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INSTITUTO UNIVERSITARIO
INGENIERIA COMPUTACIONAL

Application to Complex Solids of Adaptive Isogeometric Analysis using T-splines

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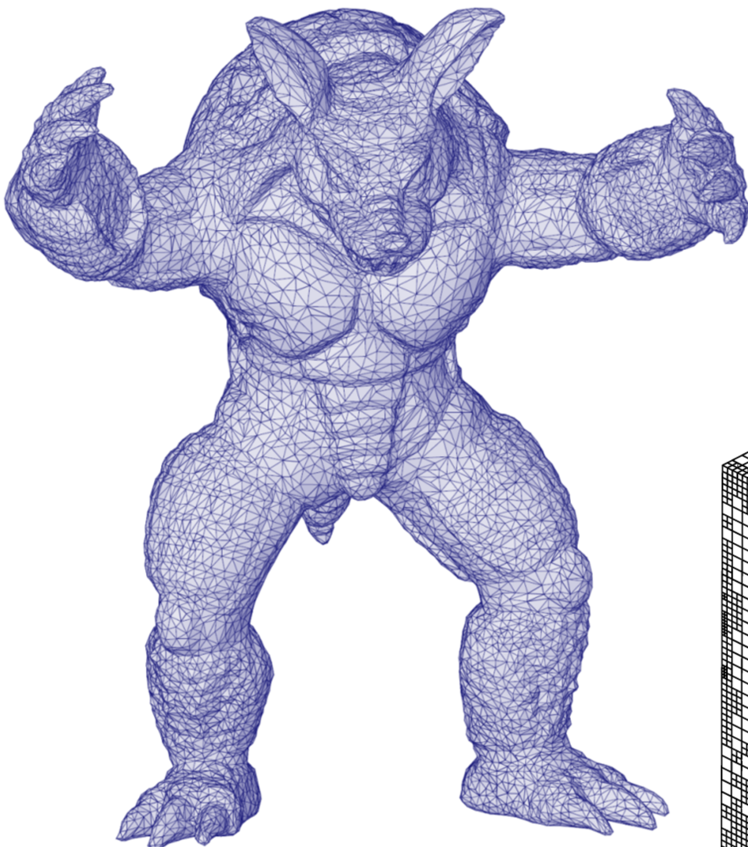
UNIVERSIDAD DE LAS PALMAS DE GRAN CANARIA

http://www.dca.iusiani.ulpgc.es/proyecto2012-2014

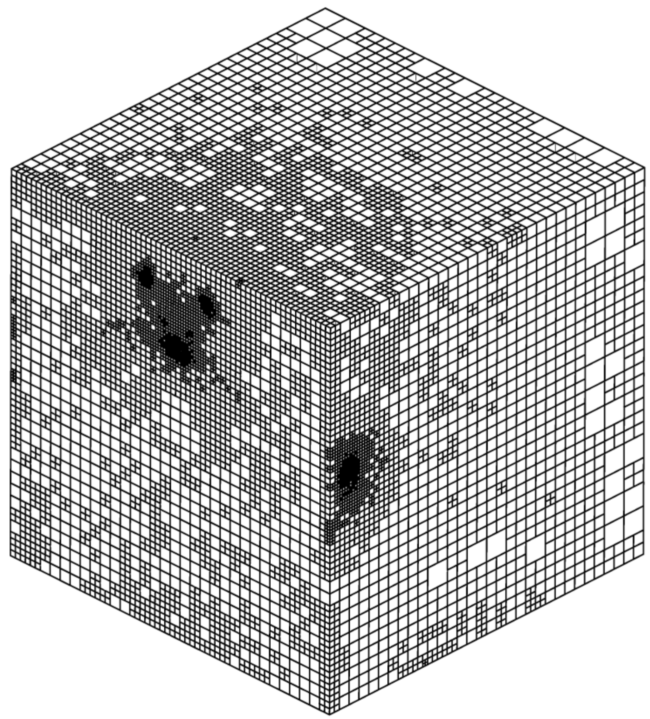
The Meccano Method for Isogeometric Solid Modeling

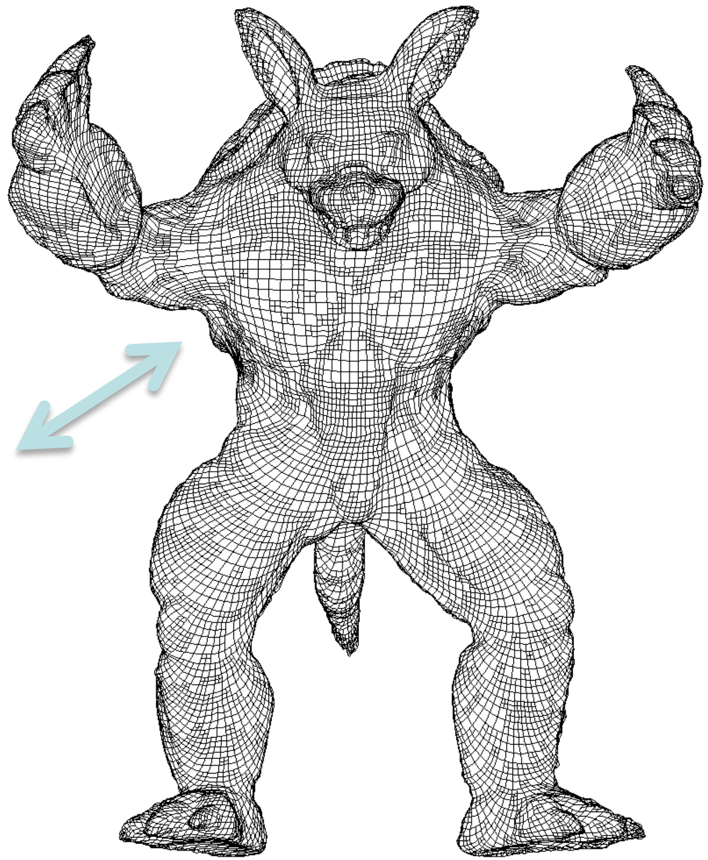
Motivation: Solid Modeling with Trivariate T-splines



- **INPUT: Surface Triangulation**

- **3-D T-Mesh of the Meccano**

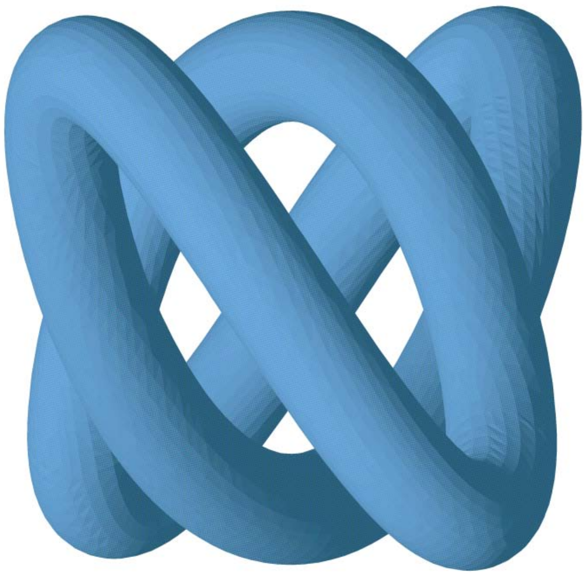




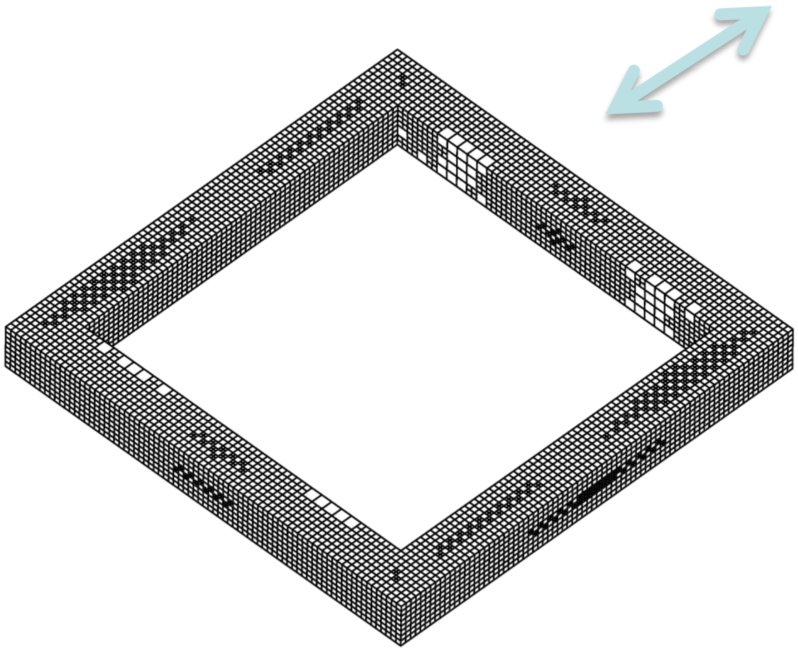
- **OUTPUT: Trivariate T-spline**

The Meccano Method for Isogeometric Solid Modeling

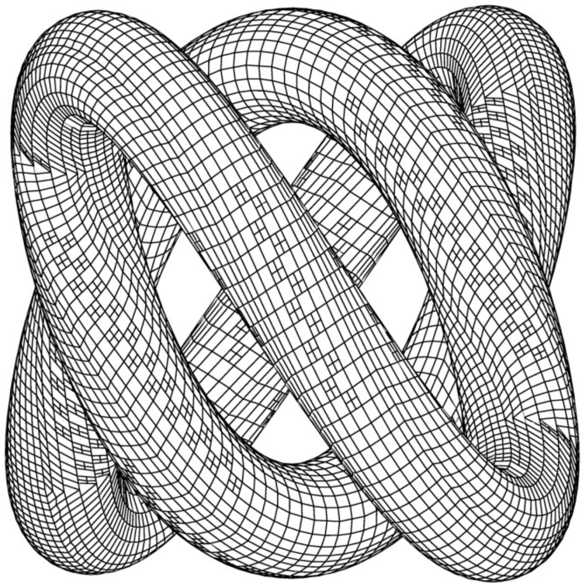
Motivation: Solid Modeling with Trivariate T-splines



• **INPUT: Solid Surface**

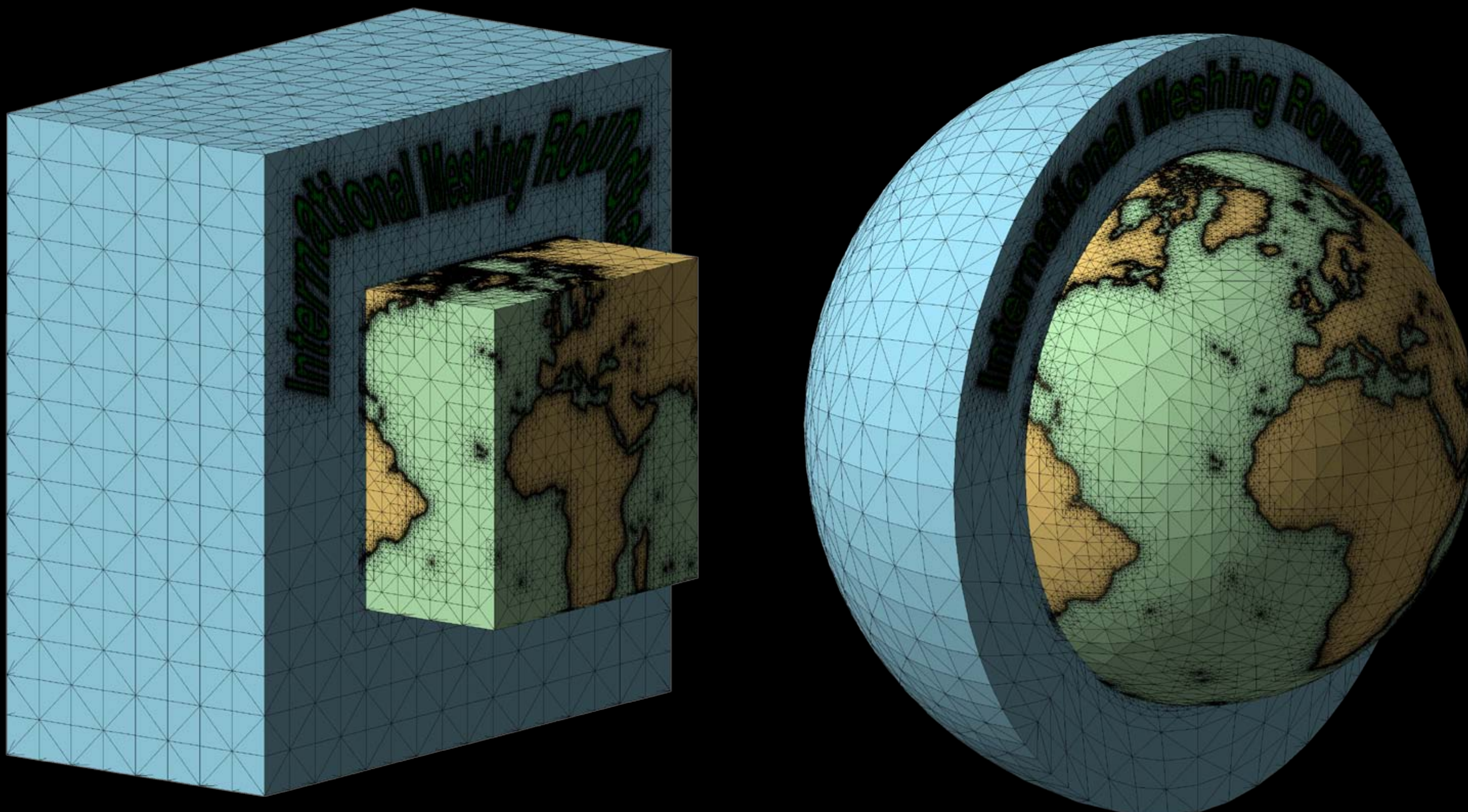


• **3-D T-Mesh of the Meccano**



• **OUTPUT: Trivariate T-spline**

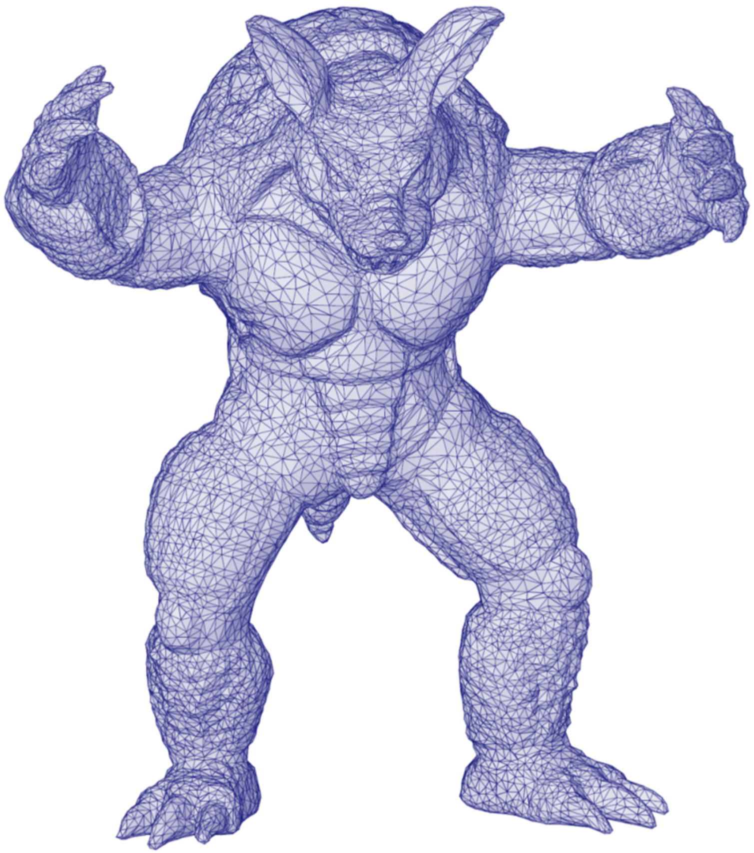
16th IMR (2007)



The Meccano Method for 3-D Mesh Generation

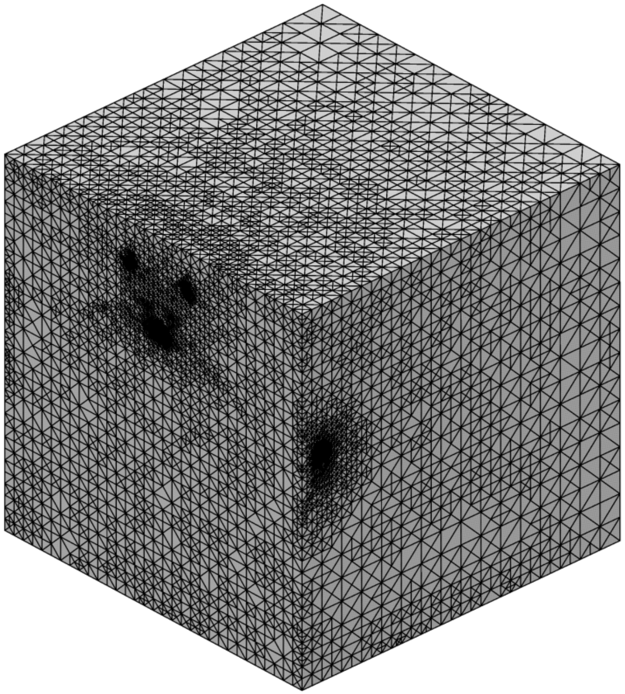
Motivation: Simultaneous Mesh Generation and Volume Parameterization

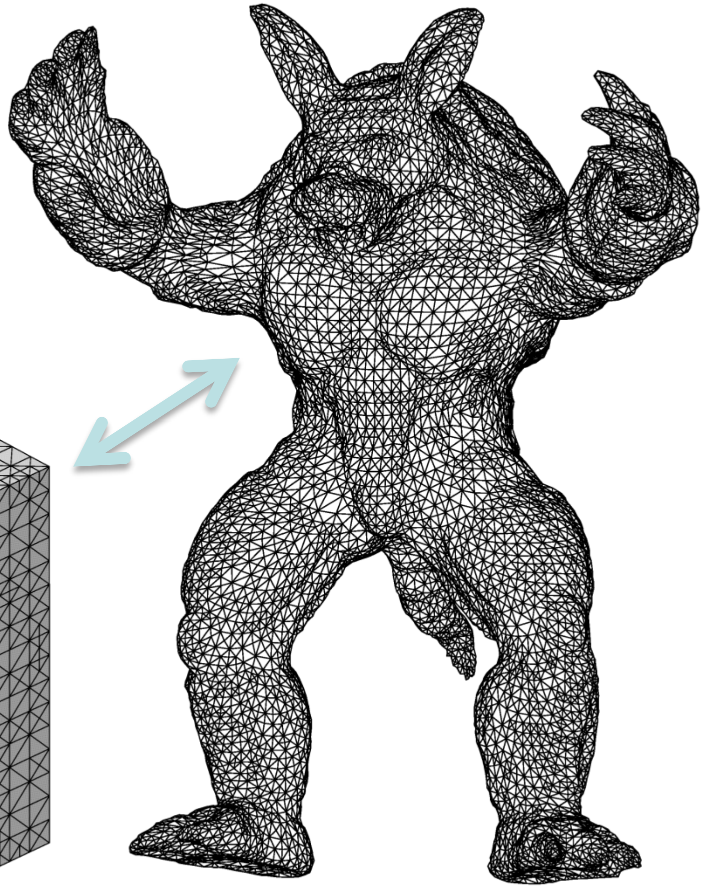
18th IMR (2009)



• INPUT: Surface Triangulation

• Meccano Tetrahedral Mesh

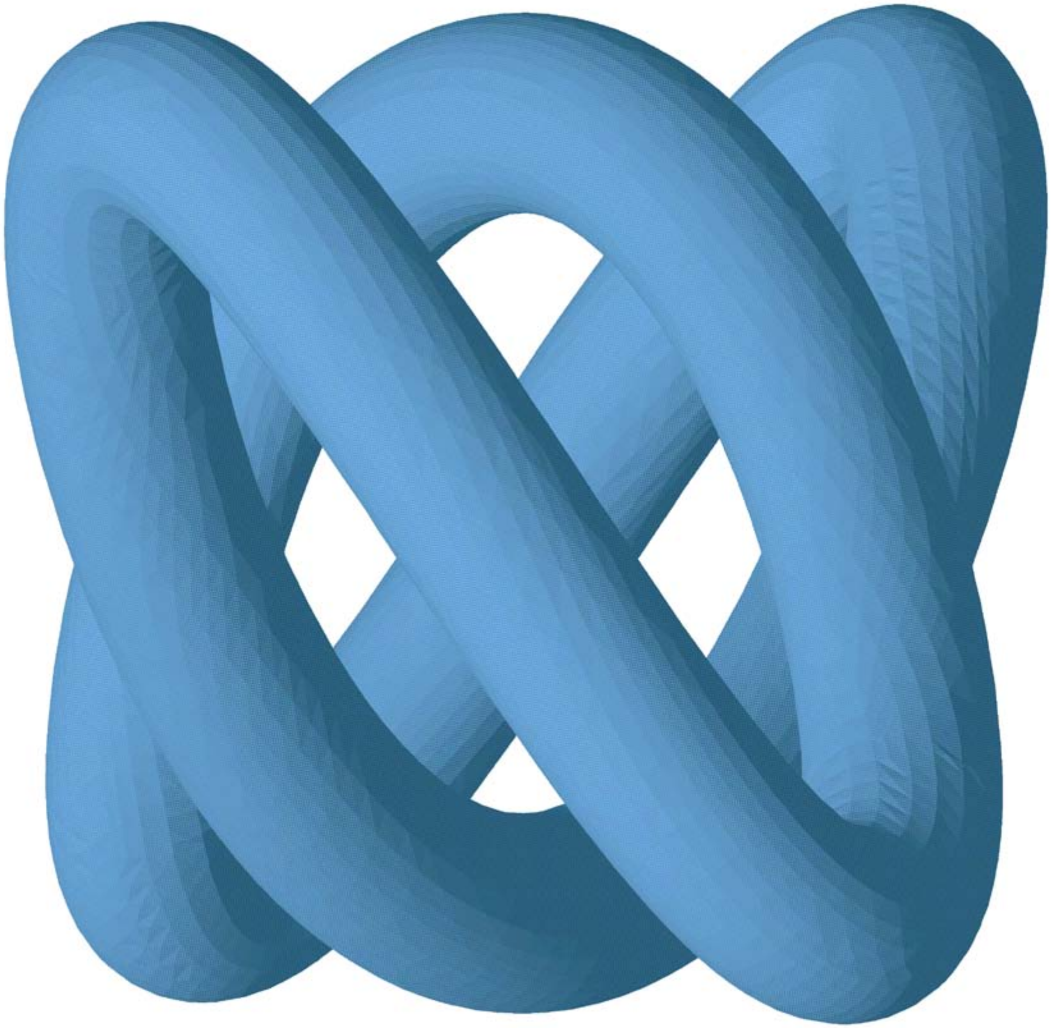




• OUTPUT: Tetrahedral Mesh

Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Surface information as input data; explicit in this case



Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: The meccano approximation

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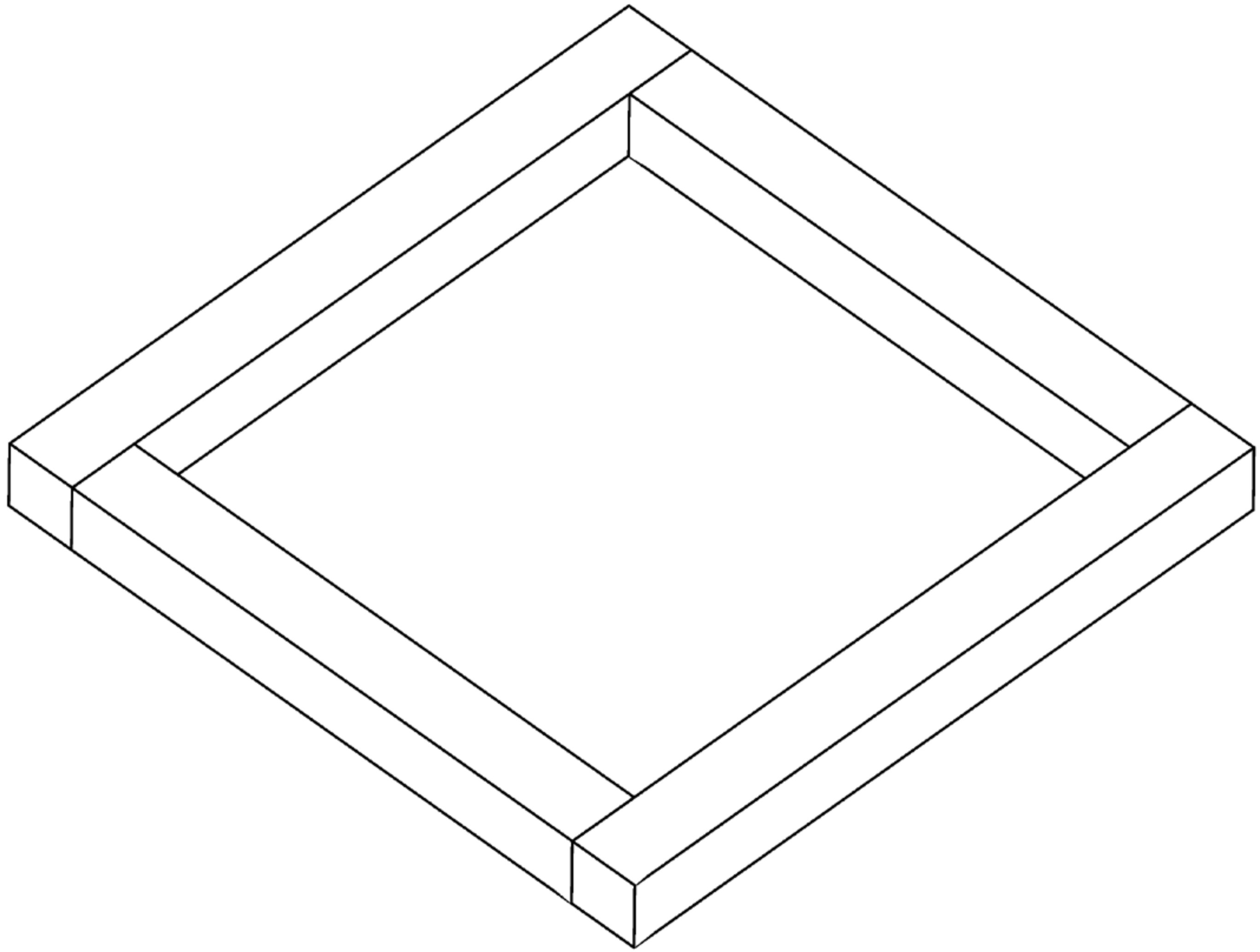

Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Coarse tetrahedral mesh (tet-subdivision of the polycube)

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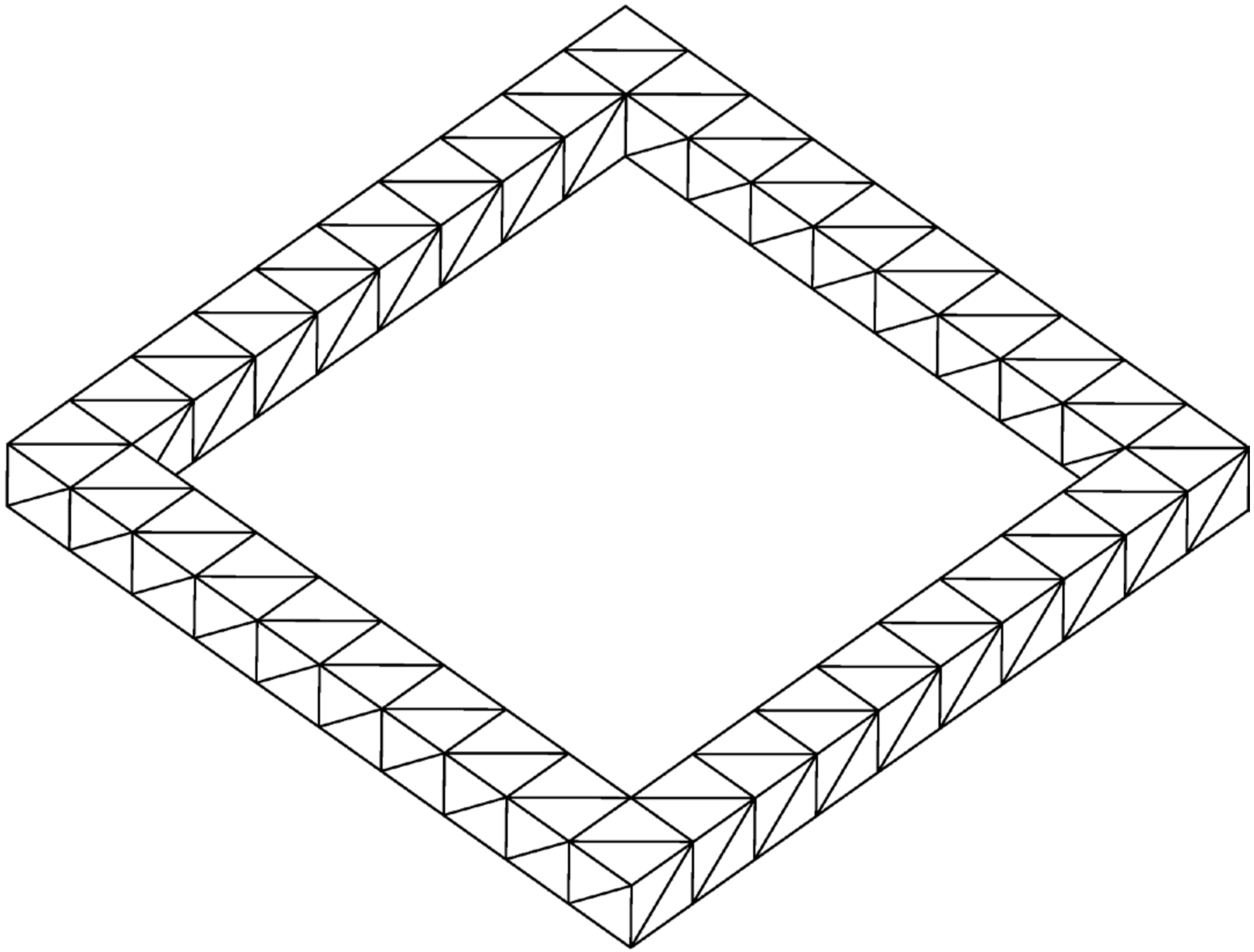

Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Local refined tetrahedral mesh

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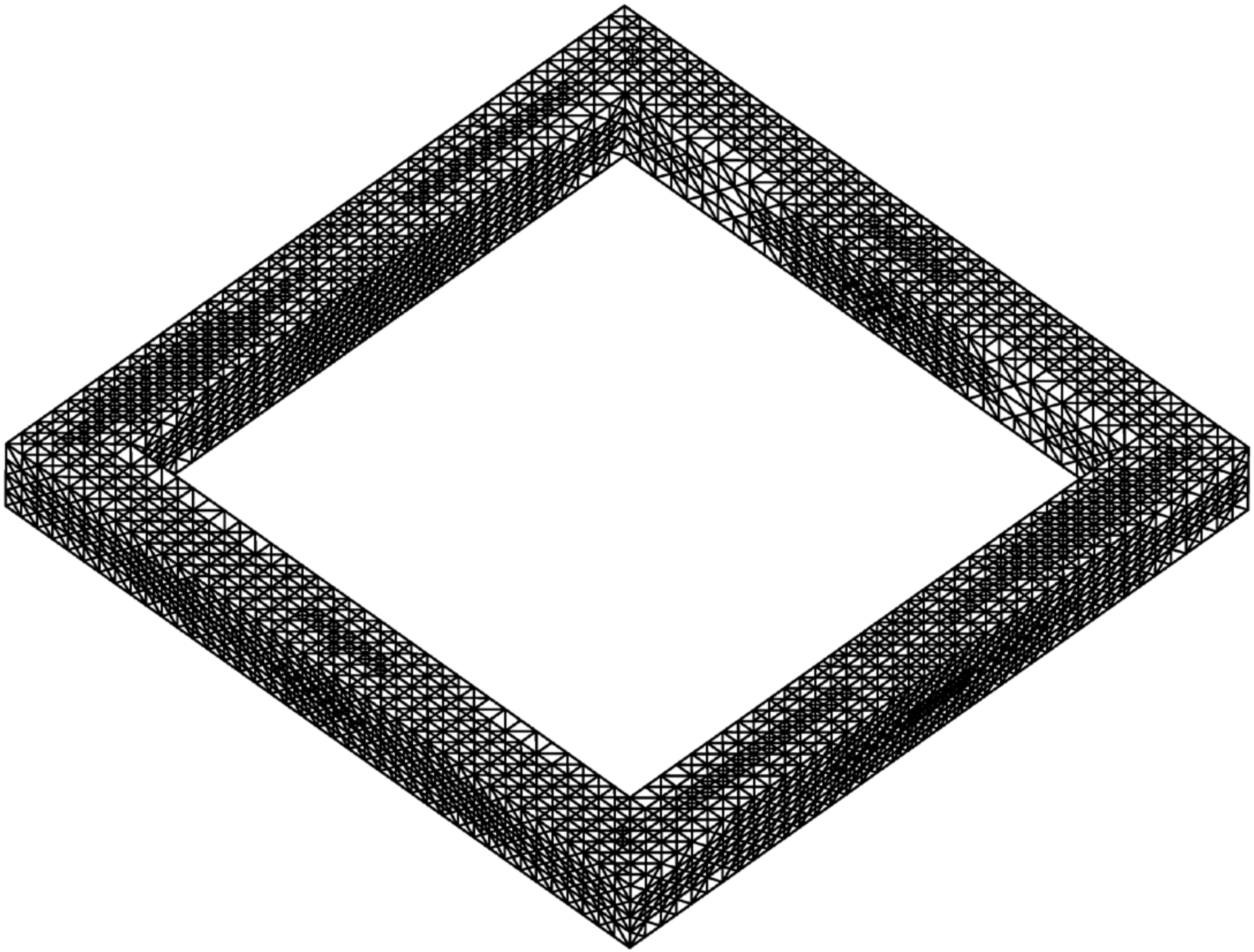
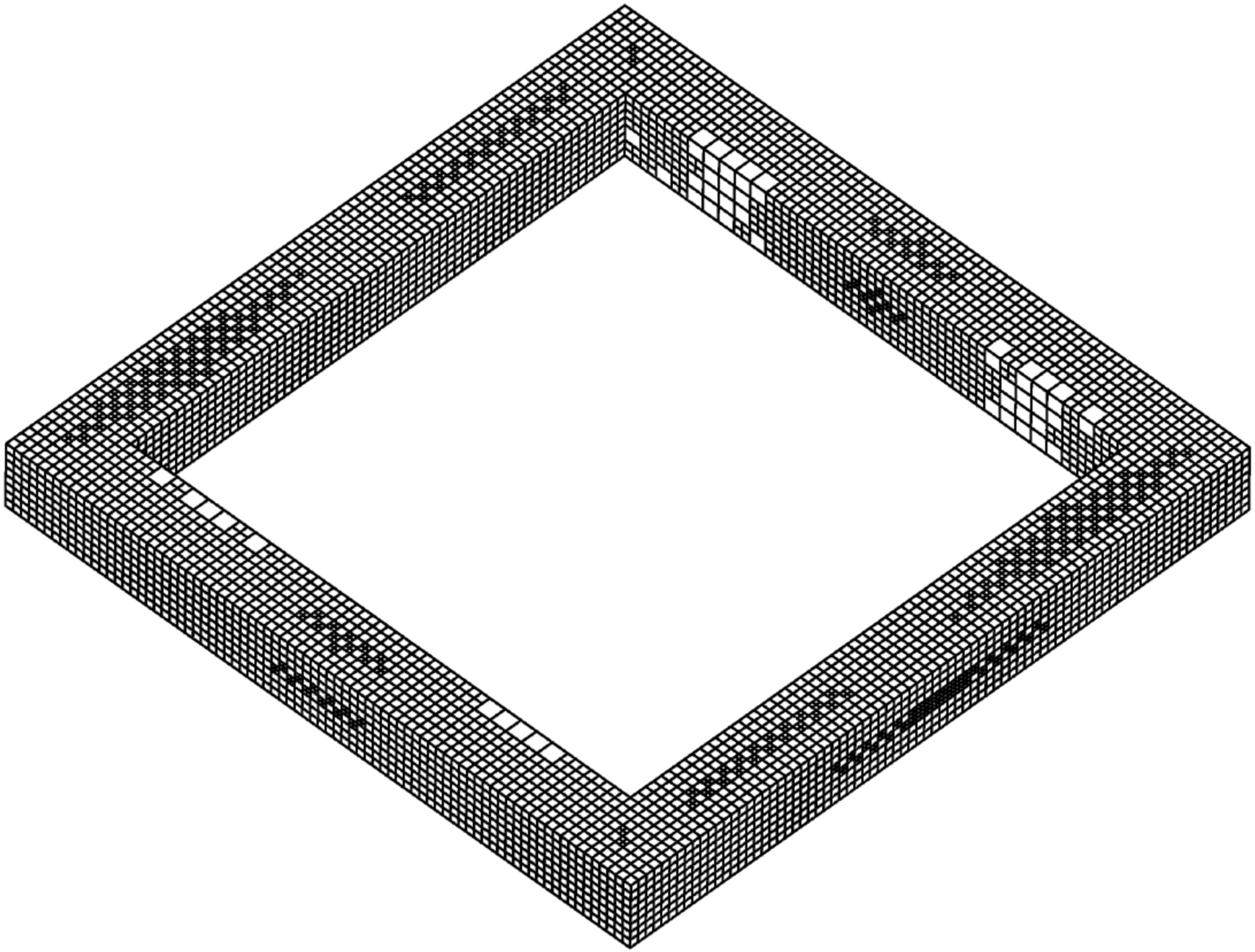


Isogeometric Modeling of a Genus-one Solid

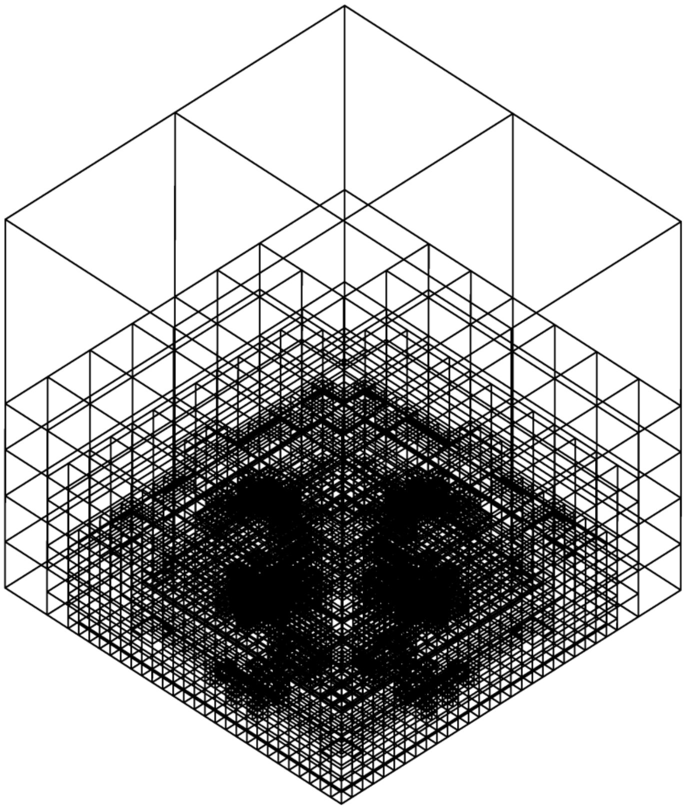
Algorithm Steps: Meccano T-mesh



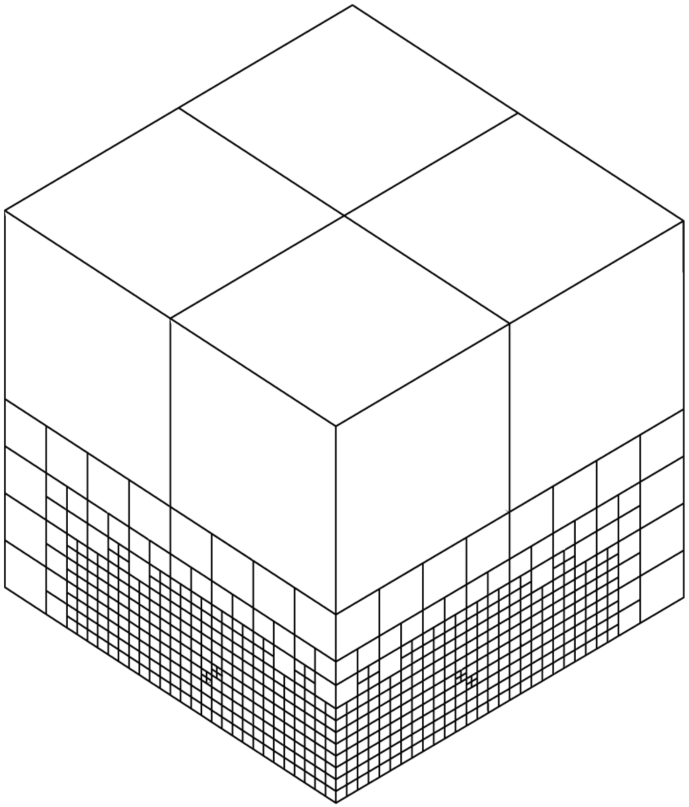
Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Meccano T-mesh

Comment: An easy solution is to include the meccano into a *compatible* cube



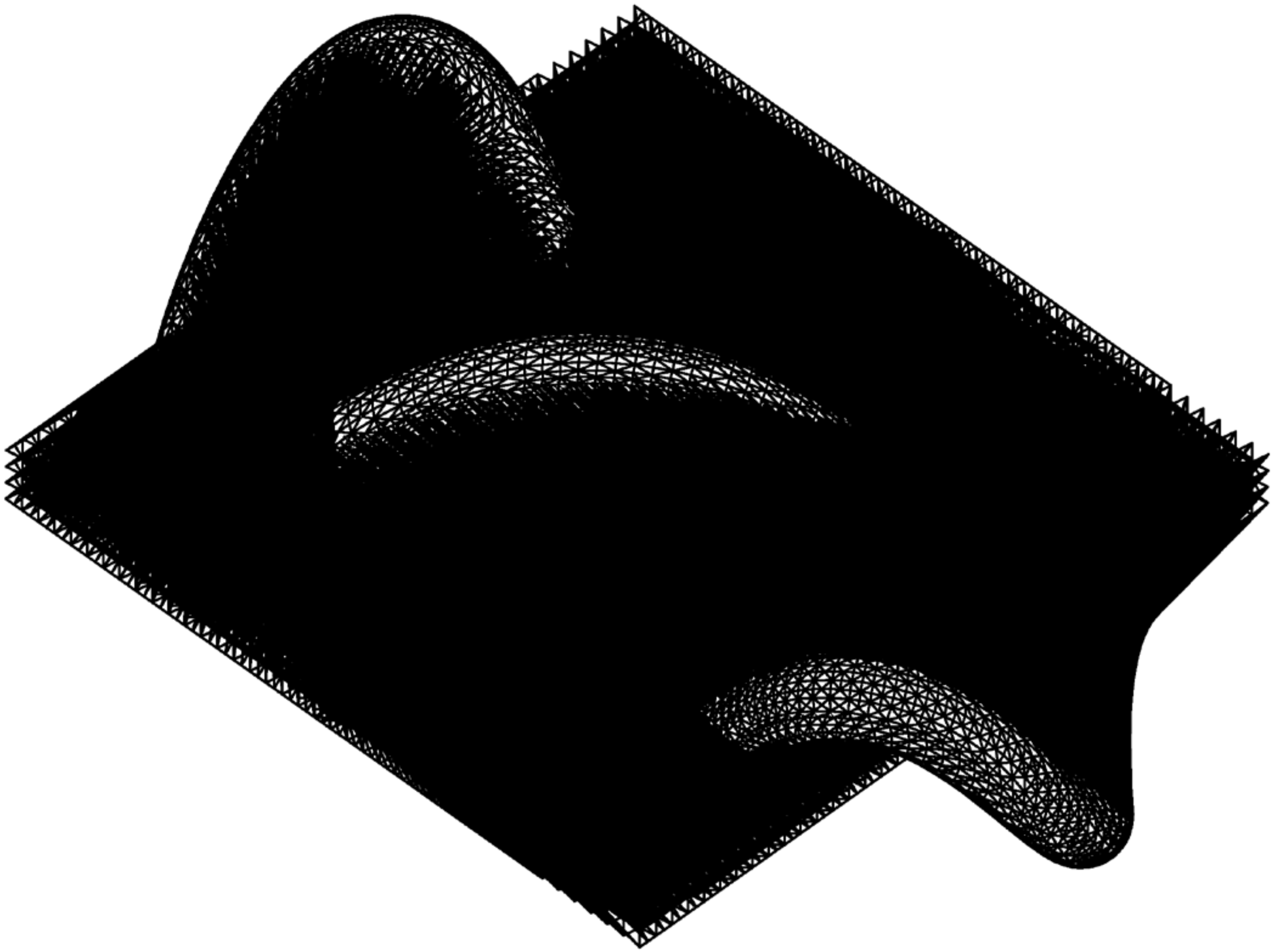
Octree subdivision of the auxiliary cube



View of external faces

Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Move the meccano boundary nodes to the solid surface



Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Final tetrahedral mesh

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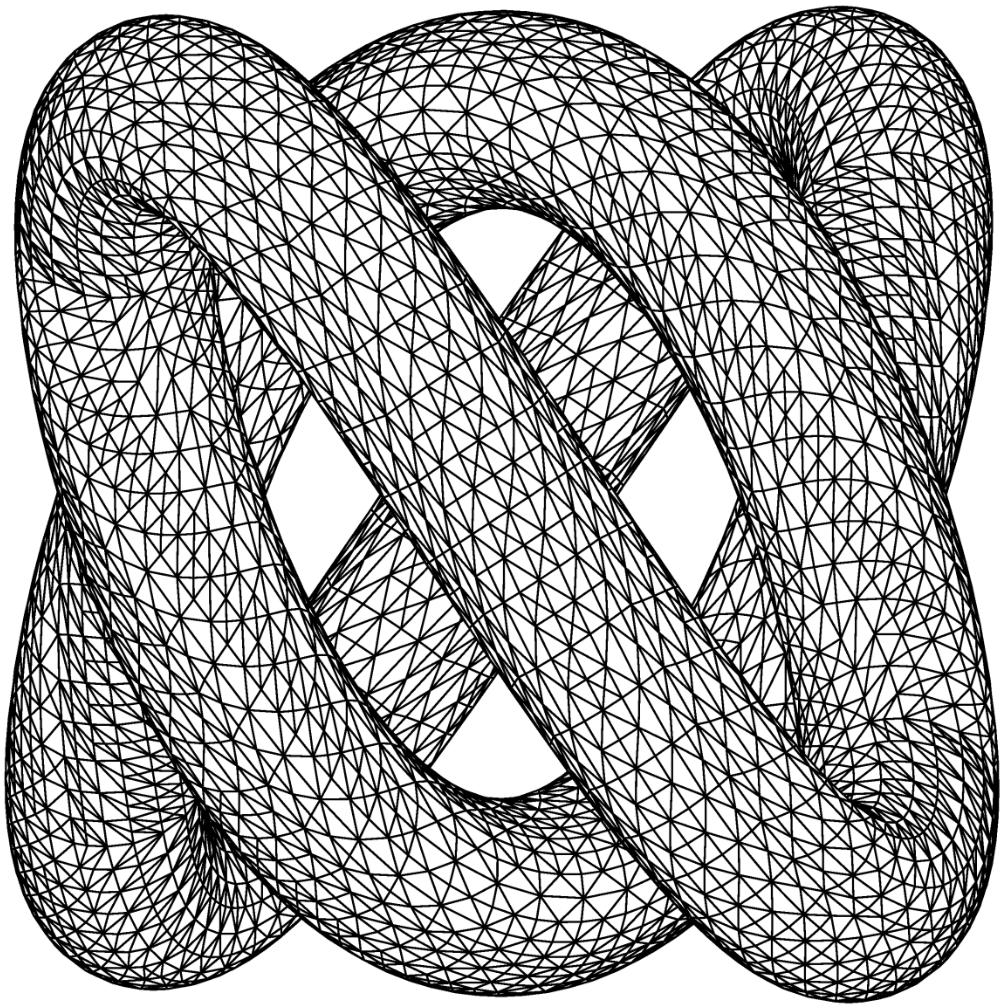
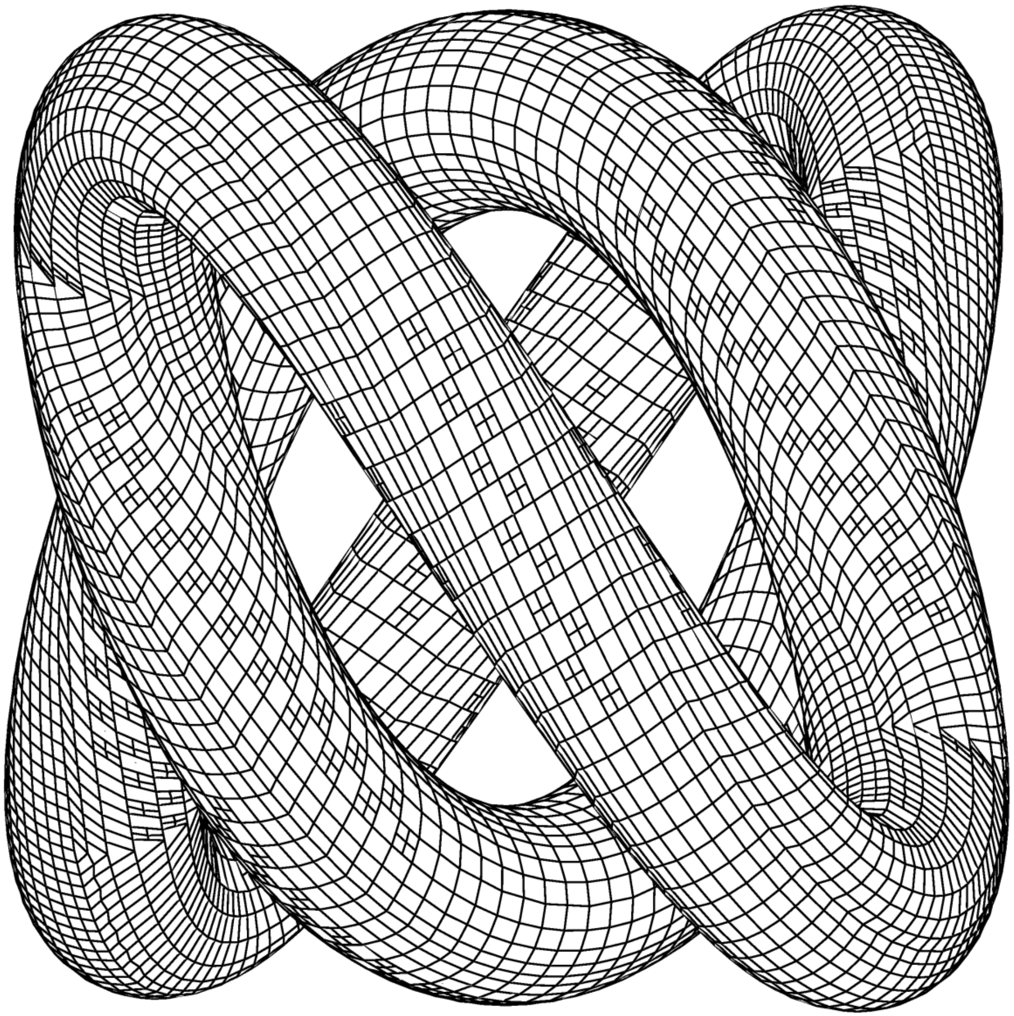


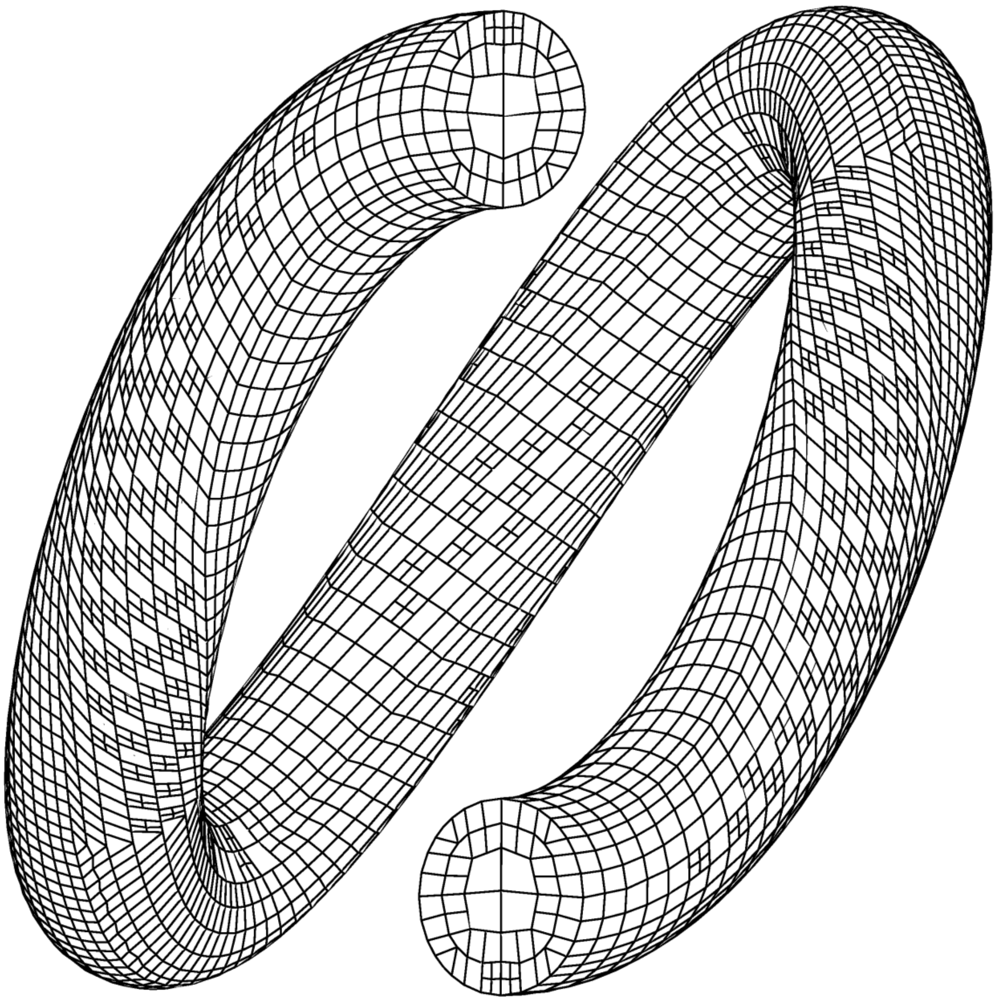
Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: T-spline representation



Isogeometric Modeling of a Genus-one Solid

Algorithm Steps: Cross-section of T-spline representation

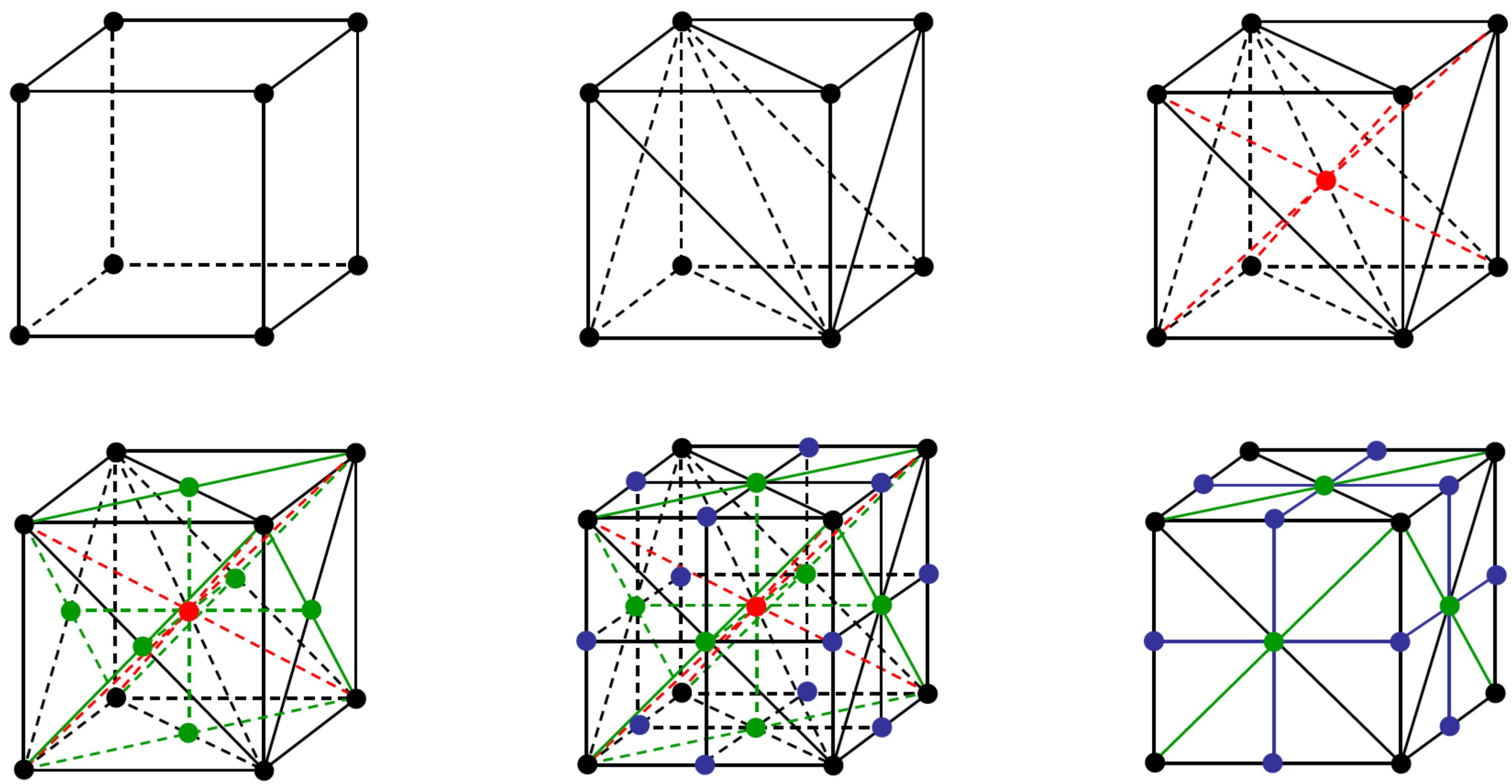


Local Refinement: Kossaczky's Algorithm (JCAM 1994)

Refinement of a cube

http://www.alberta-fem.de/, ALBERTA code

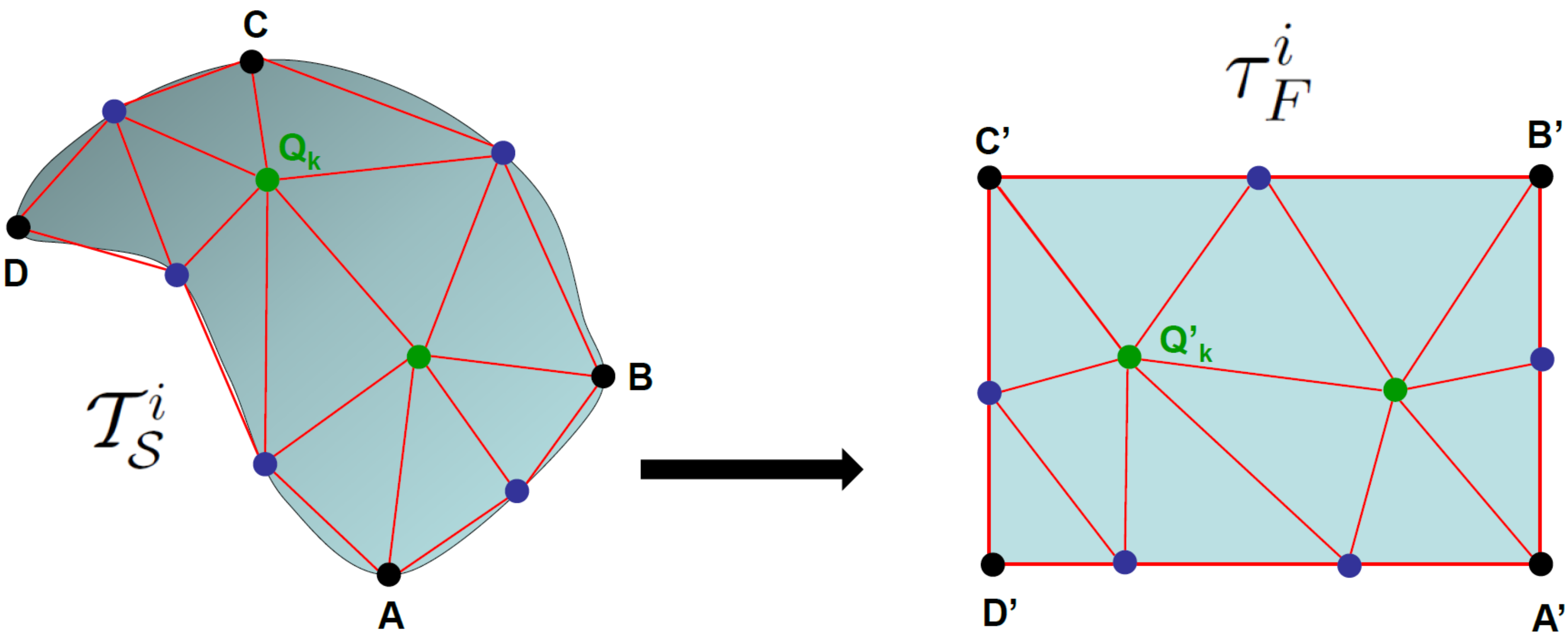
- Initial cube and its subdivision after three consecutive tetrahedron bisection



Surface Parameterization of M.S. Floater (CAGD 1997)

From a the *i-th* solid surface triangulation patch to the *i-th* meccano face

http://www.sintef.no/math software, GoTools from SINTEF ICT



Physical Space

Fixed boundary nodes,

$Q'_k = \sum_j \lambda_{j,k} Q'_j$

Parametric Space

Simultaneous Untangling and Smoothing (CMAME 2003)

Tetrahedral Mesh Optimization

SUS Code: Freely-available in http://www.dca.iusiani.ulpgc.es/proyecto2008-2011

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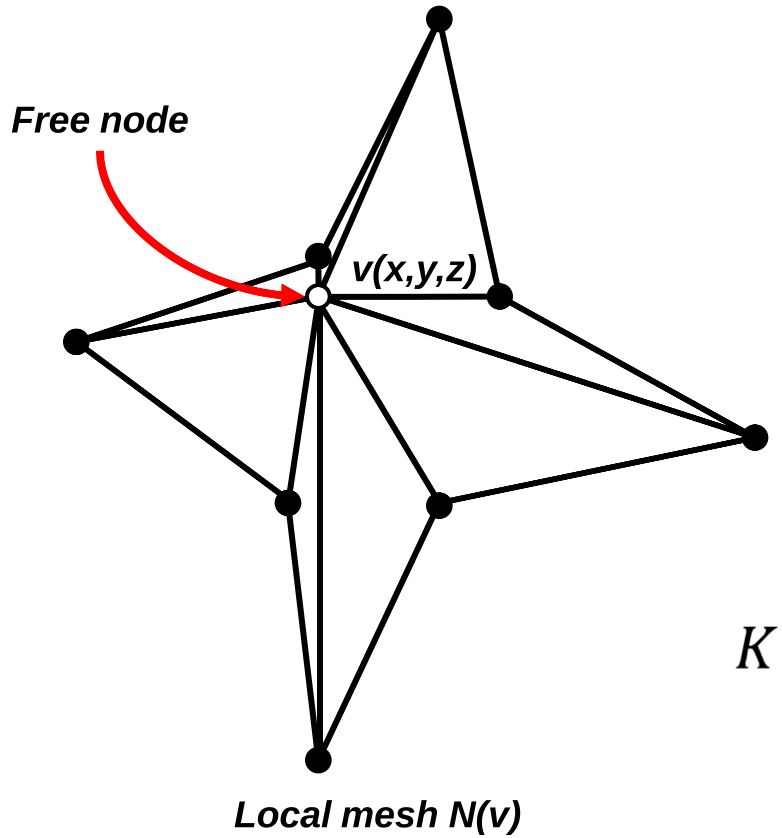
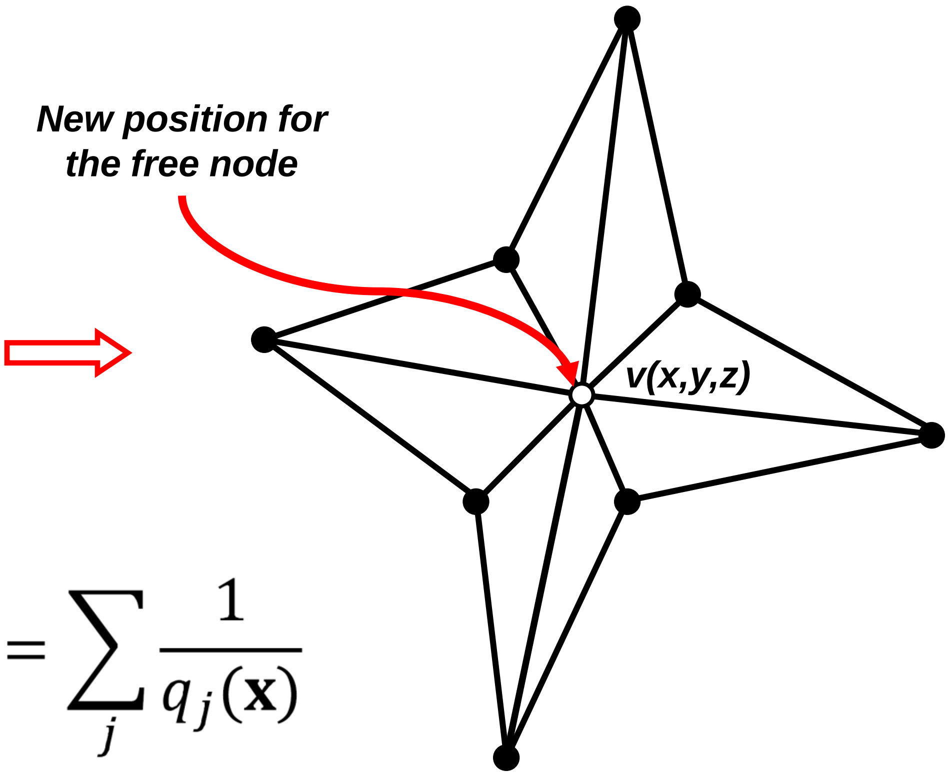
Local optimization

***Objective:* Improve the quality of the local mesh N(v) by minimising an objective function**



⇒

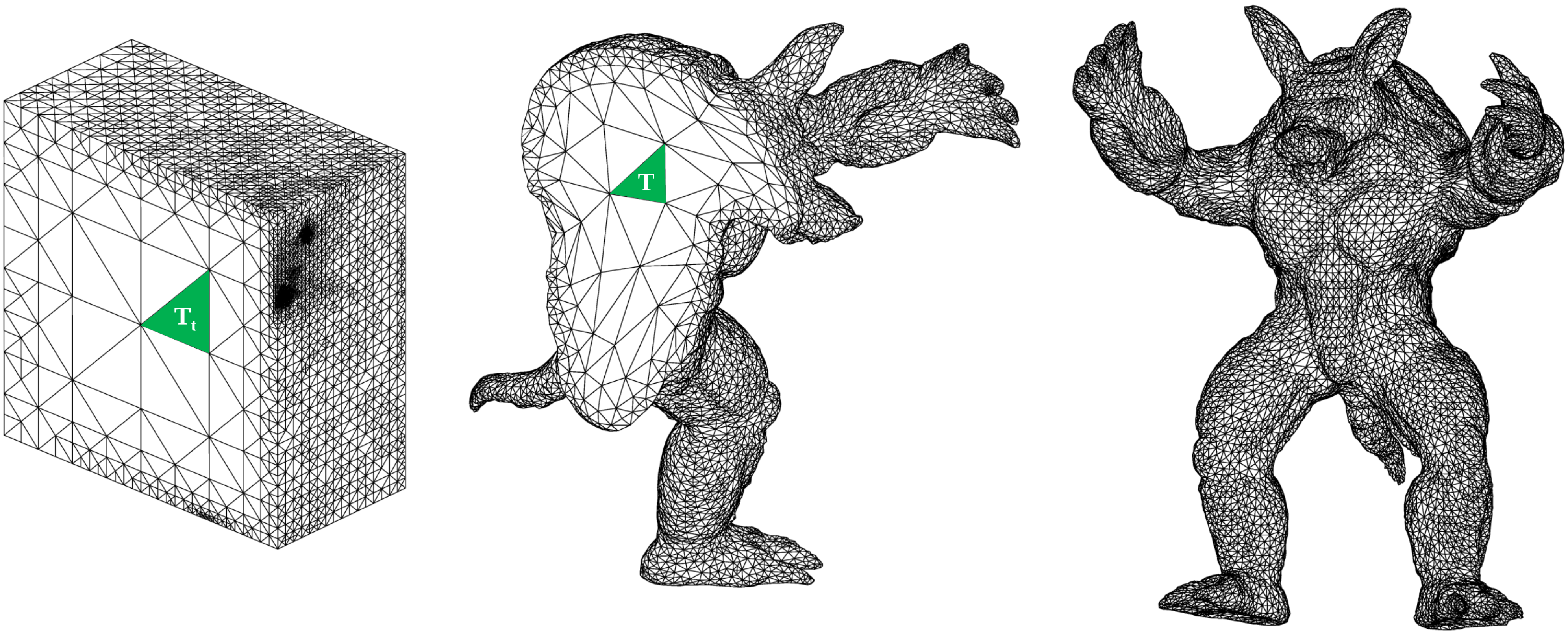


$$K(\mathbf{x}) = \sum_j \frac{1}{q_j(\mathbf{x})}$$

Meccano Method for a Complex Genus-Zero Solid

Application to the Armadillo: A surface triangulation as input datum

http://graphics.stanford.edu/data/3Dscanrep/, Stanford Computer Graphics Laboratory

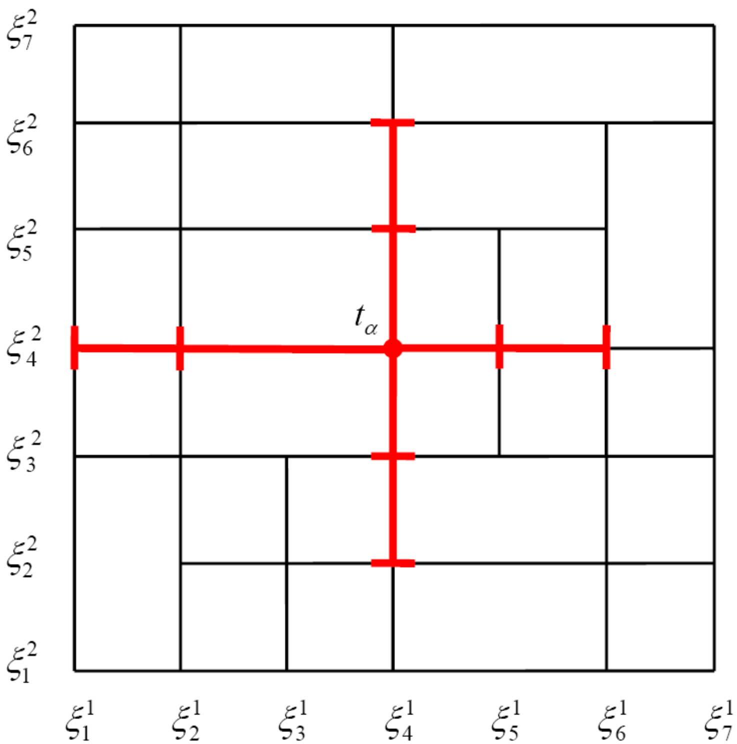


Optimization

Physical Element T → **Target Element T_t** (to get less distortion in the parameterization)

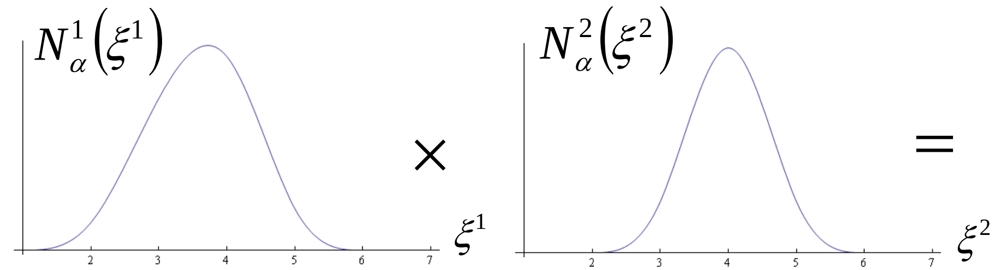
Example of T-mesh and T-splines in 2-D

T-mesh and anchor t_a



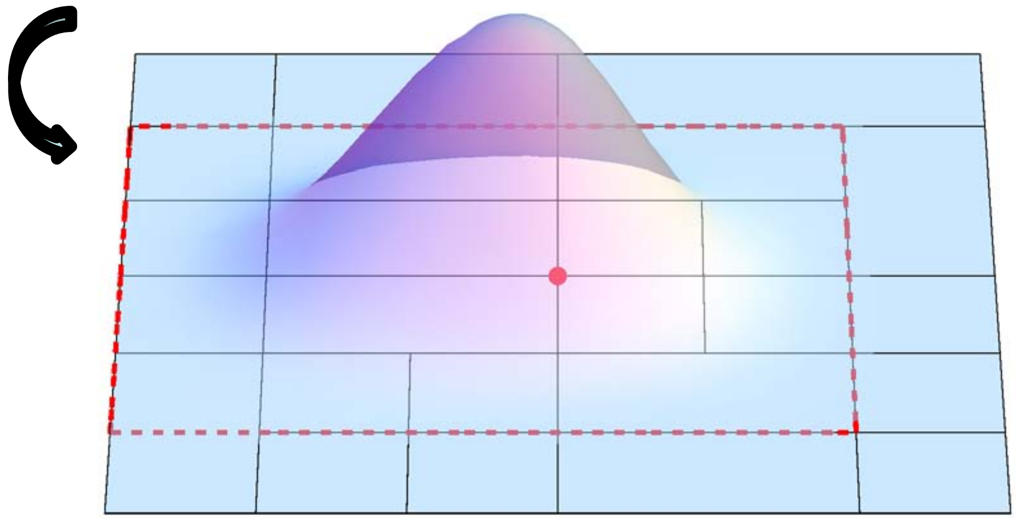
Knots associated to anchor t_a:

$$\Xi_\alpha^1 = \{\xi_1^1, \xi_2^1, \xi_4^1, \xi_5^1, \xi_6^1\} \qquad \Xi_\alpha^2 = \{\xi_2^2, \xi_3^2, \xi_4^2, \xi_5^2, \xi_6^2\}$$



$$B_\alpha(\xi^1, \xi^2) = N_\alpha^1(\xi^1) N_\alpha^2(\xi^2)$$

support of the T-spline



Bivariate Cubic T-spline Basis Function

Construction of the T-mesh for the Bunny

Automatic Adaptation of Inner and Boundary Discretizations

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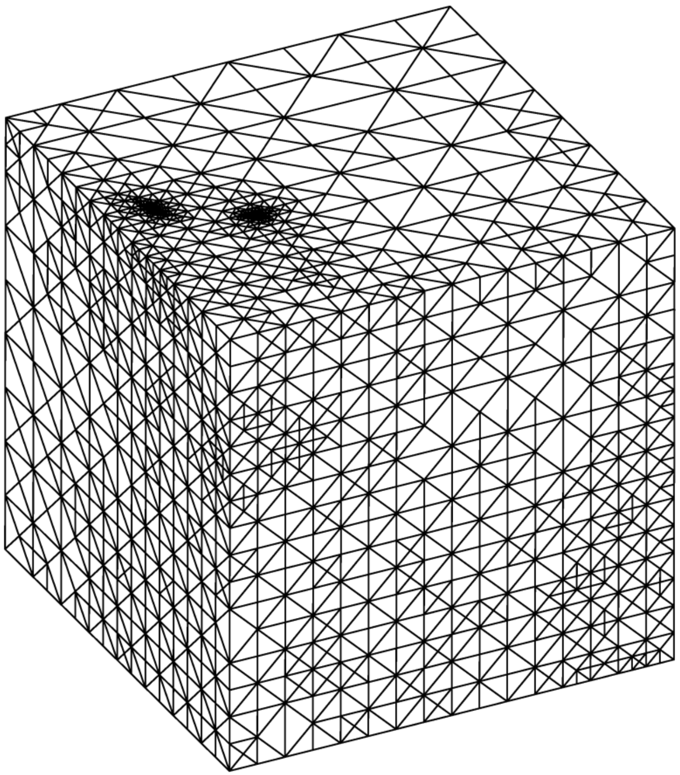
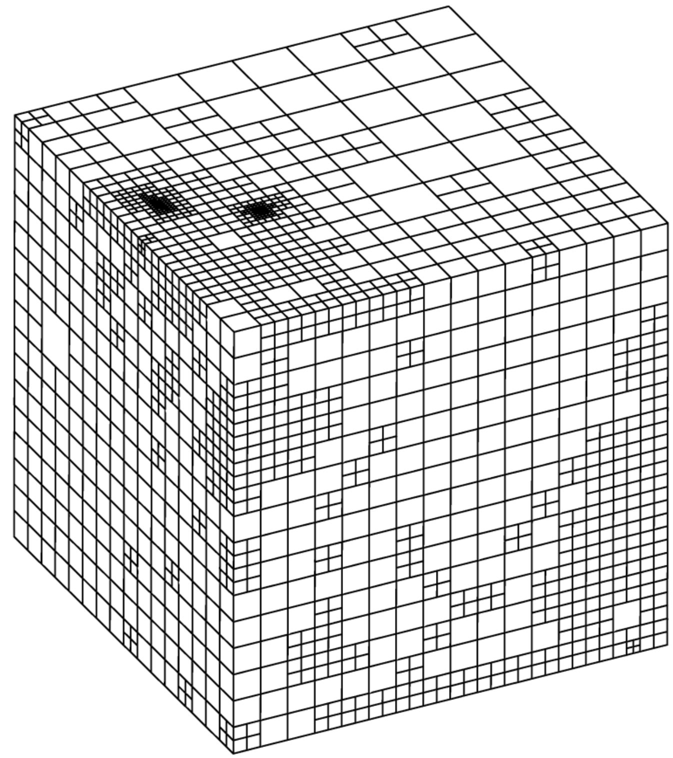
Cube tetrahedral mesh



Obtained by using the meccano method with Kossacsky refinement

Octree division of the cube

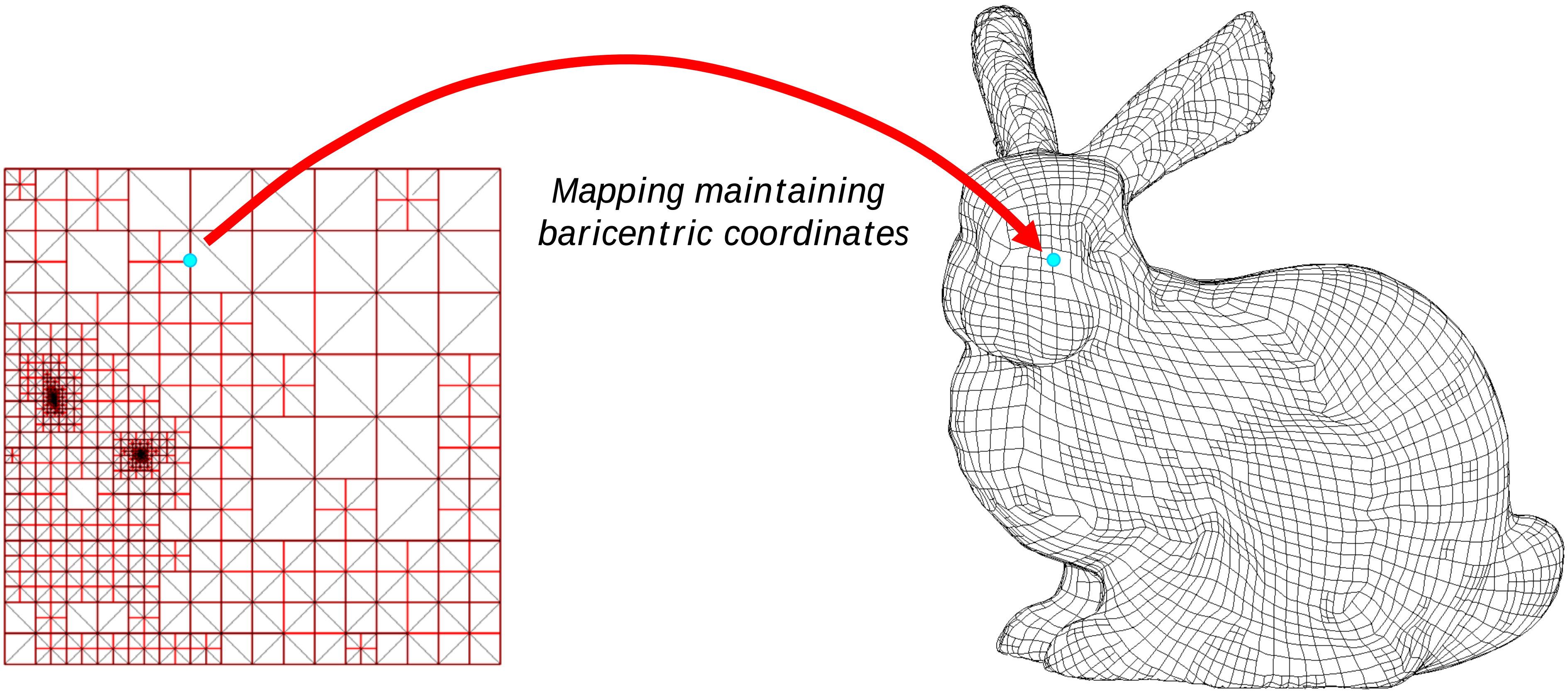
Cube T-mesh



Each cube of the octree does not contain any node of the Kossaczky mesh in its inner

Construction of the T-mesh for the Bunny

Mapping of the interpolation points



Interpolation points (the anchors) are mapped to the solid by using the volumetric parameterization that was obtained by the meccano method

The Spline Interpolation

Calculation of Control Points by Fulfilling the Interpolation Conditions

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$$\mathbf{S}\left(\xi^1,\xi^2,\xi^3\right)=\sum_{\alpha\in A}\mathbf{P}_\alpha R_\alpha\left(\xi^1,\xi^2,\xi^3\right)$$

With:

$$R_\alpha\left(\xi^1,\xi^2,\xi^3\right)=\frac{B_\alpha\left(\xi^1,\xi^2,\xi^3\right)}{\sum_{\beta\in A}B_\beta\left(\xi^1,\xi^2,\xi^3\right)}$$ *Blending functions*

$$B_\alpha\left(\xi^1,\xi^2,\xi^3\right)=N_\alpha^1\left(\xi^1\right)N_\alpha^2\left(\xi^2\right)N_\alpha^3\left(\xi^3\right)$$ *Trivariate basis splines*

Parametric space location $\mathbf{t}_\beta$ → **volumetric parameterization** → $\mathbf{S}\left(\mathbf{t}_\beta\right)$ *Physical space location*

Control points $\mathbf{P}_\alpha$ *are calculated by solving the sparse linear system*

$$\mathbf{S}\left(\mathbf{t}_\beta\right)=\sum_{\alpha\in A}\mathbf{P}_\alpha R_\alpha\left(\mathbf{t}_\beta\right)\qquad\forall\beta\in A$$

T-mesh and T-spline of the Bone

Automatic Adaptation of Inner and Boundary Discretizations

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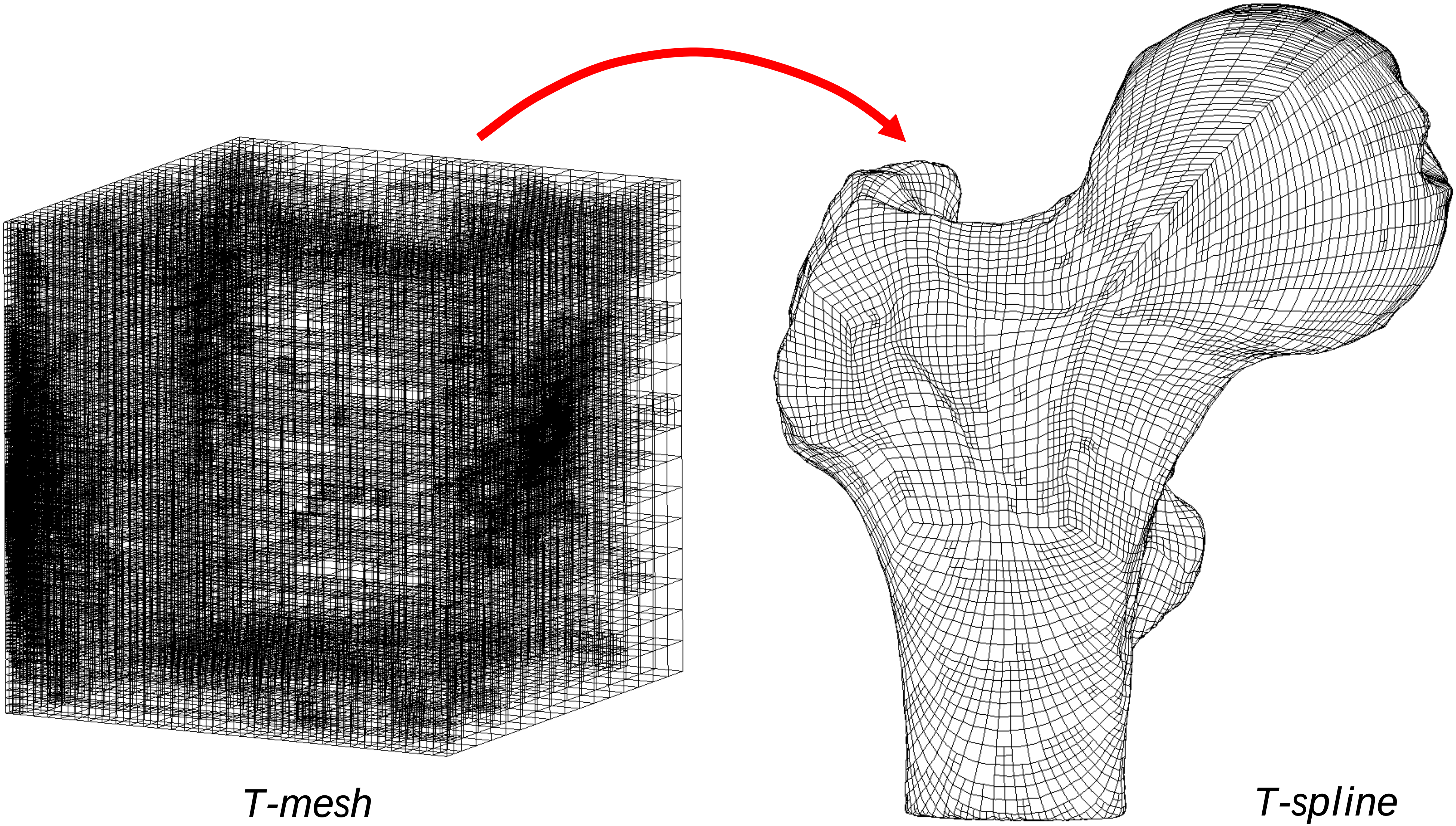

Cross-sections of the Bone T-spline

Automatic Adaptation of Inner and Boundary Discretizations

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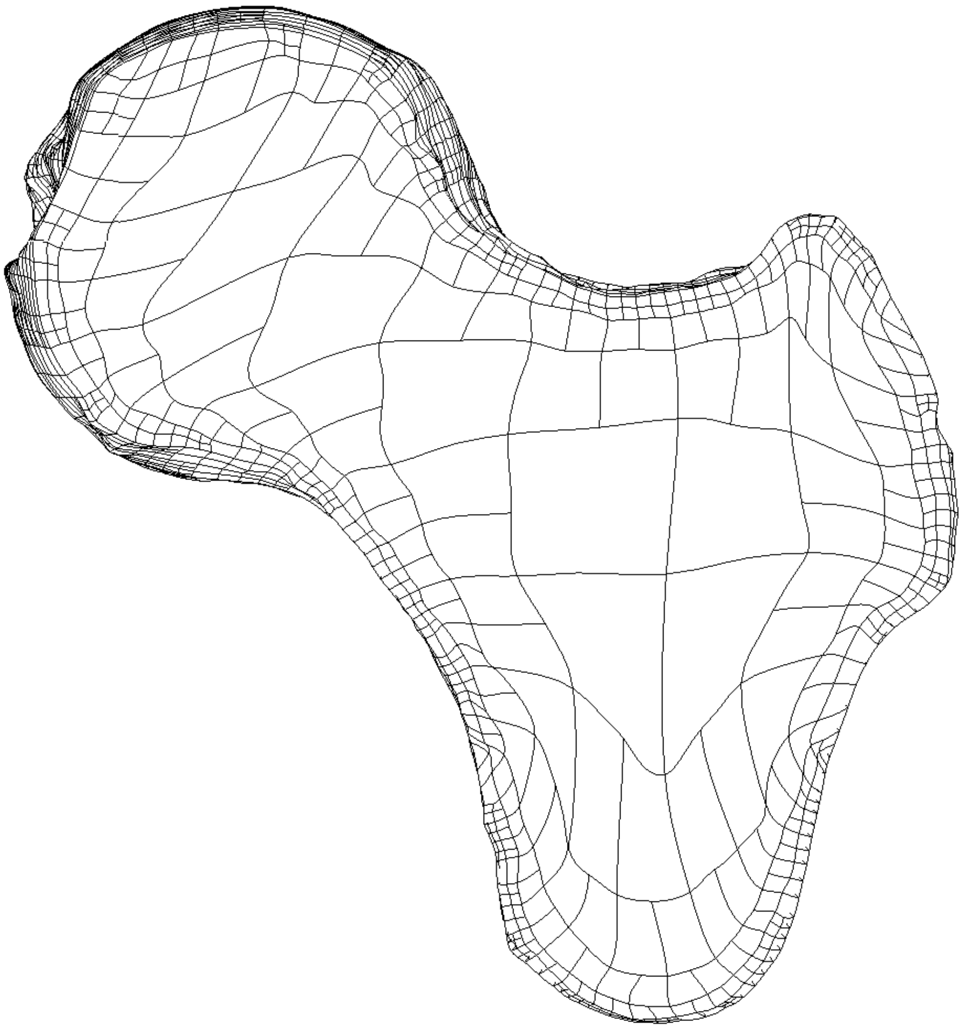
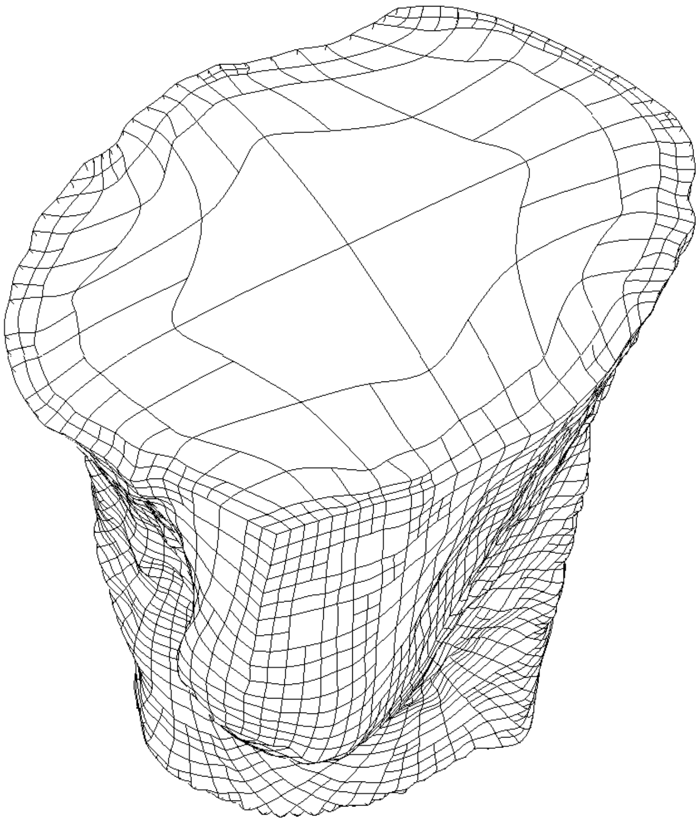




Adaptive Isogeometric Refinement

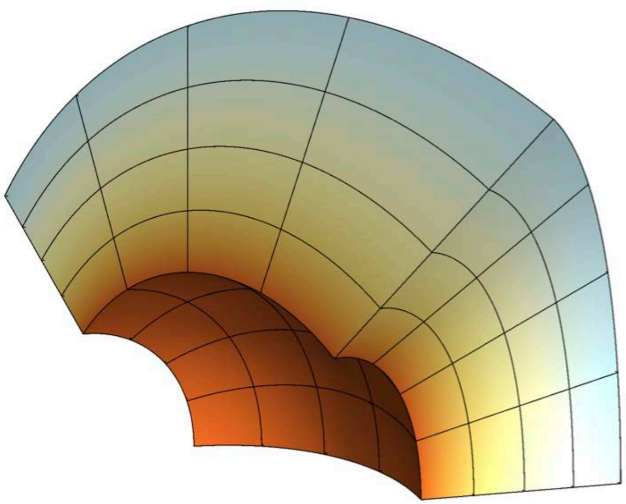
Application: Spherical region

$$\Delta u = \frac{4\left(-3+x^2+y^2+z^2\right)}{\left(1+x^2+y^2+z^2\right)^3} \quad \text{in } \Omega$$

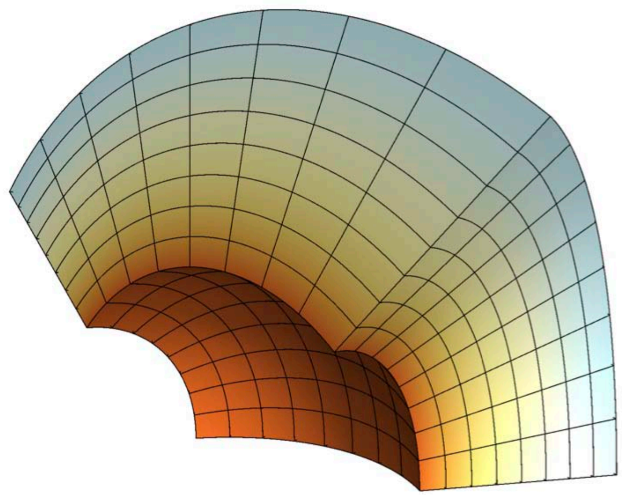
$$u\big|_{\partial\Omega} = \frac{2}{\left(1+x^2+y^2+z^2\right)}$$

Exact solution: $u \approx \dfrac{2}{\left(1+x^2+y^2+z^2\right)}$

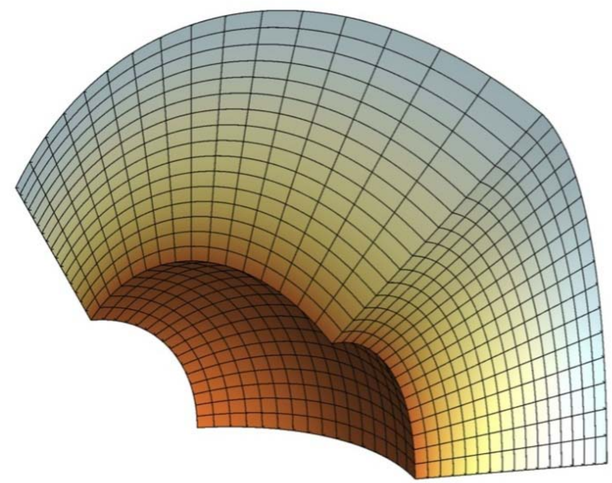
Residual-type estimator: $\eta\,(\Omega_e)^2 = \displaystyle\int_{\Omega_e} h^2\,(f+\Delta u_h)^2\; d\Omega$



Initial T-mesh
64 cells, 125 DOF



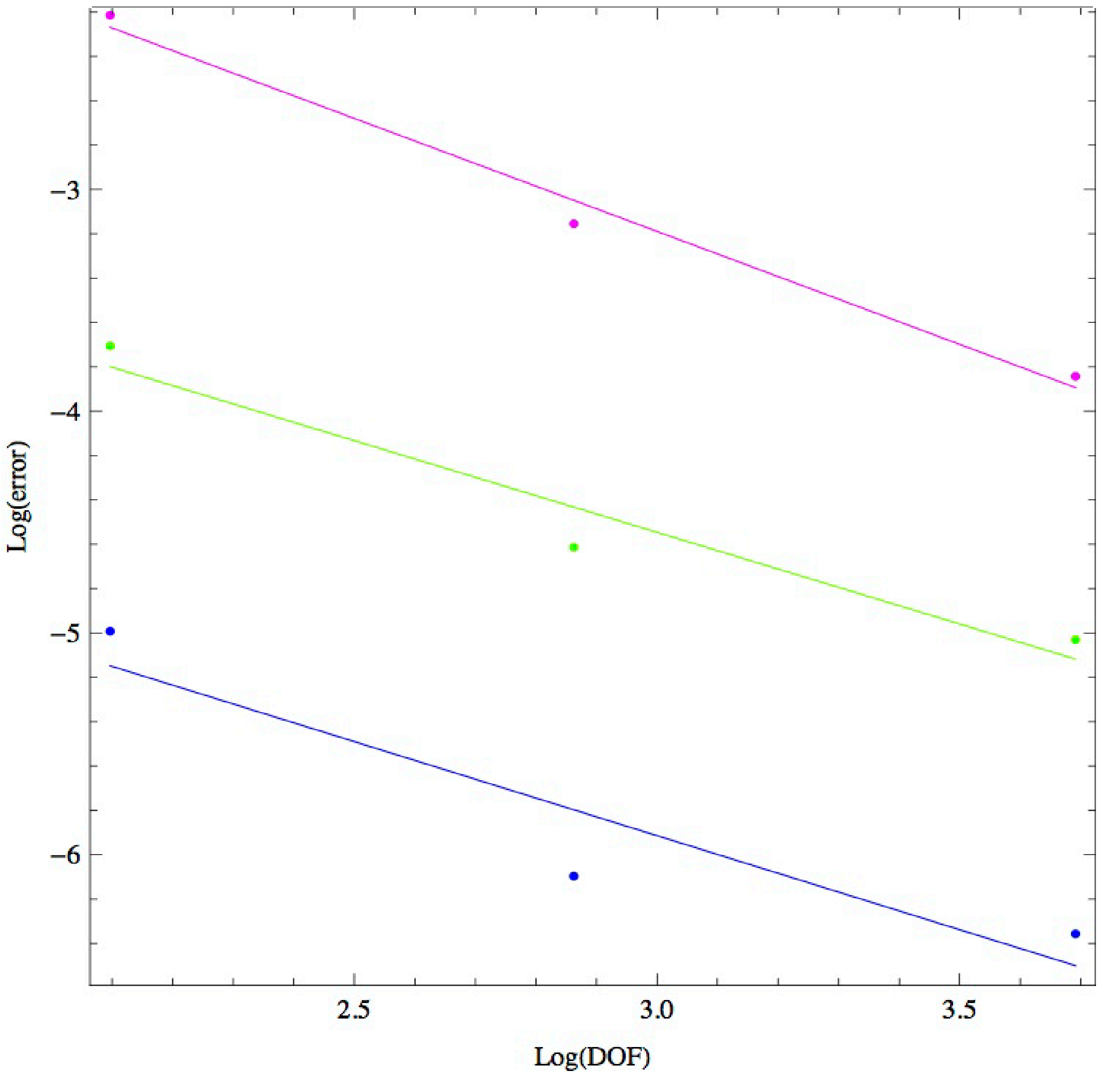
1st global refinement
512 cells, 729 DOF



2nd global refinement
4096 cells, 4913 DOF

Adaptive Isogeometric Refinement

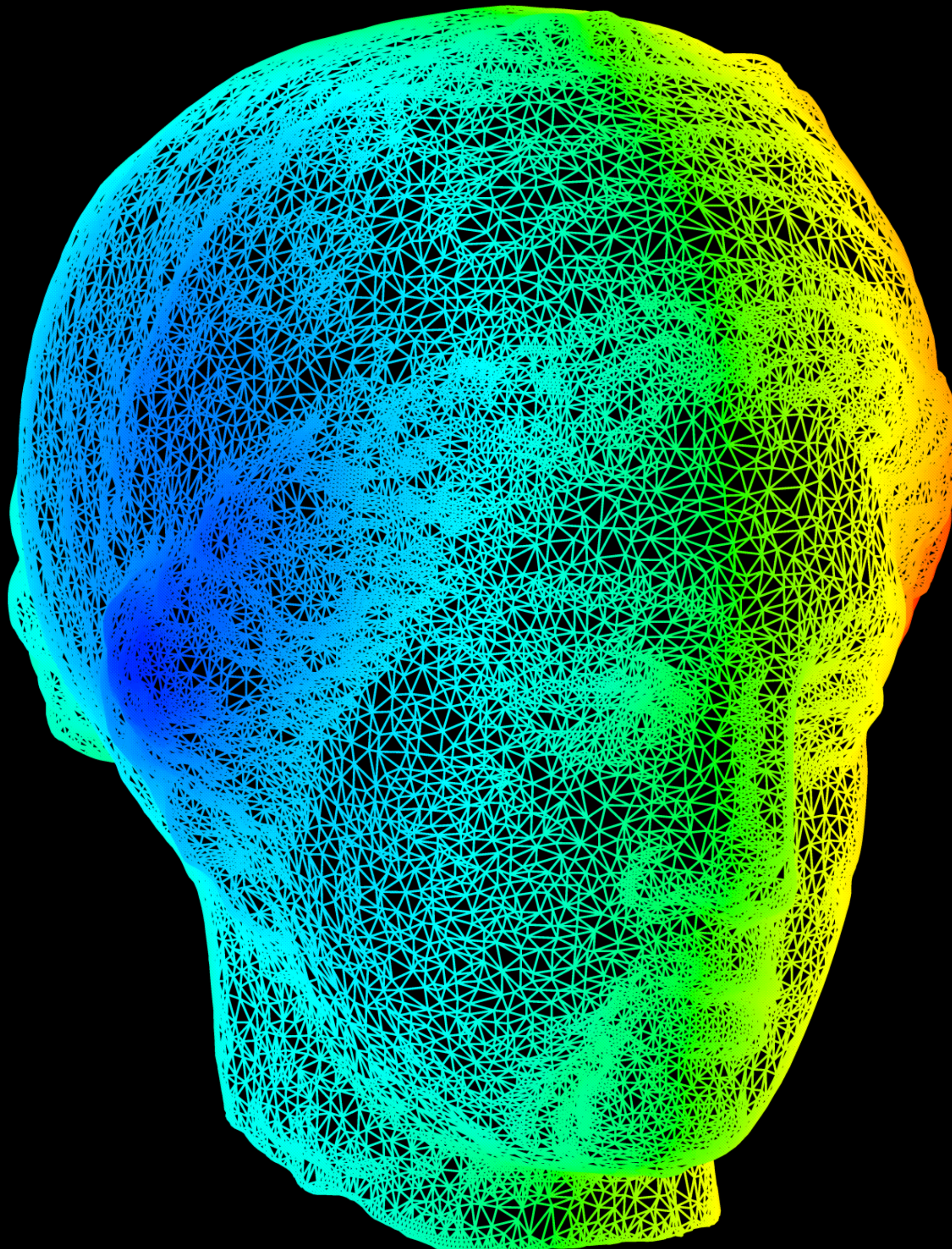
Spherical region: Rate of convergence



Error estimator (slope: -1)

Exact error in H^1 seminorm (slope: -0.8)

Exact error in L^2 norm (slope: -0.8)



http://www.cyberware.com/

Adaptive Isogeometric Refinement

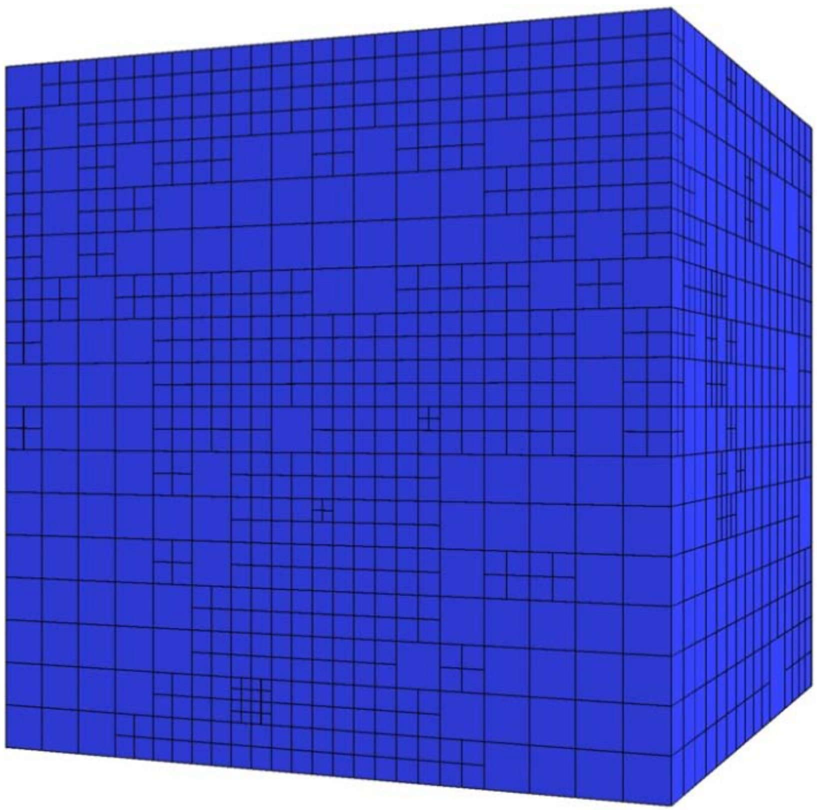
Application: Igea with a central source

$$\Delta u = \frac{1}{25} e^{-\frac{(x^2+y^2+z^2)}{10}} \left(-15 + x^2 + y^2 + z^2\right) \quad \text{in } \Omega$$

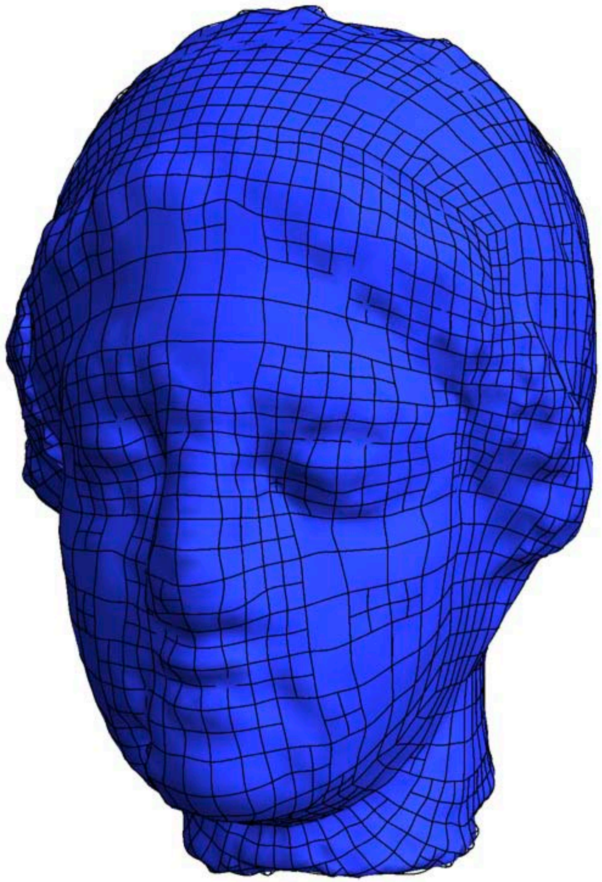
$$u\big|_{\partial\Omega} = 0$$

Exact solution:

$$u \approx e^{-\frac{(x^2+y^2+z^2)}{10}}$$



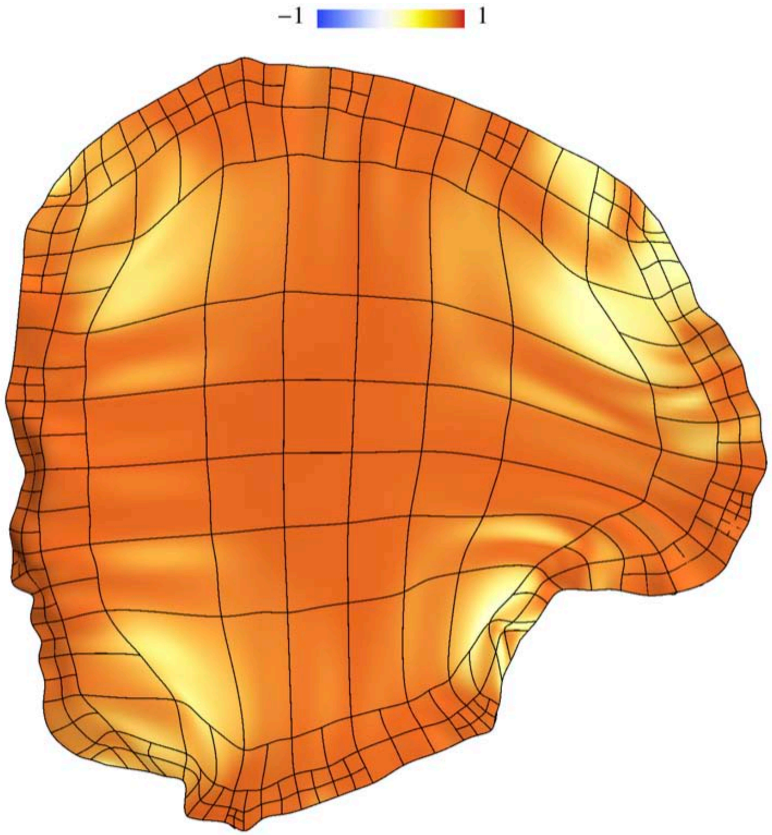
T-mesh



T-spline

Adaptive Isogeometric Refinement

Application: Igea with a central source

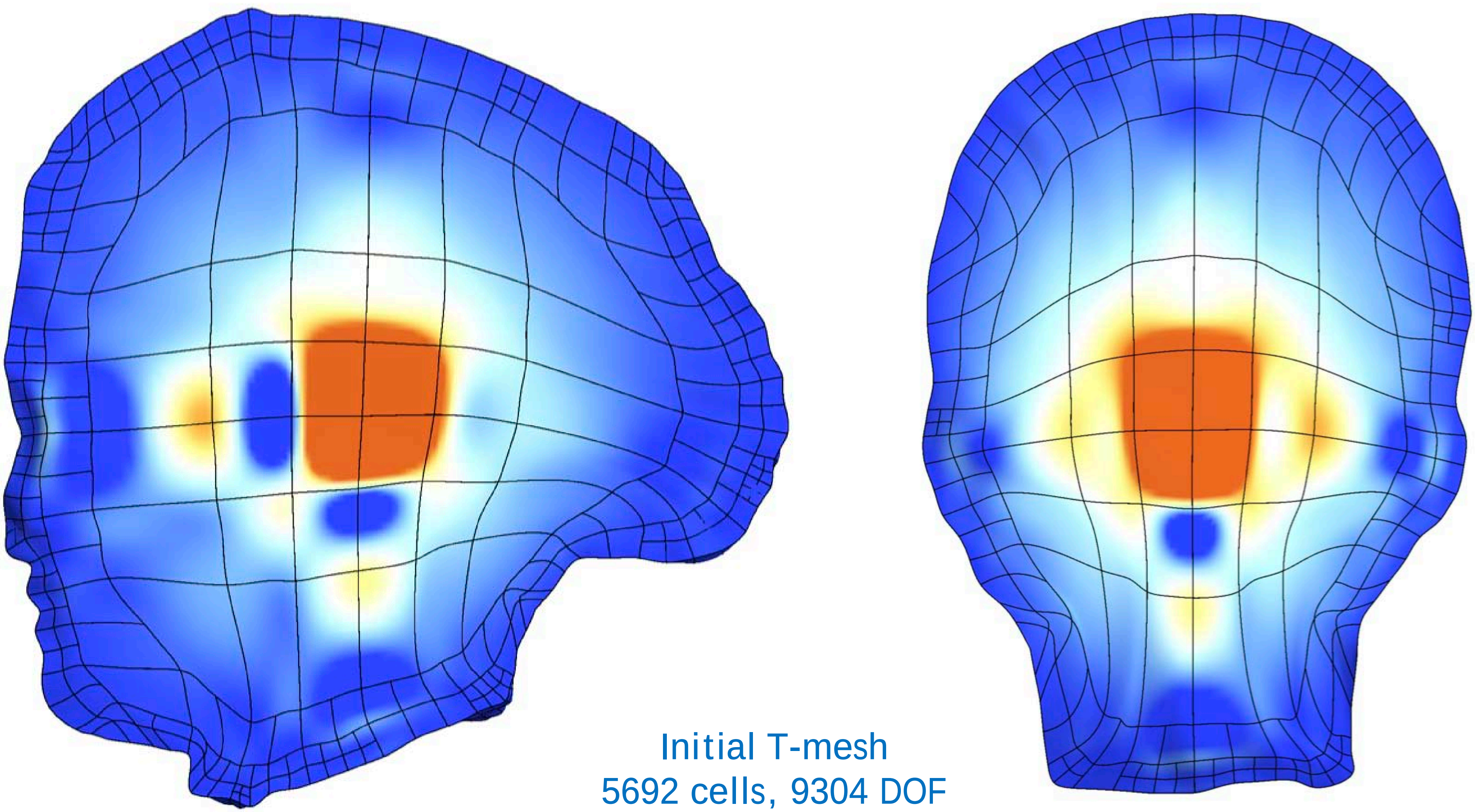


Scaled Jacobian:

$$J_s\left(\xi^1,\xi^2,\xi^3\right)=\frac{\det\left(\mathbf{S}_{\xi^1},\mathbf{S}_{\xi^2},\mathbf{S}_{\xi^3}\right)}{\left\|\mathbf{S}_{\xi^1}\right\|\left\|\mathbf{S}_{\xi^2}\right\|\left\|\mathbf{S}_{\xi^3}\right\|}$$

Adaptive Isogeometric Refinement

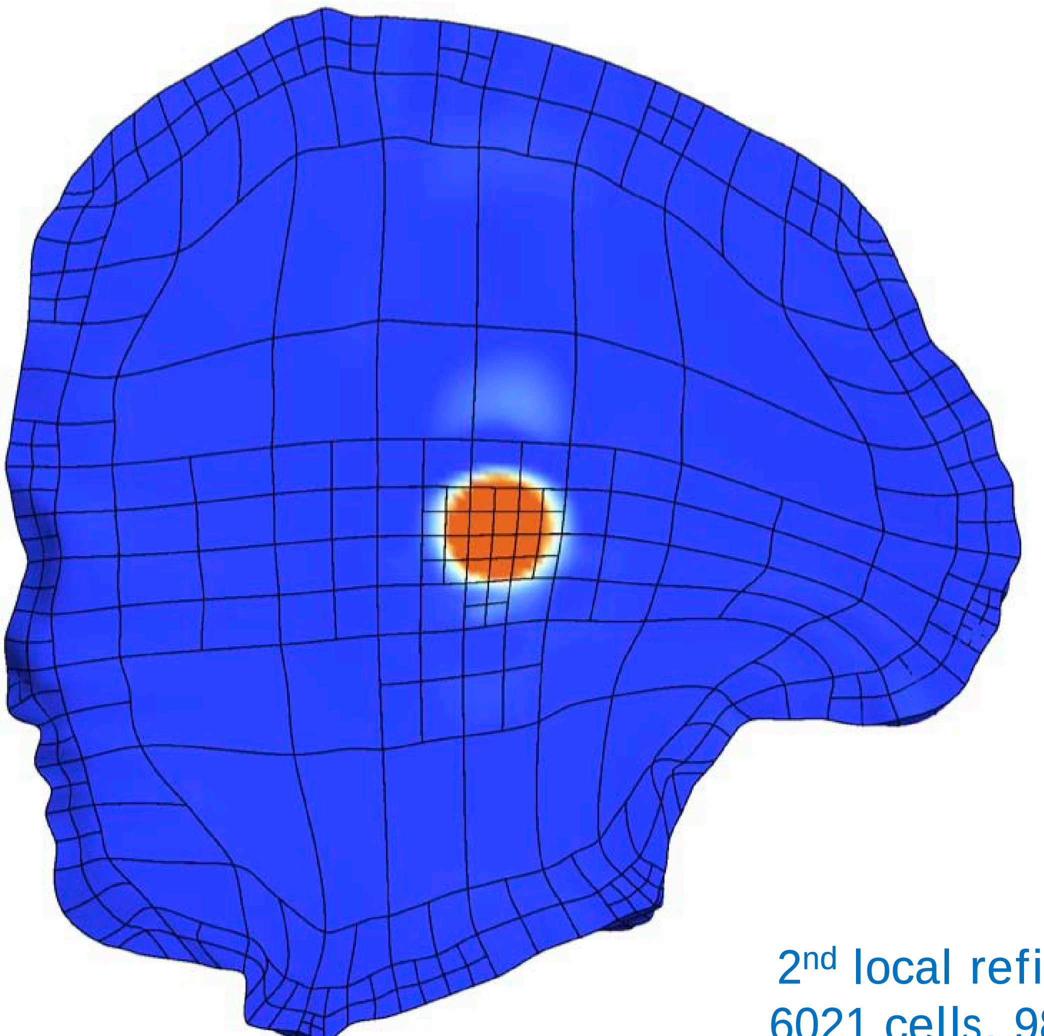
Igea: T-spline of Numerical Solution



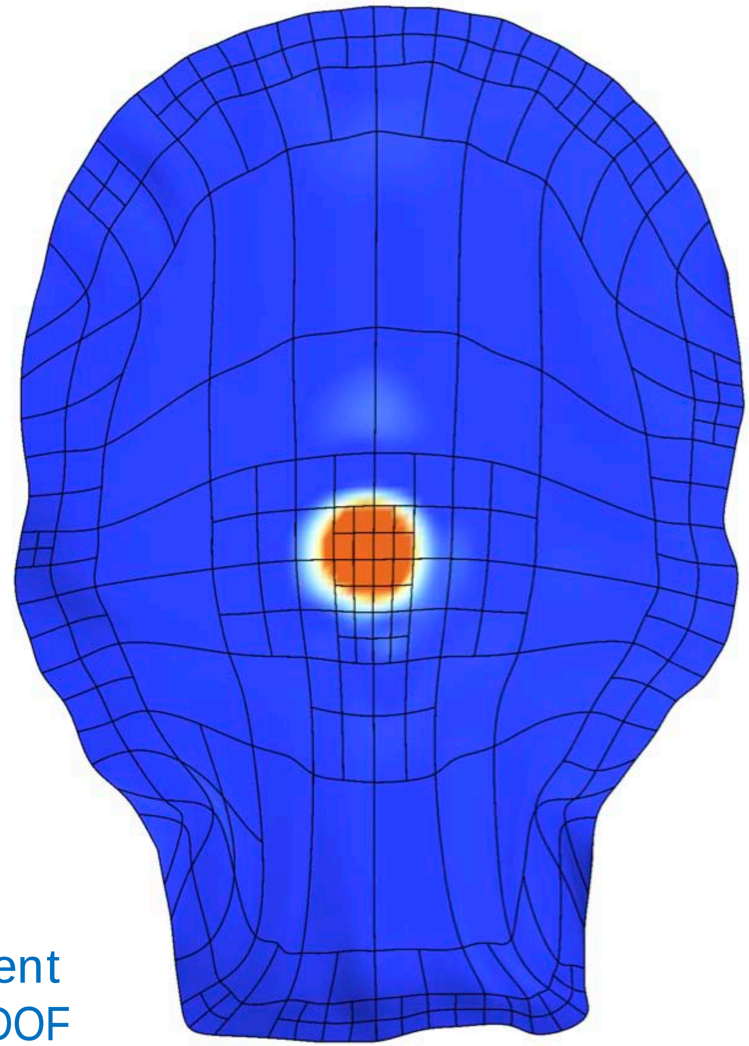
Initial T-mesh
5692 cells, 9304 DOF

Adaptive Isogeometric Refinement

Igea: T-spline of Numerical Solution



2nd local refinement
6021 cells, 9807 DOF



Adaptive Isogeometric Refinement

Igea: T-spline of Numerical Solution

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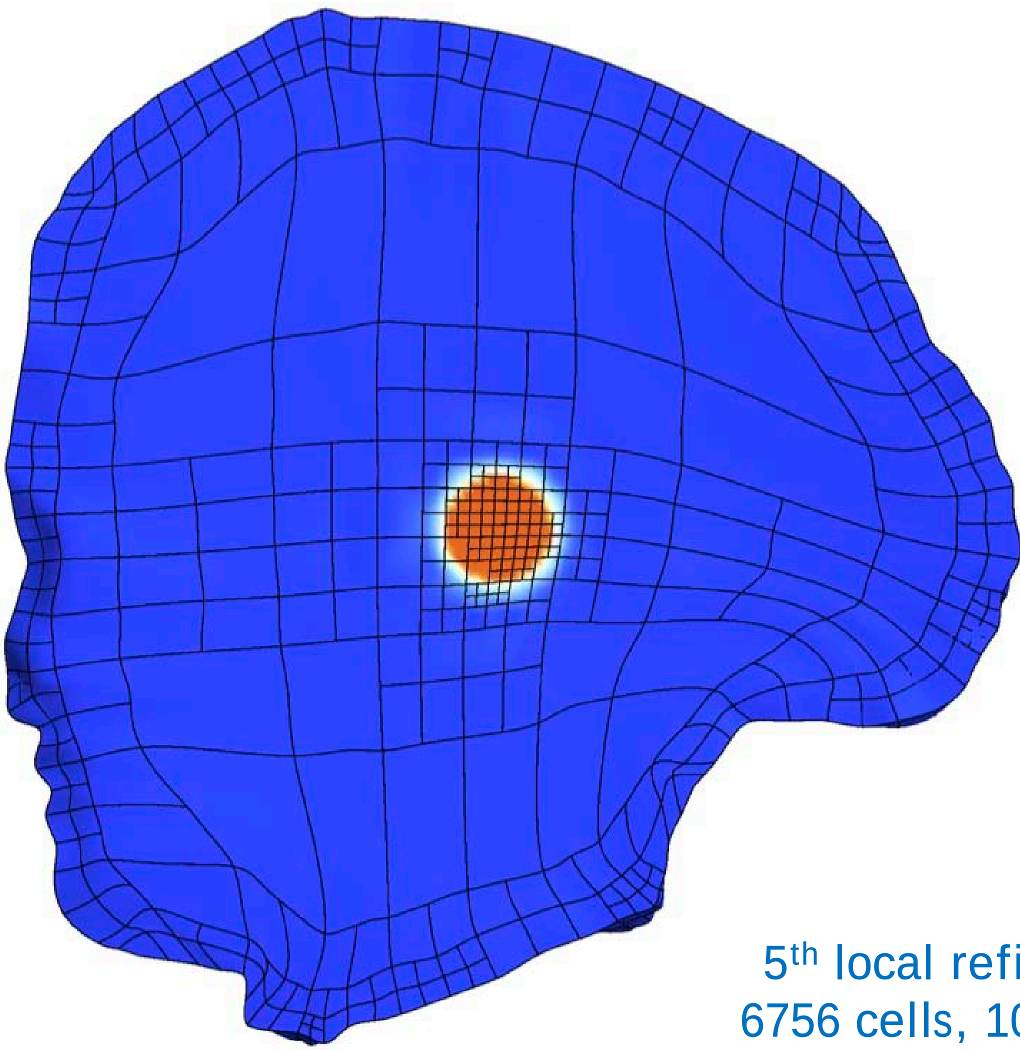
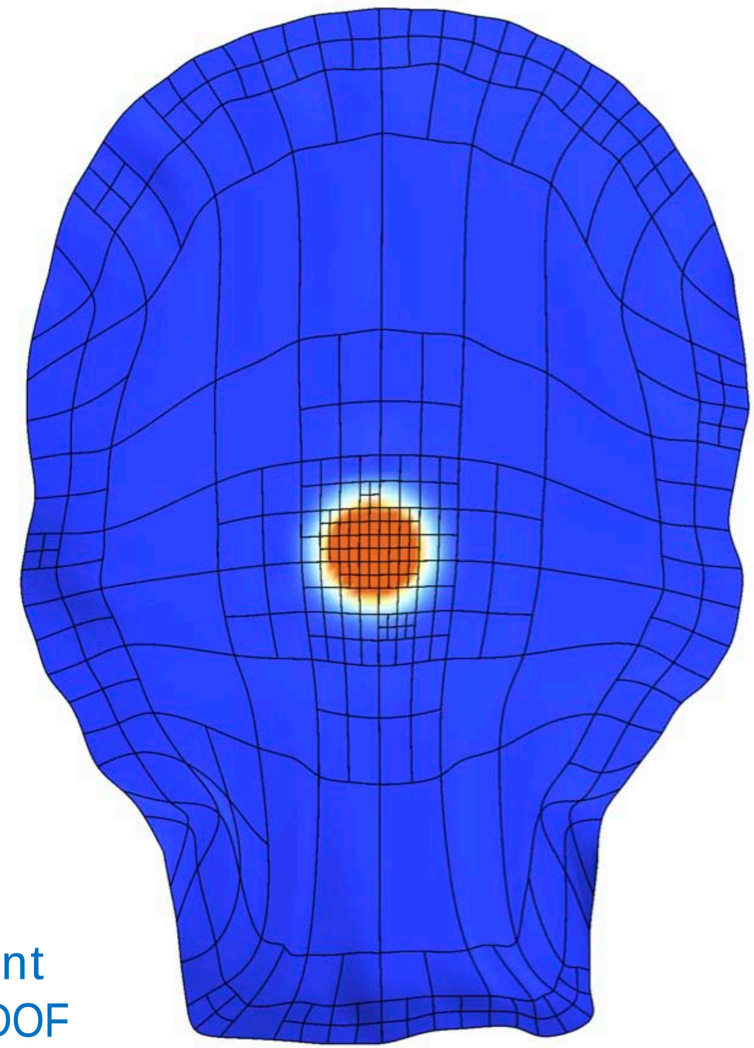
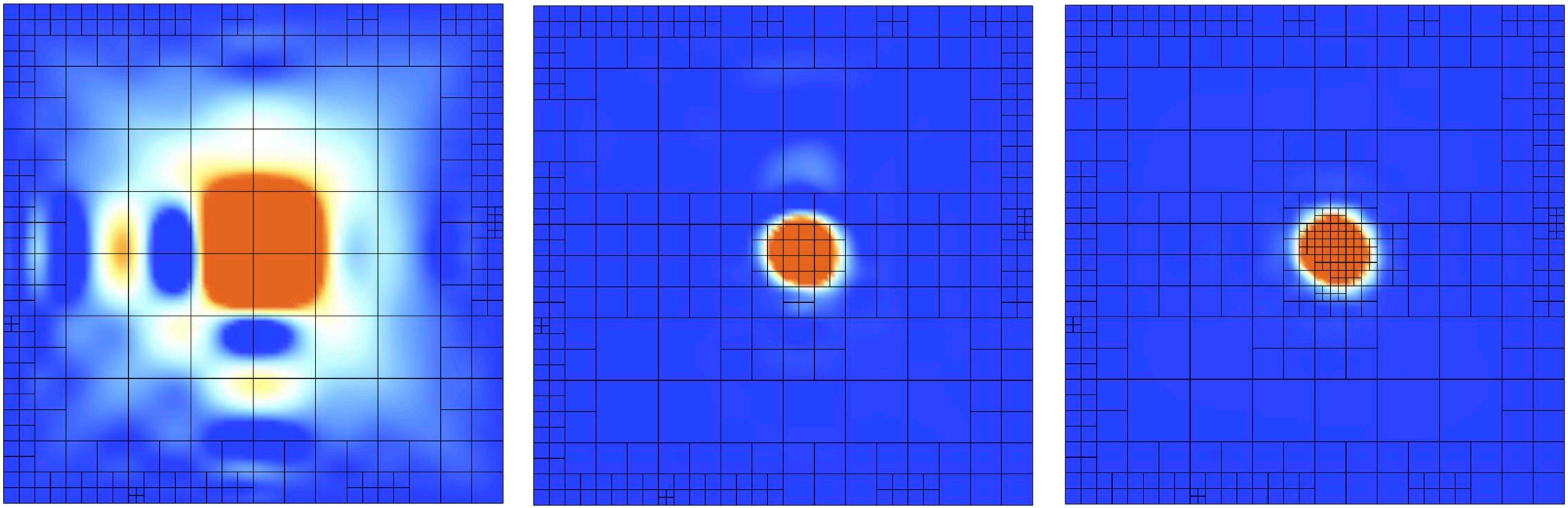




5^{th} local refinement
6756 cells, 10838 DOF

Adaptive Isogeometric Refinement

Igea: Numerical solution in a cross section of the parametric space



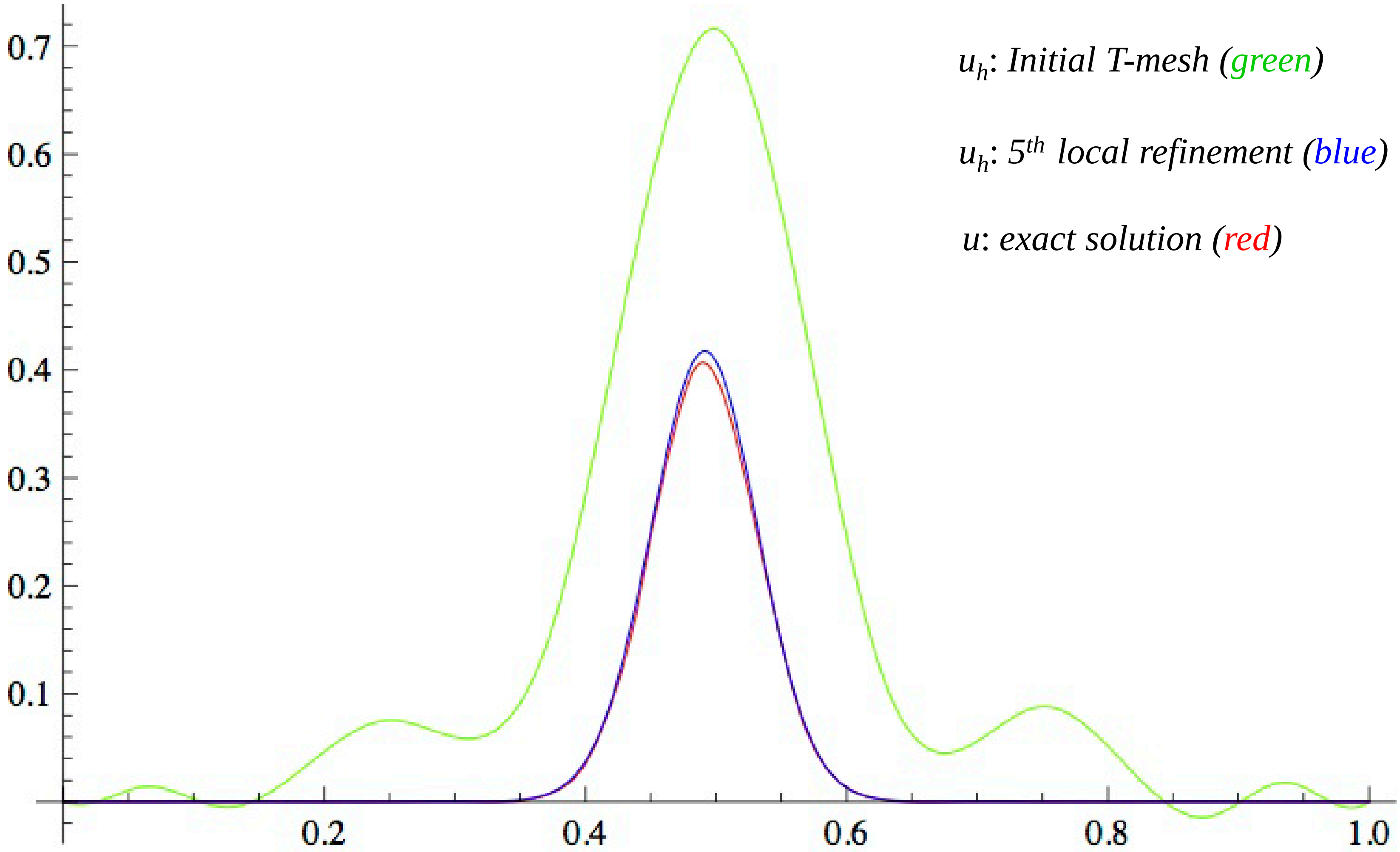
Initial T-mesh
5692 cells, 9304 DOF

2nd local refinement
6021 cells, 9807 DOF

5th local refinement
6756 cells, 10838 DOF

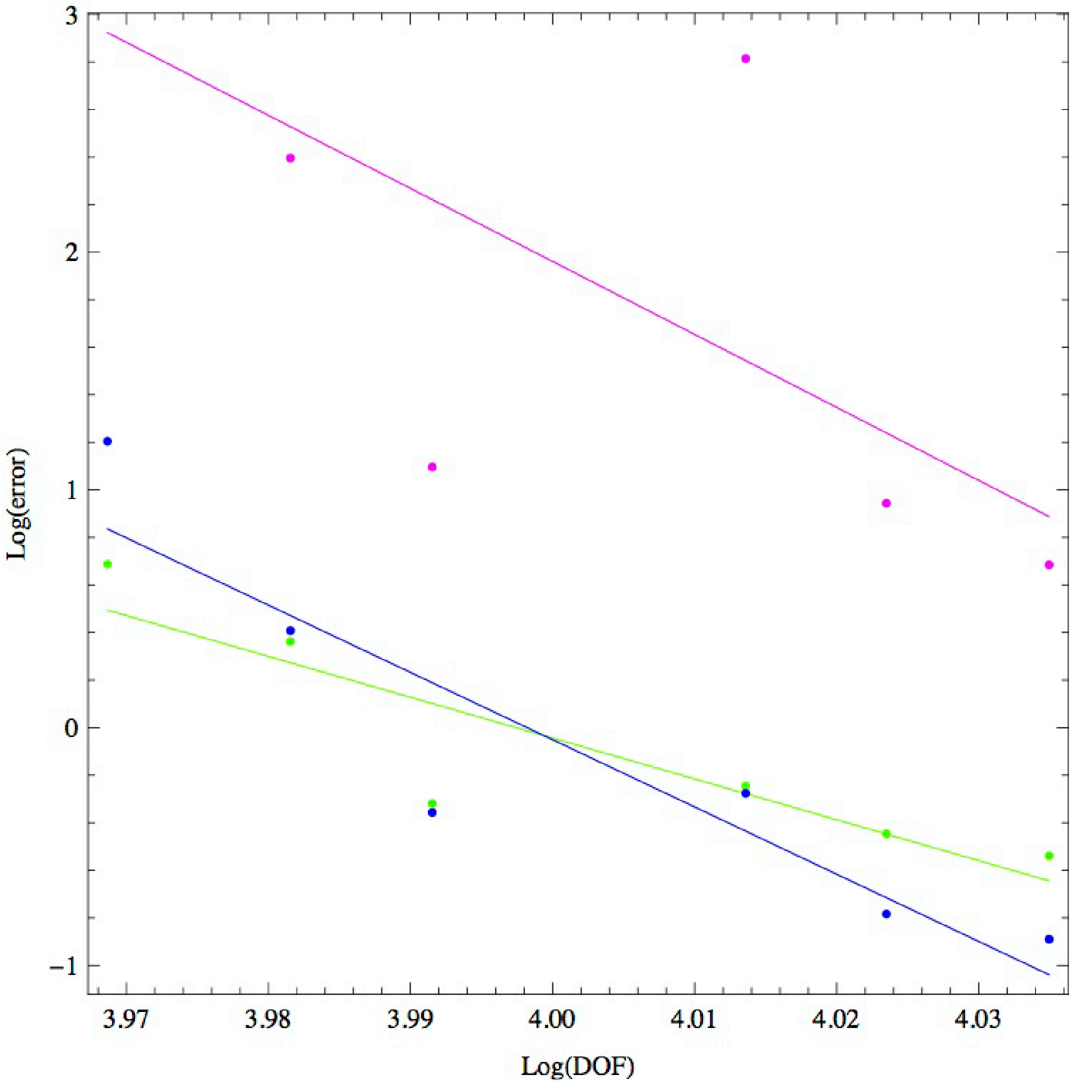
Adaptive Isogeometric Refinement

Igea: Exact and numerical solution in a cross section of the parametric space



Adaptive Isogeometric Refinement

Igea: Rate of convergence



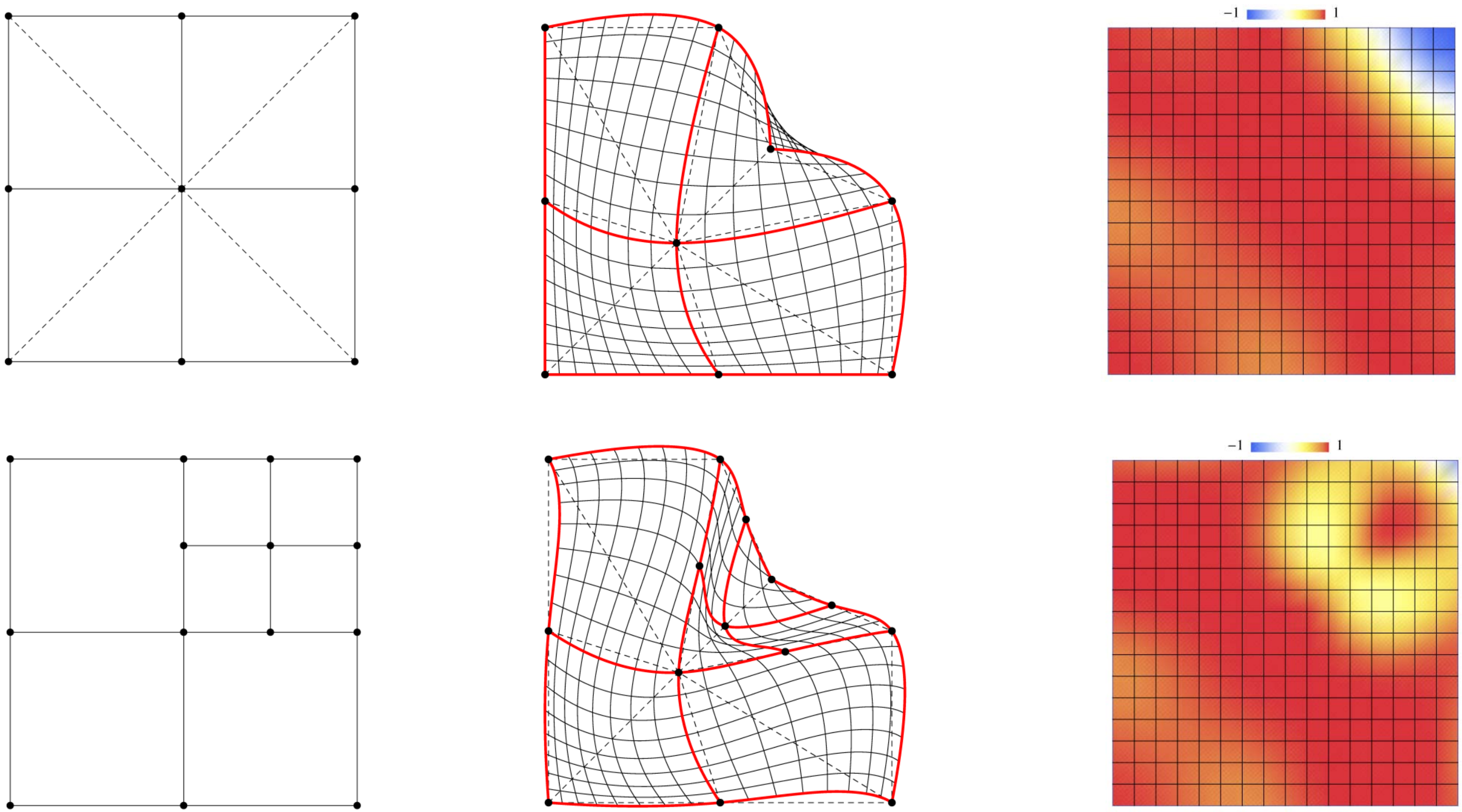
Error estimator (slope: -30.7)

Exact error in H^1 seminorm (slope: -17.2)

Exact error in L^2 norm (slope: -28.2)

Final Comments and Future Works

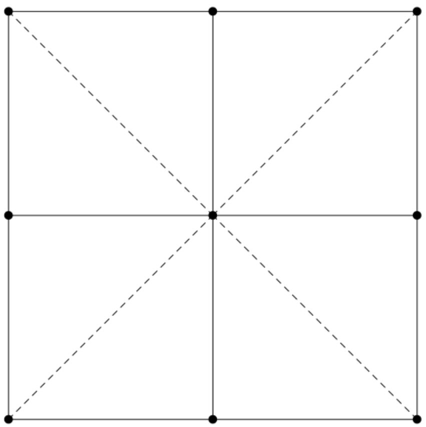
Valid and Invalid Configurations in IGA



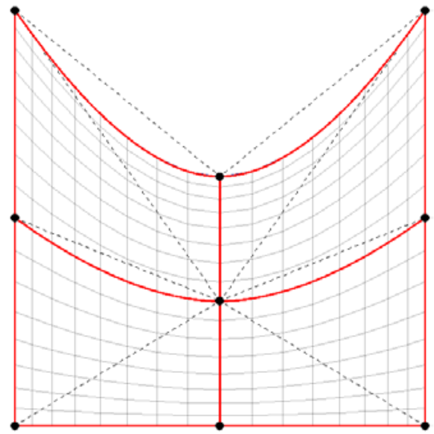
Parametric space | Physical space | Scaled Jacobian

Final Comments and Future Works

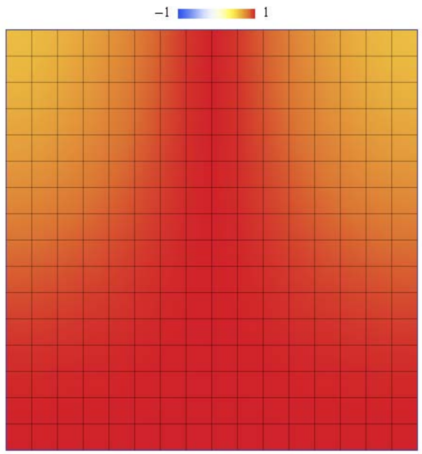
Valid and Invalid Configurations in IGA



Parametric space



Physical space



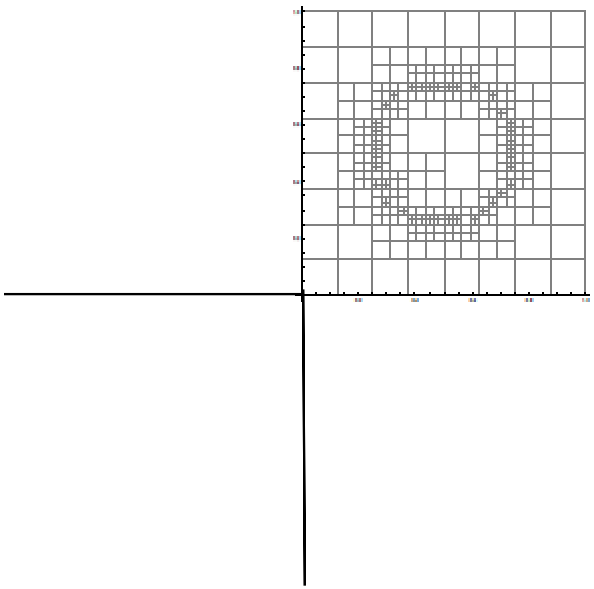
Scaled Jacobian

Remarks:

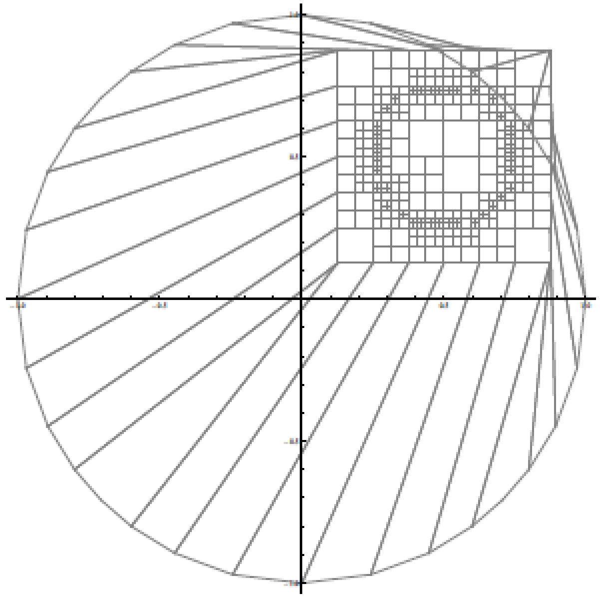
- In 2-D: Problems could appear in the corners
- In 3-D: Problems could appear in the corners and edges

Final Comments and Future Works

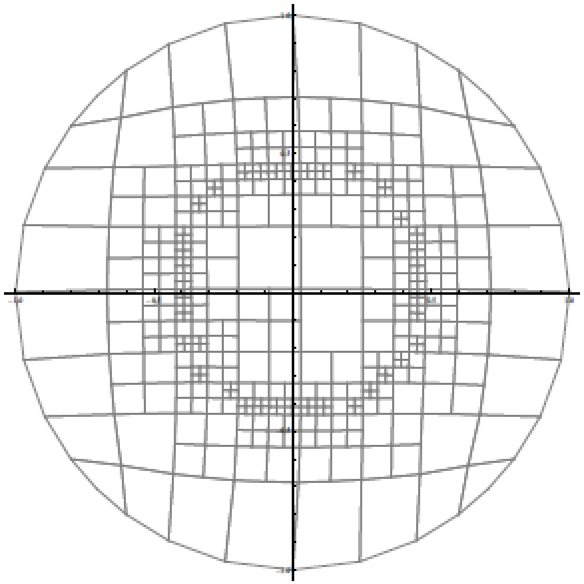
The Meccano Method for T-mesh: T-spline Optimization



Parametric space
(Meccano T-mesh)



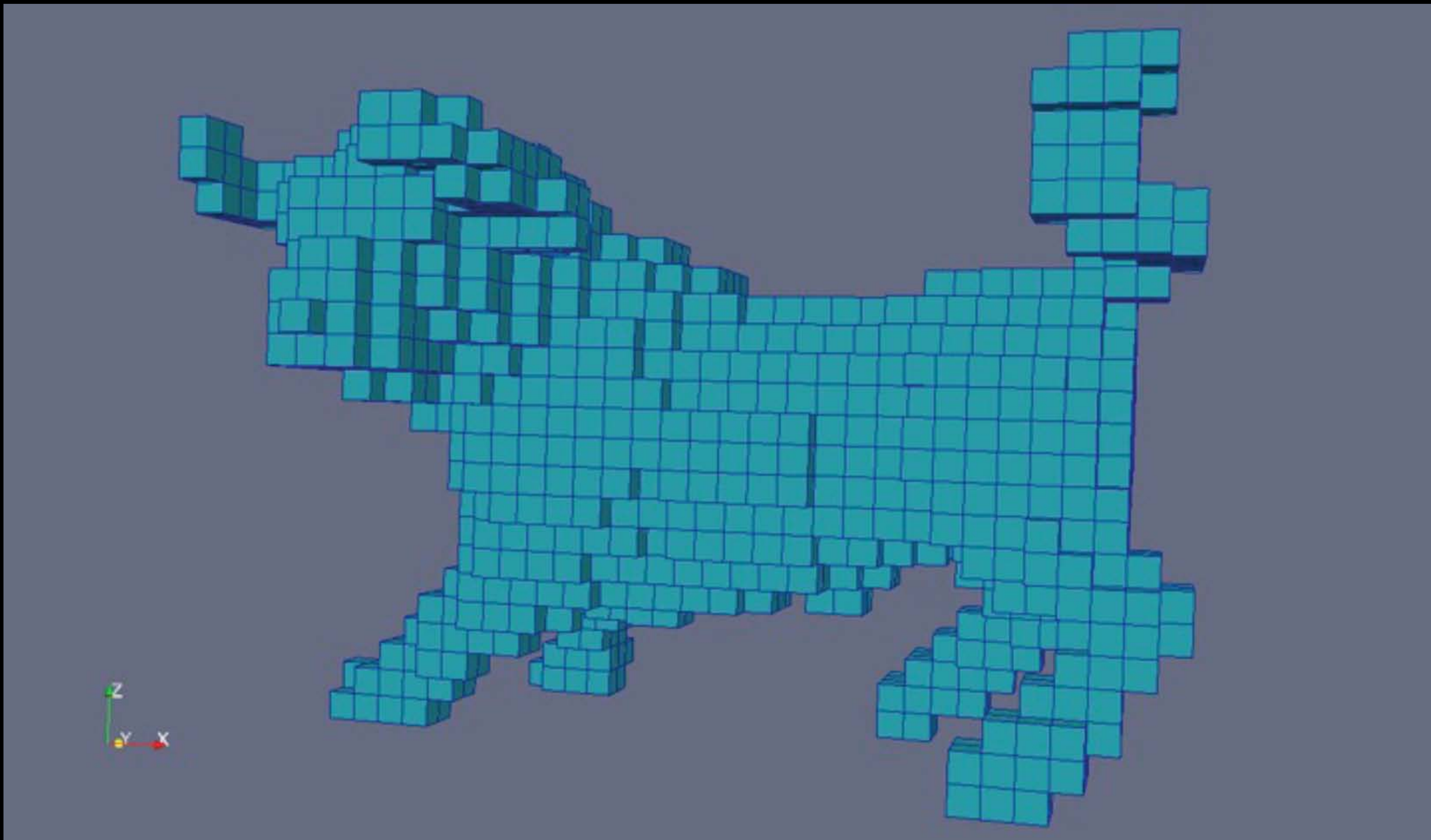
Physical space
(Tangled T-mesh)



Physical space
(Optimized T-mesh)

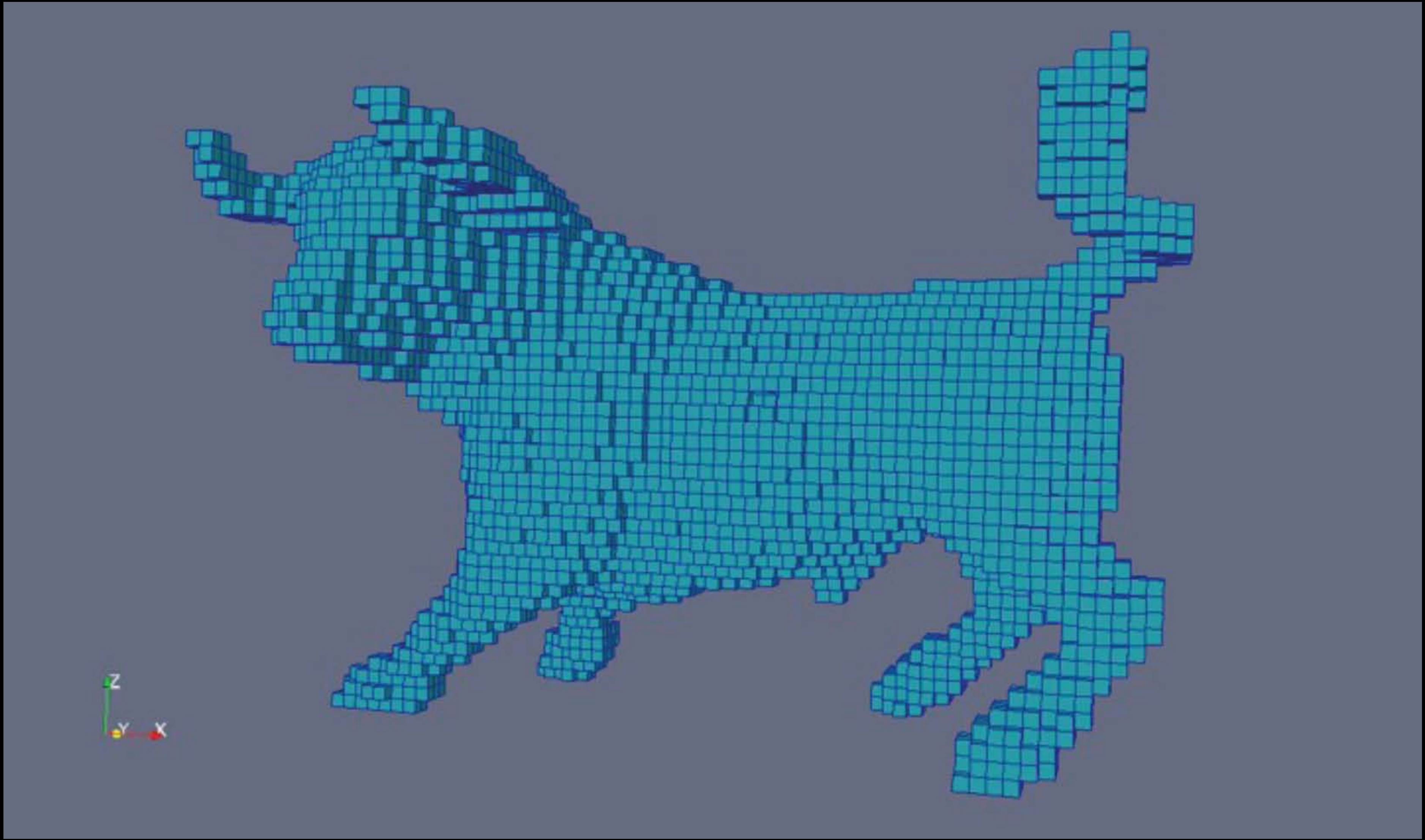
Final Comments and Future Works

Automatic Construction of the Meccano



Final Comments and Future Works

Automatic Construction of the Meccano



Conclusions

The meccano method can give a way to automatic

mesh generation

volume parameterization

adaptive isogeometric analysis

of complex solids

...????

Application to Complex Solids of Adaptive Isogeometric Analysis using T-splines

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The Meccano Method for 3-D Mesh Generation

Algorithm Steps: Given a closed surface of a solid

SIANI

1. Construct a meccano formed by polyhedral pieces; user.

2. Define an admissible mapping between the meccano boundary faces and the solid boundary; Floater, Polycube-maps, …

3. Construct a coarse tetrahedral mesh of the meccano.

4. Generate a local refined tetrahedral mesh of the meccano, such that the mapping (according step 2) of the meccano boundary triangulation approximates the solid boundary for a given precision; Kossaczky.

5. Move the boundary nodes of the meccano to the solid surface according to the mapping defined in 2.

6. Relocate the inner nodes of the meccano.

7. Optimize the current tetrahedral mesh by applying the simultaneous untangling and smoothing procedure; Escobar et al.

Meccano Technique for a Complex Genus-Zero Solid

Refinement Criterion

- Given a Kossaczky tetrahedron ***T***
- Given a significant point ***P*** of face ***abc***,

► **We refine *T* if**

Volume (a'b'c'P') > *ε*