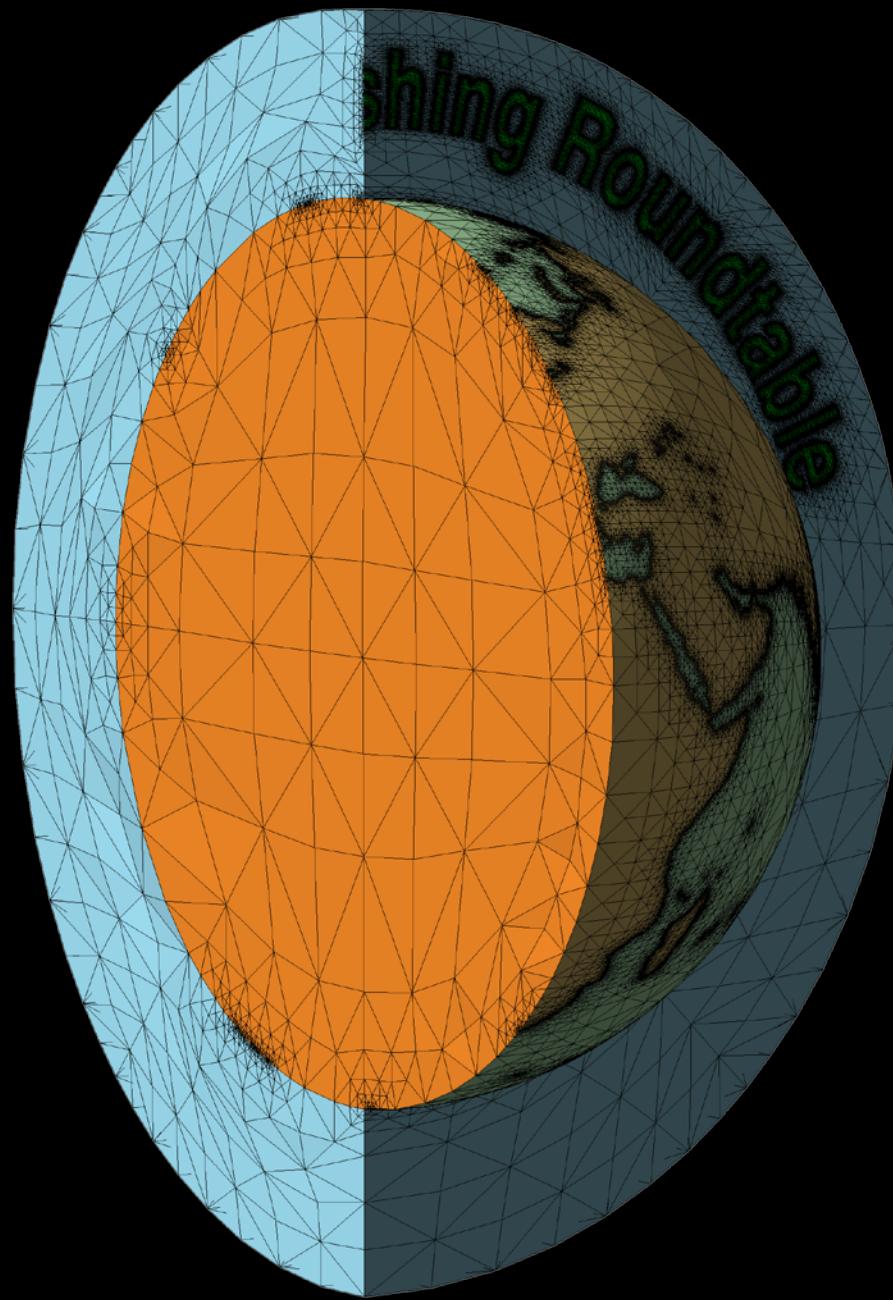












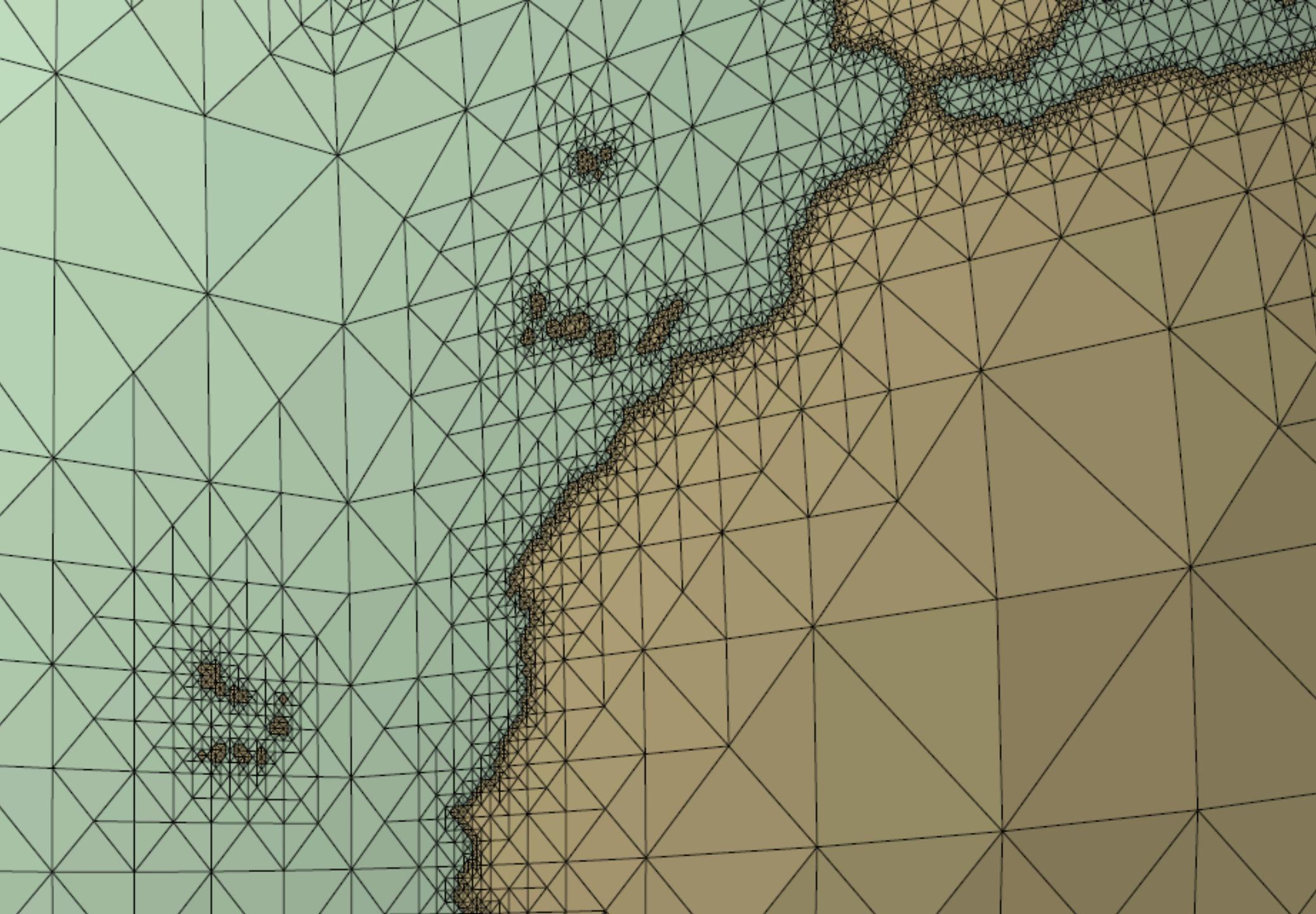






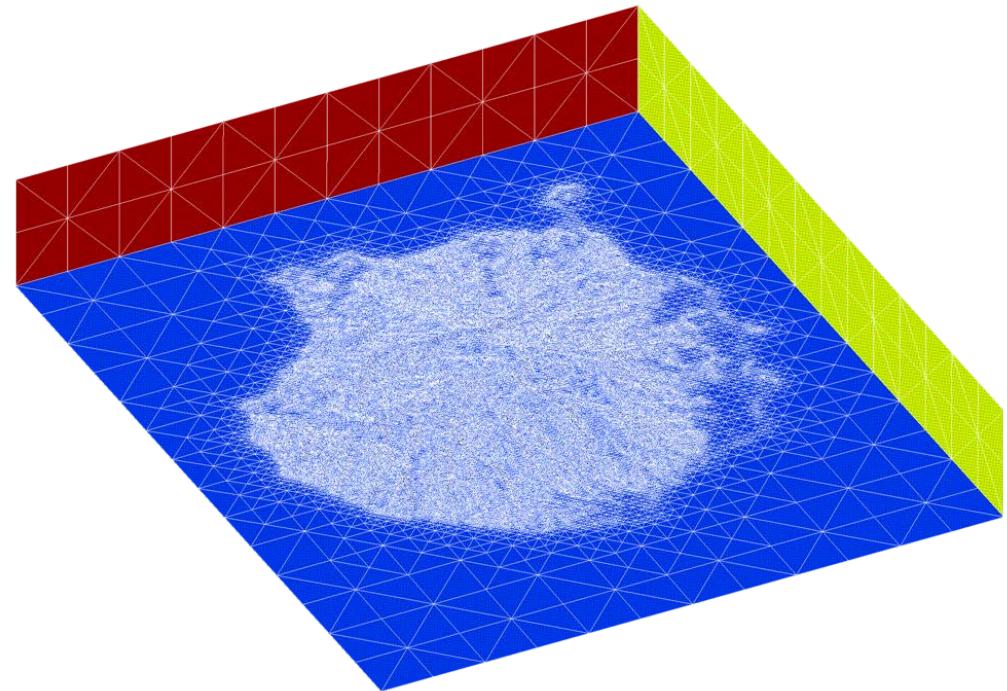
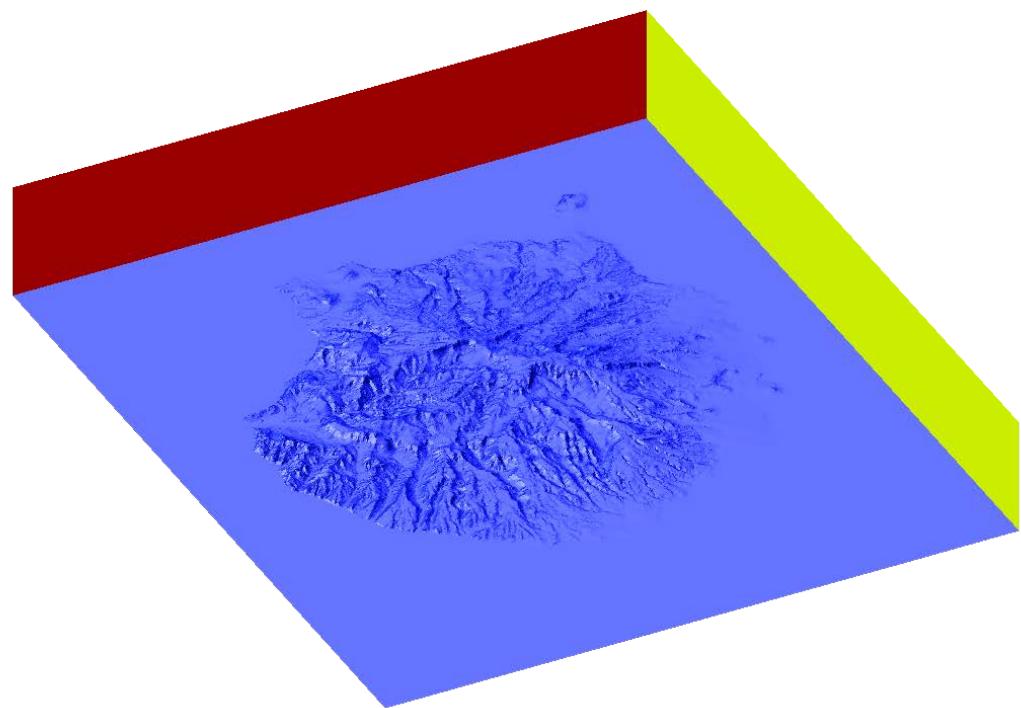






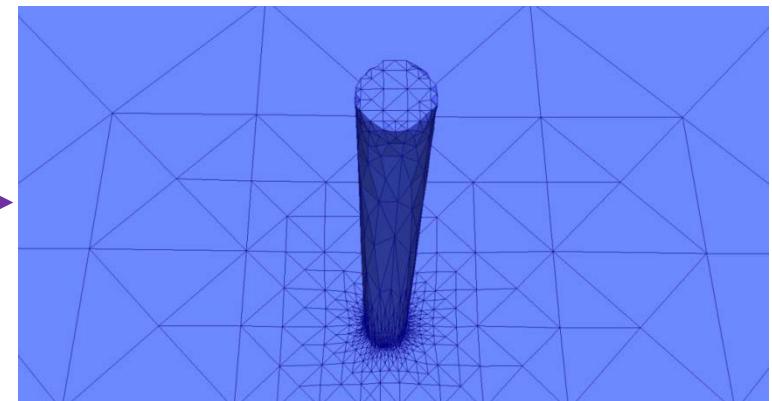
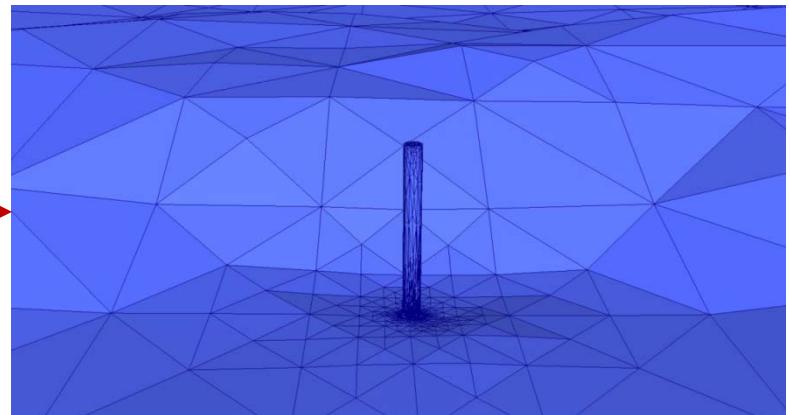
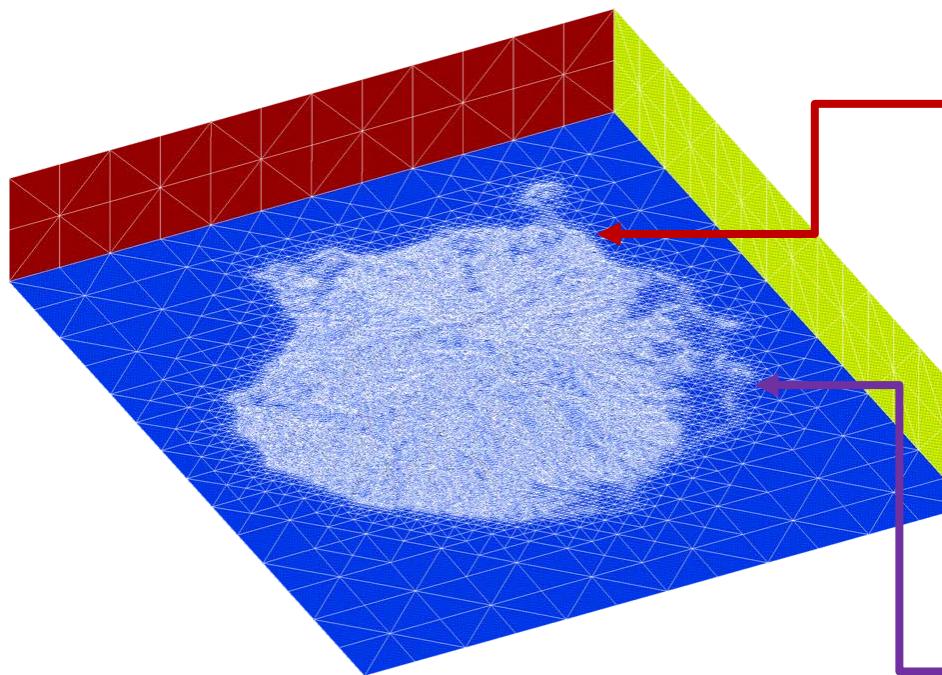
Meccano Method for Tetrahedral Mesh Generation

Application to Gran Canaria Island



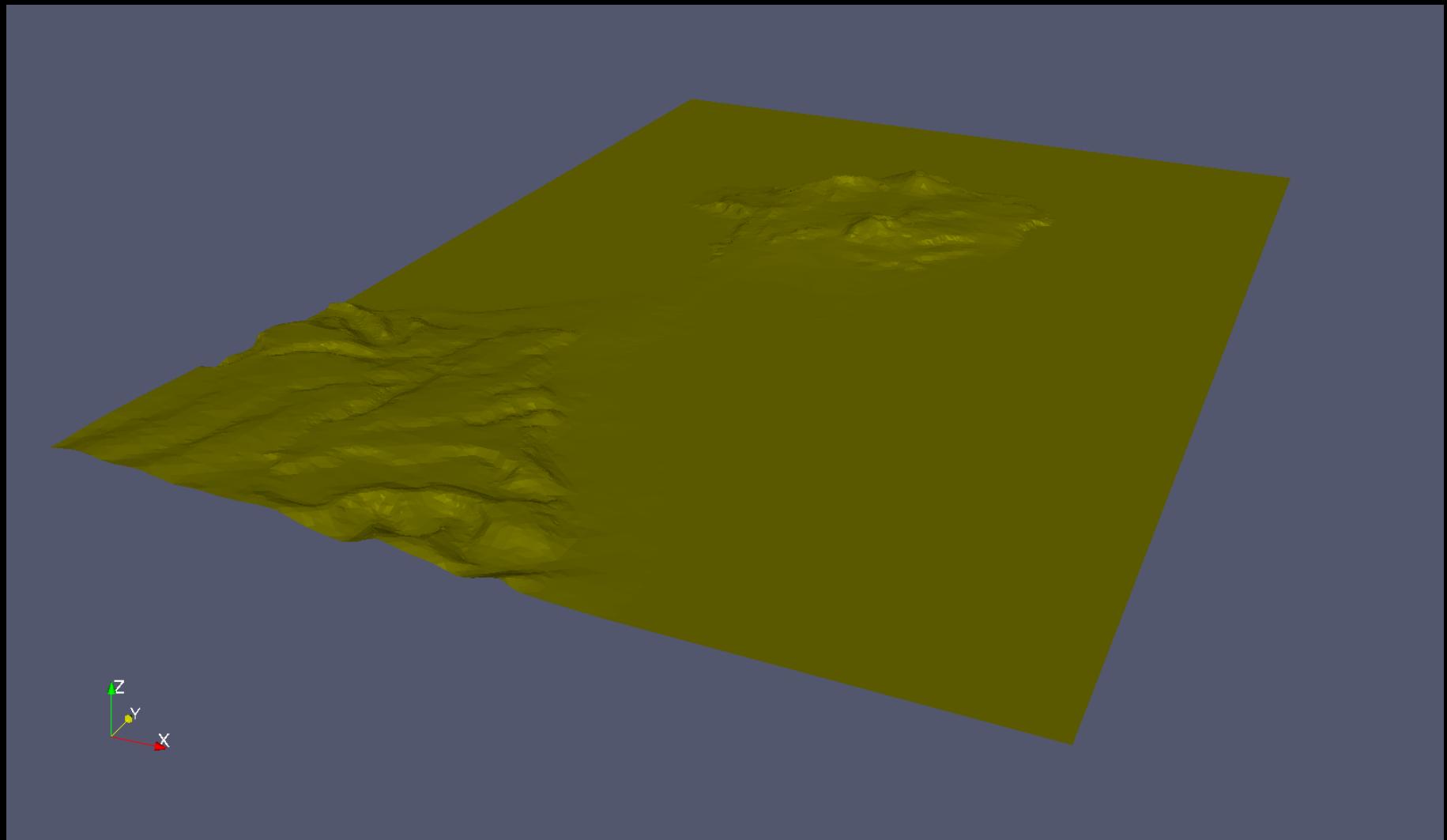
Meccano Method for Tetrahedral Mesh Generation

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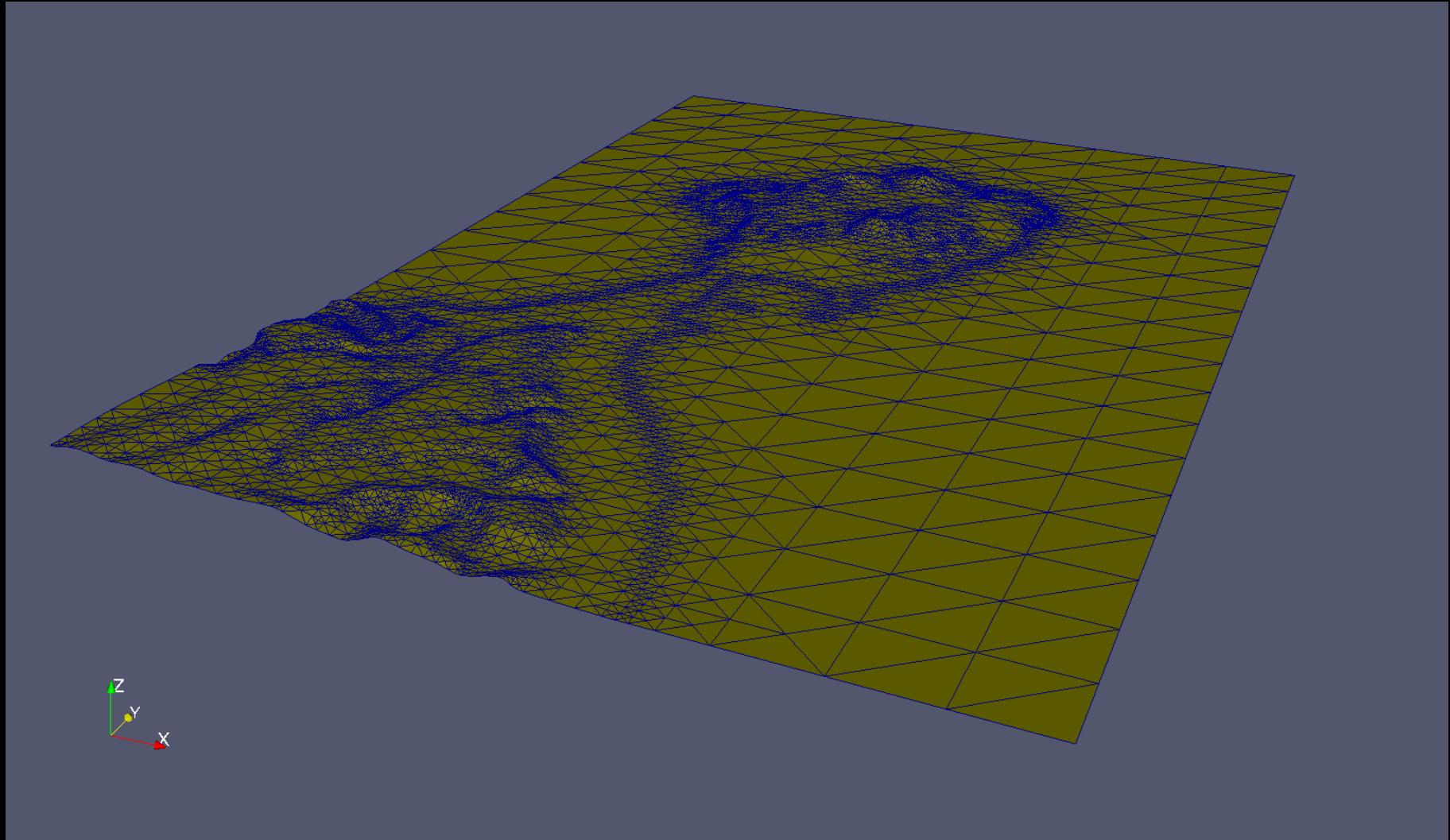
Meccano Method for Tetrahedral Mesh Generation

Gran Canaria: The city of Las Palmas de Gran Canaria (Orography)



Meccano Method for Tetrahedral Mesh Generation

Gran Canaria: The city of Las Palmas de Gran Canaria (The Mesh)



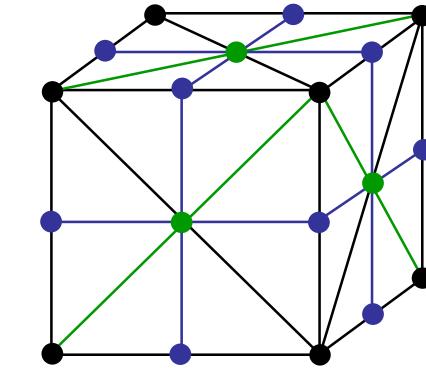
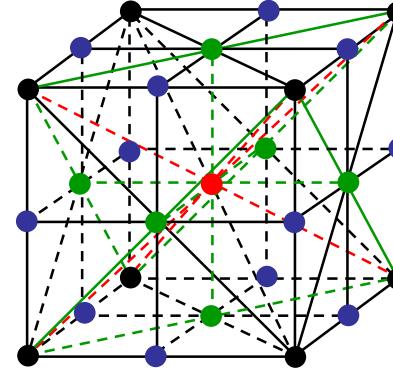
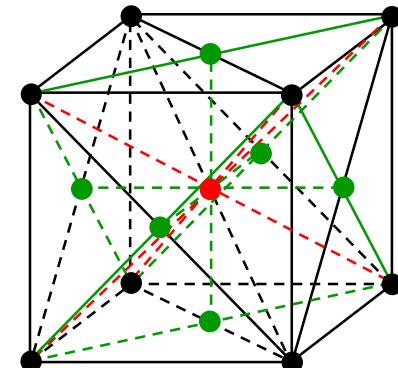
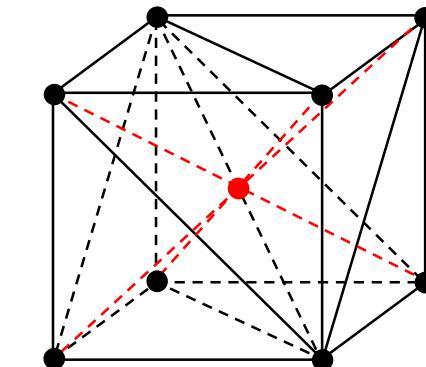
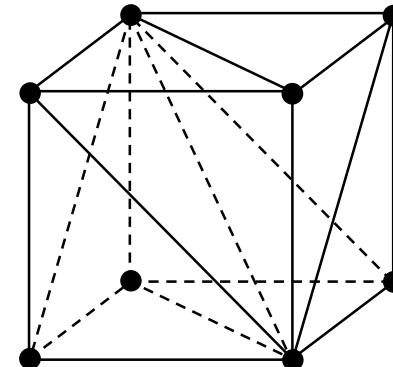
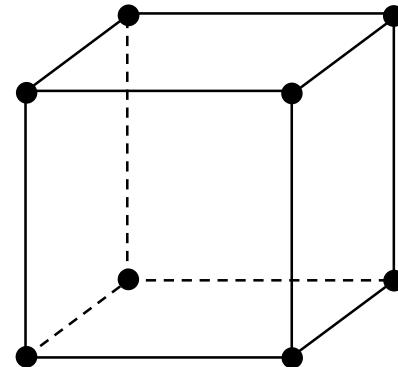
Local Refinement: Kossaczky's Algorithm (JCAM 1994)

Refinement of a cube

<http://www.alberta-fem.de/>, ALBERTA code



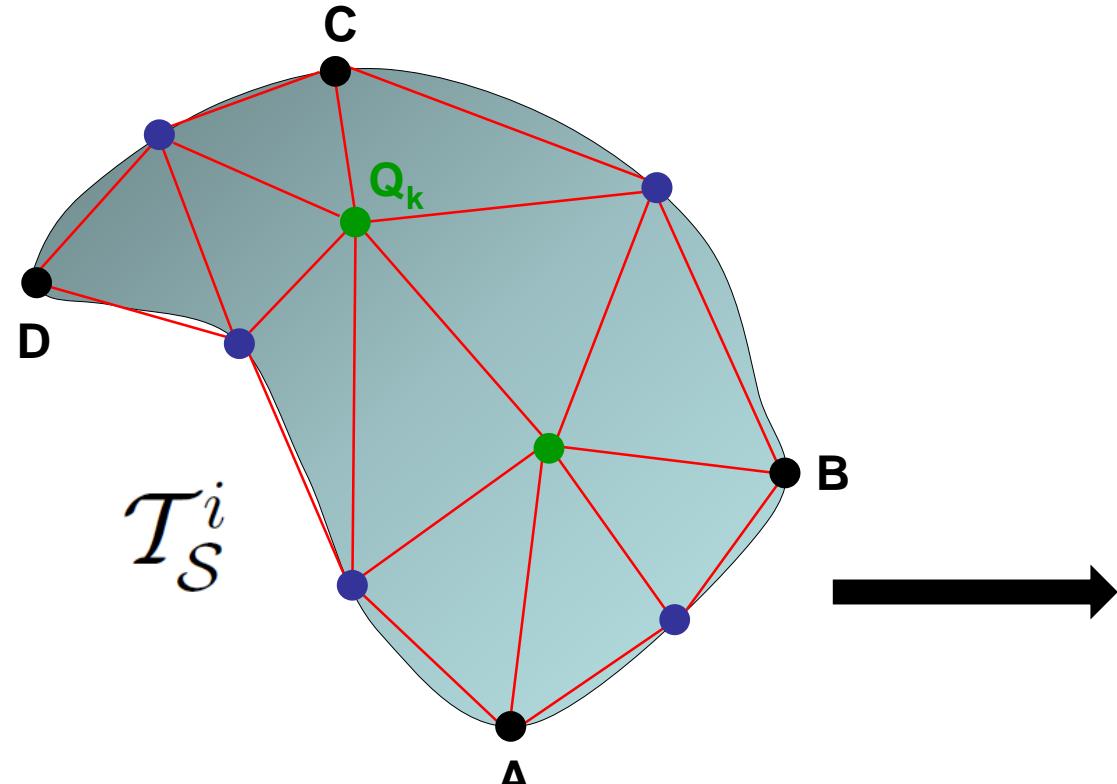
- Initial cube and its subdivision after three consecutive tetrahedron bisection



Surface Parameterization of M.S. Floater (CAGD 1997)

From a the i -th solid surface triangulation patch to the i -th meccano face

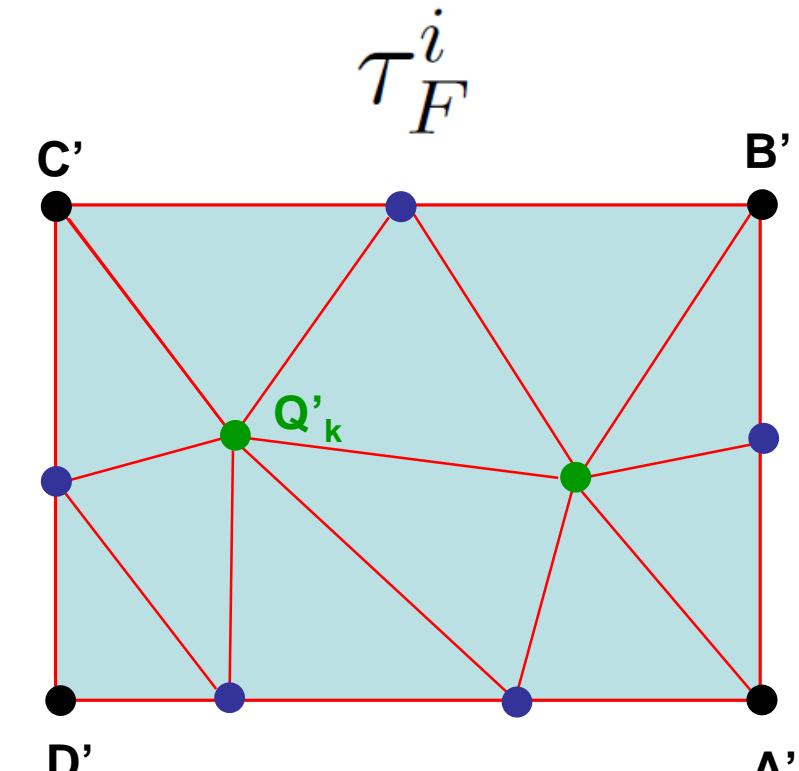
<http://www.sintef.no/math software>, GoTools from SINTEF ICT



Physical Space

Fixed boundary nodes,

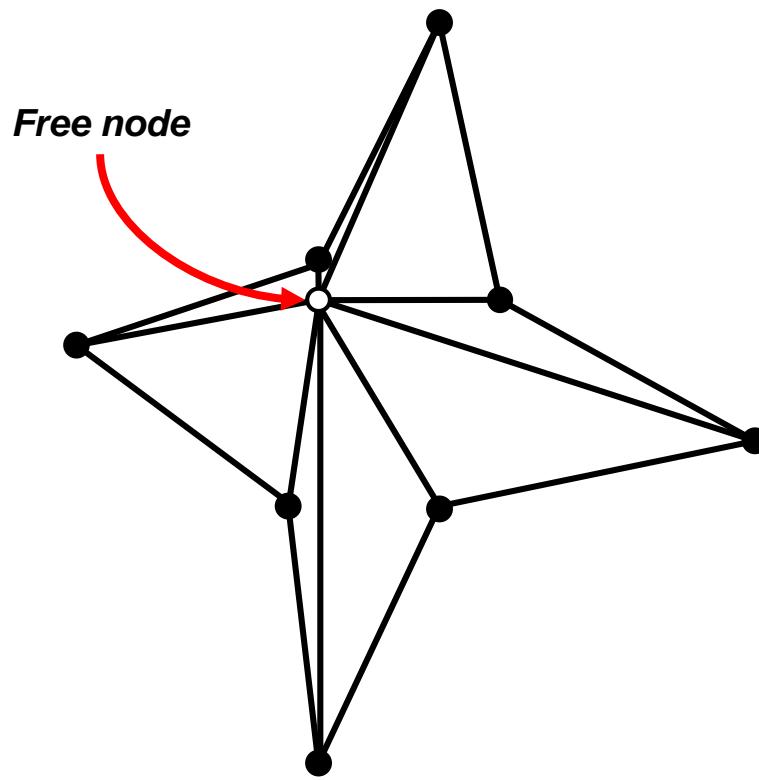
$$\mathbf{Q}'_k = \sum_j \lambda_{j,k} \mathbf{Q}'_j$$



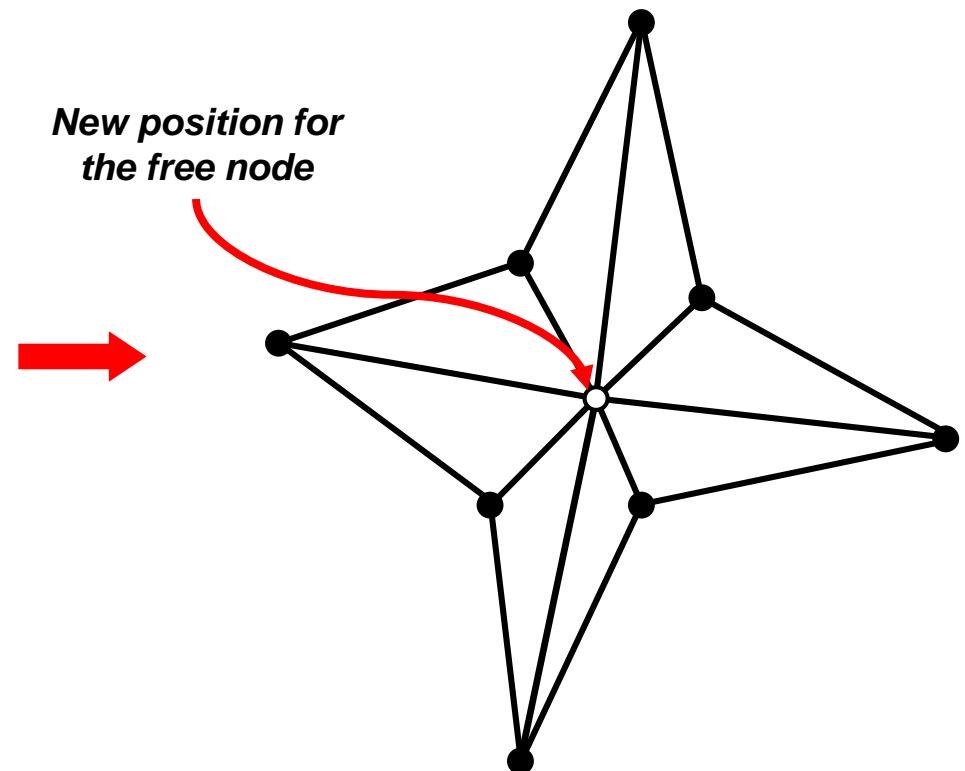
Parametric Space

Local optimization

Objective: Improve the quality of the local mesh by minimizing an objective function



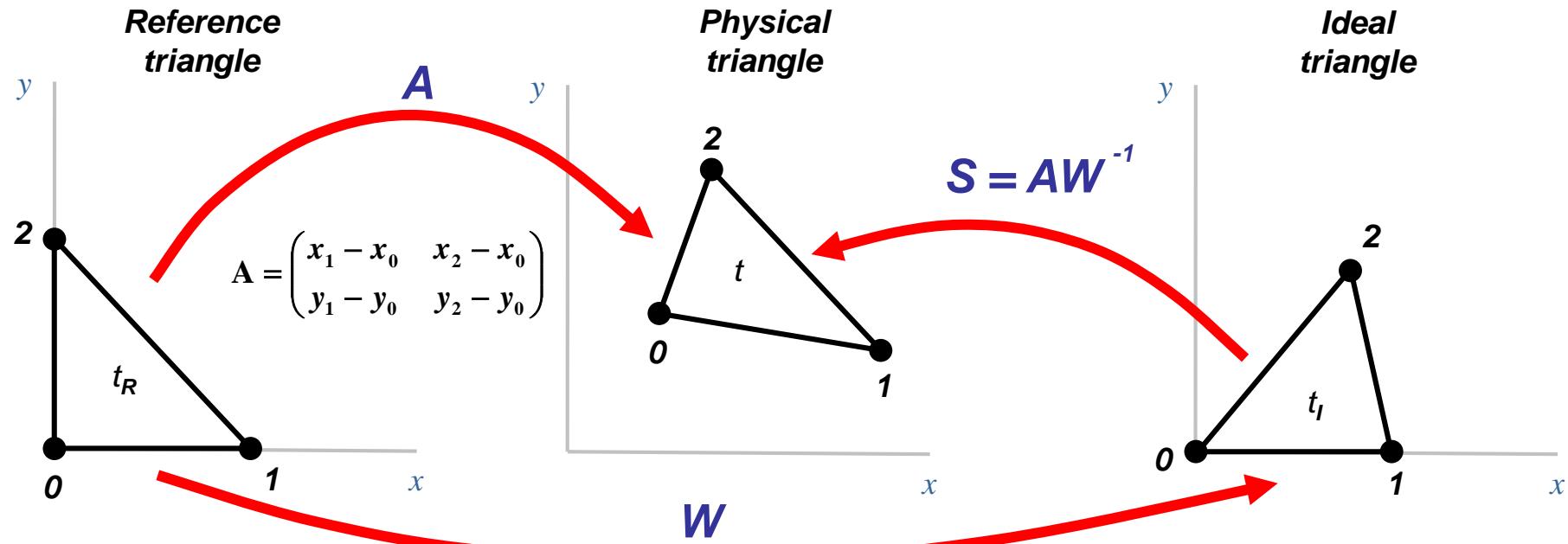
Local mesh



Optimized local mesh

Simultaneous Untangling and Smoothing (CMAME 2003)

Weighted Jacobian Matrix on a Plane



$$t_I \xrightarrow{S} t \quad S = AW^{-1}: \text{Weighted Jacobian matrix}$$

An algebraic quality metric of t
(mean ratio)

$$q = \frac{2\sigma}{\|S\|^2} = \frac{1}{\eta}$$

where:

$$\begin{aligned}\|S\| &= \sqrt{\text{tr}(S^T S)} \\ \sigma &= \det(S)\end{aligned}$$

Simultaneous Untangling and Smoothing (CMAME 2003)

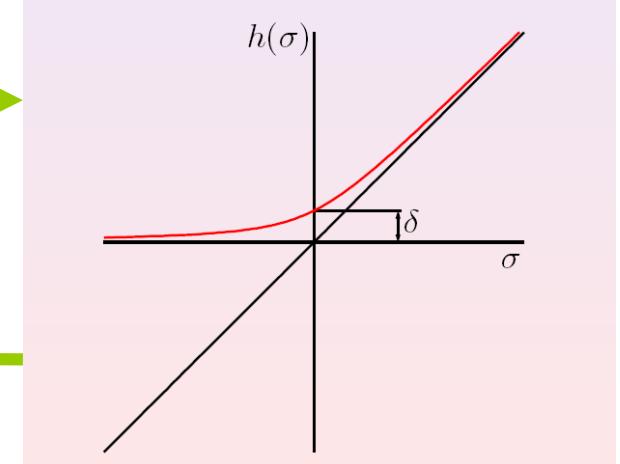
Local objective function for plane triangulations

SUS Code: Freely-available in <http://www.dca.iusiani.ulpgc.es/proyecto2012-2014>

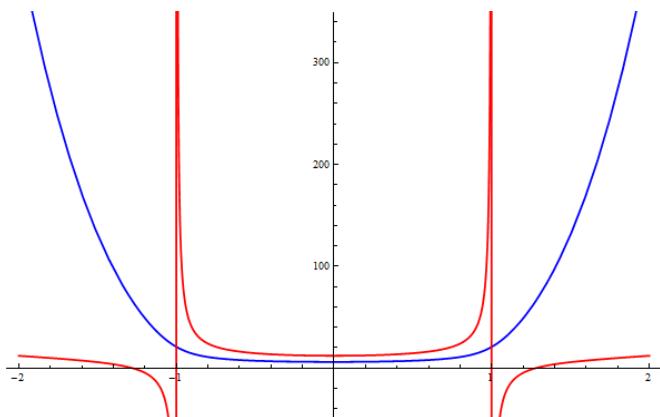


Original function: $K(\mathbf{x}) = \sum_{m=1}^M \frac{\|S_m\|^2}{2\sigma_m}$

Modified function: $K^*(\mathbf{x}) = \sum_{m=1}^M \frac{\|S_m\|^2}{2h(\sigma_m)}$



$$h(\sigma) = \frac{1}{2}(\sigma + \sqrt{\sigma^2 + 4\delta^2})$$



Modified function (blue) is regular in all \mathbb{R}^2 and it approximates the same minimum that the original function (red). Moreover, it allows a simultaneous untangling and smoothing of triangular meshes

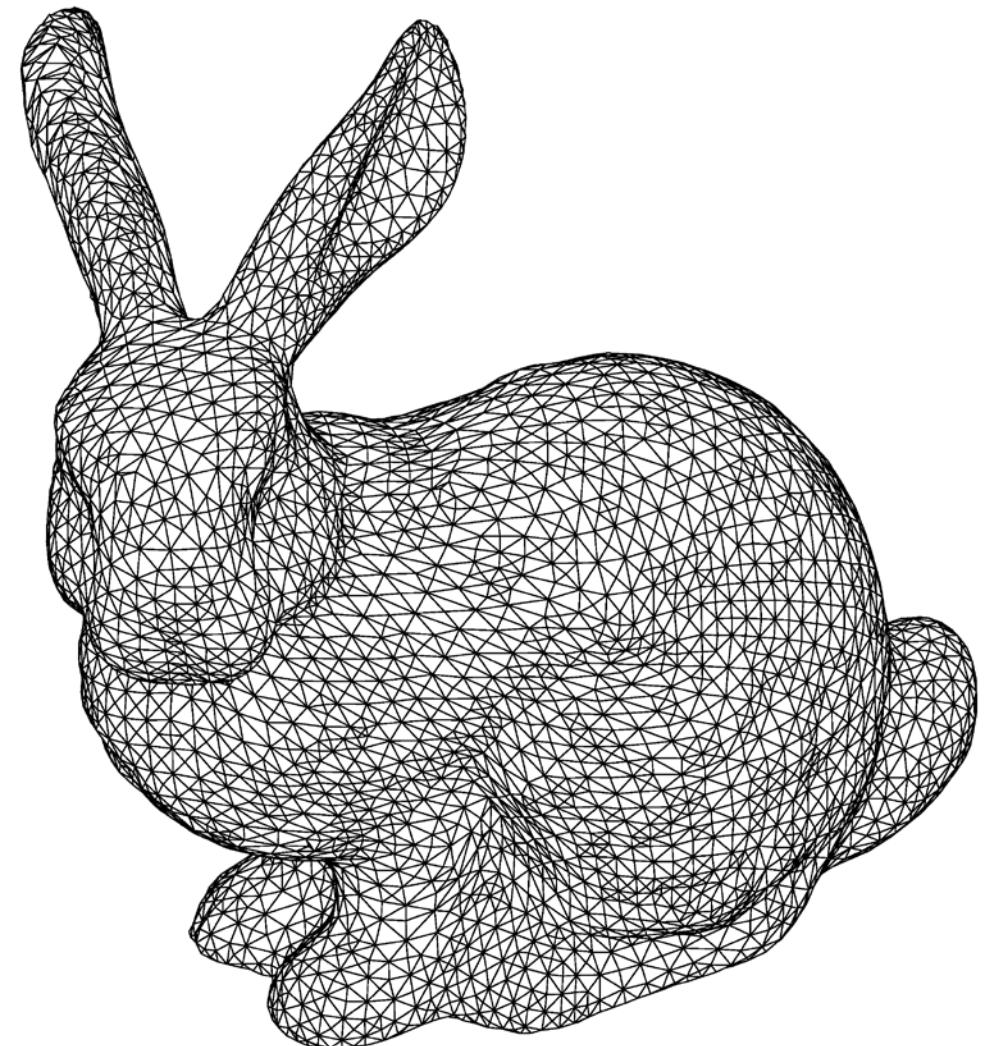
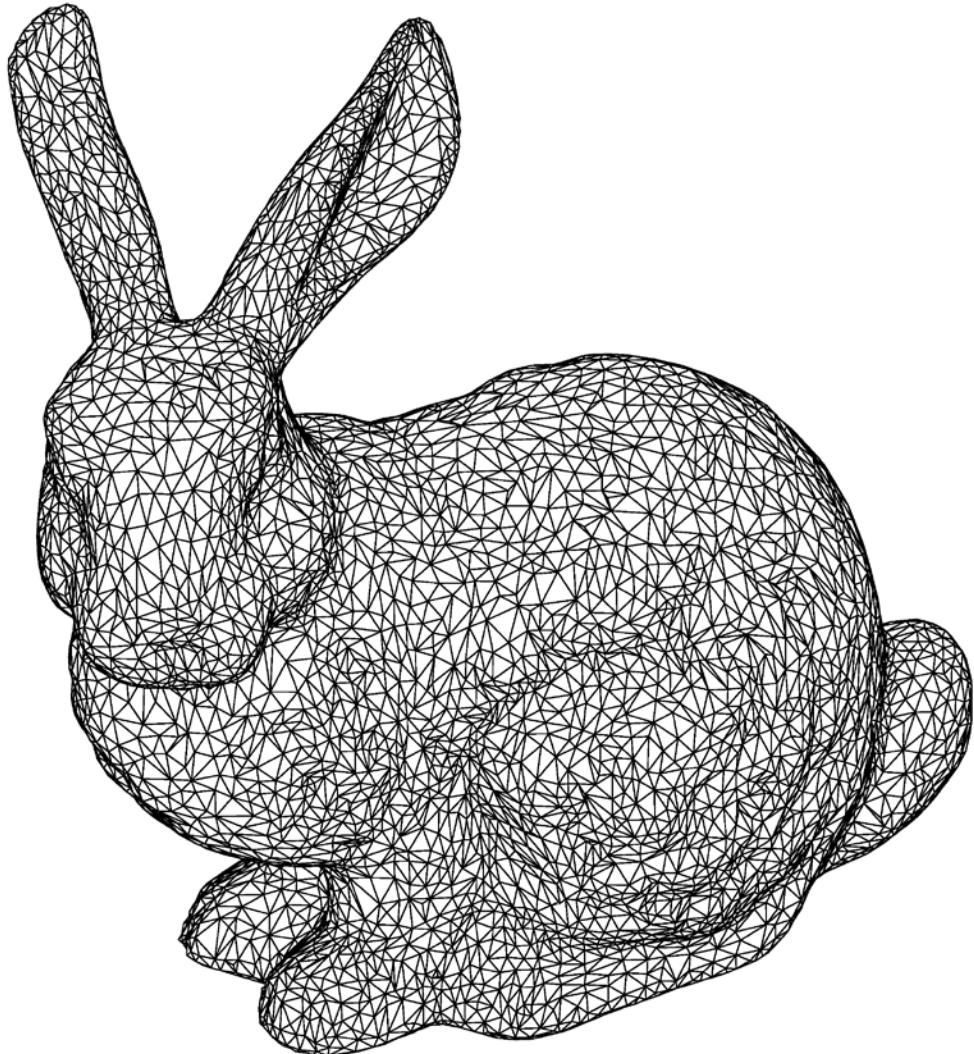
The extension to tetrahedral meshes is straightforward:

$$\eta_m = \frac{|S_m|^2}{3[h(\sigma_m)]^{\frac{2}{3}}}$$

Meccano Method for a Complex Genus-Zero Solid

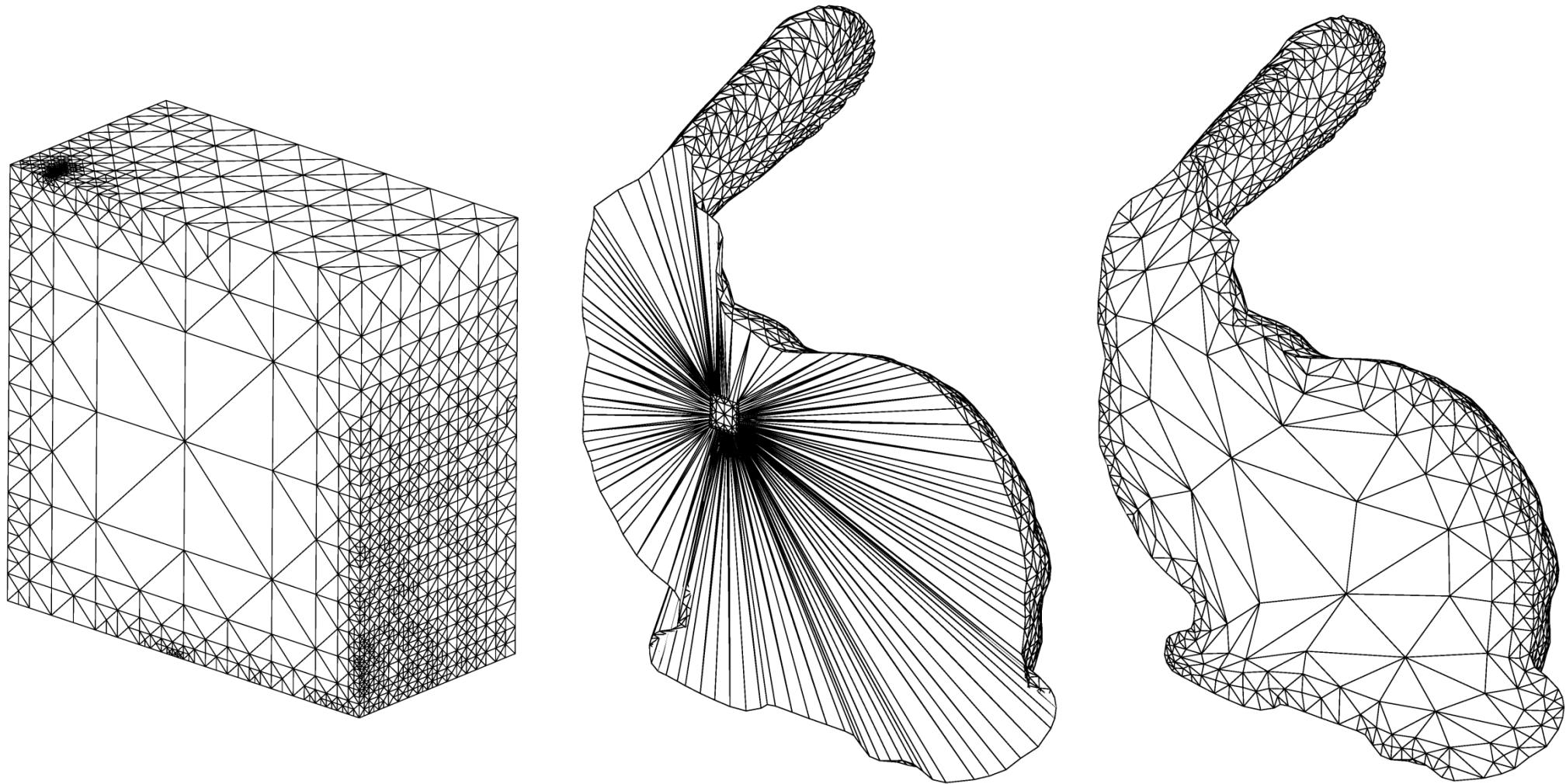
Application to the Stanford Bunny

<http://graphics.stanford.edu/data/3Dscanrep/>, Stanford Computer Graphics Laboratory



Meccano Method for a Complex Genus-Zero Solid

Application to the Stanford Bunny



Adaptive Finite Element Solution

Stanford Bunny Example

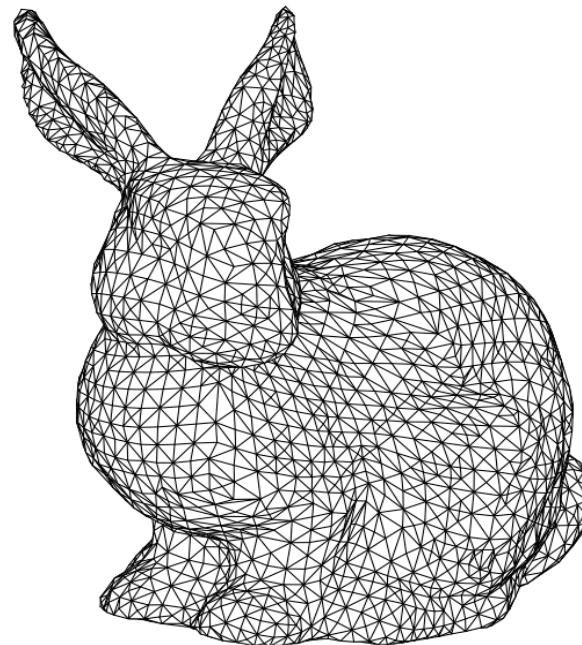


Parabolic Problem:

$$\begin{aligned} \partial_t u - \Delta u &= f && \text{in } \Omega \times (0, T), \\ u &= 0 && \text{on } \partial\Omega \times (0, T), \\ u &= 0 && \text{on } \Omega \times \{0\}, \end{aligned}$$

Adaptive Algorithm: Solve → Estimate → Mark → Refine/Derefine

3-D Domain:

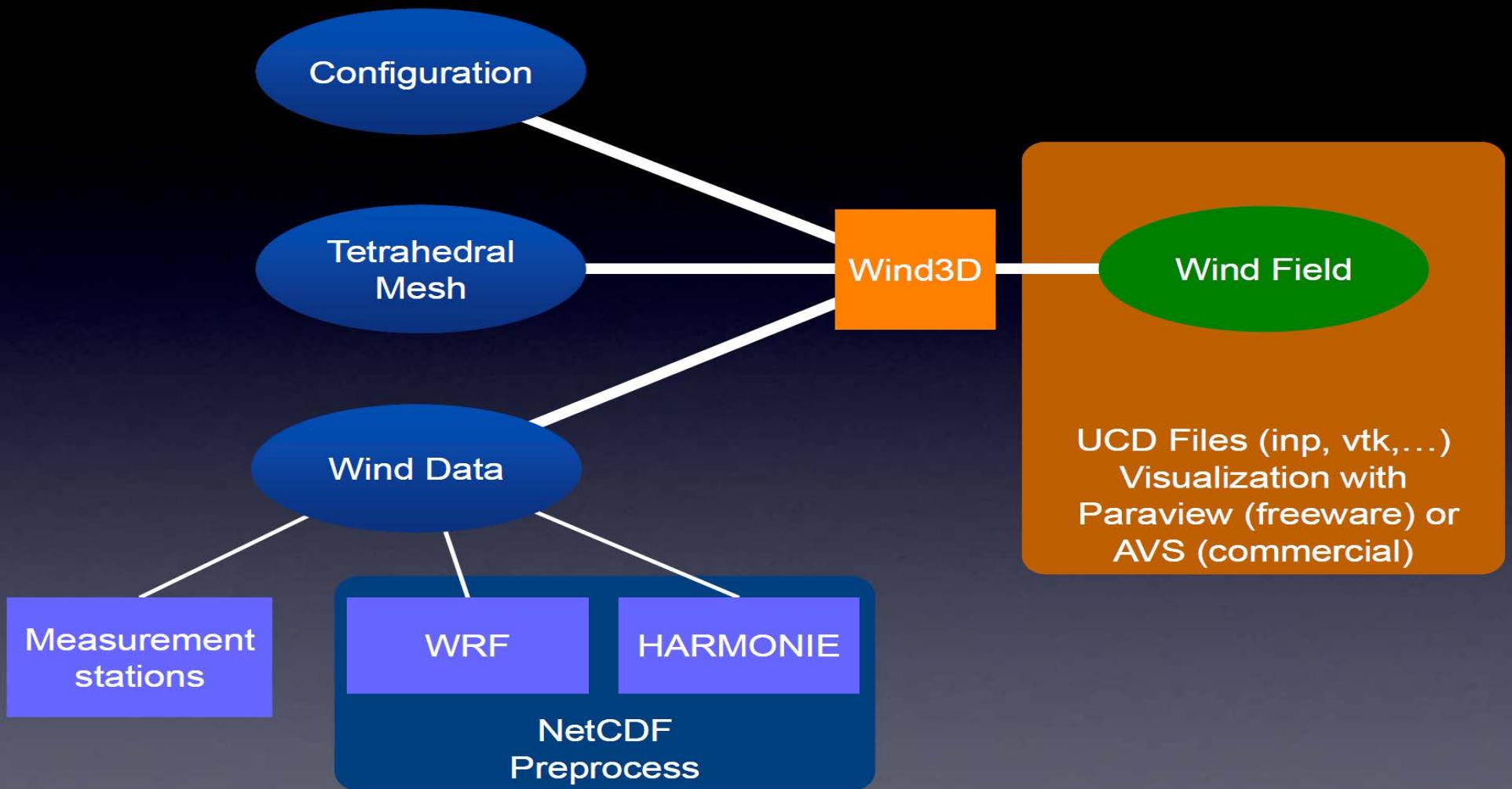


Stanford bunny

Predicción de Viento sobre Orogafía Irregular

Wind3D Code (software libre)

<http://www.dca.iusiani.ulpgc.es/Wind3D/>



Wind3D 1.0

Mass consistent wind field computation

▼ Wind3D	◀ ▶
▶ Data Structures	
▼ Files	
▶ File List	
▼ Globals	
▶ All	
▶ Functions	
Variables	
TypeDefs	
Enumerations	
Macros	

Wind3D

Introduction

Wind3D is a software that simulates a 3D wind field over complex terrains using an adaptive mass consistent model. The software can use as forecast weather models results, station measured wind data or an interpolated wind field.

Information

Title: Wind3D

Registration application number: GC-96-2012.

Registration entry: 00 / 2012 / 2506

Date: February 23rd, 2012.

Owner organization: Universidad de Las Palmas de Gran Canaria.

Authors

Eduardo Rodríguez Barrera, Gustavo Montero García, José María Escobar Sánchez, Rafael Montenegro Armas, Albert Oliver Serra

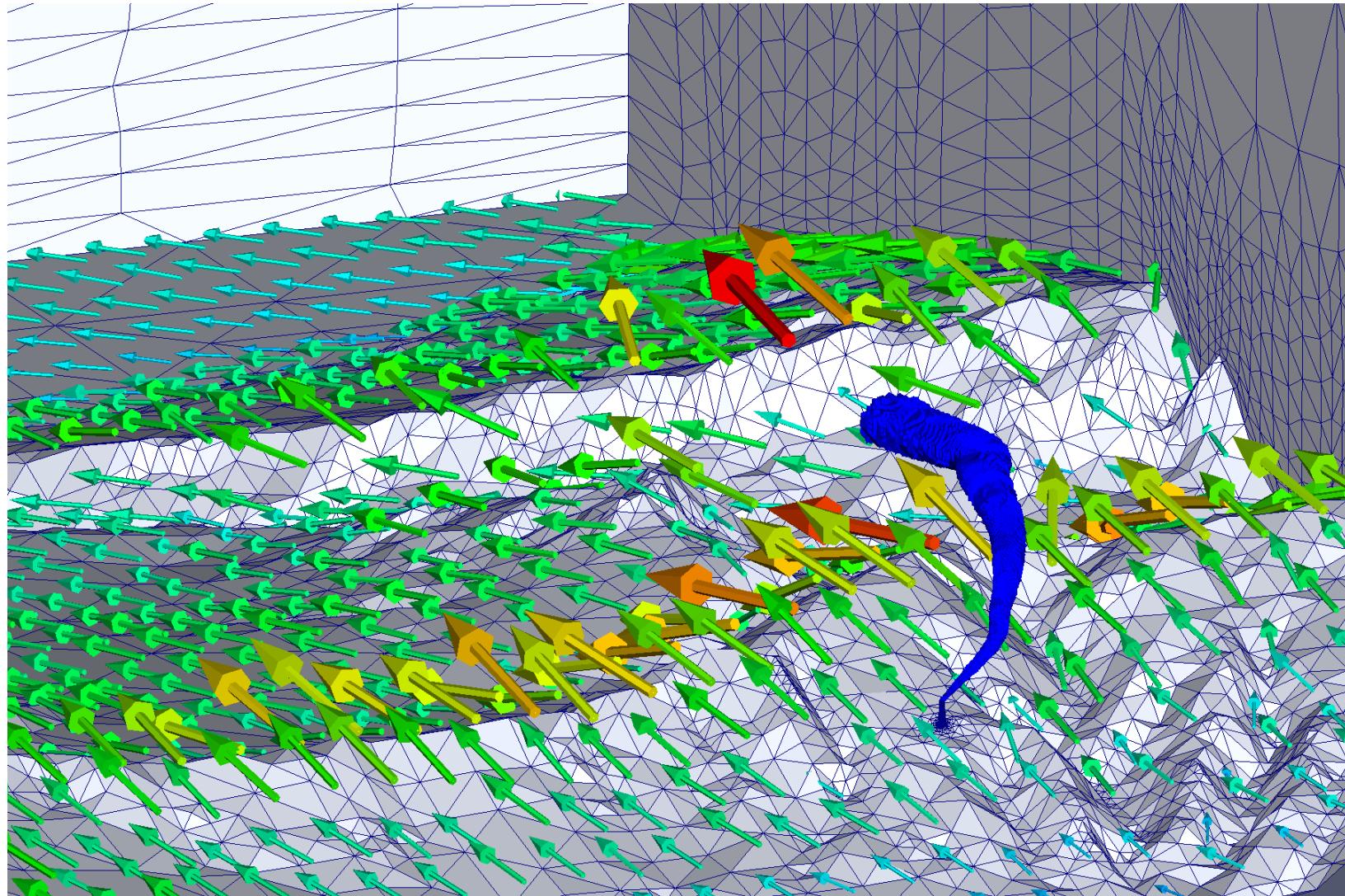
License

Table of Contents

↓ Introduction
↓ Information
↓ Authors
↓ License
↓ Download
↓ Requirements
↓ How to compile the source code
↓ Use
↓ Configuration file format
↓ Parameters
↓ Mesh file format
↓ Wind file format
↓ Stations wind file format
↓ Forecast wind file format
↓ Interpolated wind file format
↓ Rugosity file format
↓ Output points file format
↓ Acknowledgements
↓ Cite

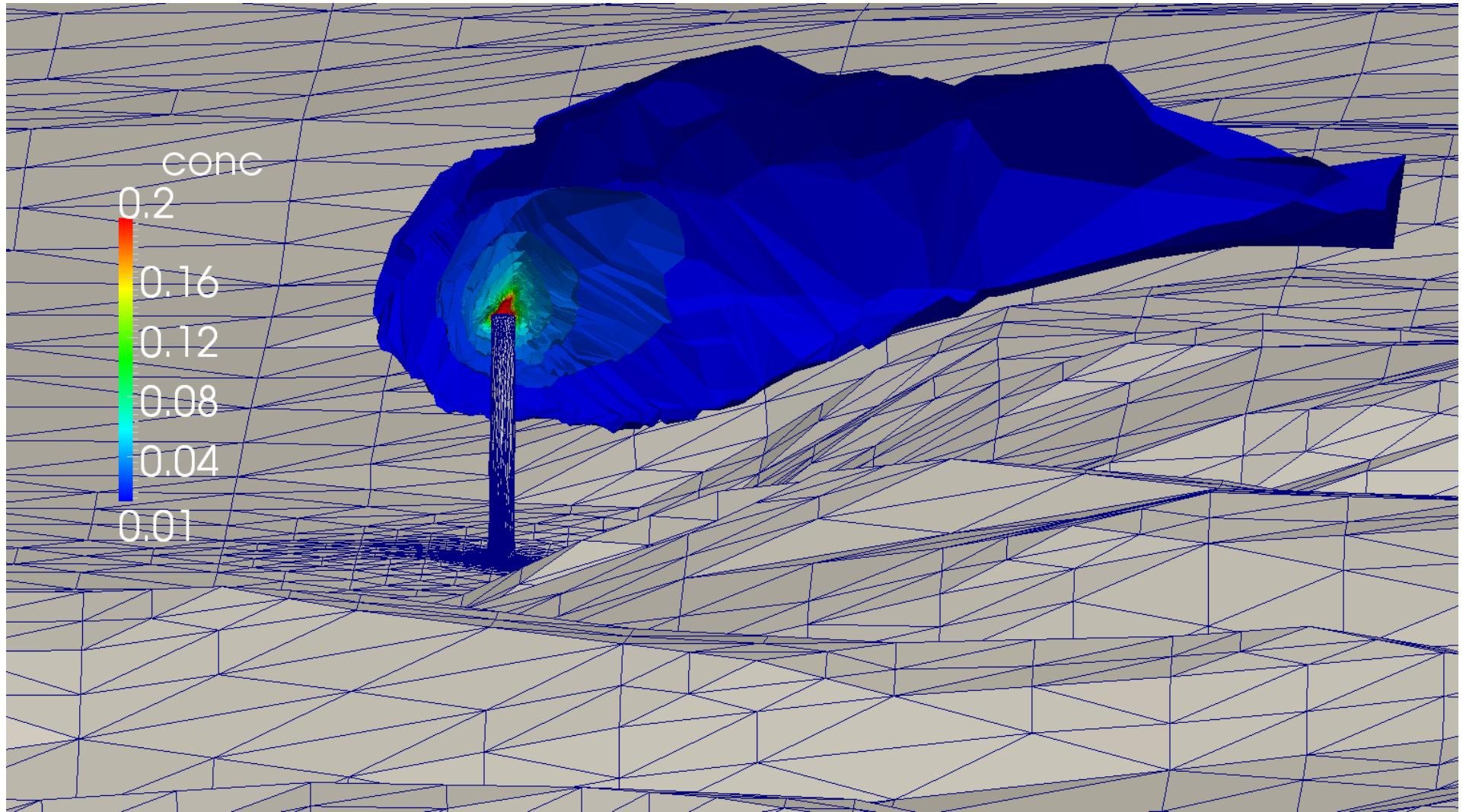
Wind Field for Air Quality Simulation

Mesh adaptation to geometry and gaussian plume



Wind Field for Air Quality Simulation

Mesh adaptation to geometry and gaussian plume



The Meccano Method for Isogeometric Modeling and Analysis



Isogeometric Analysis:

- *Exact geometry*
- C^K *continuity*

FEM:

- *Approximate geometry*
- C^0 *continuity*

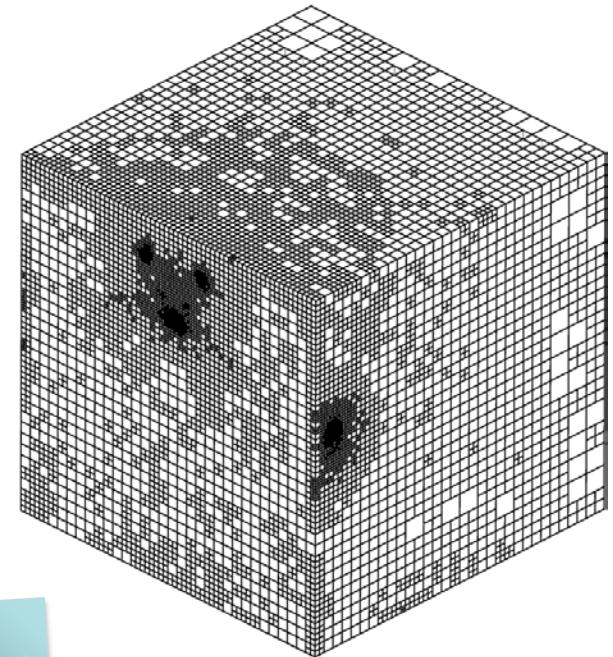
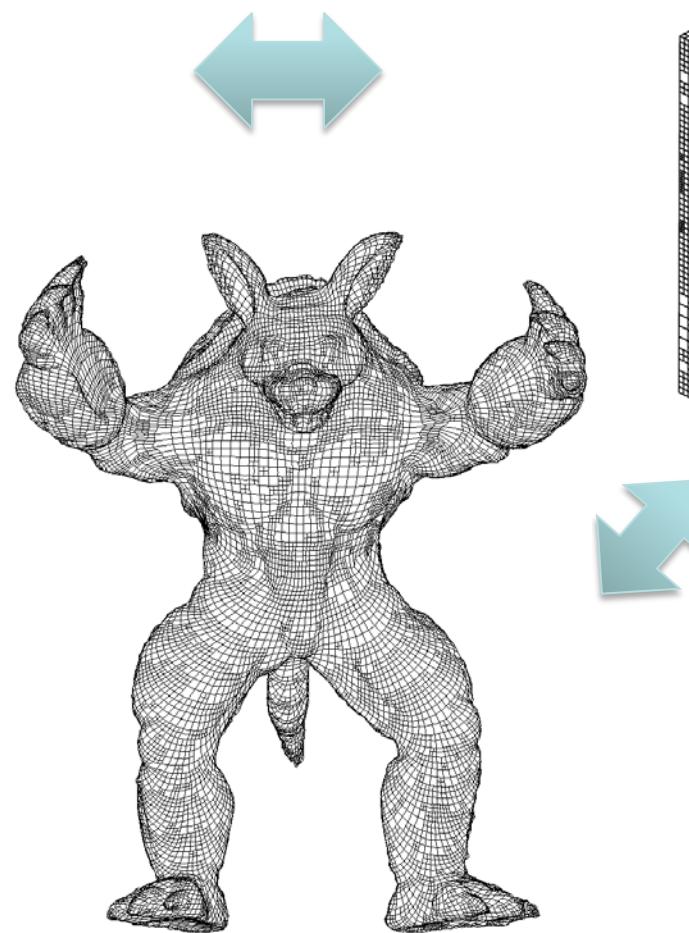
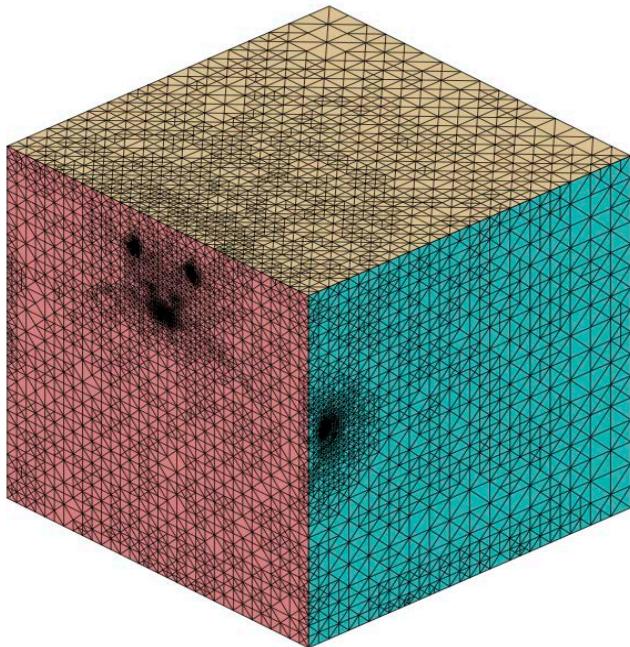
Meccano Method for Complex Solids

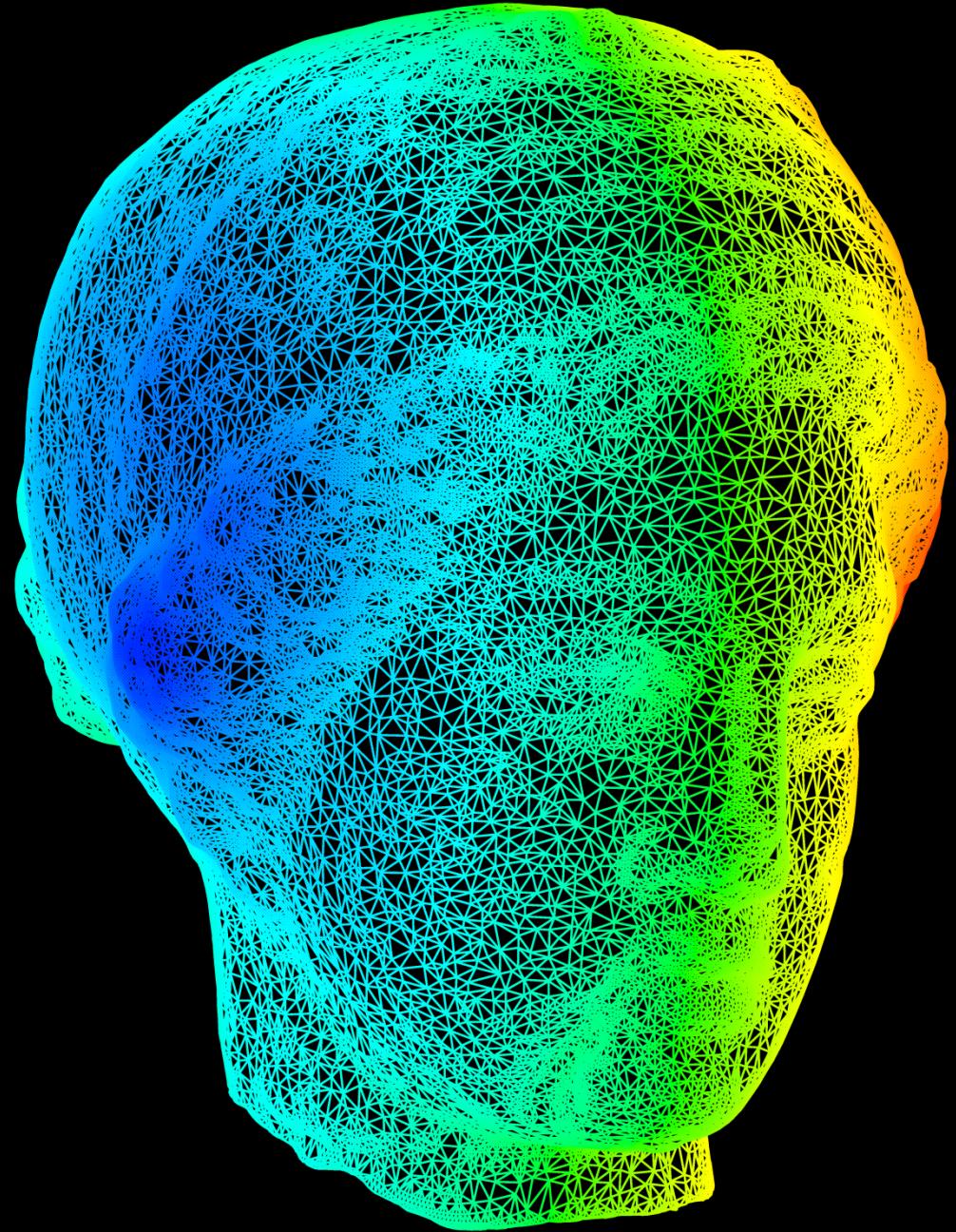
Volume parameterization based on SUS of tetrahedral meshes

Solid T-spline representation

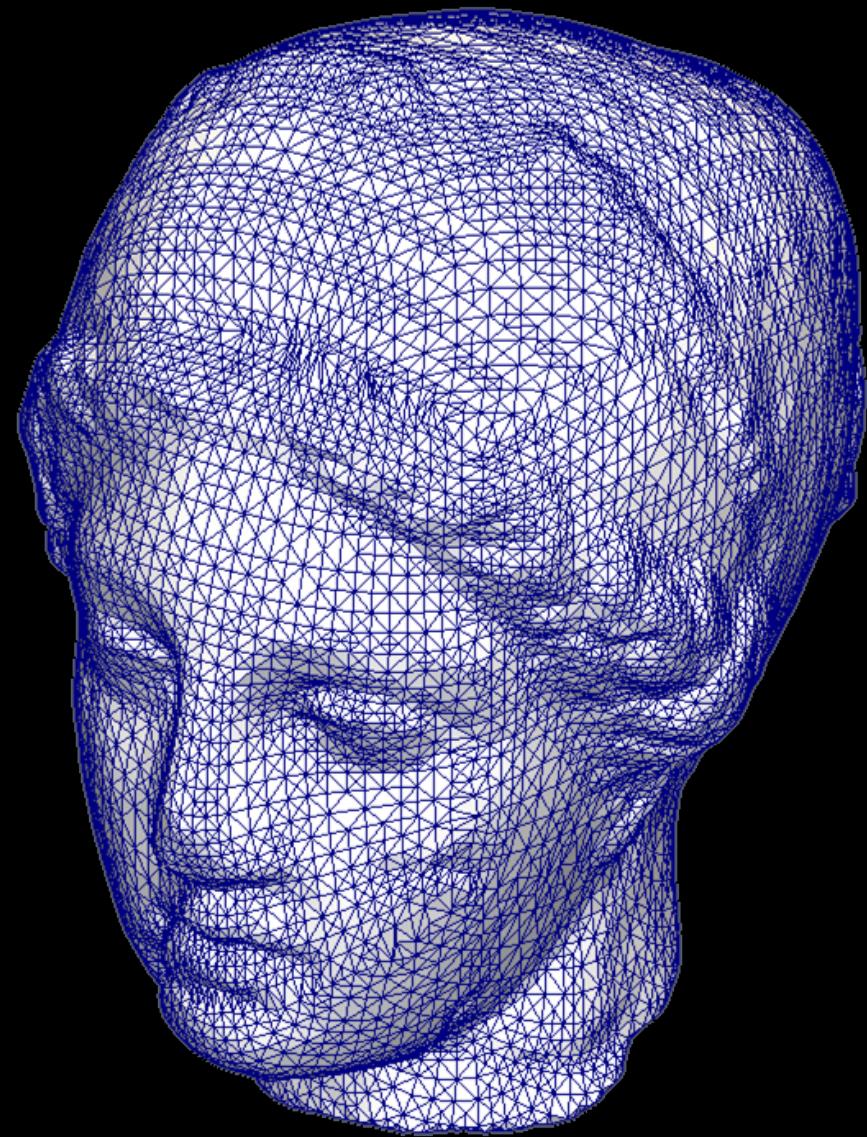
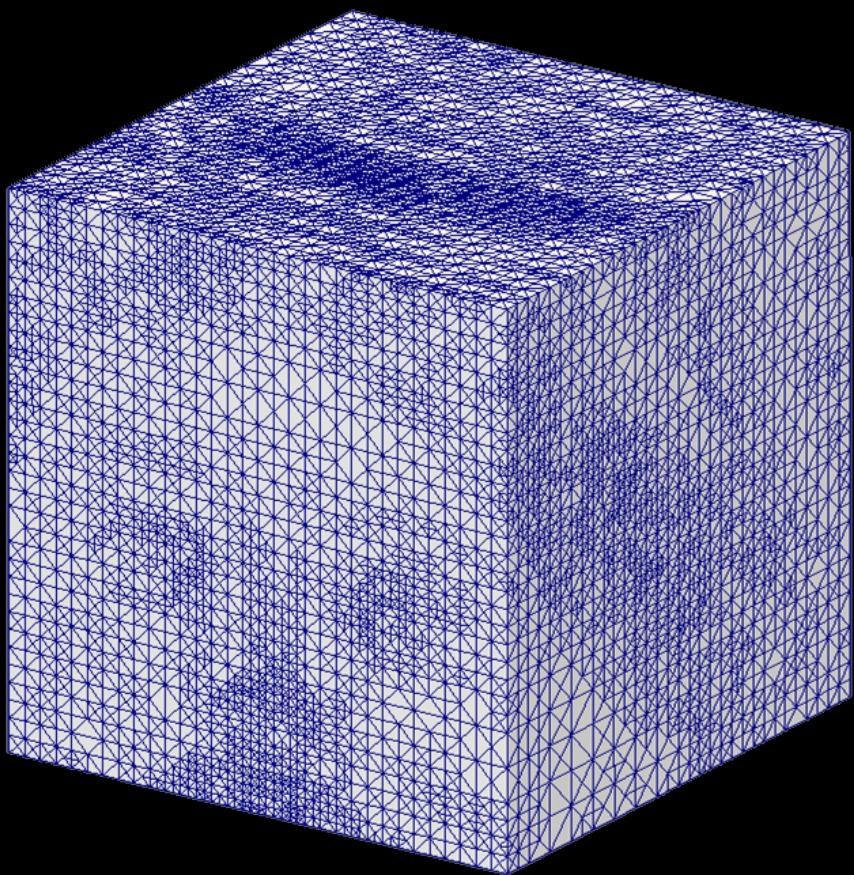


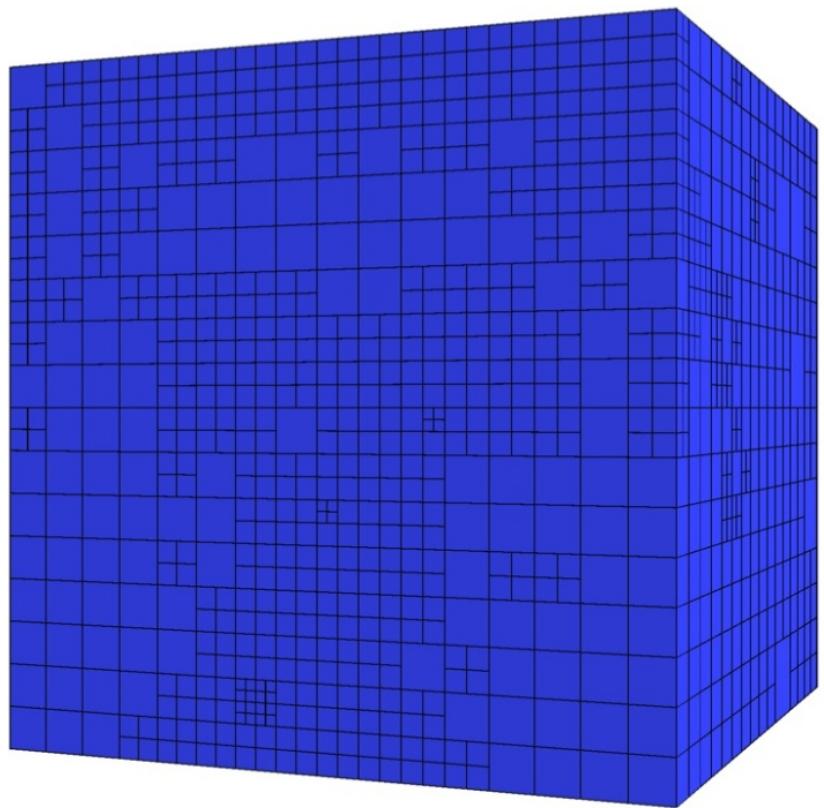
- Octree subdivision





INPUT DATA: Surface Triangulation
<http://www.cyberware.com/>



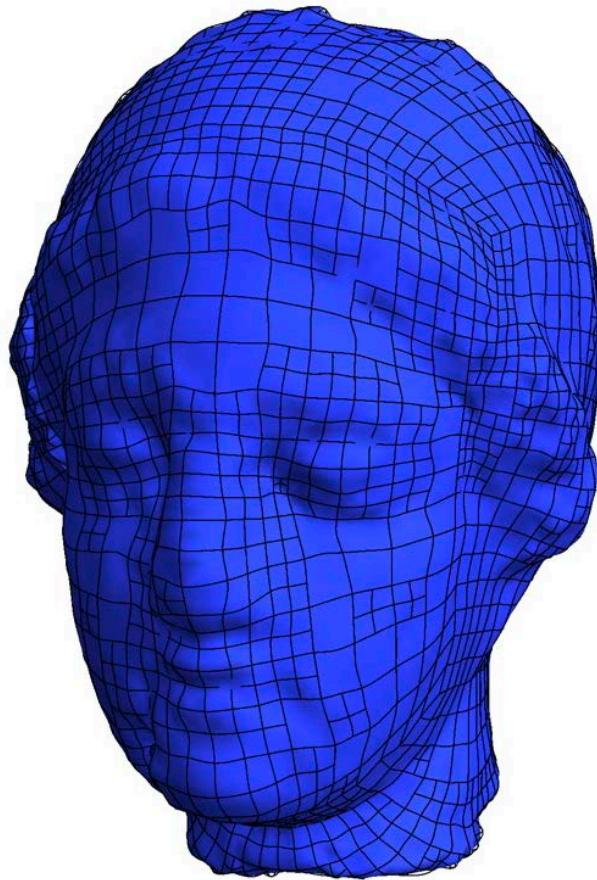


T-mesh

$S(\xi)$



T-spline



The Spline Interpolation

Calculation of Control Points by Fulfilling the Interpolation Conditions



$$\mathbf{S}(\xi^1, \xi^2, \xi^3) = \sum_{\alpha \in A} \mathbf{P}_\alpha R_\alpha(\xi^1, \xi^2, \xi^3)$$

With:

$$R_\alpha(\xi^1, \xi^2, \xi^3) = \frac{B_\alpha(\xi^1, \xi^2, \xi^3)}{\sum_{\beta \in A} B_\beta(\xi^1, \xi^2, \xi^3)}$$

Blending functions

$$B_\alpha(\xi^1, \xi^2, \xi^3) = N_\alpha^1(\xi^1)N_\alpha^2(\xi^2)N_\alpha^3(\xi^3)$$

Trivariate basis splines

Parametric space location \mathbf{t}_β volumetric
parameterization \longrightarrow $\mathbf{S}(\mathbf{t}_\beta)$ Physical space location

Control points \mathbf{P}_α are calculated by solving the sparse linear system

$$\mathbf{S}(\mathbf{t}_\beta) = \sum_{\alpha \in A} \mathbf{P}_\alpha R_\alpha(\mathbf{t}_\beta) \quad \forall \beta \in A$$

Adaptive Isogeometric Refinement (EWC 2012)

Application in Igea: Poisson problem with a central source

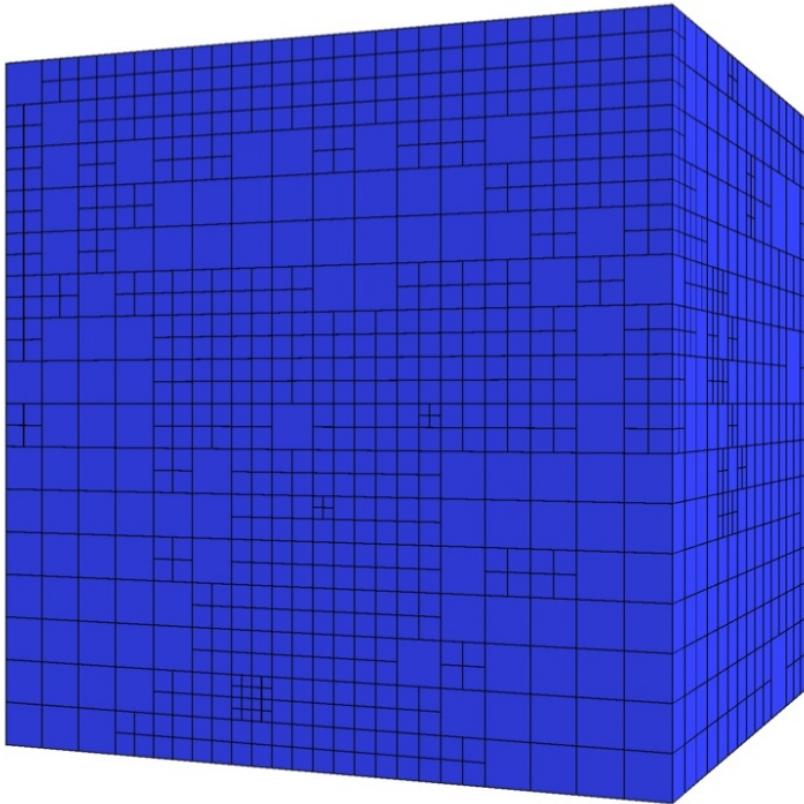


$$\Delta u = \frac{1}{25} e^{-\frac{(x^2+y^2+z^2)}{10}} (-15 + x^2 + y^2 + z^2) \quad \text{in } \Omega$$

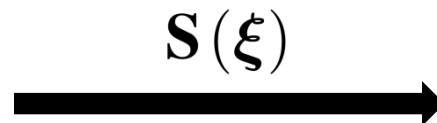
$$u|_{\partial\Omega} = 0$$

Exact solution:

$$u \approx e^{-\frac{(x^2+y^2+z^2)}{10}}$$

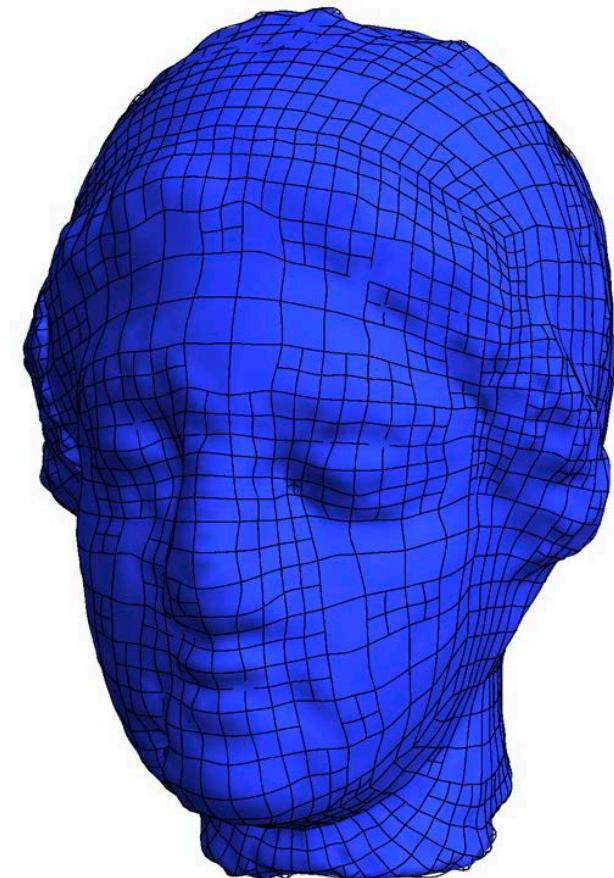


T-mesh



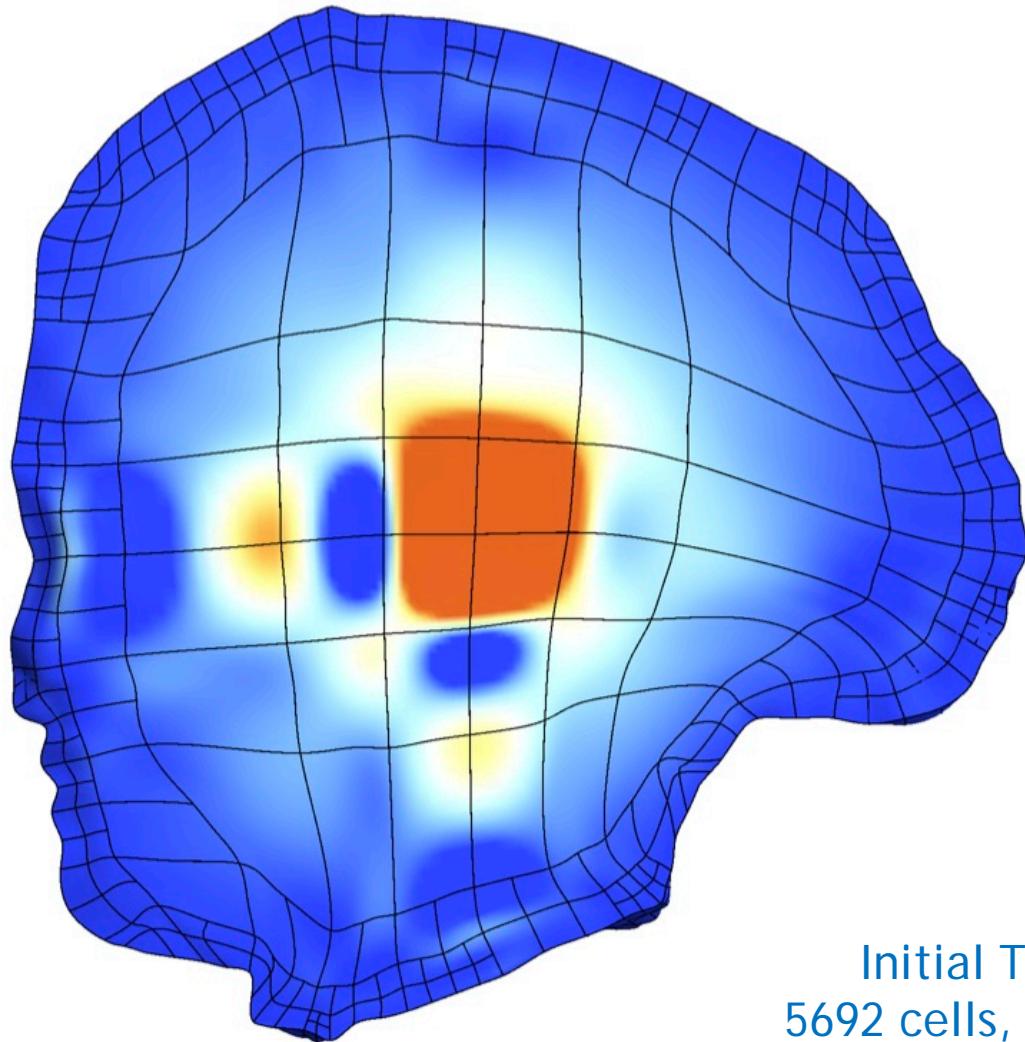
$S(\xi)$

T-spline

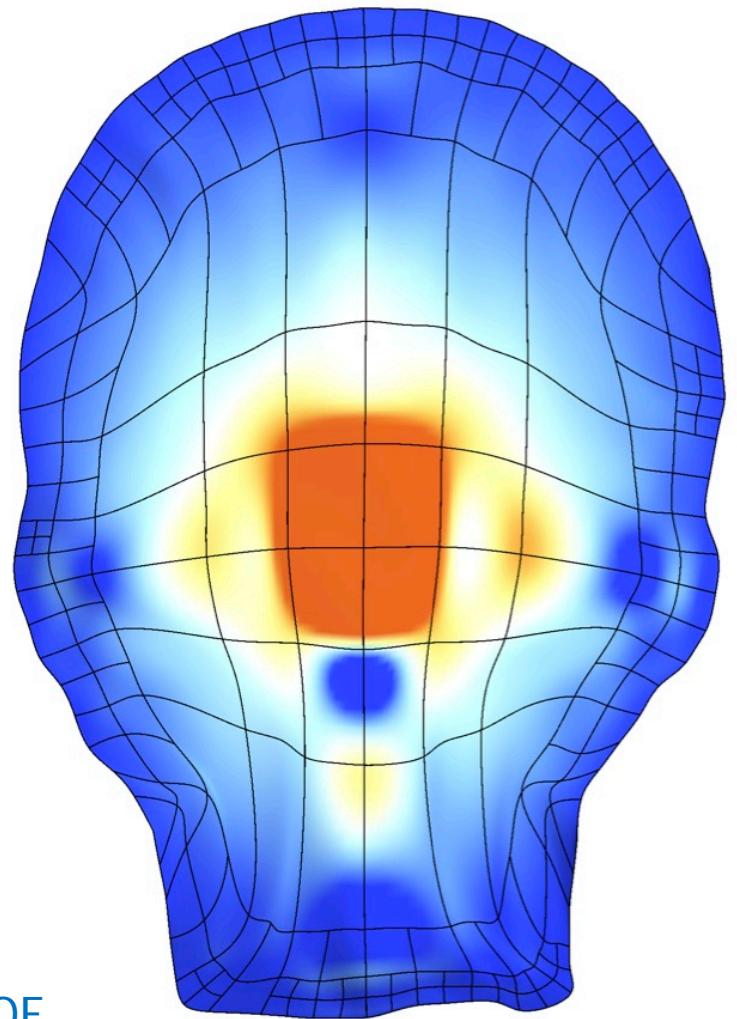


Adaptive Isogeometric Refinement (EWC 2012)

Igea: T-spline of Numerical Solution



Initial T-mesh
5692 cells, 9304 DOF

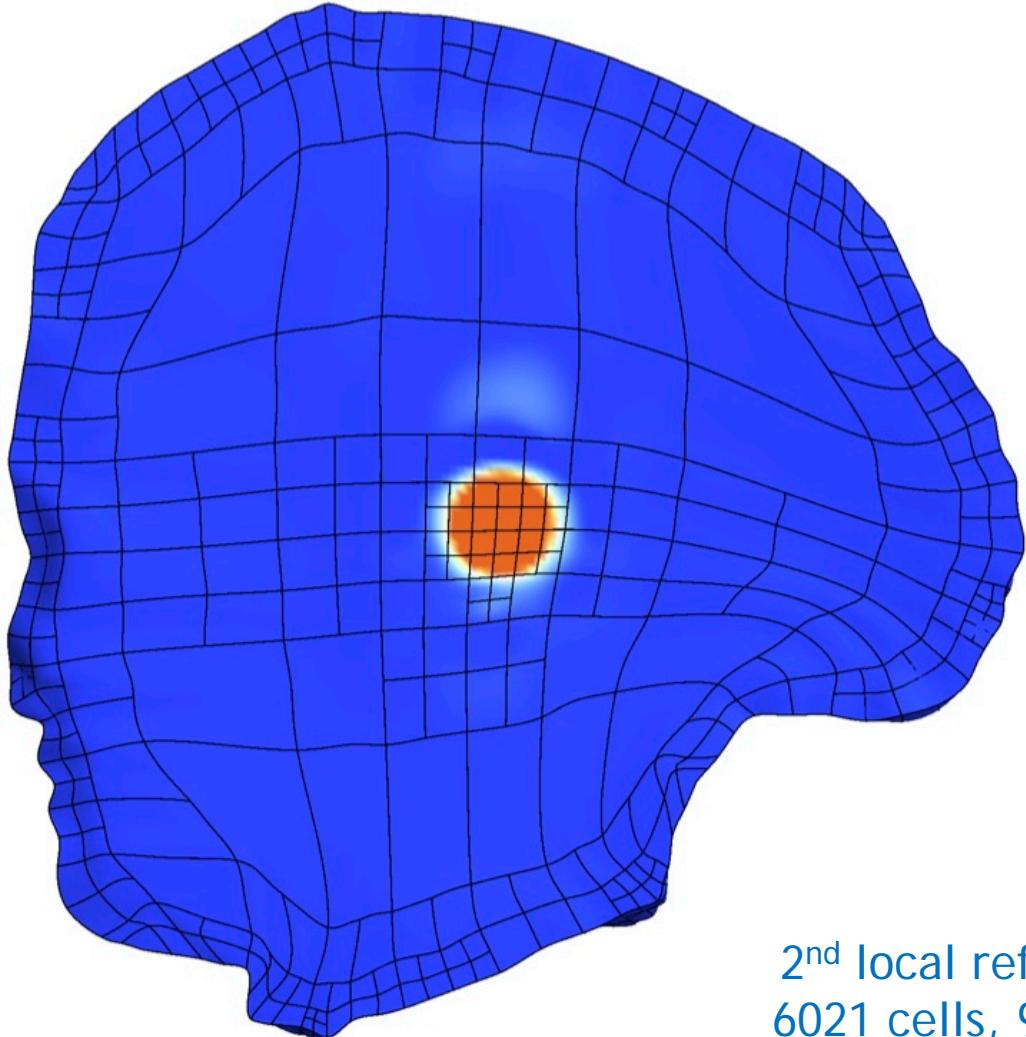


Adaptive Isogeometric Refinement (EWC 2012)

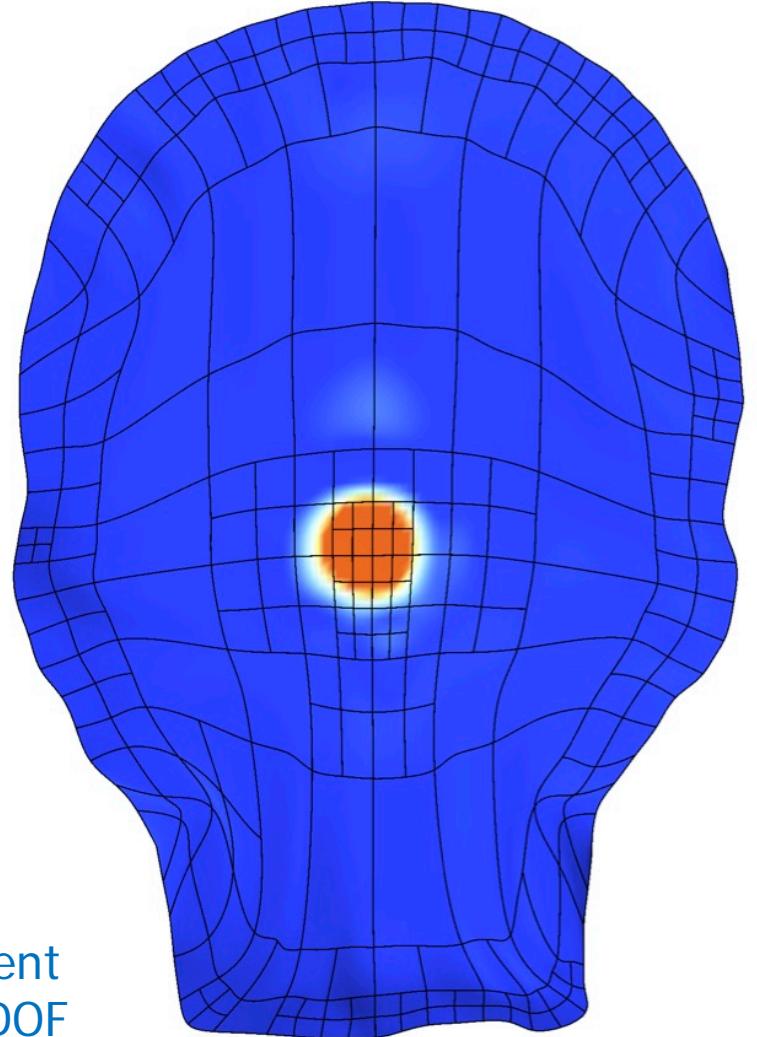
Igea: T-spline of Numerical Solution



$$\text{Error indicator : } \eta(\Omega_e)^2 = \int_{\Omega_e} h^2 (f + \Delta u_h)^2 d\Omega$$



2nd local refinement
6021 cells, 9807 DOF

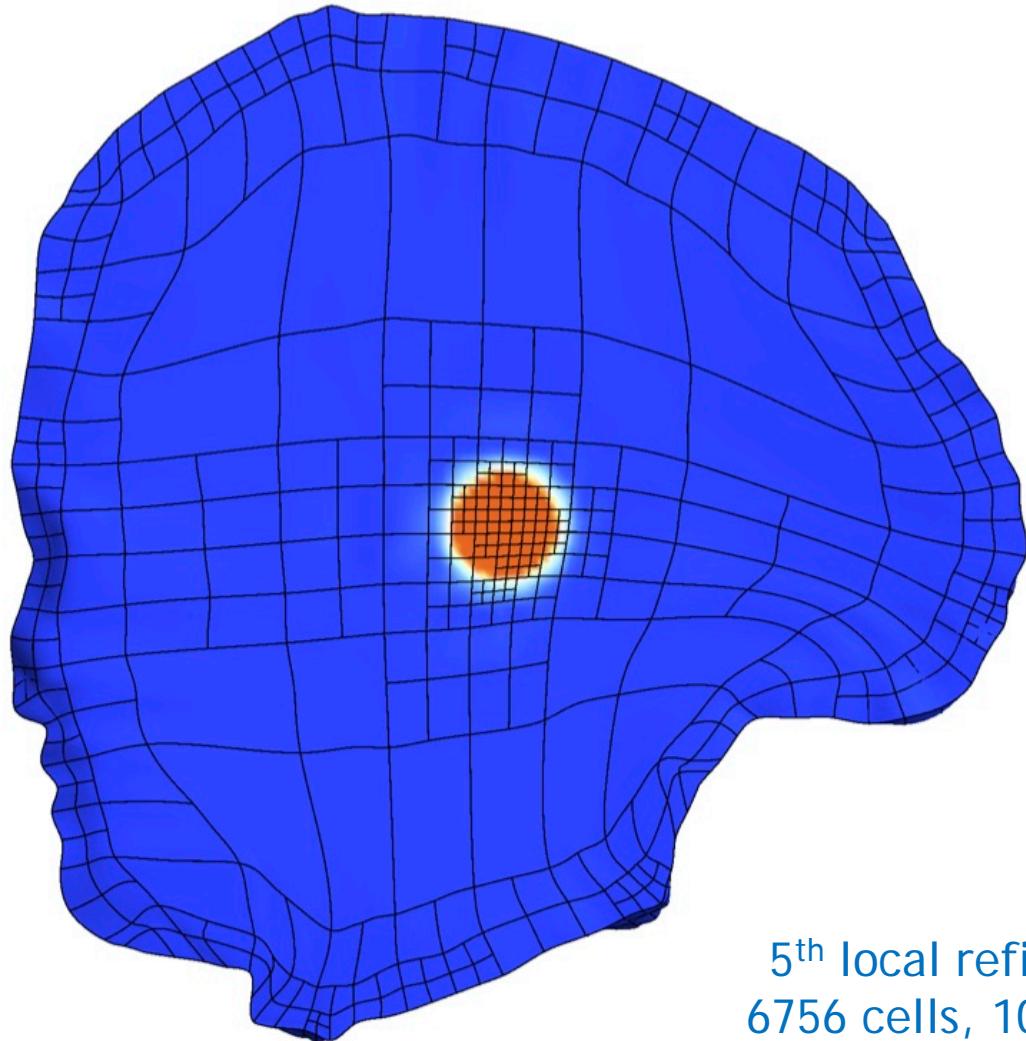


Adaptive Isogeometric Refinement (EWC 2012)

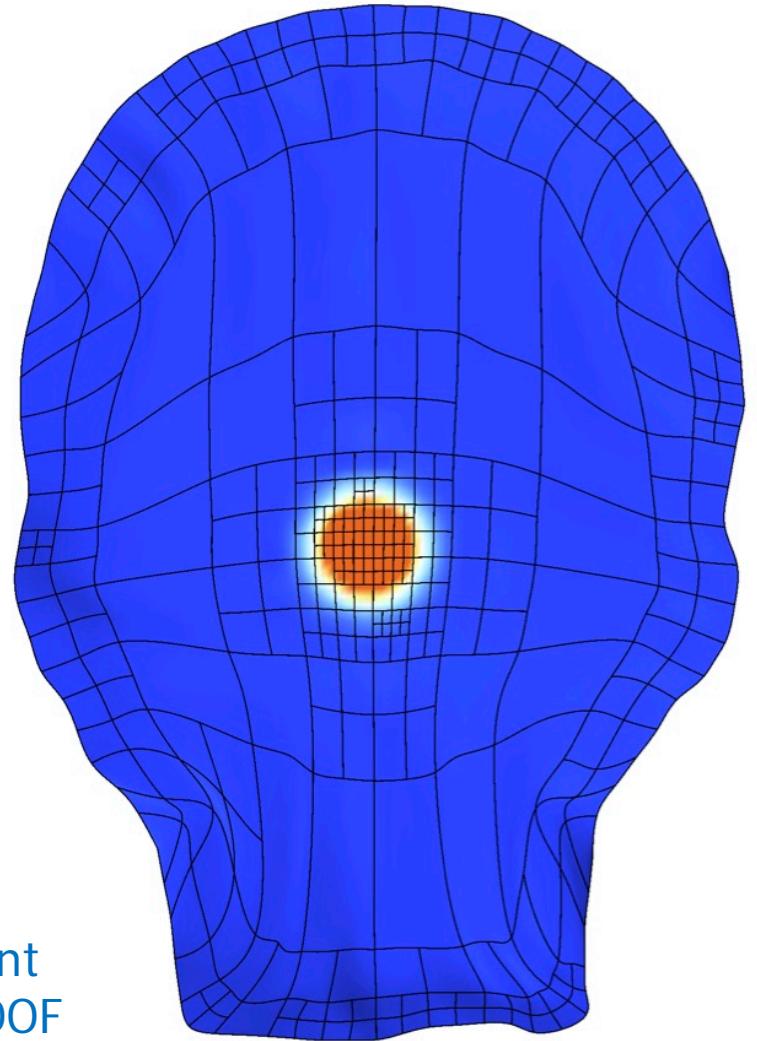
Igea: T-spline of Numerical Solution



$$\text{Error indicator : } \eta(\Omega_e)^2 = \int_{\Omega_e} h^2 (f + \Delta u_h)^2 \, d\Omega$$



5th local refinement
6756 cells, 10838 DOF



Meccano Method on T-meshes for Complex Solids

Volume parameterization based on SUS of T-meshes



1. The key of the meccano method is the simultaneous untangling and smoothing (SUS) procedure.
2. The quality of the T-spline mapping (i.e., positive Jacobian, good uniformity and orthogonality of the isoparametric curves) depends on the quality of the T-mesh in the physical space. We have to fix a quality metric for this mapping.
3. In order to simplify the procedure and to get less distortion in the volume parameterization, it should be interesting to **directly apply the meccano method on T-meshes instead of tetrahedral meshes**.
4. We have started analysing the problem in 2-D.

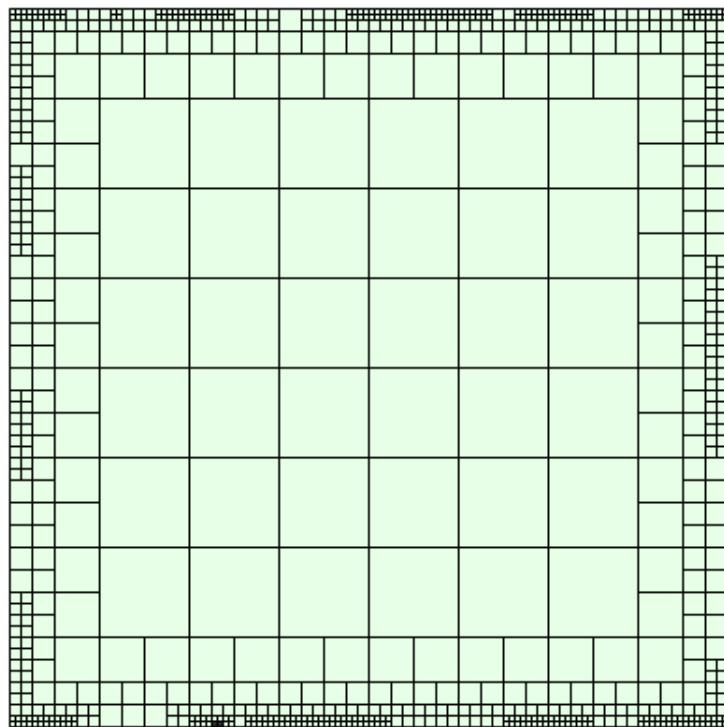
The Meccano Method on T-meshes in 2-D



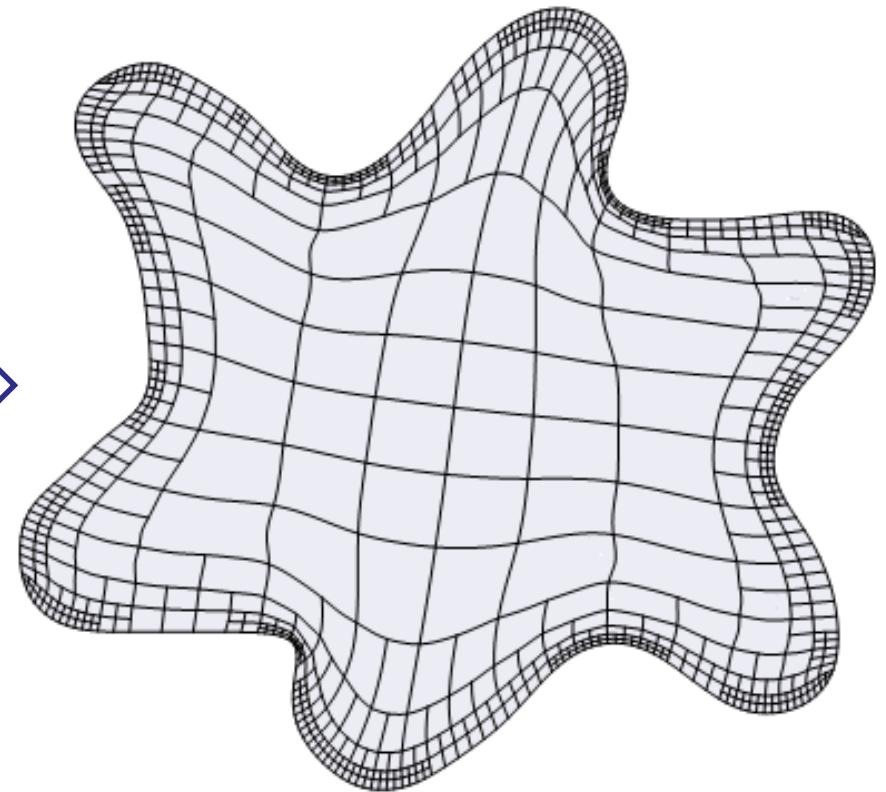
Input data: Boundary representation of the object

Objective: Construction of a high quality T-spline parameterization

T-mesh



T-spline mesh



$$\mathbf{S}(\xi)$$

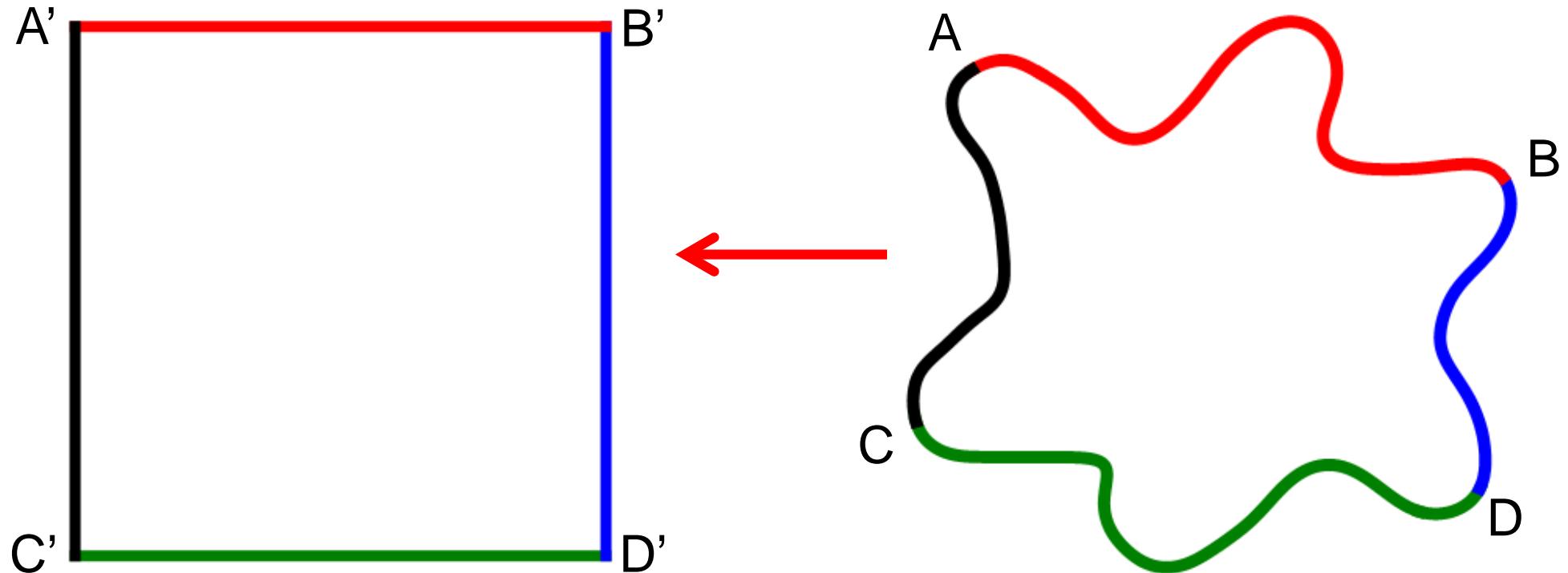


Parameter space

Physical space

The Meccano Method on T-meshes in 2-D

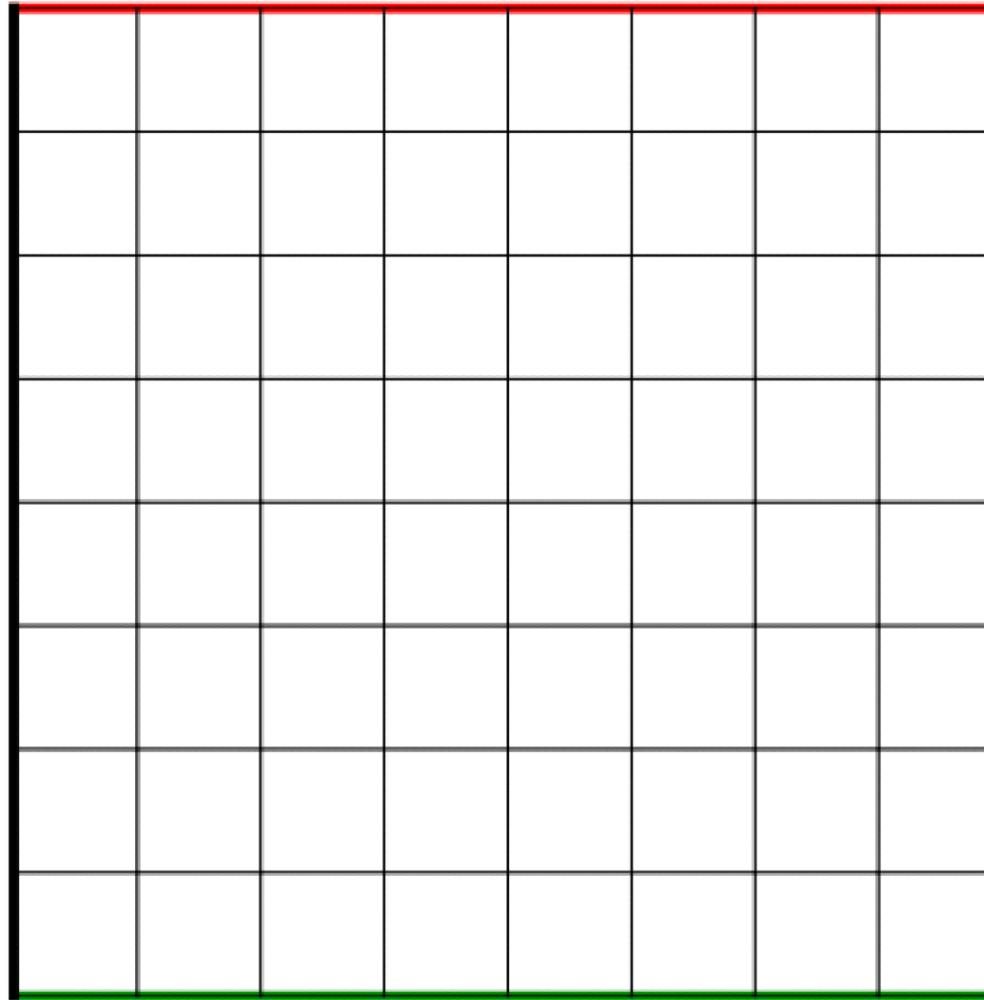
Step 1: Input boundary (image, polyline, curve, etc.) and boundary mapping



- Select four points (A, B, C, D) of the input boundary
- Boundary parameterization via chord-length

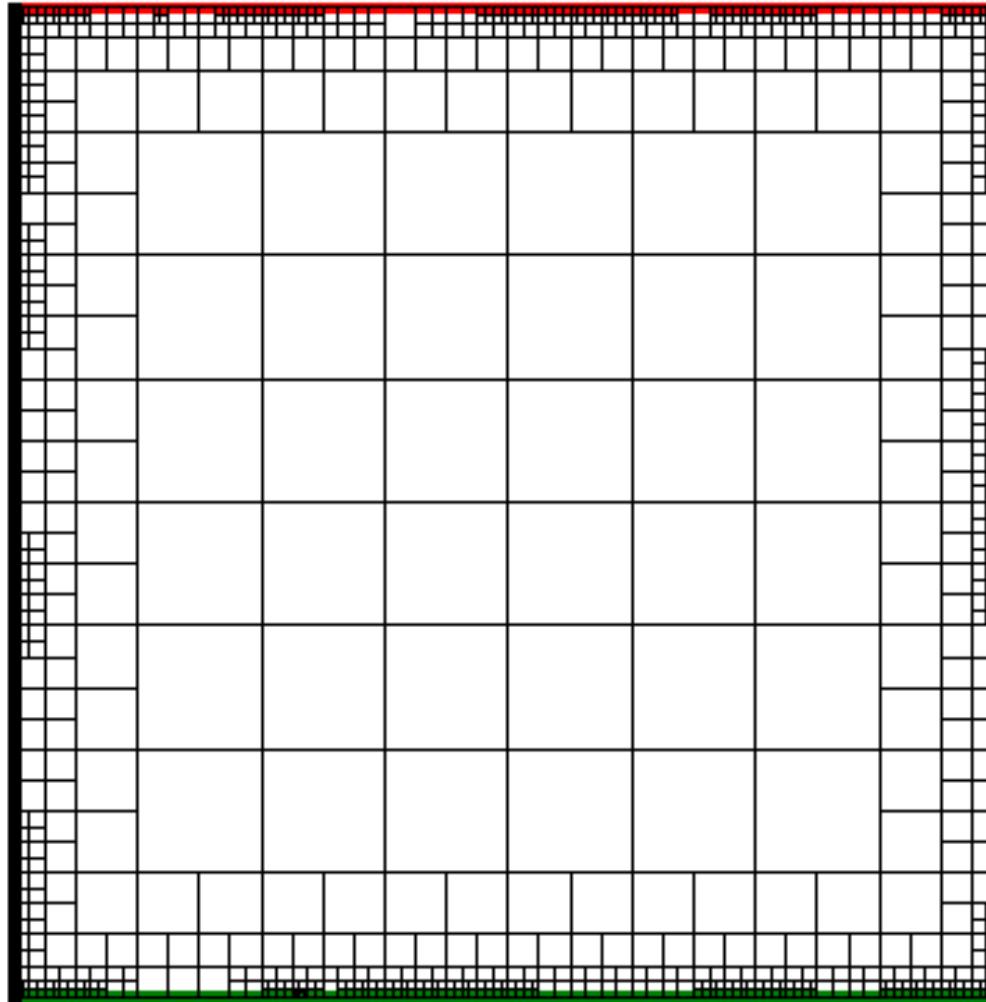
The Meccano Method on T-meshes in 2-D

Step 2: Coarse quadrilateral mesh of the meccano (parameter space)



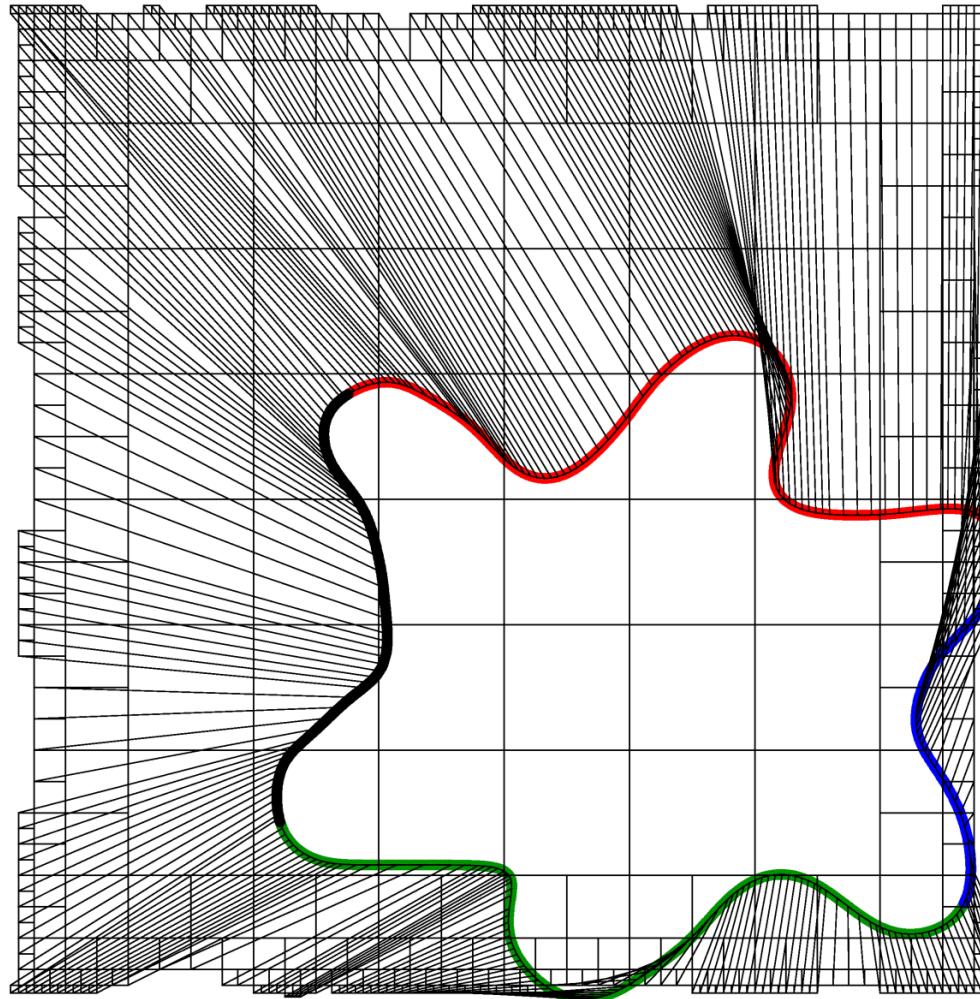
The Meccano Method on T-meshes in 2-D

Step 3: Refine mesh with quadtree subdivisions to approach the boundary



The Meccano Method on T-meshes in 2-D

Step 4: Move the meccano boundary nodes to the object boundary

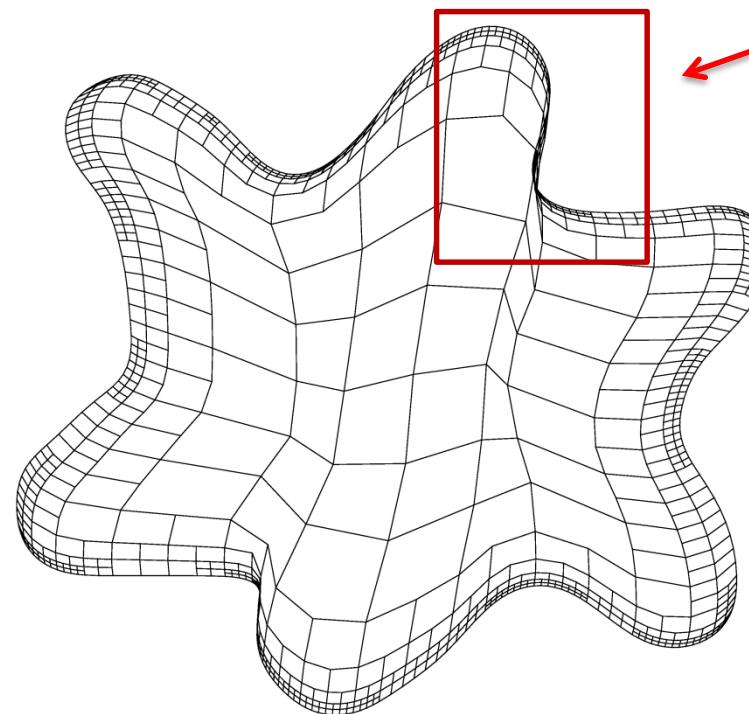
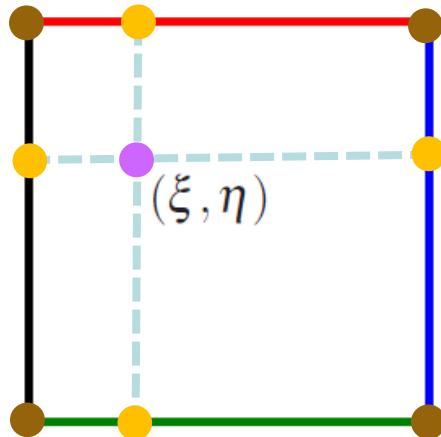


The Meccano Method on T-meshes in 2-D

Step 5: Inner node relocation with Coons patch to facilitate the optimization



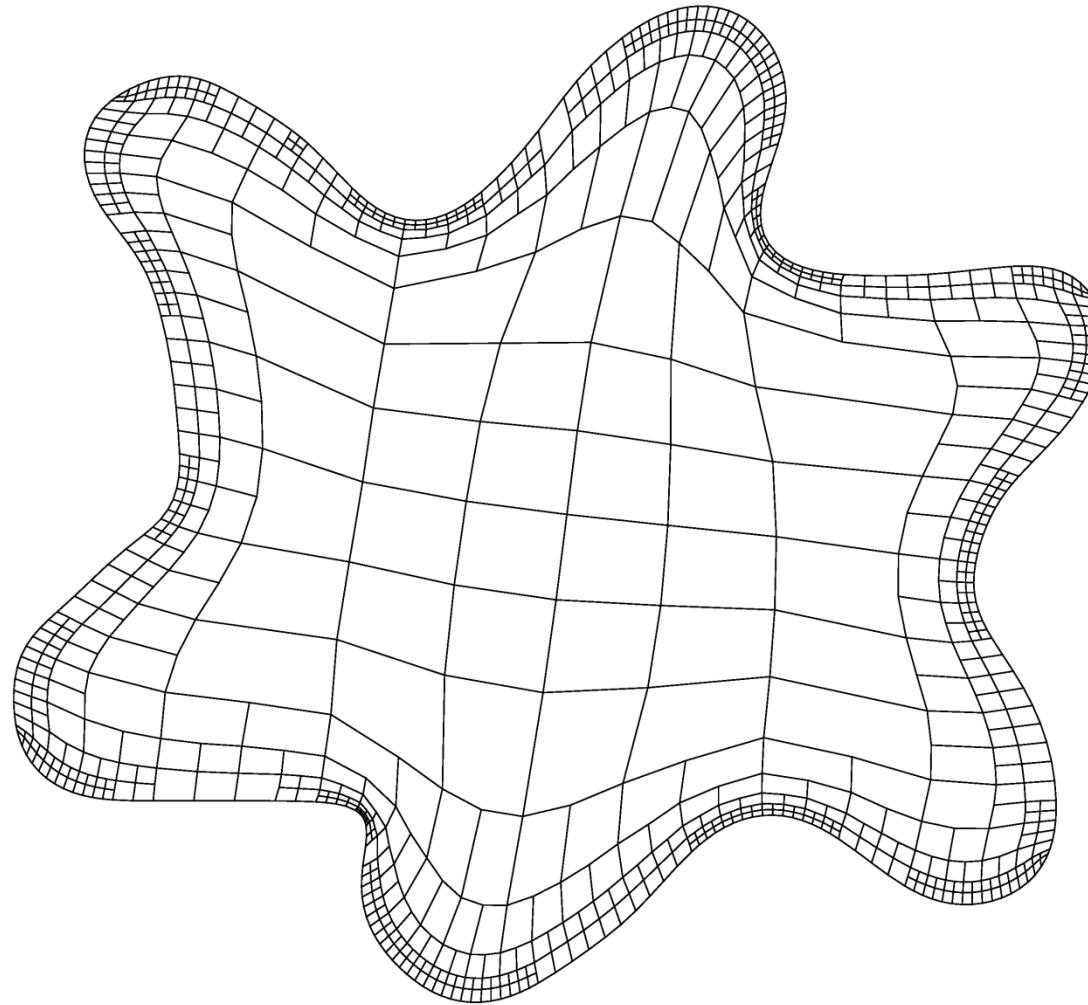
$$\begin{aligned}\mathbf{x}(\xi, \eta) = & (1 - \xi)\mathbf{x}(0, \eta) + \xi\mathbf{x}(1, \eta) \\ & + (1 - \eta)\mathbf{x}(\xi, 0) + \eta\mathbf{x}(\xi, 1) \\ & - [1 - \xi \ \xi] \begin{bmatrix} \mathbf{x}(0,0) & \mathbf{x}(0,1) \\ \mathbf{x}(1,0) & \mathbf{x}(1,1) \end{bmatrix} \begin{bmatrix} 1 - \eta \\ \eta \end{bmatrix}\end{aligned}$$



Mesh folder

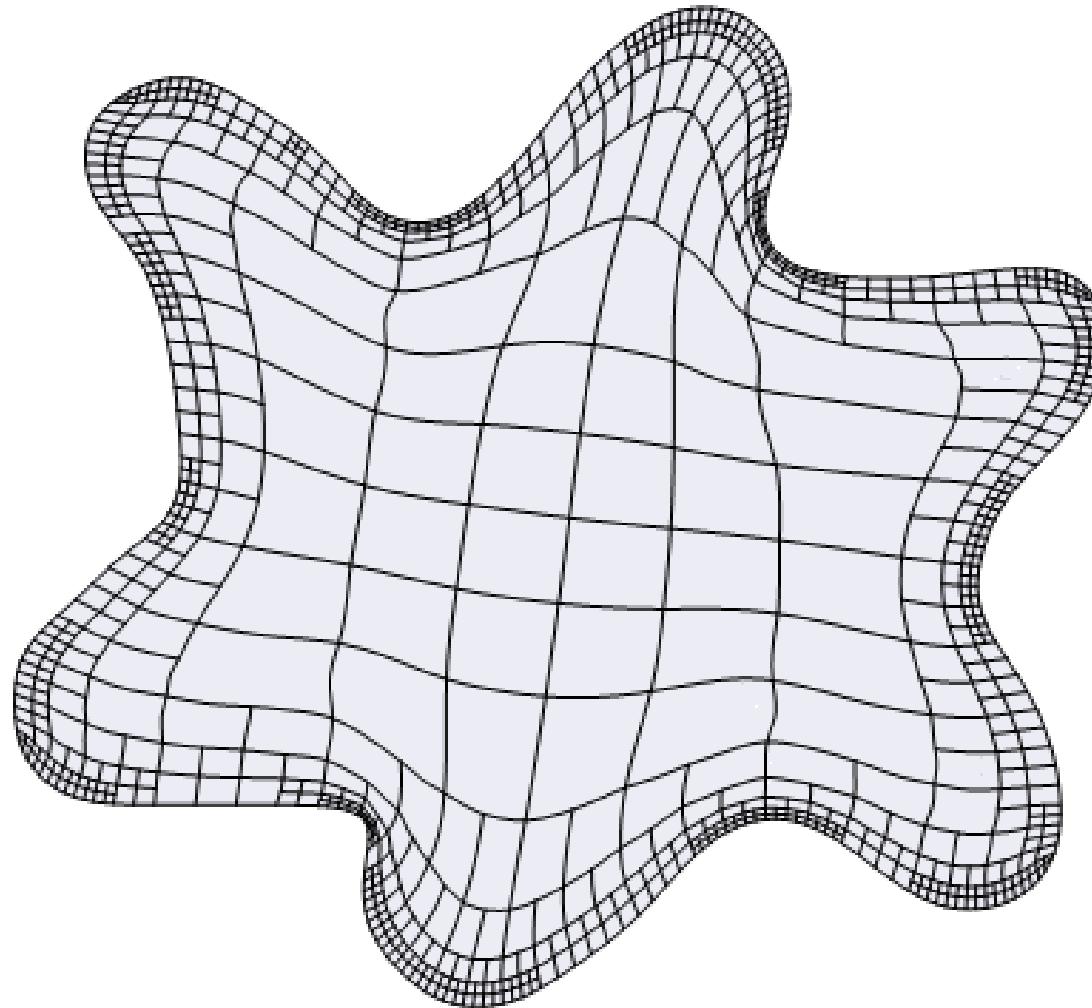
The Meccano Method on T-meshes in 2-D

Step 6: Simultaneous Untangling and Smoothing (SUS) of the T-mesh



The Meccano Method on T-meshes in 2-D

Step 7: T-spline representation of the spot

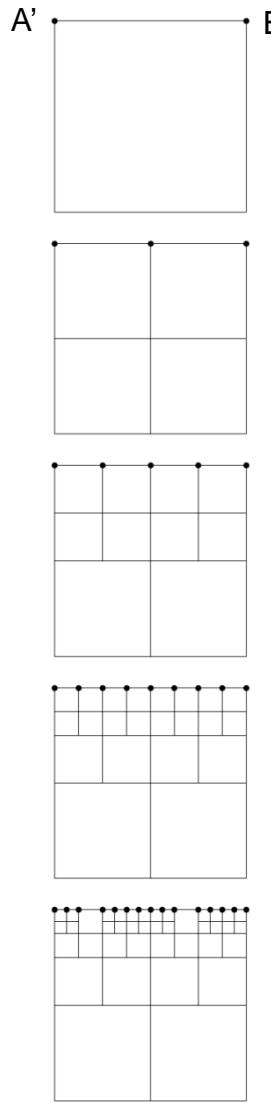


Boundary Approach in 2-D

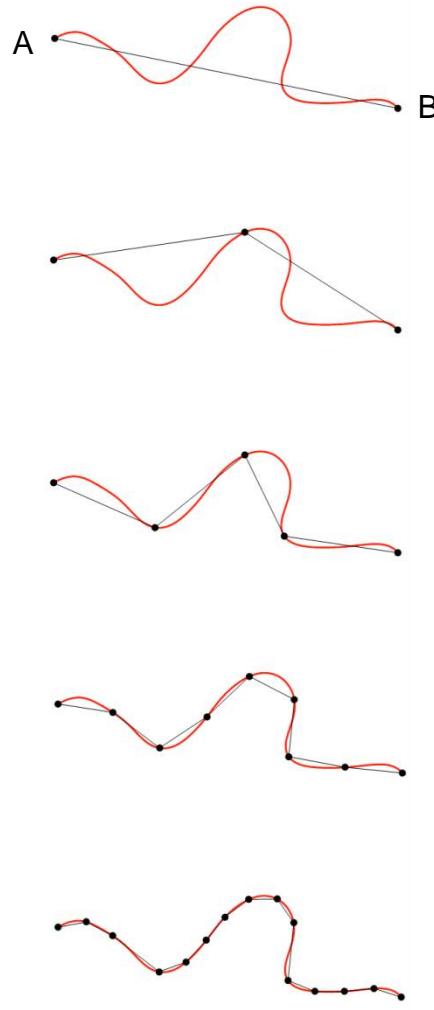
Input data: Boundary polyline approximation (red color line)



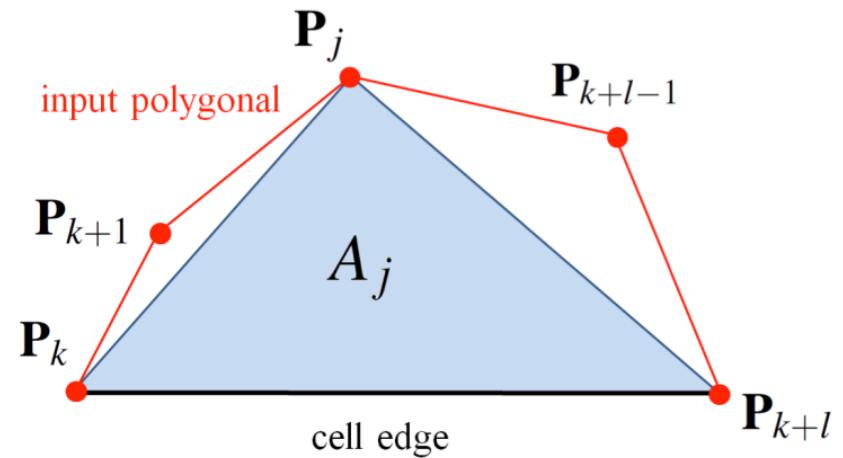
T-mesh adaptation in parameter space



Boundary approximation in physical space



Boundary edge refinement criterion:



$$\exists j : A_j > \varepsilon \Rightarrow \text{refine}$$

$$A_j = \text{Area}(\mathbf{P}_k, \mathbf{P}_j, \mathbf{P}_{k+l})$$

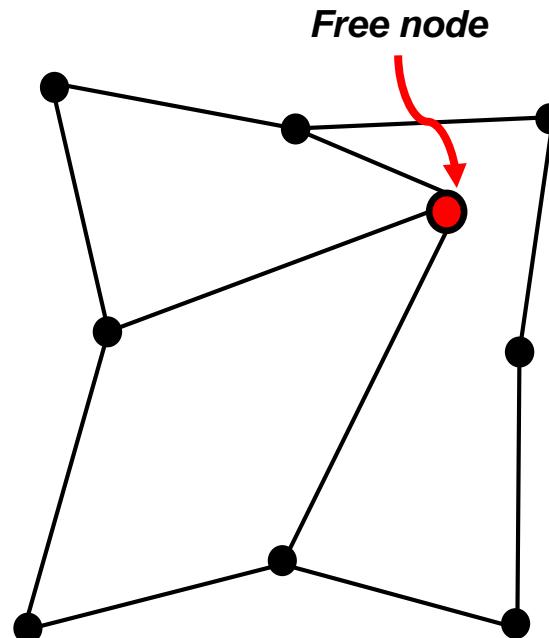
$$k + 1 \leq j \leq k + l - 1$$

Simultaneous Untangling and Smoothing

Case of plane T-meshes (EWC 2013)

Local optimization

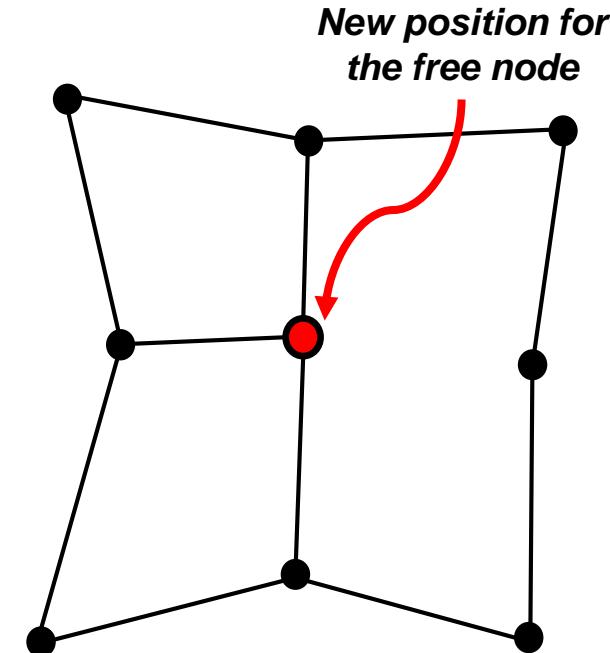
Objective: Improve the quality of the local mesh by minimizing an objective function



Local mesh



?



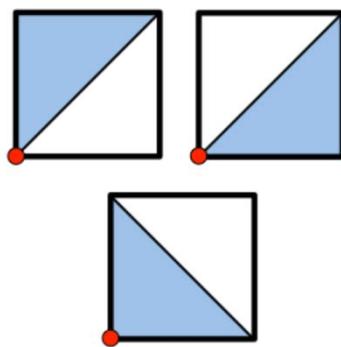
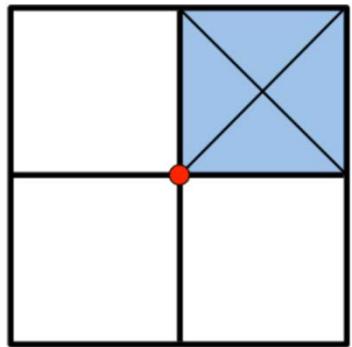
Optimized local mesh

Simultaneous Untangling and Smoothing of T-meshes

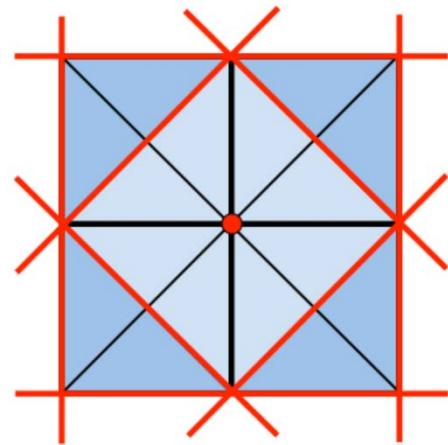
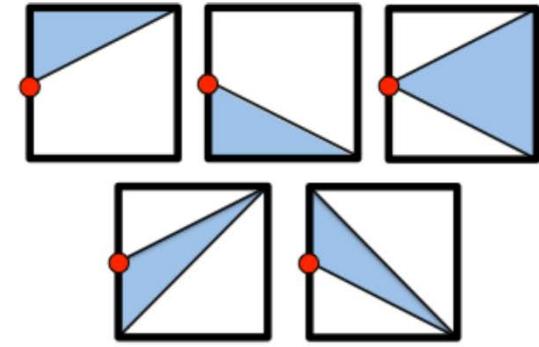
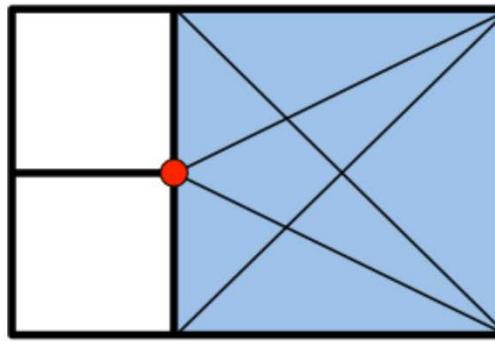
Triangle decomposition of the T-mesh cells



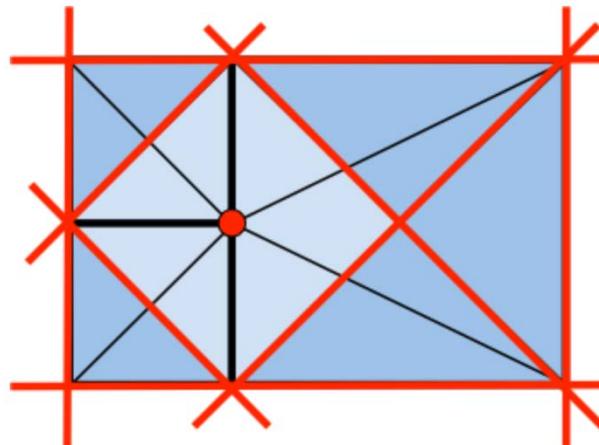
Case 1: Free node is a regular node



Case 2: Free node is a hanging node



*Barriers and feasible
region for a regular node*



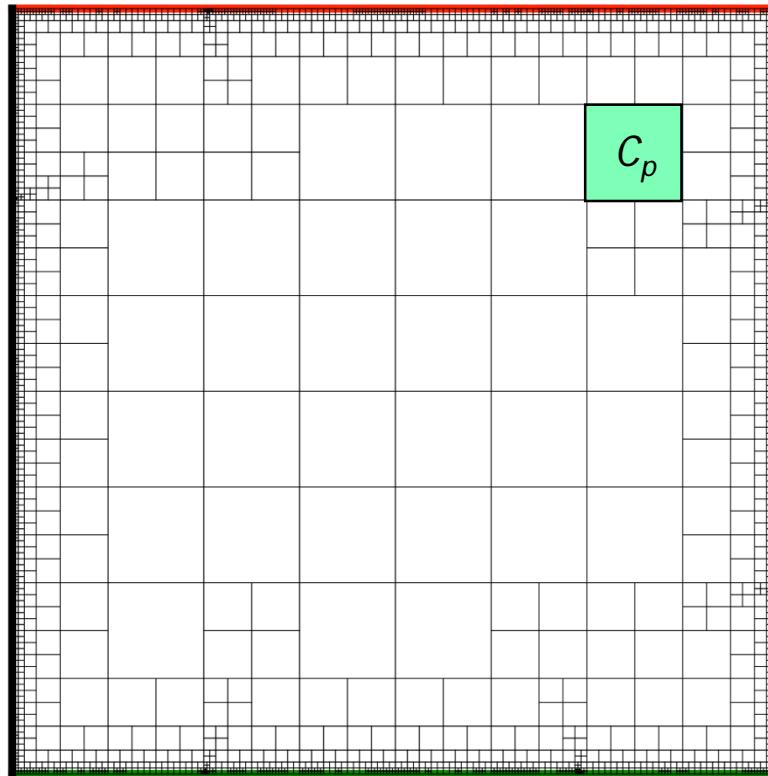
*Barriers and feasible
region for a hanging node*

Simultaneous Untangling and Smoothing of T-meshes

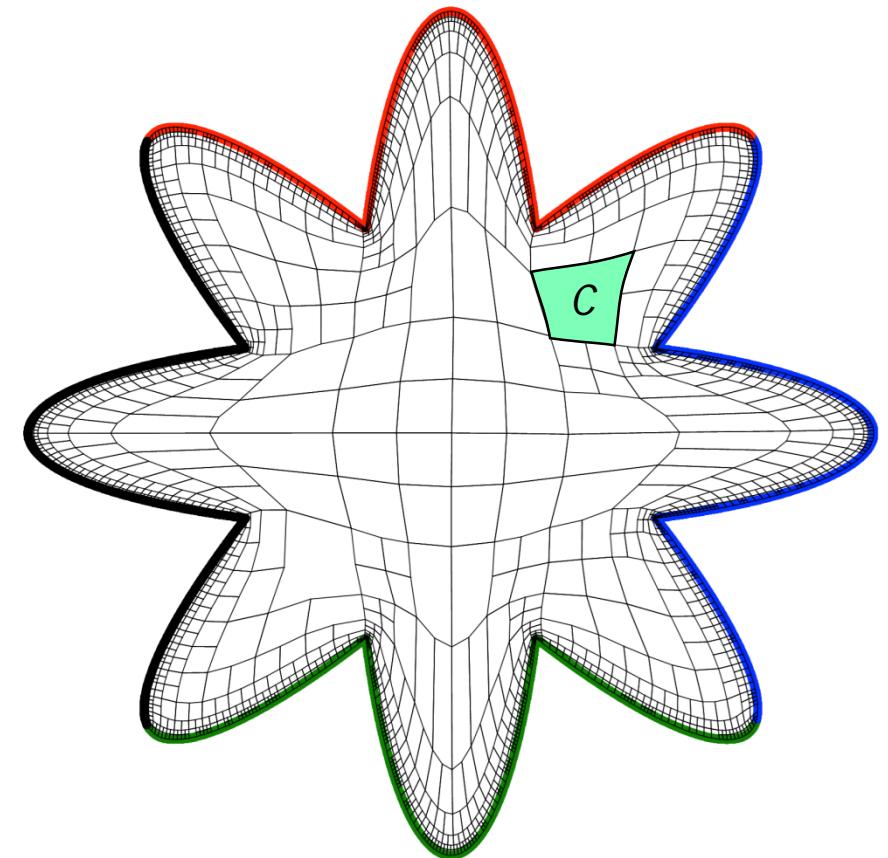
Optimization is guided by the parametric T-mesh



Physical cell C must be as similar as possible to the counterpart in the parametric space C_p



Parametric T-mesh



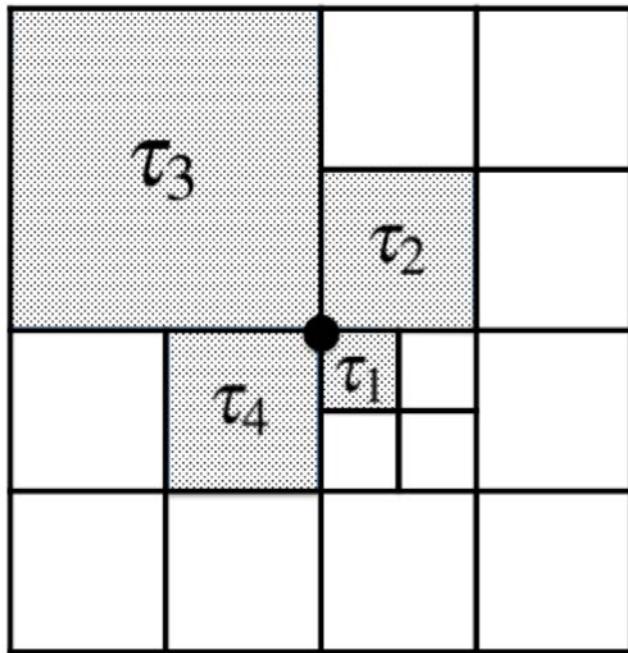
Optimized physical T-mesh

Simultaneous Untangling and Smoothing of T-meshes

Weighted objective functions (regular node)

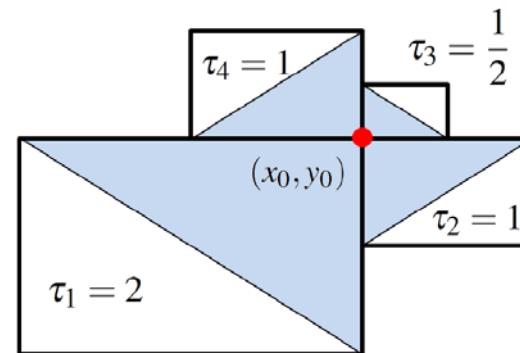


$$K_{\tau}^{*}(\mathbf{x}) = \tau_1 \sum_{m=1}^3 \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_2 \sum_{m=4}^6 \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_3 \sum_{m=7}^9 \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_4 \sum_{m=10}^{12} \frac{\|S_m\|^2}{2h(\sigma_m)}$$

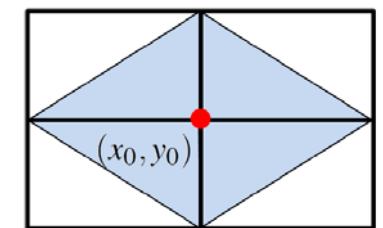


All possible weights for balanced quadtrees:

$$\rightarrow \tau_1 = 1, \tau_2 = \tau_4 = 2 \text{ y } \tau_3 = 4$$



Non-conformal mesh



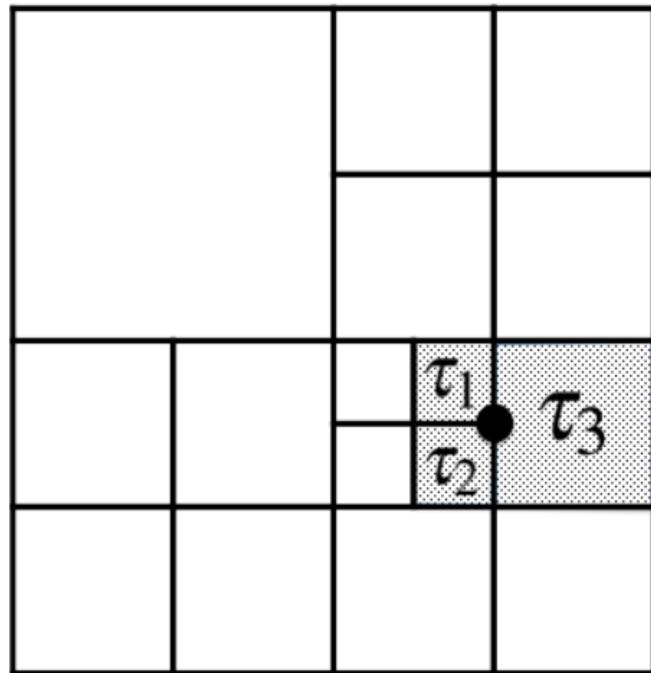
Conformal mesh

Simultaneous Untangling and Smoothing of T-meshes

Weighted objective functions (hanging node)

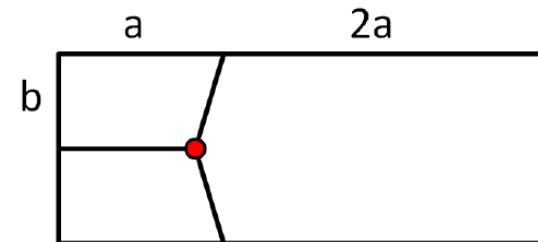


$$K_{\tau}^{*}(\mathbf{x}) = \tau_1 \sum_{m=1}^3 \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_2 \sum_{m=4}^6 \frac{\|S_m\|^2}{2h(\sigma_m)} + \tau_3 \sum_{m=7}^{11} \frac{\|S_m\|^2}{2h(\sigma_m)}$$

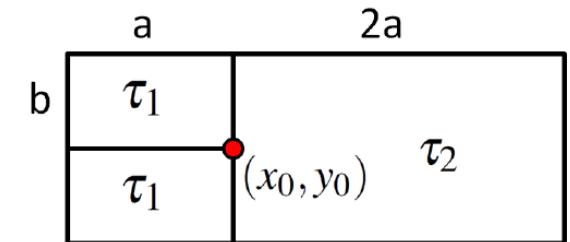


All possible weights for balanced quadtrees:

$$\longrightarrow \tau_1 = \tau_2 = 1 \text{ y } \tau_3 = \frac{8}{5}$$



Optimized mesh
without weights



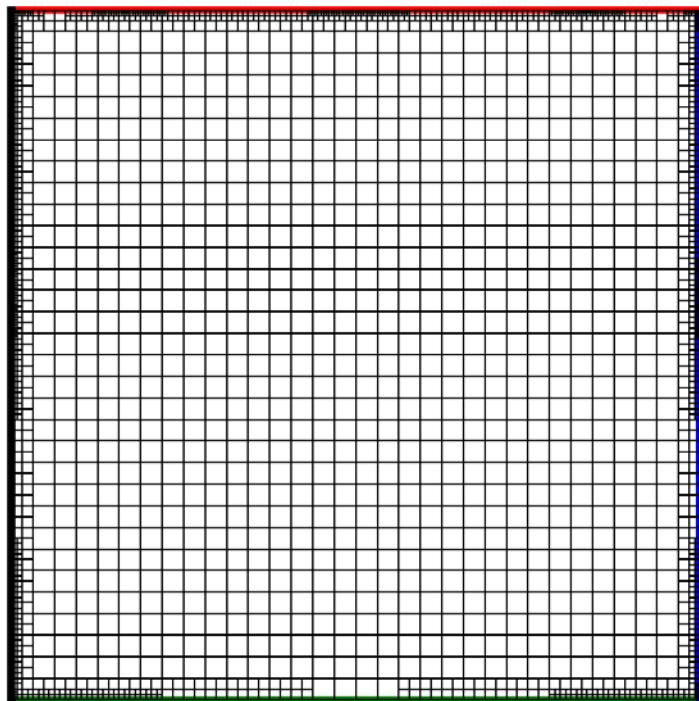
Optimized mesh
with weights

Simultaneous Untangling and Smoothing of T-meshes

T-mesh transformation along the SUS process: Example



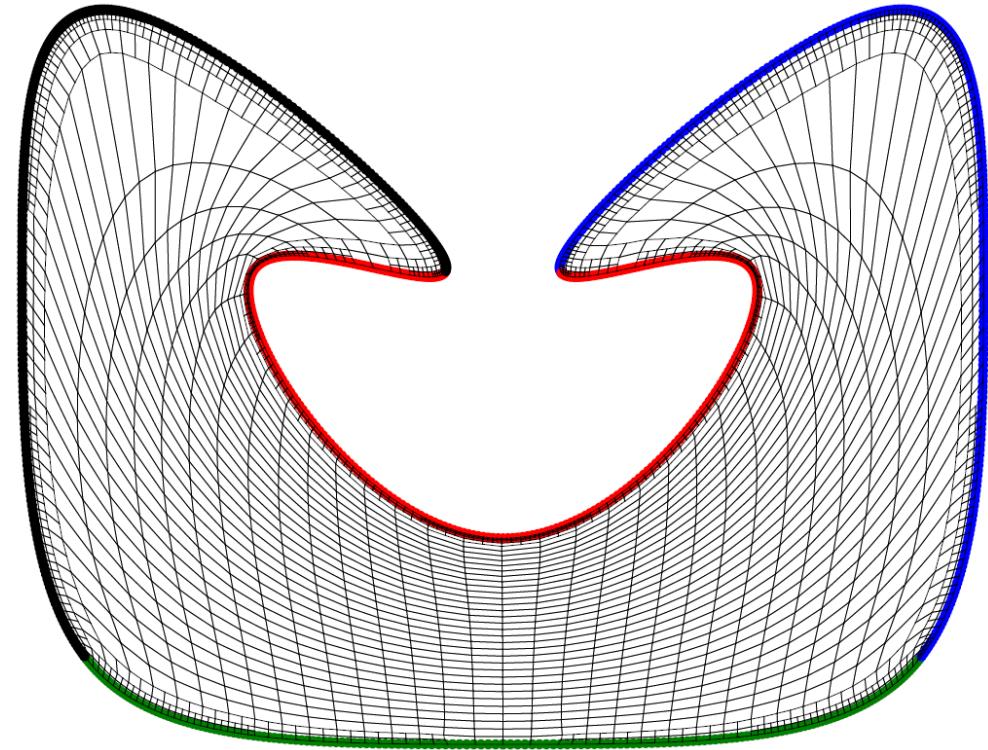
T-mesh



$$S(\xi)$$

A blue arrow pointing from the T-mesh in parameter space to the T-spline in physical space, indicating the mapping function $S(\xi)$.

T-spline

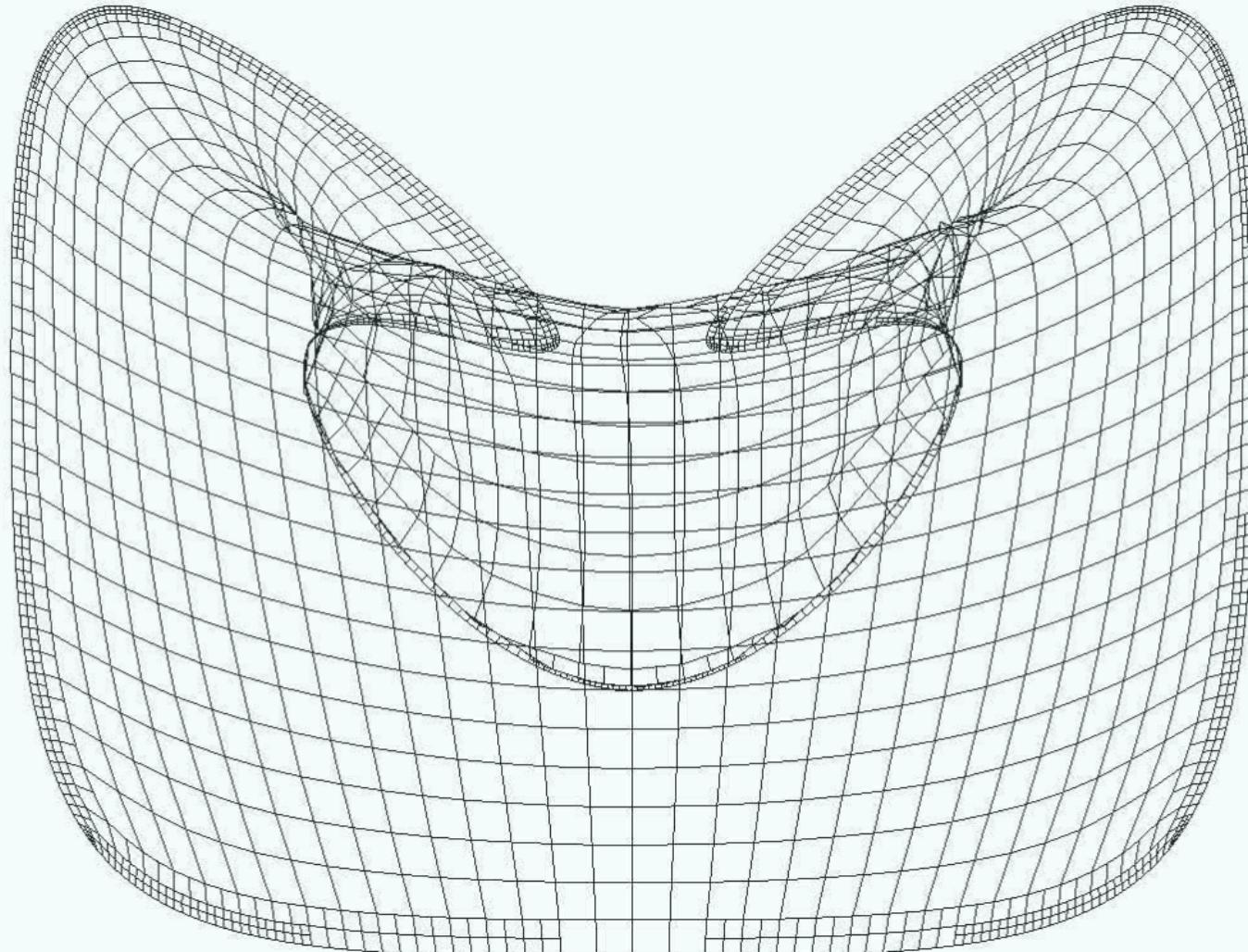


Parameter space

Physical space

Simultaneous Untangling and Smoothing of T-meshes

T-mesh transformation along the SUS process: Video

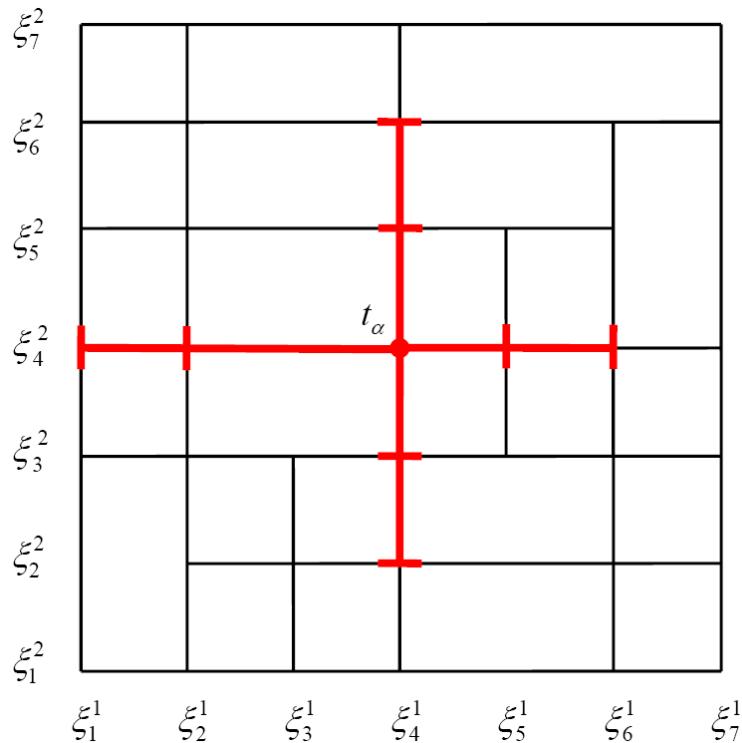


T-spline Basis Functions in 2-D

Example of a bivariate basis on a T-mesh



T-mesh and anchor t_a



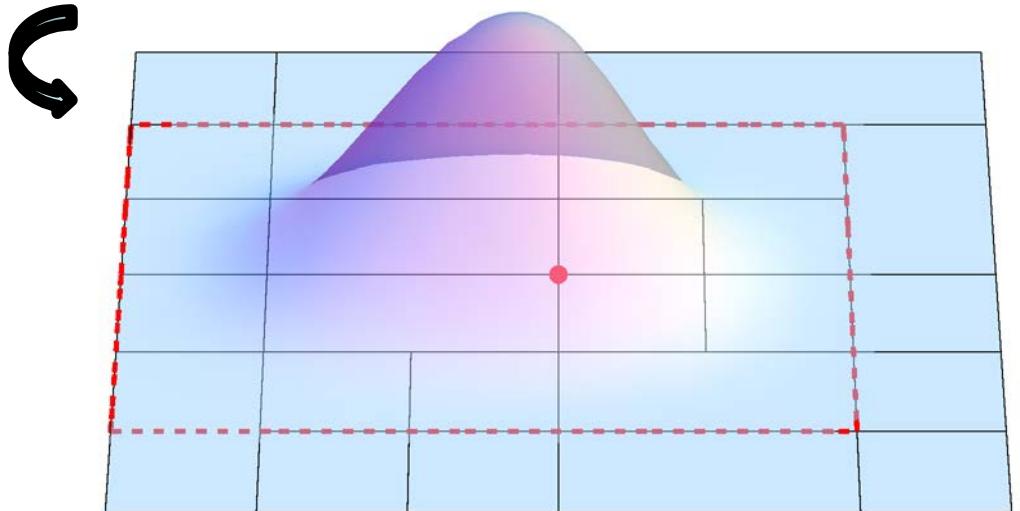
Knots associated to anchor t_a :

$$\Xi_\alpha^1 = \{\xi_1^1, \xi_2^1, \xi_4^1, \xi_5^1, \xi_6^1\} \quad \Xi_\alpha^2 = \{\xi_2^2, \xi_3^2, \xi_4^2, \xi_5^2, \xi_6^2\}$$

$$N_\alpha^1(\xi^1) \times N_\alpha^2(\xi^2)$$

$$B_\alpha(\xi^1, \xi^2) = N_\alpha^1(\xi^1)N_\alpha^2(\xi^2)$$

support of the T-spline



Bivariate Cubic T-spline Basis Function

T-spline Parameterization

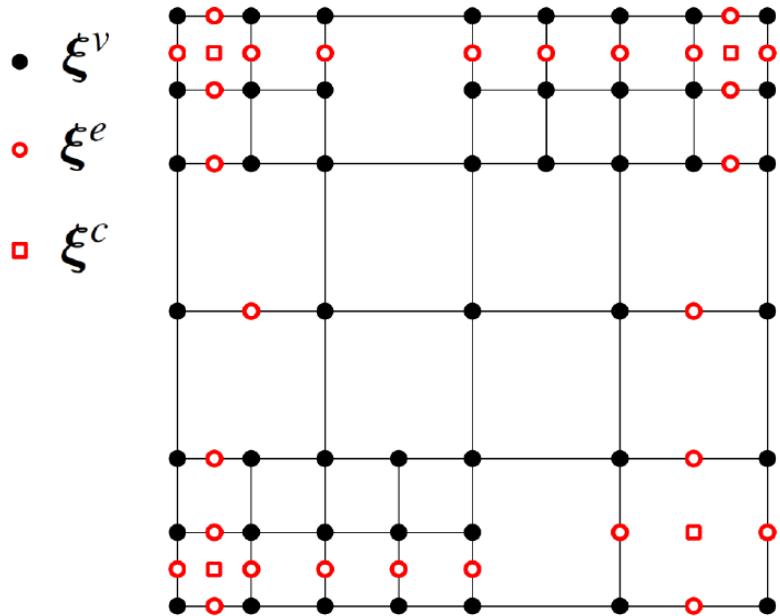
Determination of control points by imposing interpolation conditions



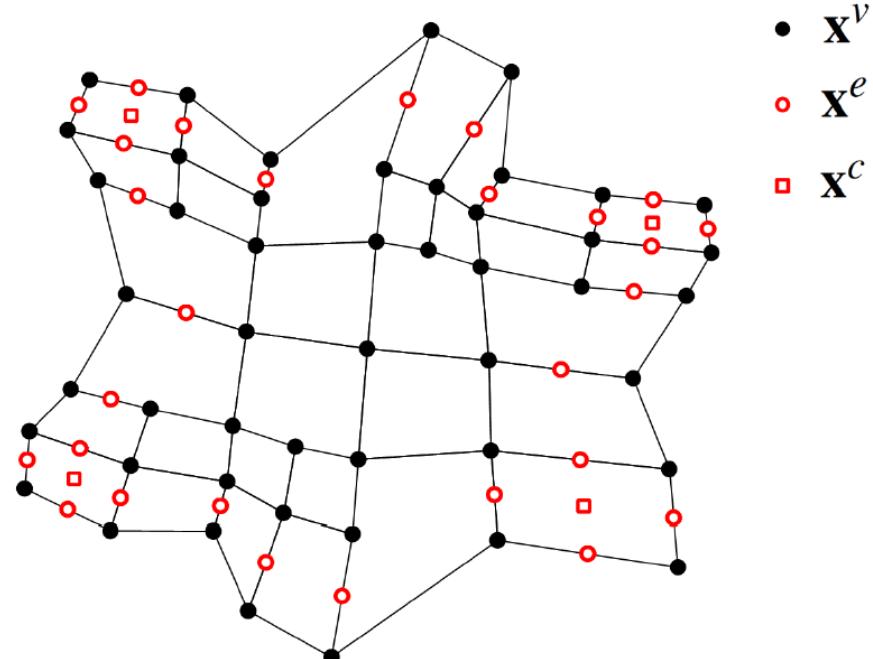
$$S(\xi) = \sum_{\alpha \in A} P_\alpha R_\alpha(\xi)$$

where

$$R_\alpha(\xi) = \frac{w_\alpha B_\alpha(\xi)}{\sum_{\beta \in A} w_\beta B_\beta(\xi)}$$



$S(\xi)$



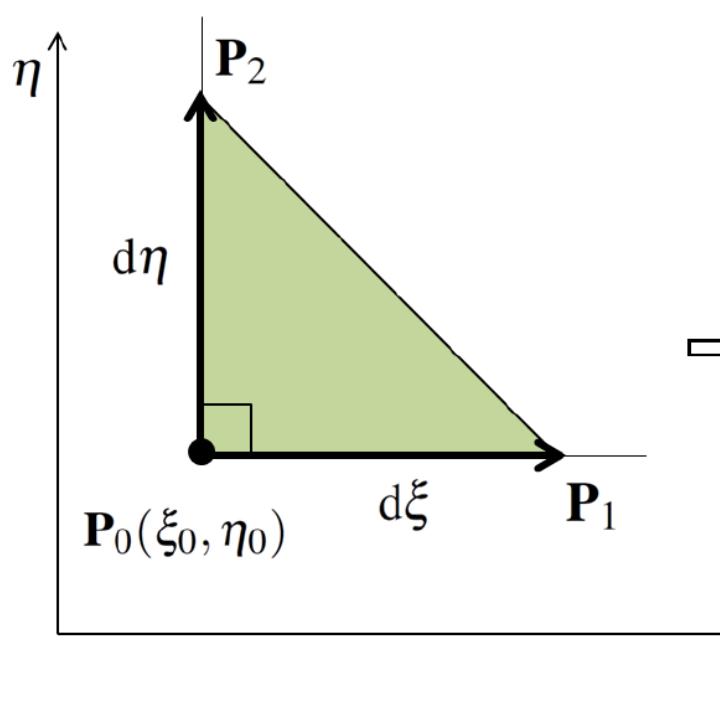
Mean Ratio Jacobian $J_r(\xi)$ of Parametric Transformation

A quality metric of the T-spline mapping at any point P_0

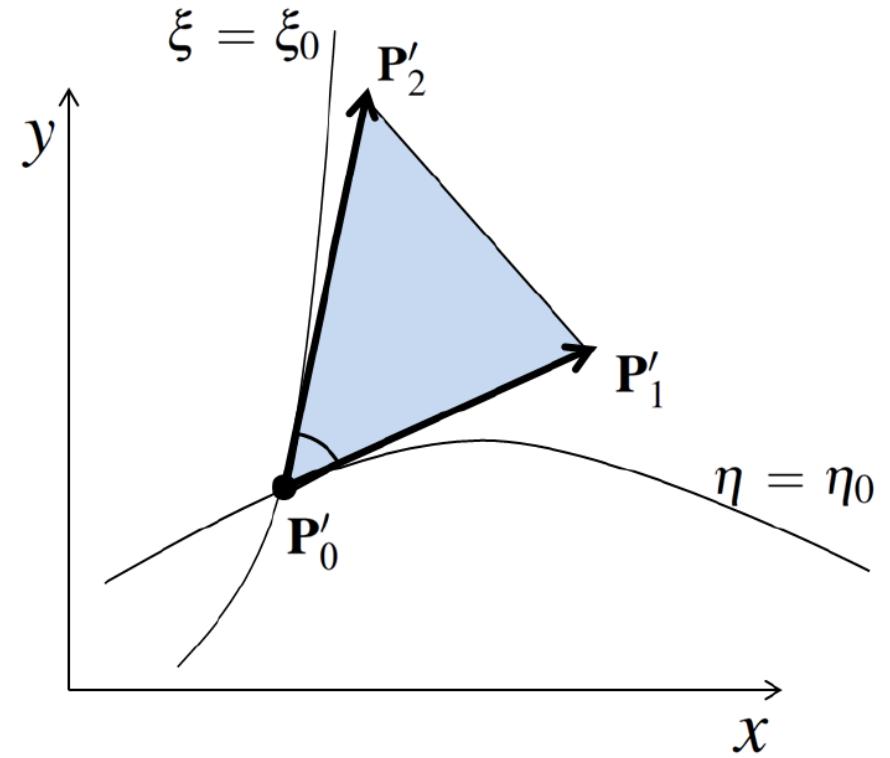


$$-1 \leq J_r(\xi) = \frac{2 \det(J)}{\|J\|^2} \leq 1$$

where \mathbf{J} is the jacobian matrix of the T-spline mapping \mathbf{S}

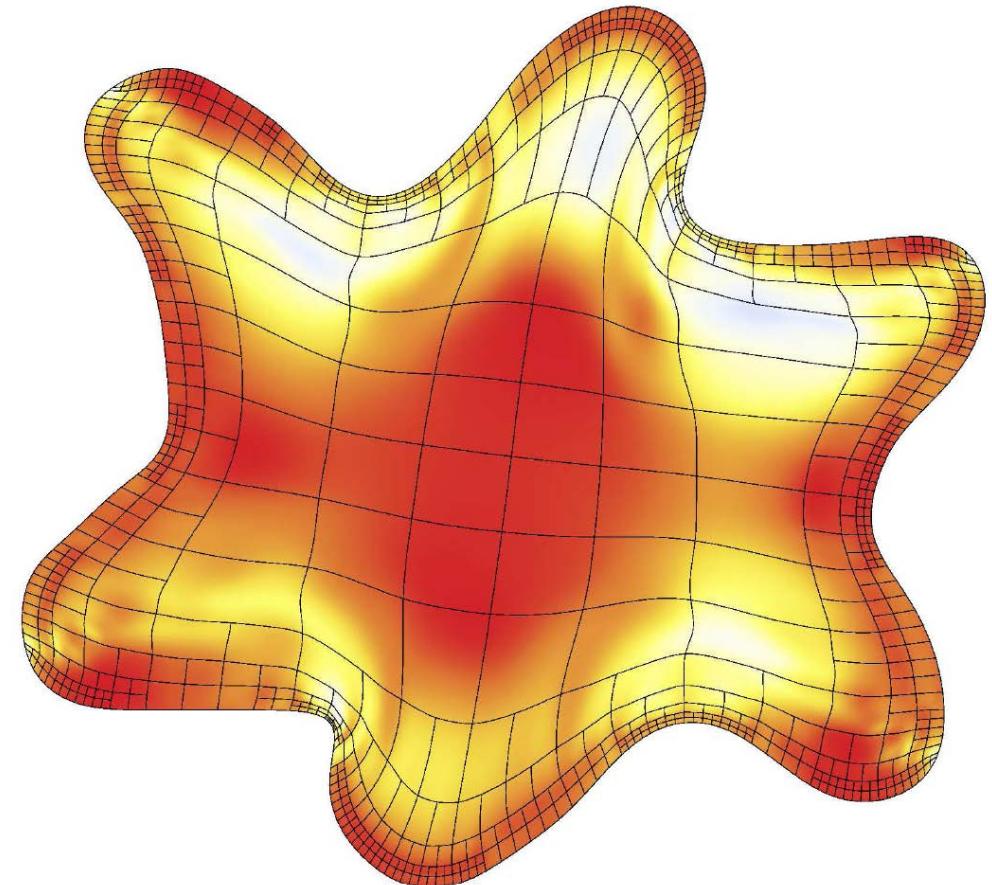
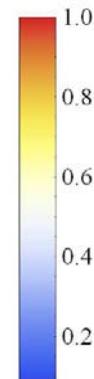
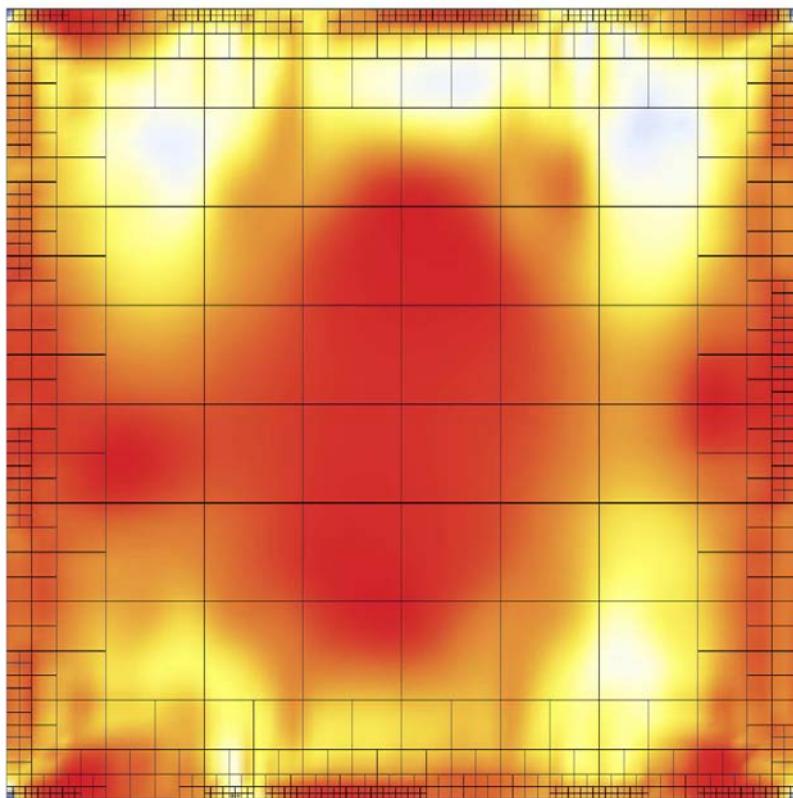


$$\mathbf{S}(\xi, \eta)$$



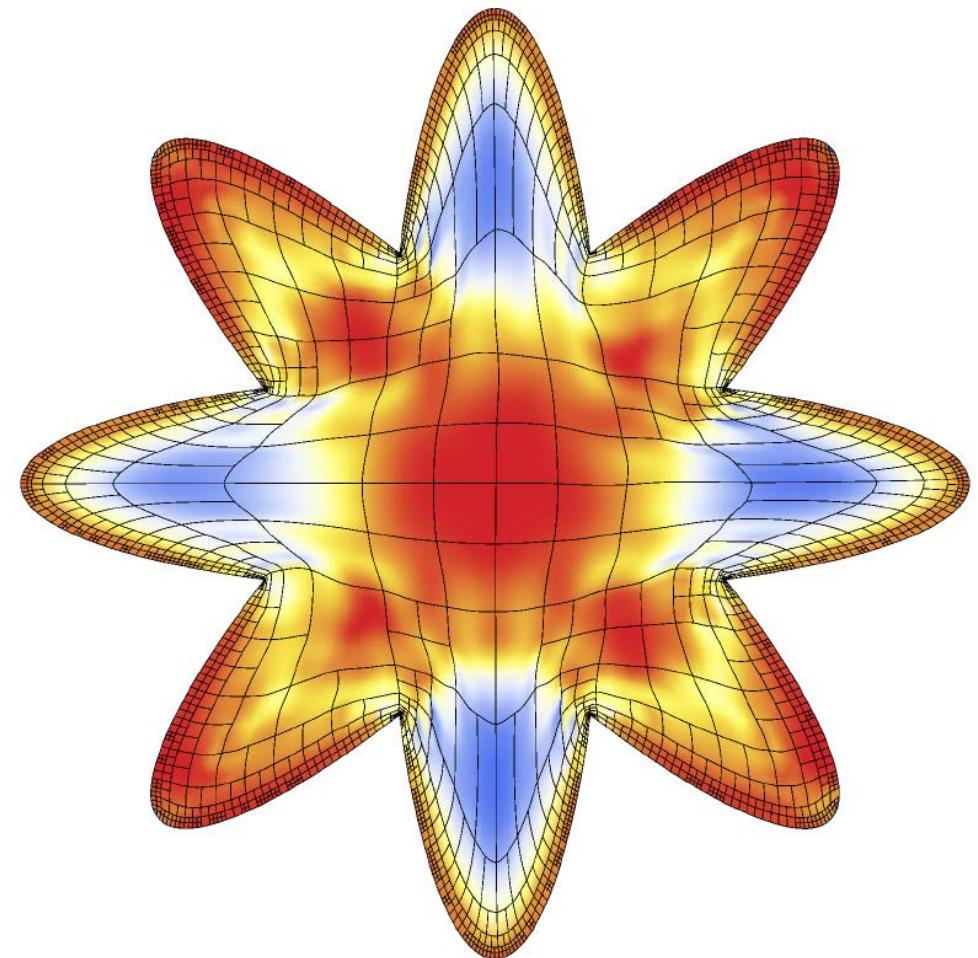
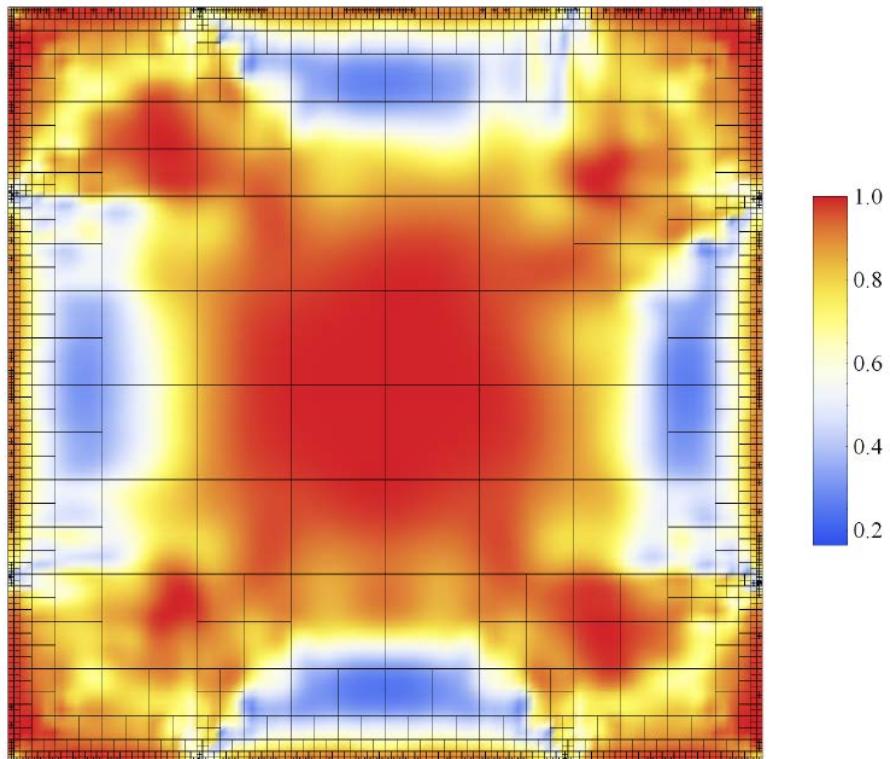
Applications: Isogeometric Modeling

The Spot (Mean Ratio Jacobian)



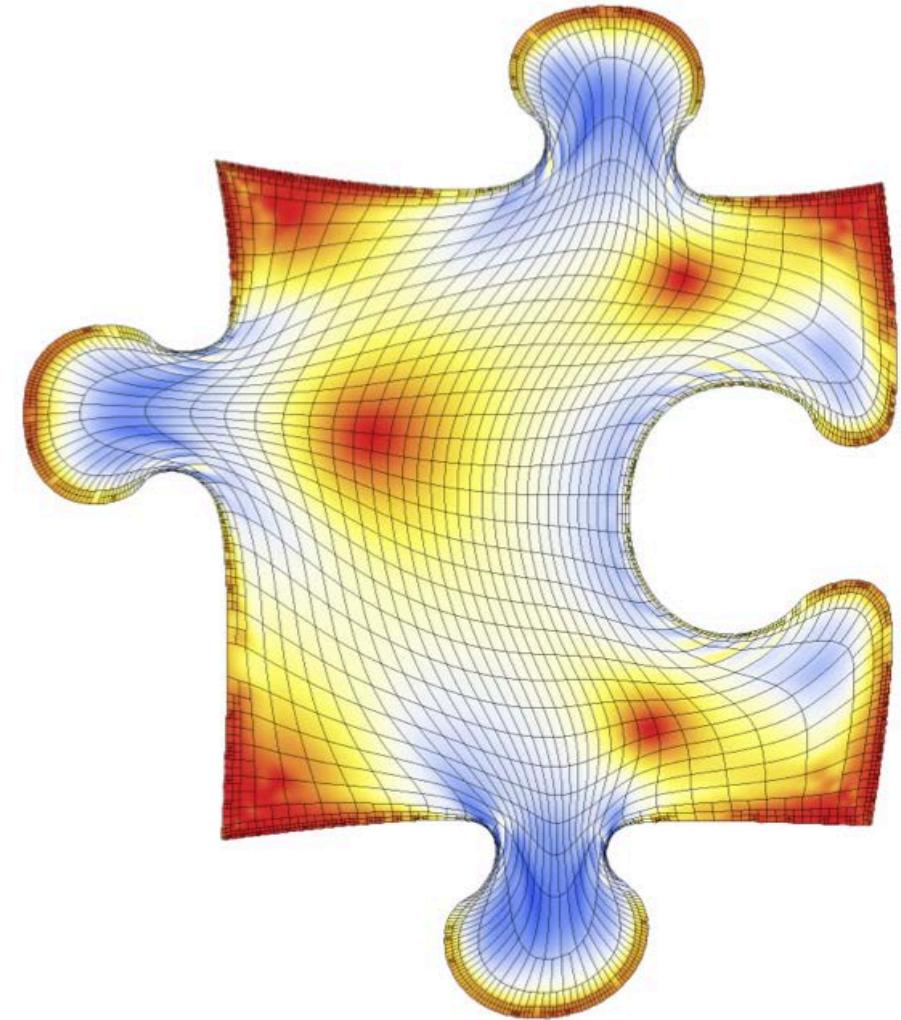
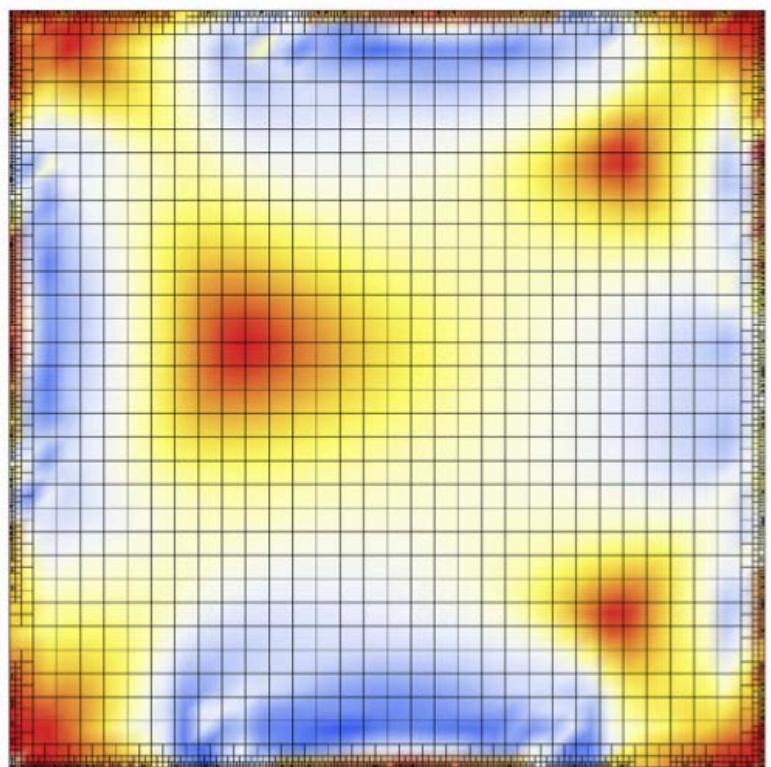
Applications: Isogeometric Modeling

The Flower (Mean Ratio Jacobian)



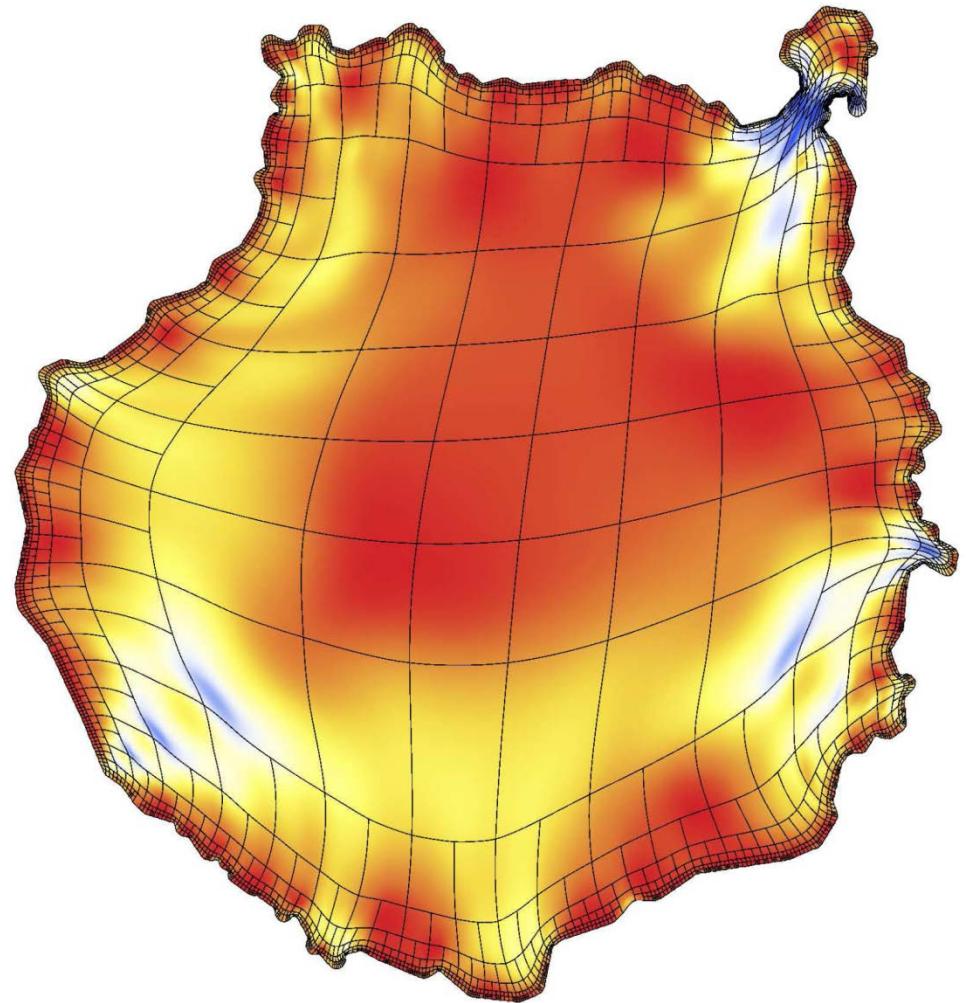
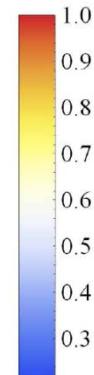
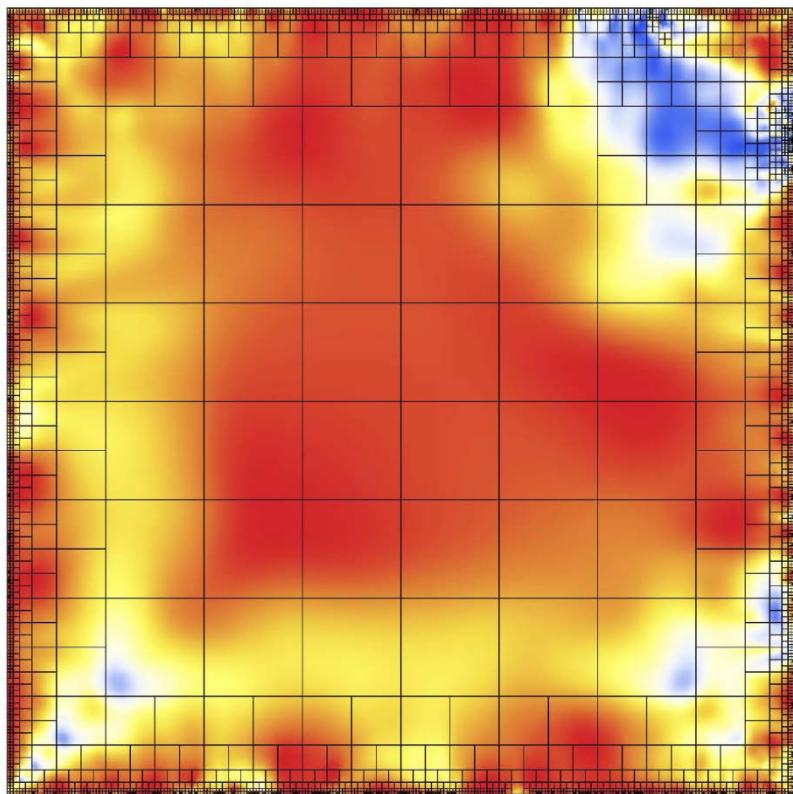
Applications: Isogeometric Modeling

Puzzle Piece (Mean Ratio Jacobian)



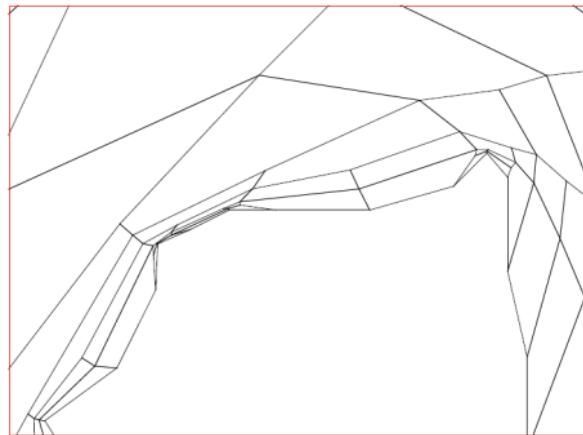
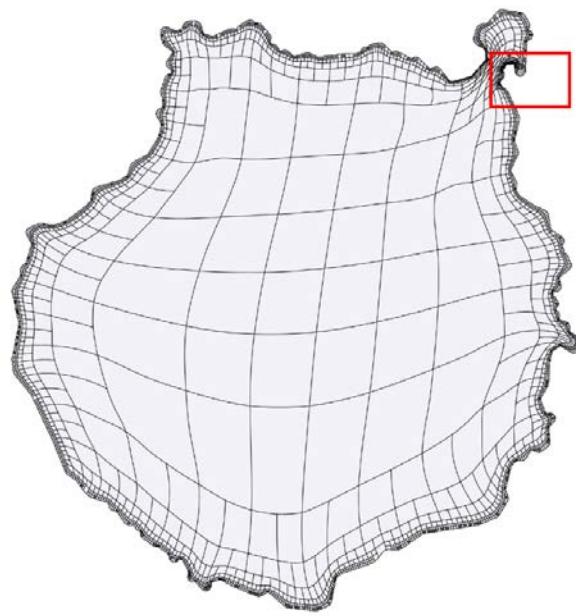
Applications: Isogeometric Modeling

Gran Canaria Island (Mean Ratio Jacobian)

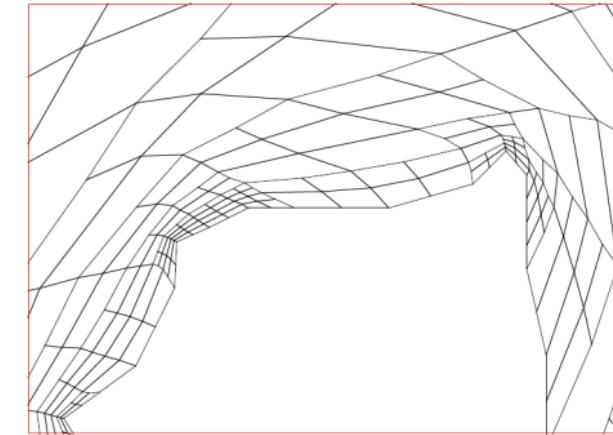


Applications: Isogeometric Modeling

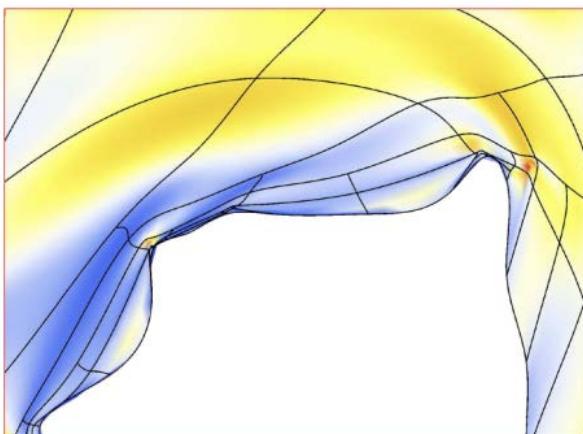
Gran Canaria Island (adaptive refinement to improve the mean ratio Jacobian)



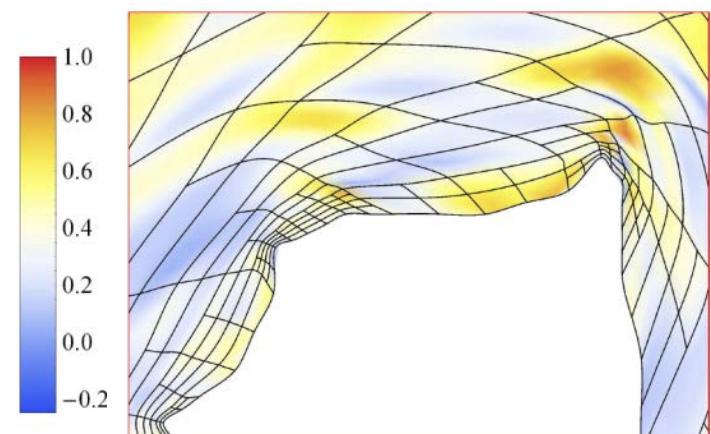
Initial T-mesh



Refined T-mesh



Initial T-spline & Mean ratio Jacobian

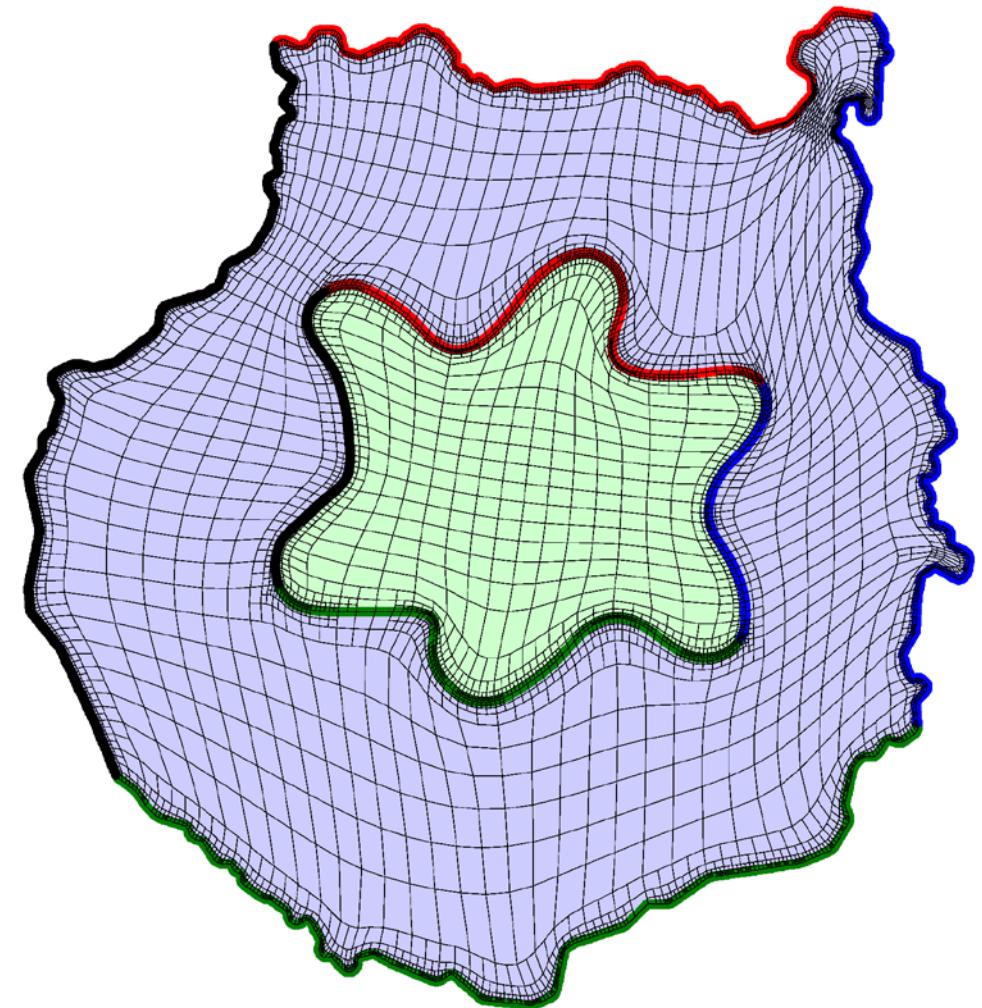
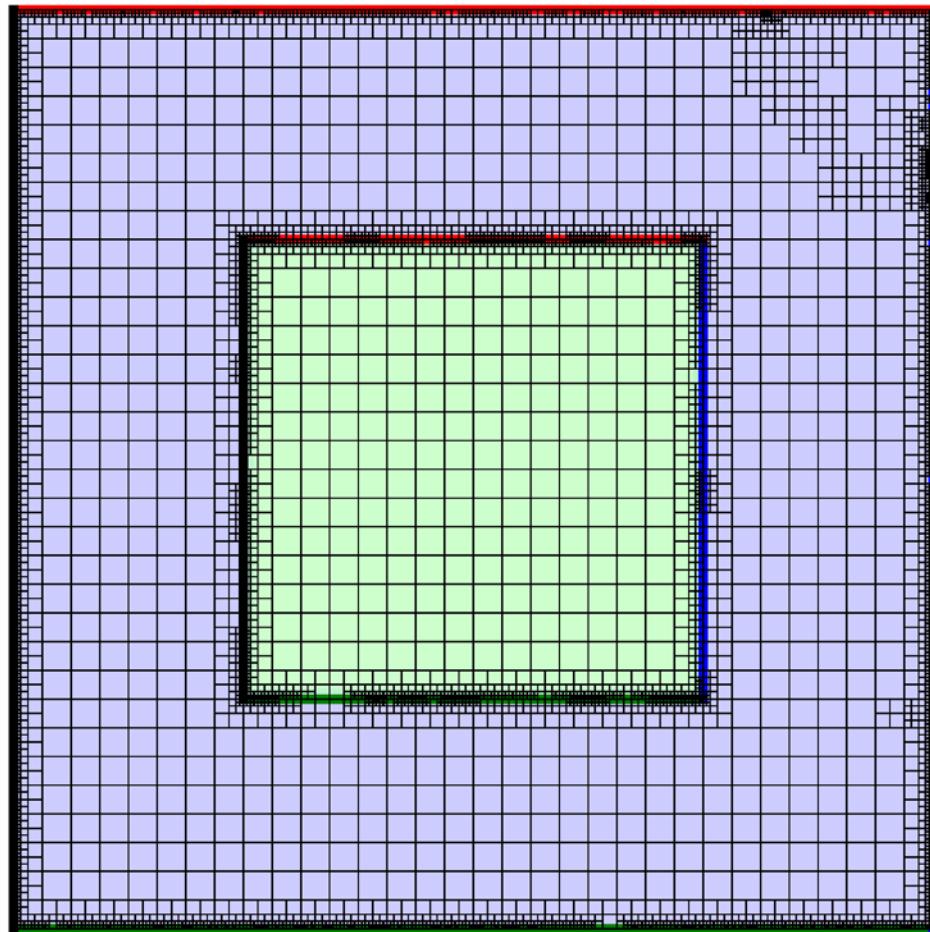


Refined T-spline & Mean ratio Jacobian

No negative Jacobian
after refinement!

Applications: Isogeometric Modeling

Geometries with several materials

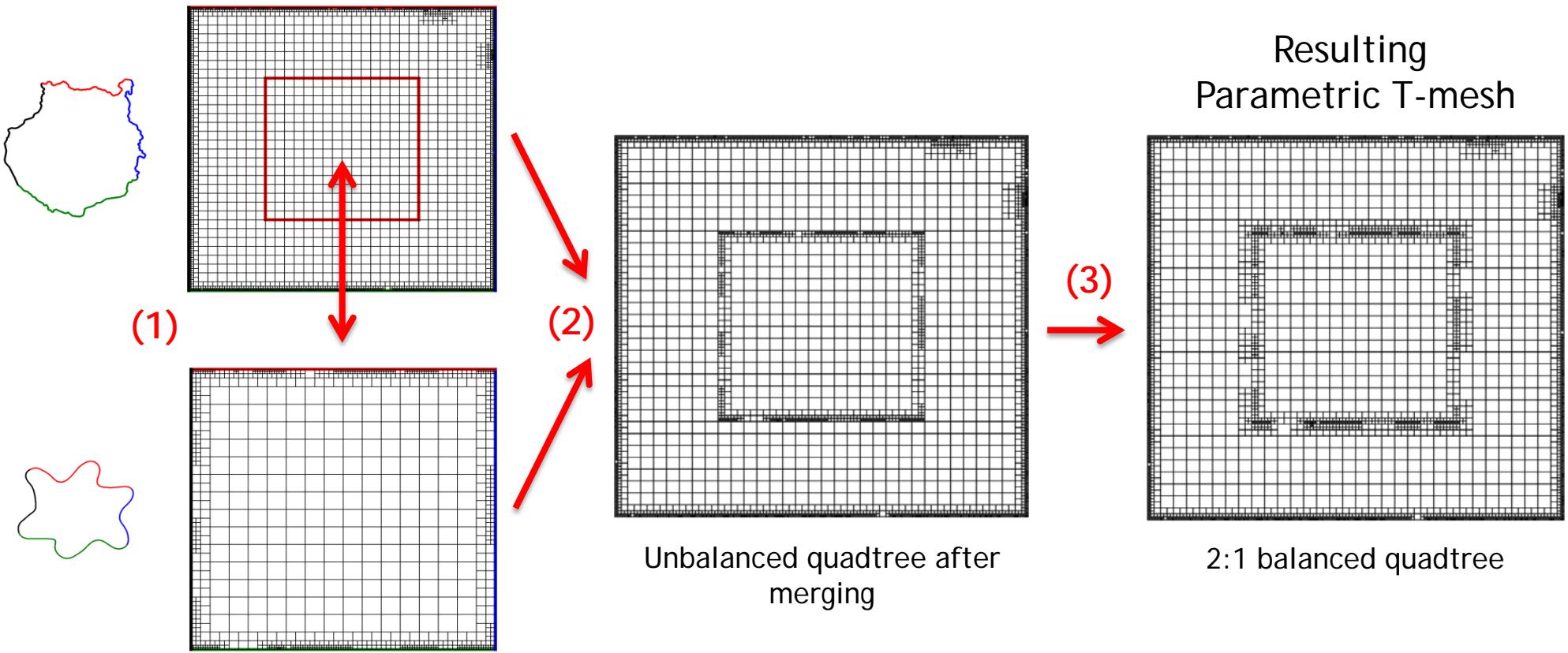


Applications: Isogeometric Modeling

Strategy for embedded planar geometries: Three steps

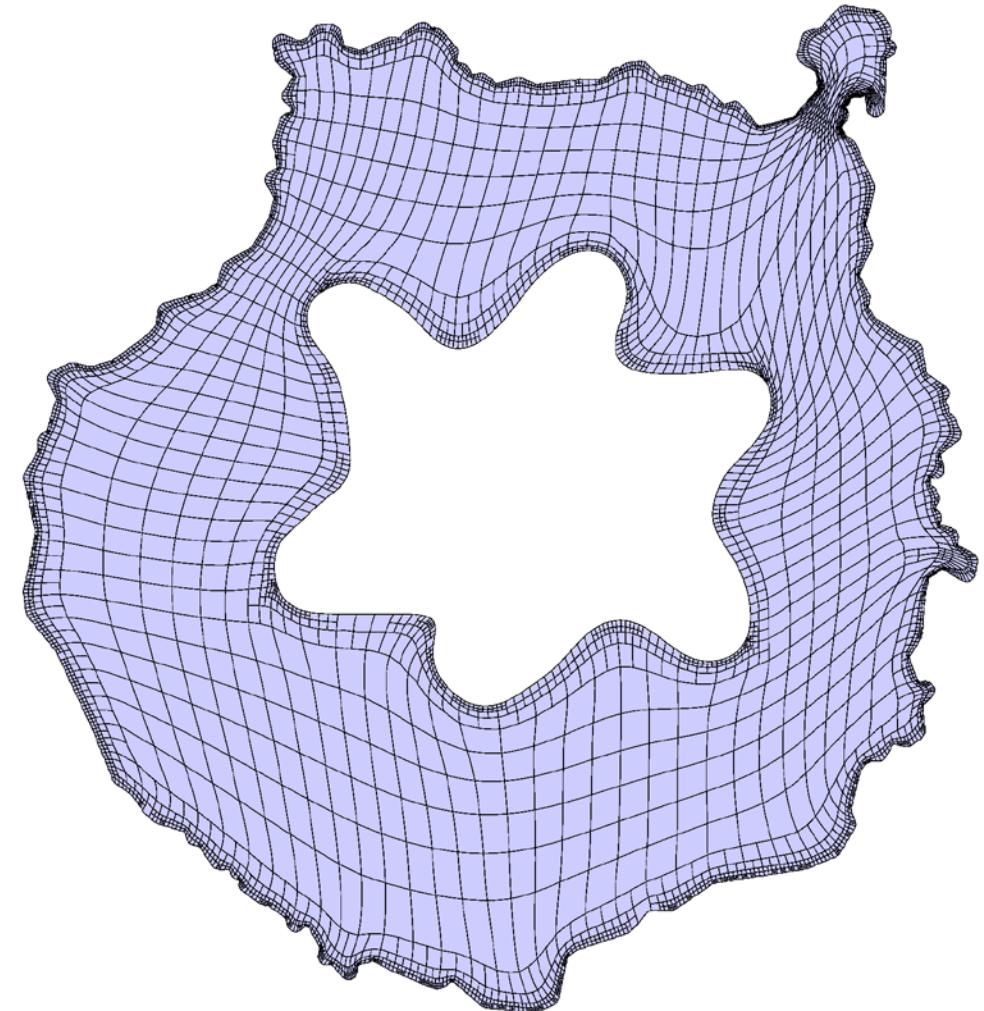
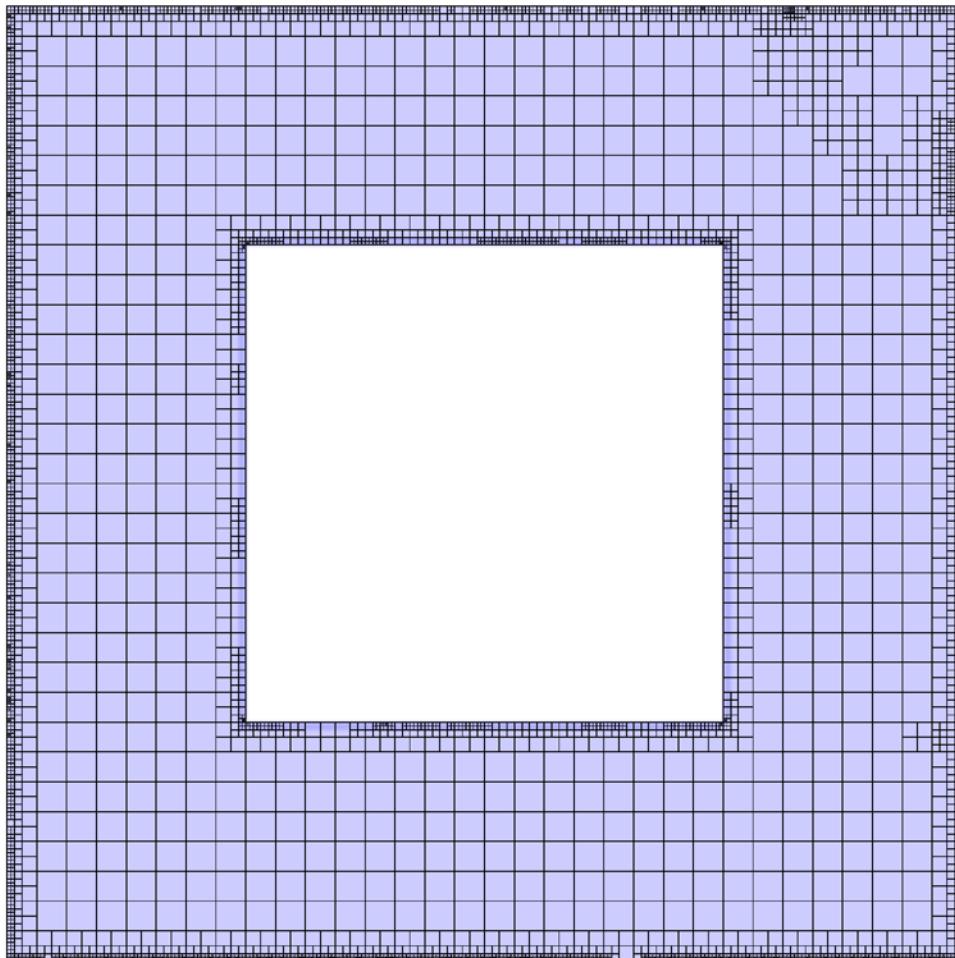


1. Individual quadtree approximation for each geometry (they can be processed in parallel)
2. Insert a quadtree in a region of another quadtree
3. Balance the resulting quadtree



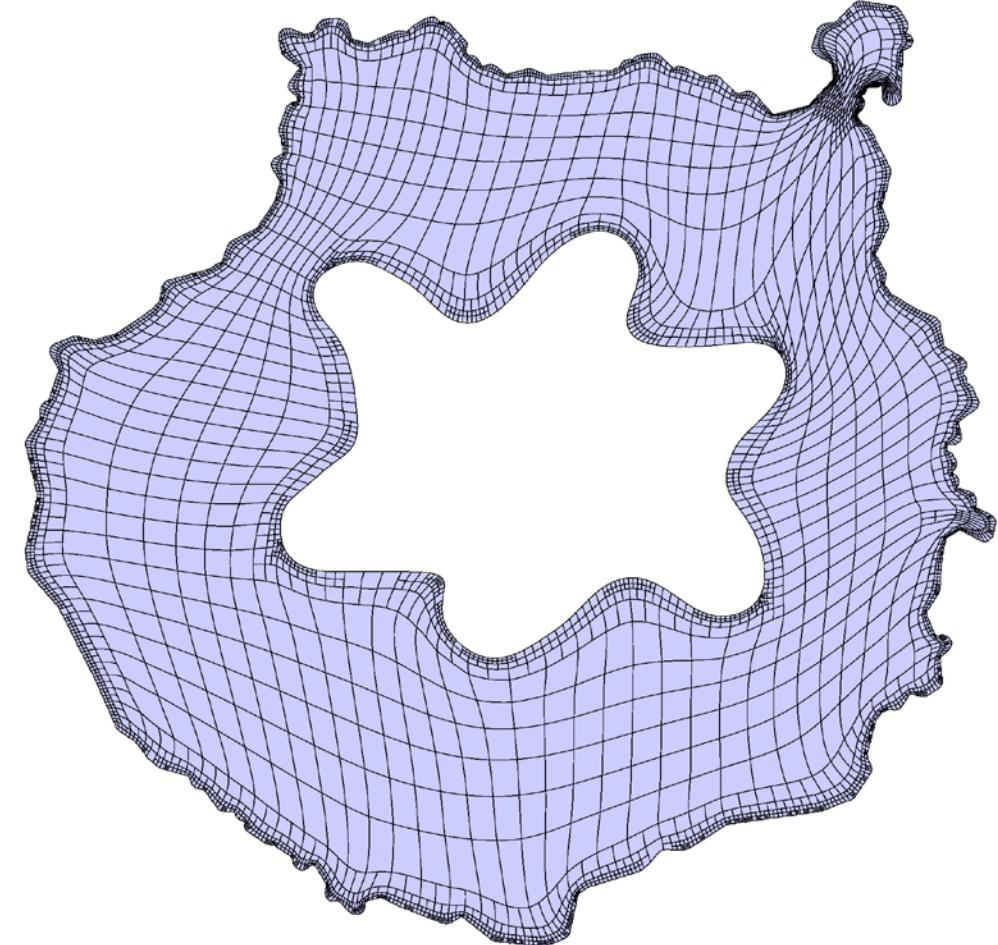
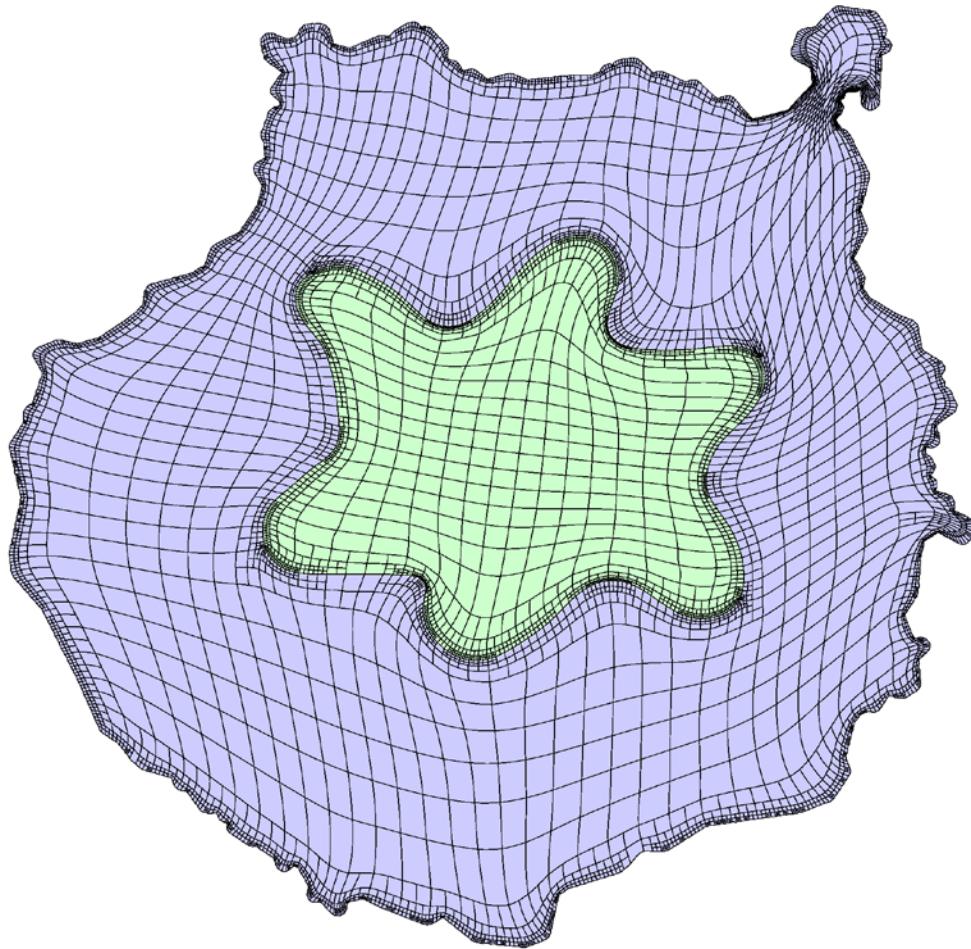
Applications: Isogeometric Modeling

Geometries with holes



Applications: Isogeometric Modeling

Embedded planar geometries (T-spline representation in physical space)

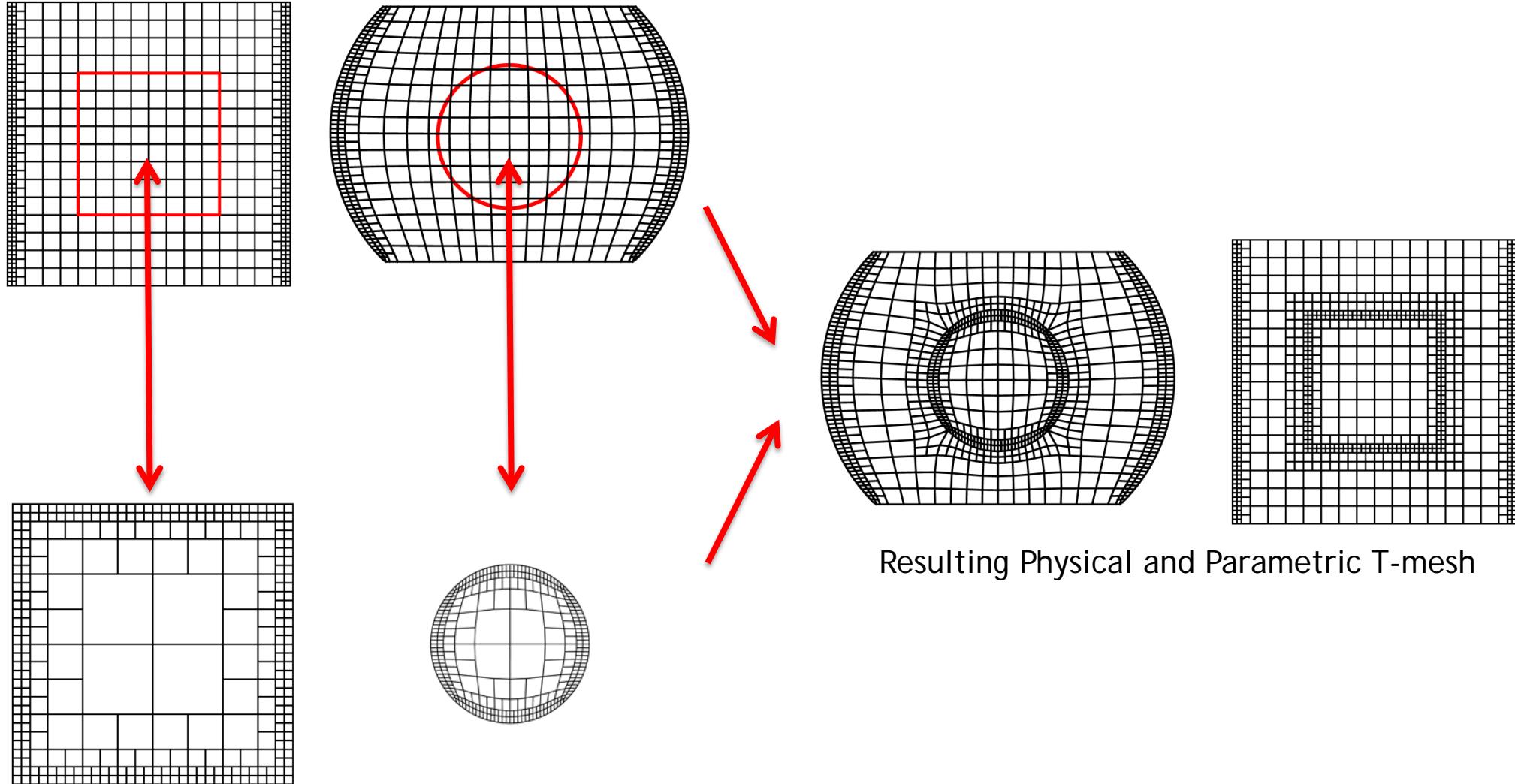


T-spline representation with different materials

T-spline representation with a hole

Applications: Isogeometric Modeling

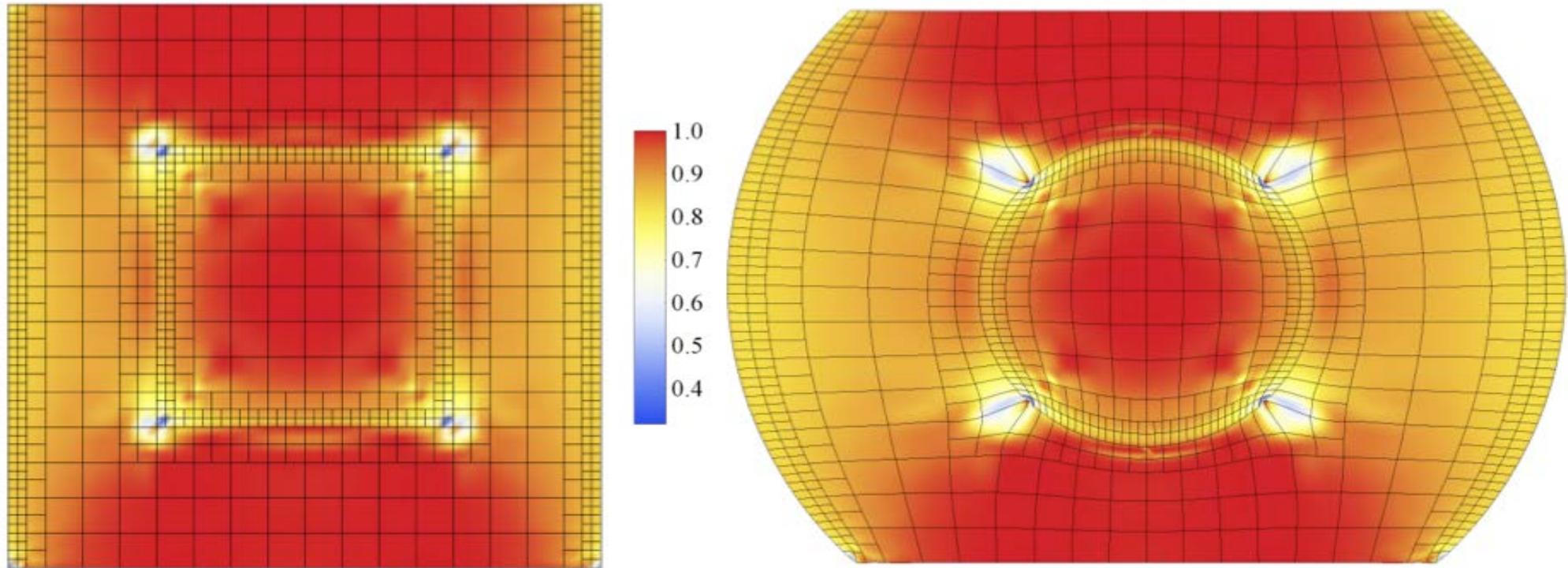
Strategy for embedded planar geometries: Other example



Applications: Isogeometric Modeling

Strategy for embedded planar geometries: Other example

Mean ratio Jacobian



Local Nested Adaptive Refinement

Numerical solution of a Poisson problem

Concentrate source in relation to the initial mesh size



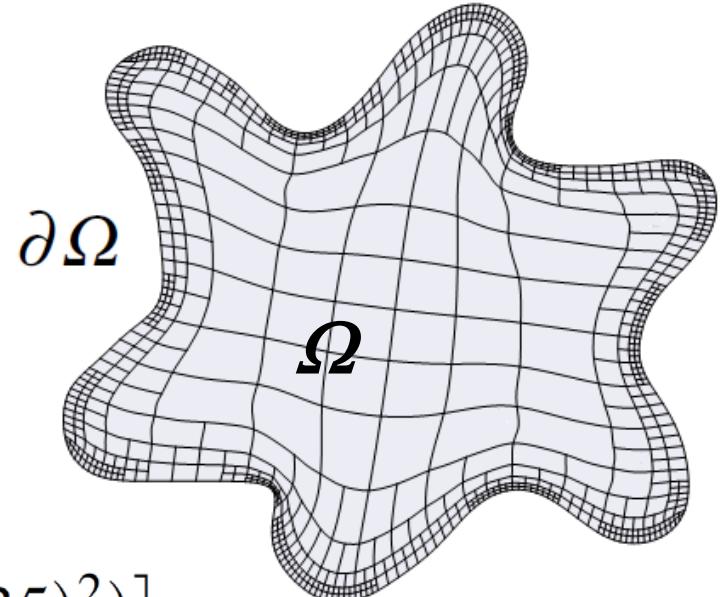
$$\begin{aligned} -\Delta u &= f && \text{in } \Omega, \\ u &= g && \text{on } \partial\Omega. \end{aligned}$$

Exact solution:

$$u(x, y) = \exp \left[-10^3((x - 0.6)^2 + (y - 0.35)^2) \right]$$

Residual-type error indicator:

$$\eta(\Omega_e)^2 = \|h(f + \Delta u_h)\|_{0,\Omega_e}^2 = \int_{\Omega_e} h^2 (f + \Delta u_h)^2 \, d\Omega$$

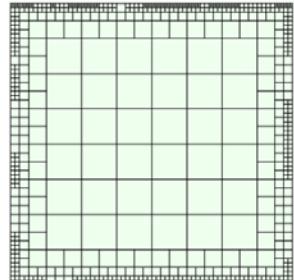


Local Nested Adaptive Refinement

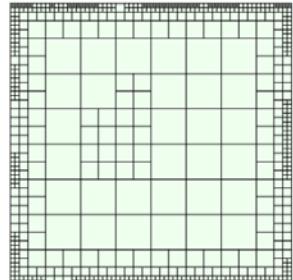
Numerical solution of a Poisson problem



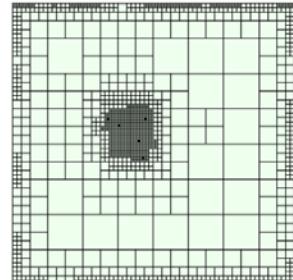
$$\text{Error indicator : } \eta(\Omega_e)^2 = \int_{\Omega_e} h^2 (f + \Delta u_h)^2 d\Omega$$



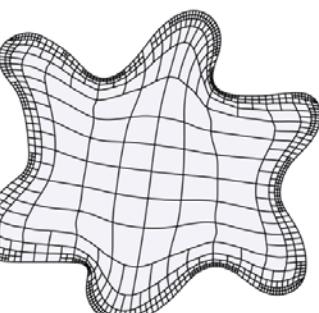
(a) 844 cells, 1456 DOF



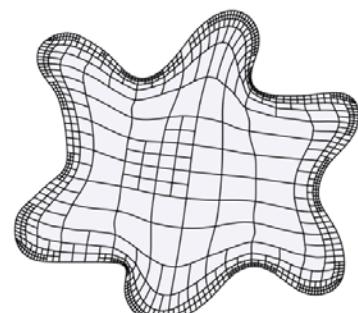
(b) 859 cells, 1476 DOF



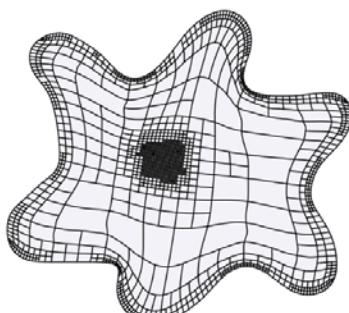
(c) 1552 cells, 2233 DOF



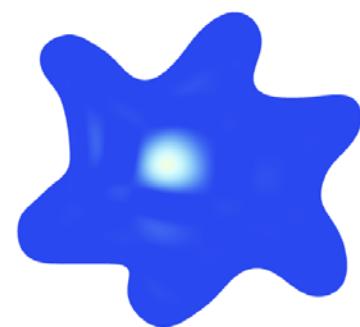
(d) Initial mesh



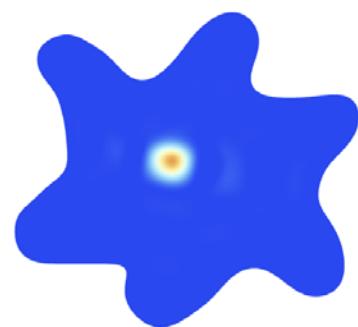
(e) 1-st refinement



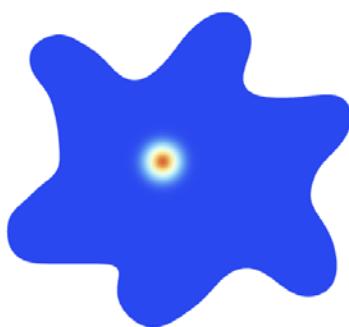
(f) 14-th refinement



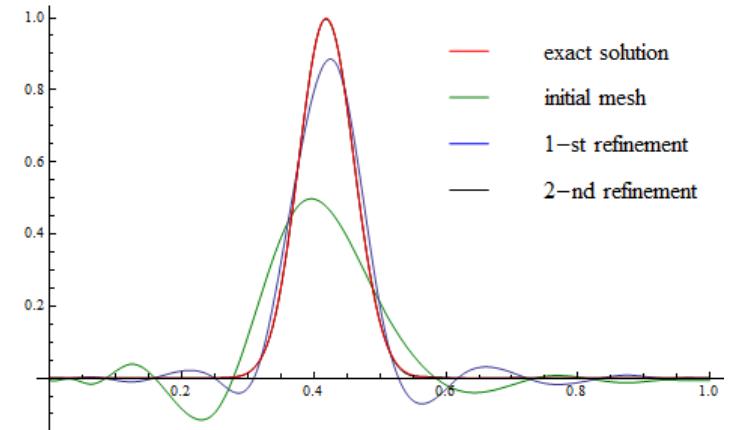
(g) Initial solution



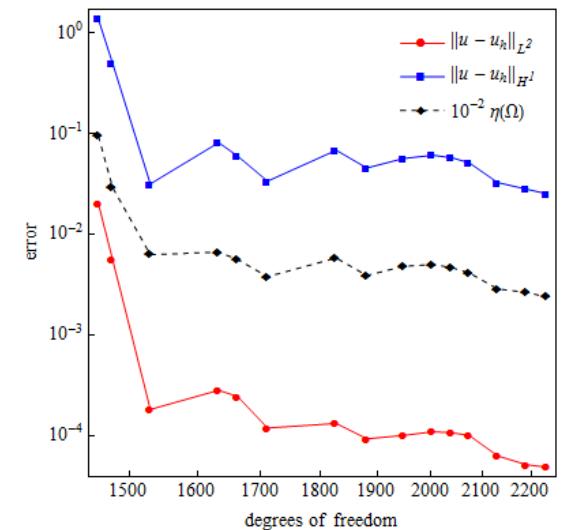
(h) 1-st refinement



(i) 14-th refinement



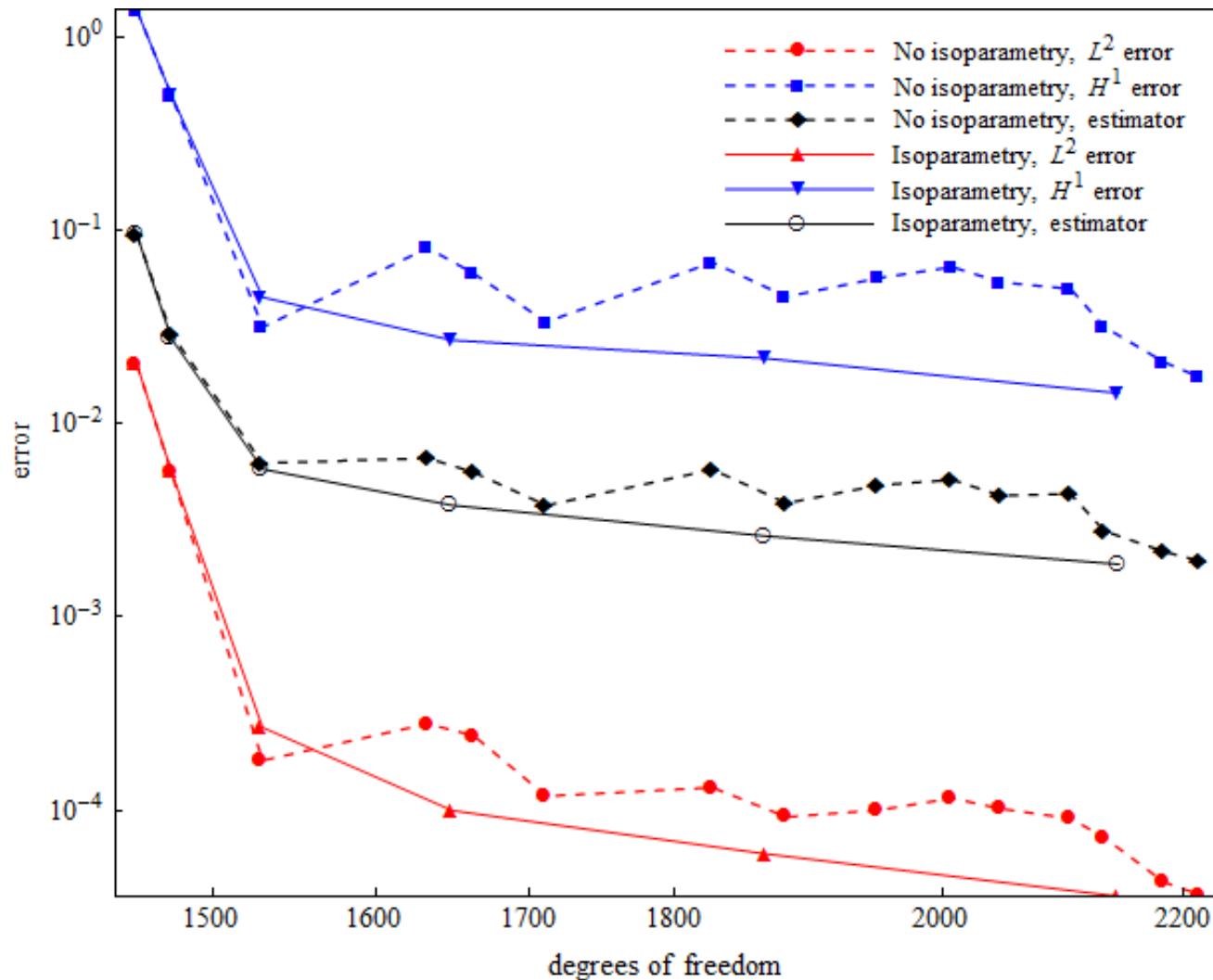
Numerical solution across a section



Convergence behavior

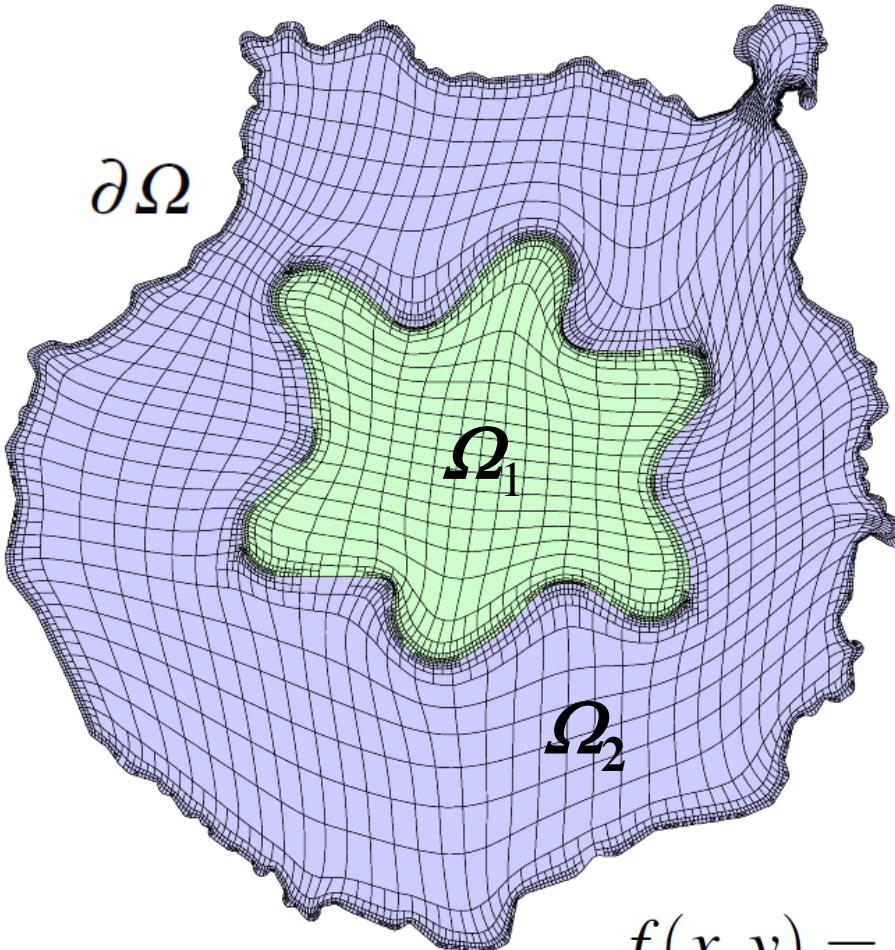
Local Nested Adaptive Refinement

Influence of isoparametry in the convergence behavior



Poisson Problem for a Domain with Two Materials

Statement of the problem



$$\begin{aligned} -\nabla(k(\mathbf{x})\nabla u) &= f && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega \end{aligned}$$

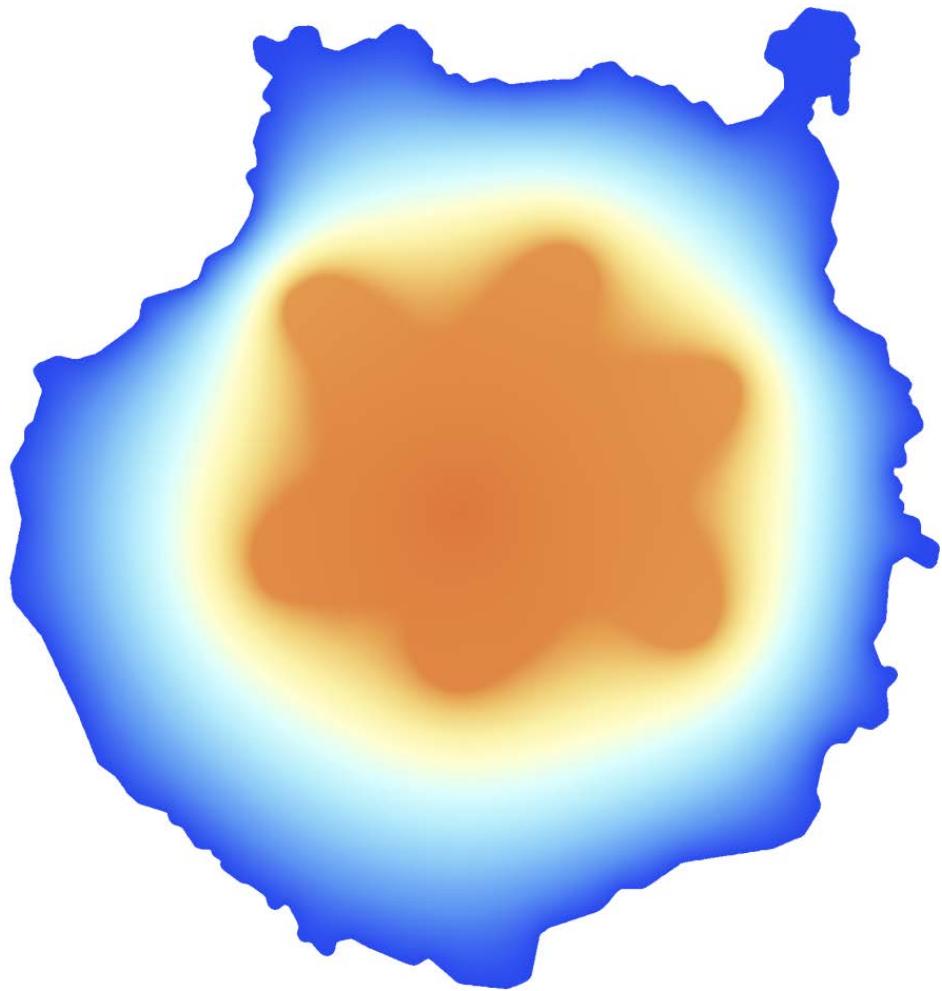
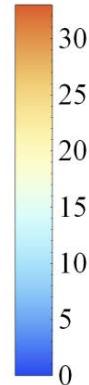
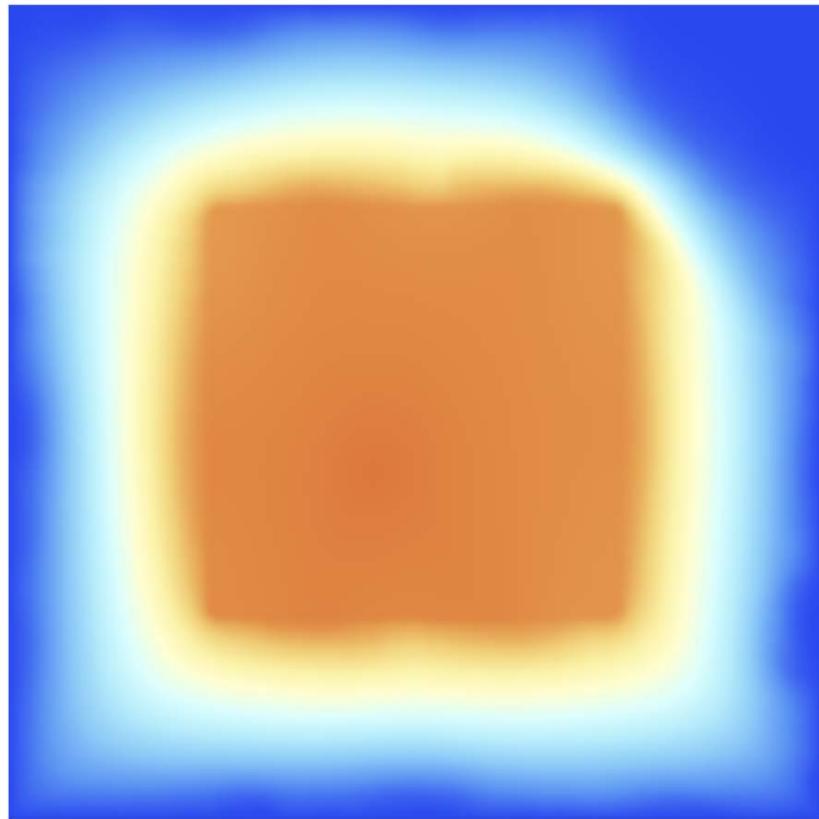
$$k(\mathbf{x}) = \begin{cases} k_1 & \text{if } \mathbf{x} \in \Omega_1 \\ k_2 & \text{if } \mathbf{x} \in \Omega_2 \end{cases}$$

$$k_1 = 1 \gg k_2 = 0.01$$

$$f(x, y) = 10^3 \exp[-10^3((x - 0.5)^2 + (y - 0.5)^2)]$$

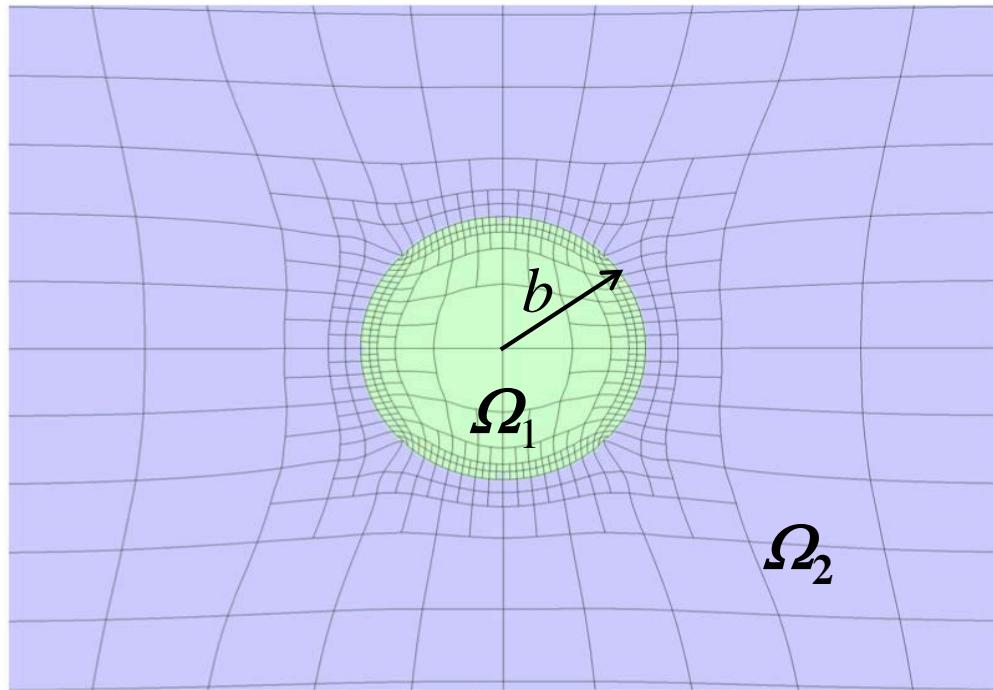
Poisson Problem for a Domain with Two Materials

Numerical solution in parametric and physical domain



Poisson Problem for a Domain with Two Materials

Dielectric cylinder in an uniform horizontal electric field E_0



T-spline detail

$$\begin{aligned} -\nabla(k(\mathbf{x})\nabla u) &= 0 && \text{in } \Omega, \\ u &= g && \text{on } \partial\Omega. \end{aligned}$$

$$k(\mathbf{x}) = \begin{cases} \varepsilon_0 \varepsilon_r & \text{if } \mathbf{x} \in \Omega_1 \quad \rho < b \\ \varepsilon_0 & \text{if } \mathbf{x} \in \Omega_2 \quad \rho \geq b \end{cases}$$

The analytic solution in cylindrical coordinates (ρ, φ) is given by:

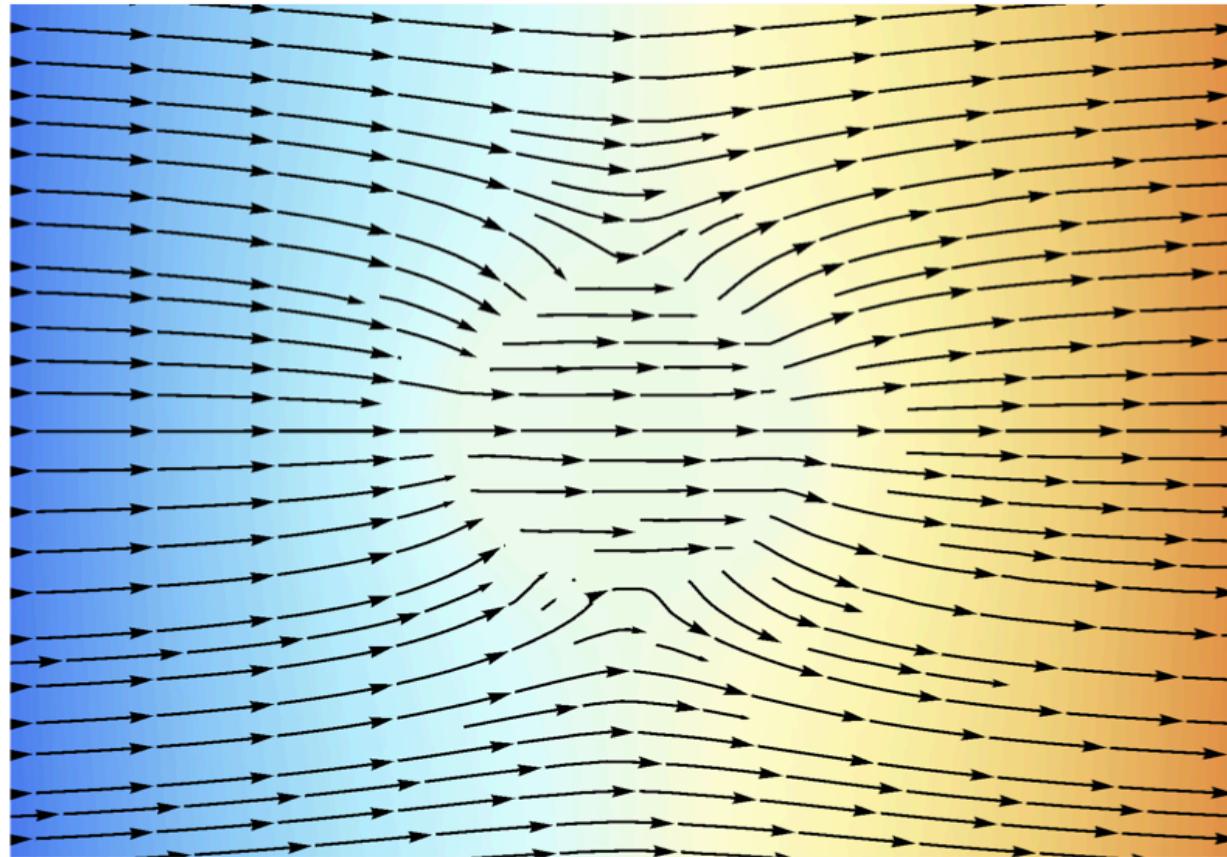
$$u_{\rho < b} = \frac{-2E_0 \rho \cos \varphi}{(\varepsilon_r + 1)}$$

$$u_{\rho \geq b} = E_0 \cos \varphi \left(-\rho + \frac{b^2(\varepsilon_r - 1)}{\rho(\varepsilon_r + 1)} \right)$$

Poisson Problem for a Domain with Two Materials

Dielectric cylinder in an uniform electric field

Numerical solution detail (potential and electric field) in physical domain

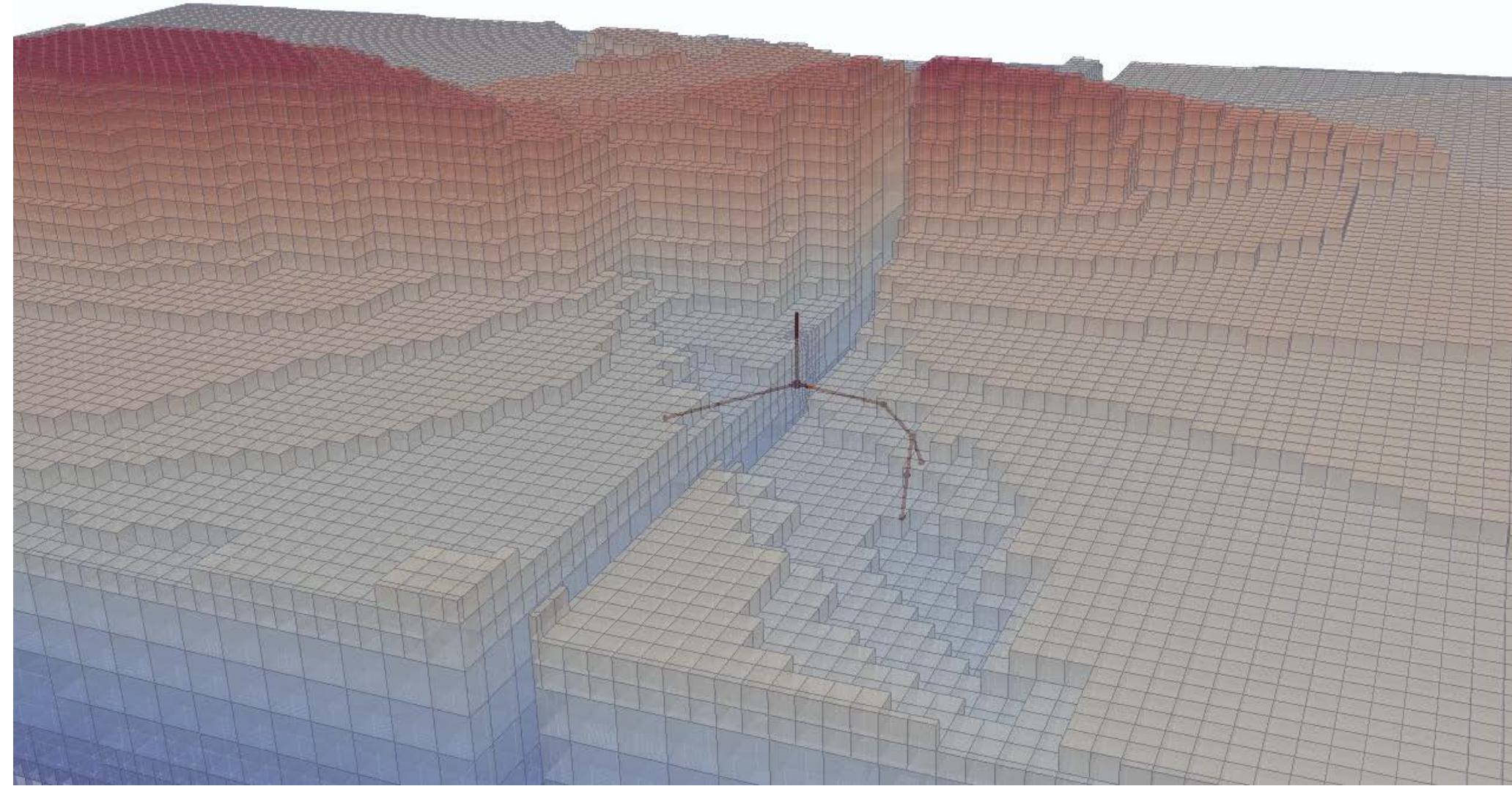


Comparing with the analytic solution, we have measured a maximum error in the potential of 0.88%.

Final Comments and Future Works

Automatic Construction of the Meccano

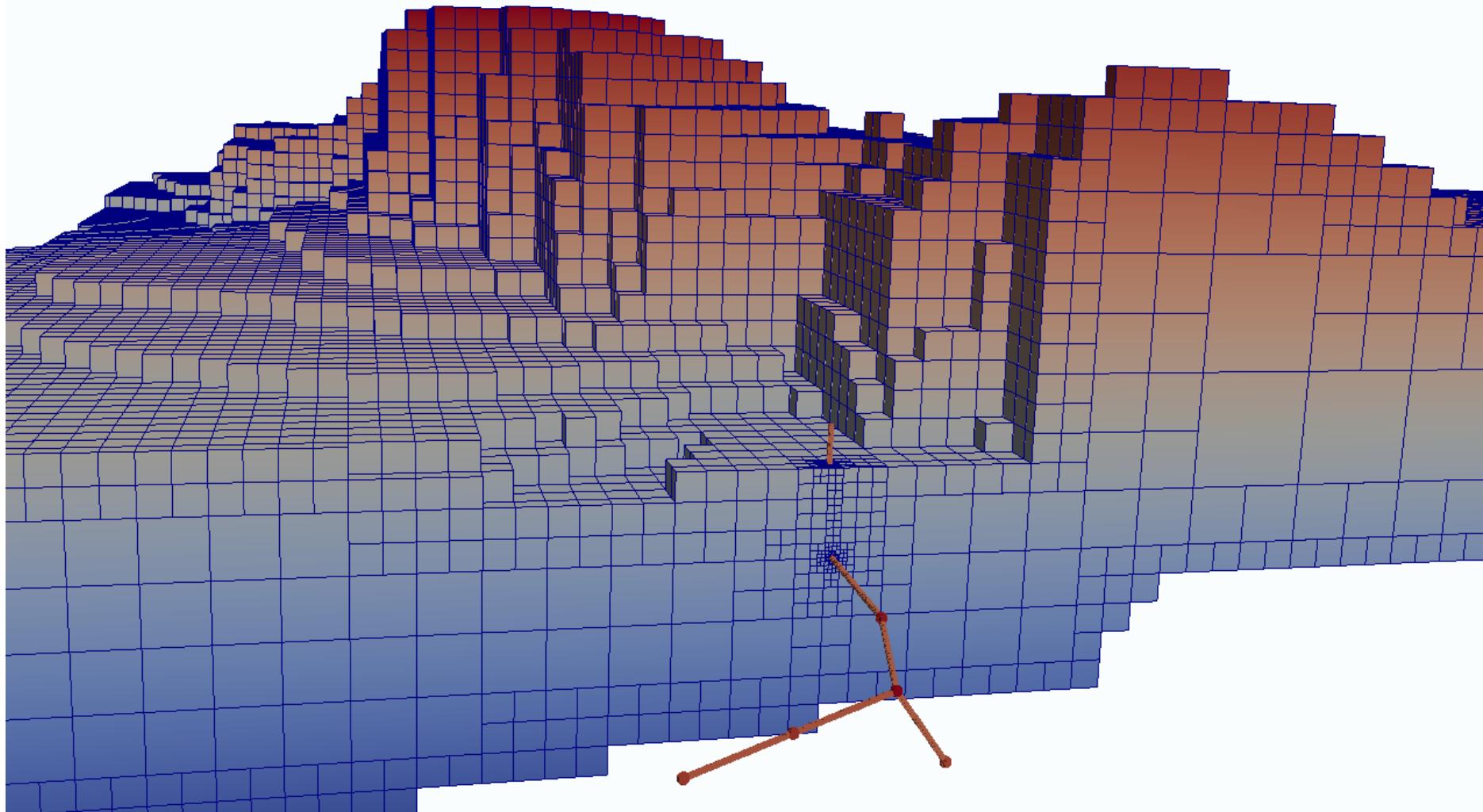
Octree Subdivision – Petrossoft (México)



Final Comments and Future Works

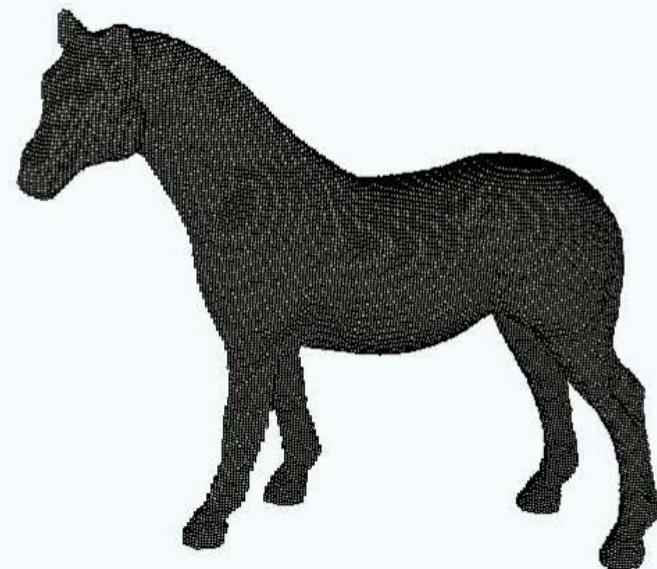
Automatic Construction of the Meccano

Octree Subdivision – Petrossoft (México)



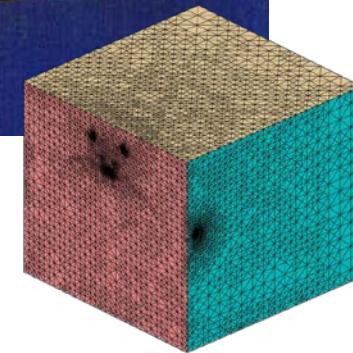
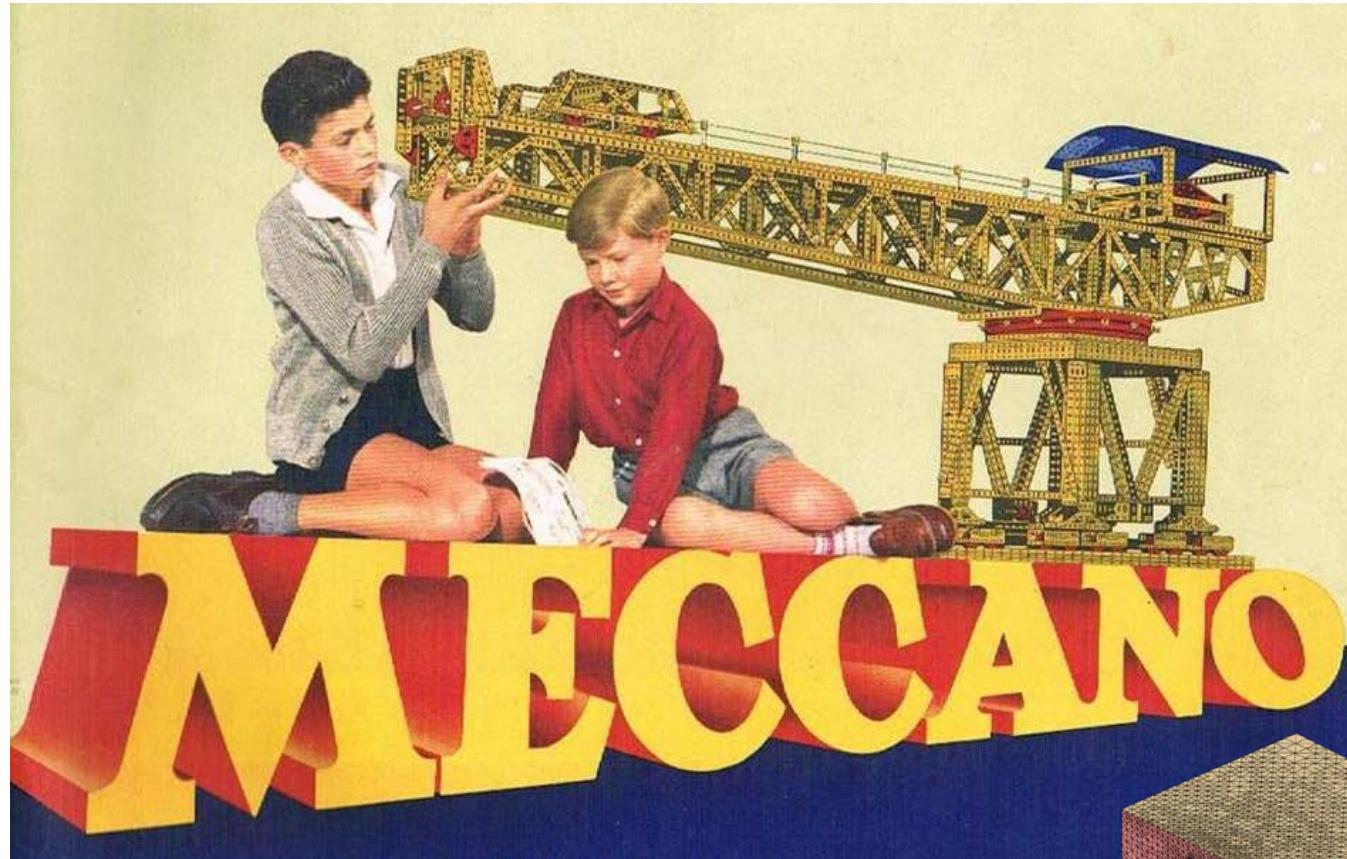
Final Comments and Future Works

Automatic Construction of the Meccano



Final Comments and Future Works

Automatic Construction of the Meccano



Video of University Institute SIANI



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Mesh Generation and Isogeometric Analysis

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<http://www.dca.iusiani.ulpgc.es/proyecto2012-2014>



Meccano Technique for a Complex Genus-Zero Solid

Refinement Criterion

