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Cooperation in collective dilemmas under opinion-based risk perceptions

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ABSTRACT

Collective risk dilemmas are evolutionary games in which every player can contribute some amount to avoid a certain risk of failure. The main goal of this study is to integrate, within a collective dilemma, the influence, evolution, and formation of the perceived risk by individuals of the population. In order to understand the effects of subjective evolving opinions about risk perception in the evolutionary game, we pair a traditional collective game model of homogeneous groups with a network of players evolving their opinions or perceptions about the risk of common failure. We study the evolution of the players' perception about the risk and how different network topologies and opinion models, with and without considering the outcome of the game, affect the output of the evolutionary game. We show that cooperation generally increases when the evolution of opinions leads to consensus under unimodal and polarized initial opinions, for all the evaluated scenarios. Even when the population has similar mean final opinions, the transition of the opinions affects the final cooperation of the game. These findings highlight the practical relevance of peer opinion exchange between agents in real dilemmas as a mechanism to increase cooperation and avoid collective failures.

1. Introduction

Preventing catastrophic events often requires individuals to cooperate for the collective good. Large-scale collective action is particularly difficult due to the presence of stressors or factors counteracting cooperation, such as anonymity of contributors, lack of accountability and uncertainty [1]. This is challenging in climate social dilemmas, as there is a need for a contribution, tempting free riders to take advantage of the efforts of others [2]. One of the most significant element is to find a trade-off between selfish interests and a common good for the entire population [3]. An example of mitigating climate change is the decision to reduce carbon emissions, which is one of the most important applications of climate dilemmas [3–6].

Specifically, public good games are evolutionary game models in which players interact in groups and constitute a convenient modeling framework to obtain valuable information on how to cooperate to prevent and mitigate the effects of climate change and disasters influenced by climate conditions [7]. Evolutionary game theory provides a comprehensive mathematical framework for studying such social interactions, especially in the context of social dilemmas [8], where individuals can choose whether or not

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to cooperate with their peers, defining the population's preferences to unilaterally or mutually defect or cooperate [9]. The set of applications of evolutionary games is immense, from ecological applications [10], to pandemics [11] and economic problems [12,13].

Within public good games, there is a theoretical framework specifically designed to avoid collective risks, called the collective risk dilemma (CRD) [14,15]. CRDs are multiplayer public good games in which players are distributed in groups and can contribute some amount to avoid a certain risk of failure in their group. In these games, the players' perception of risk condition the levels of cooperation achieved by the groups. Cooperation (that is, obtaining an acceptable group achievement in the population) is easier when the risk is high than when it is low. Different mechanisms such as punishment, local institutions, or seeding punishment players have been proposed to increase cooperation with low risk level [15–17].

All of the latter models considered a fixed risk value for the whole evolution of the game. However, risk is only a perception of the players about the likelihood of failure if the total contribution in a group does not achieve a certain threshold. The perceived risk by players is subjective and a form of opinion that changes over time about the risk of a given hazard [18]. Individuals recursively review their perception of risk by processing these different sources of information through their own risk sensitivity [19]. In addition and in more practical and realistic applications, players can be conditioned to other different sources of information (or misinformation), such as local risk perception shared by the dynamic social influence network [20,21].

In the case of the risk perception evolution, the spread of misinformation or fake news is a key phenomenon. In this regard, the structure of the interaction network influences the spread of misinformation, favoring the creation of echo chambers, in other words, positive correlation of opinions and network segregation [22,23]. Some experiments show that misinformation is more likely spread in ideologically segregated networks than in more integrated social networks [24]. This may influence climate change risk dilemmas, where correlation of opinions and ideology is observed [25]. Some recent models represent the diffusion of misinformation by adapting classical epidemiological models [26]. Other models analyzes the effect of misinformation in an evolving cooperative game on a spatial network with migration and finds that misinformation impedes the formation of stable clusters of collective cooperation [27].

Opinion dynamics (OD) is an interdisciplinary field that analyzes the mechanism of adopting and changing opinions [28]. In this context, agents change their opinions when interacting with each other and some macroscopic phenomena can emerge, such as consensus, fragmentation, and polarization. One interesting behavior of agents is stubbornness or the reluctance to change their opinion. In the presence of such agents, the system drives to fragmentation, where opinions are concentrated in the neighborhood of stubborn agents' opinions [29]. Aging or the inertia to maintain past opinions is a milder form of stubbornness that favors consensus, as shown in the case of the voter model [30].

Some recent studies have included a dynamic mechanism for players' perception of risk in CRDs. This is the case, to the best of our knowledge, of the first work in which a feedback mechanism between cooperation and risk perception has been studied [31]. In this study, the risk of collective failure is lower in a highly cooperative society but becomes significant in the opposite case. Other studies also incorporated this dynamic evolution of risk perception, related to the level of cooperation between groups [32,33]. Another recent study proposed a framework in which individuals revise their perception of risk by processing information broadcast by the institution and shared by peers, and accounts for heterogeneity in terms of individuals' trust in institutions, peers, and in their own risk sensitivity, but without playing an evolutionary game [18]. However, an OD process has not been used to represent the risk perception yet. Only a few works have combined OD and evolutionary games in contexts other than CRDs [34–37].

The general goal of this article is to integrate, within a mathematical CRD framework, the influence, evolution, and formation of perceived risk by individuals of the population following an OD process. The risk perception is a continuous value and both the initial and stabilized patterns of opinions can be obtained [38] through the OD process by having a consensus, polarization, or fragmentation of the opinion of the population. Each own player's risk perception will be the one used when playing the public game in the group and calculating the payoffs of the game. The perception of risk of every player can evolve from their initial value, influenced by peers through the relationships of peers and social networks [39–41], and by the outcome or payoff of the player in her/his group.

The experiments of the paper involve different OD models (with and without payoffs dependence) and parameterization to generalize our results as well as population opinion profiles (e.g., non-polarization and polarization in the initial players' perception). Furthermore, we study how real data-based networks and synthetic homogeneous and heterogeneous network topologies impact the evolution of OD during the simulation of the evolutionary game. The main contributions and novel aspects of this work are as follows:

1. In this paper, three OD models are considered: the well-known Hegselmann–Krause (HK) [42] model, the HK model with stubbornness [43,44], and a novel HK with payoffs-dependent stubbornness. The three models help to generalize our results and determine the evolution of the risk perception of each player, linked to the CRD evolutionary dynamics. We build a CRD with homogeneous groups and an independent social network to model the OD evolution of the players and the strategy update rule to understand the effects of subjective evolving opinions about risk perception in the evolutionary game.
2. A complete set of experiments under 11 different network topologies and extensive parameter configurations is used to compare the static CRD, where risk is fixed during the whole simulation, with respect to the dynamic evolution of the players' perception or opinion about the risk. The found patterns can affect the outcome of the evolutionary game and how players interact in groups and achieve higher or lower cooperation in a CRD.
3. This analysis is particularly relevant for real applications of CRDs, such as in the case of climate dilemmas and natural disasters, where agents (countries, institutions, or stakeholders) have heterogeneous interests and risk perceptions. In that way, the CRD, instead of setting an exogenous fixed value for the risk of a disaster, is enriched with more realistic features. Unlike previous studies that assumed a dynamic but identical risk perception of all players, we propose to provide game players with heterogeneous risk perceptions, modeled as real-valued evolving opinions.

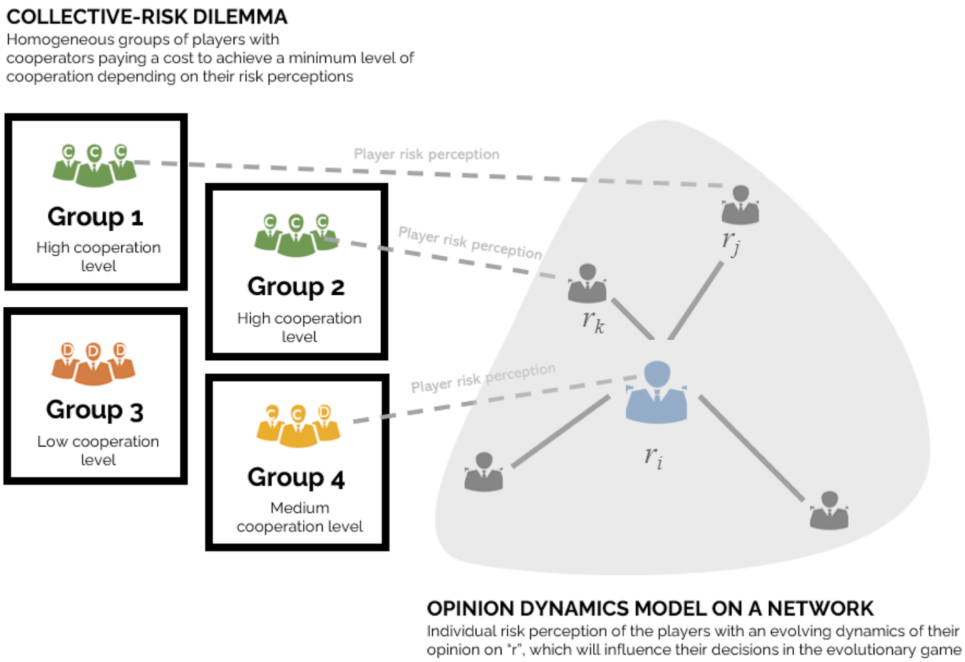

Fig. 1. Illustration of the opinion-based CRD with risk perceptions. Groups are homogeneous and opinions evolve in a separate social network layer having a with the obtained payoffs in the group.

The mathematical definitions of the CRD model with heterogeneous risk and the OD model are described in Section 2. Section 3 defines the simulation-based methods and the experimental setup used for the experiments. Later, Section 4 presents the main results for the proposed methods. The paper ends with the final remarks and practical implications in Section 5; and main limitations together with future works in Section 6.

2. Methods for CRD and risk opinions dynamics

Our proposal is to pair a traditional CRD with homogeneous groups for public-good games with a network of players who evolve their opinions on their perceptions of risk, also coupled with the payoffs obtained by the players. Fig. 1 shows an illustrative example of the formation of groups with heterogeneous perceptions of risk together with the evolving risk opinions network. In the following paragraphs, we will give the details of each of the elements of the proposed model. First, Section 2.1 describes the CRD with heterogeneous risk perceptions by players, and Section 2.2 explains how OD models are used to evolve opinions on risk perception. Later, the evolutionary update strategy is described in Section 2.3.

2.1. CRD definition with heterogeneous risk perceptions

The CRD model considers a finite population of Z players who, at each time-step t, participate in a public goods game, continuing for up to T rounds. The population is partitioned into homogeneous groups of size N, with Z being an integer multiple of N [45]. Every player i belongs to exactly one group k and can choose between two strategies: cooperation (C) or defection (D). Cooperators incur a cooperation cost c to sustain the public good in their group k. Defectors, in contrast, do not contribute but still enjoy the benefits generated by the group, effectively free-riding on the cooperators' efforts.

CRDs rely on a risk of failure that arises when the number of cooperators j_C in a group fails to reach a minimum threshold defined by the parameter n_{pg}. Consequently, players' payoffs strongly depend on the perceived risk of failure $r \in [0, 1]$, where 0 represents an almost negligible risk in the near future and 1 corresponds to the maximum perceived likelihood of failure (i.e., cooperation is more strongly justified, making the game effectively easier for cooperators). However, this risk perception is subjective, so players may disagree about the probability of failure. Accordingly, our CRD model incorporates heterogeneous individual risk perceptions r. Each player is thus characterized by an individual risk perception r_i, which may differ from that of other players, even within the same group k.

Given these definitions of the CRD, the expected payoff π_i (or fitness, in the context of the evolutionary process) for a player i depends on whether they adopt strategy C or D. The payoff expressions for the two strategies, Π_i^C for cooperation and Π_i^D for defection, are the same as those introduced in Vasconcelos et al. [16] and are written as follows:

$$\Pi_i^C = -c + \Theta(j_C - n_{pg}N) + (1 - r_i)(1 - \Theta(j_C - n_{pg}N)), \tag{1}$$

$$\Pi_i^D = \Pi_i^C + c, \tag{2}$$

where N is the size of the specific group and r_i is the perception or opinion of risk by the player i. $\Theta(x)$ is the Heaviside step function, equal to 0 whenever $x < 0$ and equal to 1 otherwise. Finally, we assume that the initial endowment of each player is set to 1.

2.2. OD models for risk perception

OD models deal with the evolution of continuous opinions, such as risk perception $r_i \in [0, 1]$. These models also offer a way to achieve consensus, polarization (that is, two main clusters or groups of opinions) and fragmentation (that is, more than two clusters). The DeGroot model [46] is considered the classical one and explicitly describes the process of reaching consensus by a linear combination of the initial opinions of the agents.

Let $r_i^t \in [0, 1]$ be the opinion or risk perception of an agent $i \in \{1, \dots, Z\}$ at time-step t, and let w_{ij} be the weight agent i assigns to agent j, satisfying $w_{ij} \geq 0$ and $\sum_{j=1} w_{ij} = 1$. Then, the update rule for the opinion of agent $i \in \{1, \dots, Z\}$ is given by:

$$r_i^{t+1} = \sum_{j=1}^{Z} w_{ij}\, r_j^t, \qquad t = 1, 2, \dots, T \tag{3}$$

The compact form can be written as $R^{t+1} = W \times R^t$, $t = 0, 1, \dots, T$, where $W = (w_{ij})_{Z\times Z}$ and $R^t = (r_j)_{Z\times 1}$. In the DeGroot model, the set of weights W is constant. This model is also used in a social network to determine when consensus is reached from the linear combination of the opinions of the agents in the network [47].

2.2.1. The HK model

We adopt here the HK model [42], a well-known extension of the DeGroot model, for the OD. This model is framed in the bounded confidence models [42], where the weight matrix $W^t = (w_{ij}^t)$ changes over time through interactions between agents. The main assumption is that agents interact with others when their opinions are similar, and this similarity is given by a threshold or confidence level. Having again r_i^t the opinion of an agent i in the time-step t, the parameter ε is the bounded confidence level, normally homogeneous for all agents in the population Z.

The HK model [42], compared to the linear DeGroot model, is a general non-linear model, in which the weights depend on the evolution of opinions. In the HK model, each agent i only modifies its opinion taking into account those agents having similar opinions while ignoring agents with sufficiently different opinions. Agents synchronously update their opinions by averaging all opinions in their confidence sets (i.e., a set formed by agents with similar opinions).

Then, according to the HK model, the weight w_{ij}^t an agent i gives to the opinion of agent j in time-step t is defined by Eq. (4):

$$w_{ij}^t = \begin{cases} 1/||S_i^t||, & \text{if } j \in S_i^t. \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

where S_i^t is the set of agents belonging to the set of bounded confidence levels of agent i ($S_i^t = \{j,\ |r_i^t - r_j^t| \leq \varepsilon\}$) and ε the confidence level. $|.|$ denotes the absolute value of a real number and $||.||$ denotes the number of elements in a set. Please take into account that the agent itself is included in this set S_i. Then, the agent's opinion i is updated by Eq. (5):

$$r_i^{t+1} = \sum_{j\in S_i^t} w_{ij}^t r_j^t. \tag{5}$$

2.2.2. The HK model with stubbornness

A significant extension of the HK model is that it allows agents to exhibit a certain degree of stubbornness [44], or attachment to their own opinion or perception of risk [29,43]. The parameter $\alpha \in [0, 1]$ is a weight of the influence of the agent i current opinion r_i^t relative to that of its neighbors. We adopt the model where α is time-invariant and global to all agents to simplify the dynamics. The higher values of α indicate stronger self-persistence and the lower values correspond to a greater susceptibility to social influence. In particular, $\alpha = 1$ produces a fully stubborn agent whose opinion remains unchanged, while $\alpha = 0$ represents an agent who updates its opinion solely based on those of its neighbors.

Agents define confidence sets and interaction weights as in the standard HK model (see Eq. (4)), but update their opinions according to Eq. (6):

$$r_i^{t+1} = \begin{cases} \alpha r_i^t + (1-\alpha)\dfrac{\sum_{j\in S_i^t\setminus\{i\}} r_j^t}{||S_i^t||-1}, & \text{if } ||S_i^t|| > 1 \\ r_i^t, & \text{if } ||S_i^t|| = 1 \end{cases} \tag{6}$$

Please note that, in this formulation, the agent's own risk perception is excluded from the bounded confidence averaging step. As a consequence, setting $\alpha = 0$ does not recover the standard HK model, defined in the previous sub-section.

2.2.3. The HK model with payoff-dependent stubbornness

Building on the HK model with stubbornness, we propose a novel variant in which the stubbornness of each specific agent evolves over time based on their payoff outcomes in the CRD model. Thus, we extend the parameter mentioned above α to α_i^t. This dependence between payoff realization and stubbornness introduces a feedback loop where both the OD and the CRD dynamics influence each other.

Specifically, agents become more attached to their risk perception (that is, more stubborn) when their payoff is higher. The update of the opinion still follows Eq. (6), but at the end of each time-step t, the stubbornness α_i^t is updated according to the payoff of the current agent π_i^t, as in Eq. (7):

$$\alpha_i^t = 1 - \frac{\pi_{i,t}^{\max} - \pi_i^t}{\pi_{i,t}^{\max} - \pi_{i,t}^{\min}}, \tag{7}$$

where $\pi_{i,t}^{\max}$ and $\pi_{i,t}^{\min}$ denote the maximum and minimum possible payoffs that can be obtained by agent i at time step t in her/his group, given their current perception of risk r_i^t. Consequently, $\alpha_i^t = 0$ when $\pi_i^t = \pi_{i,t}^{\min}$ and $\alpha_i^t = 1$ when $\pi_i^t = \pi_{i,t}^{\max}$.

2.3. Strategy update using focal player risk perception

After completing a round at time-step t, player i revises their strategy based on the payoff obtained from their group. The evaluation and updating of the strategy follow Fermi function [48]. The Fermi rule is a stochastic pairwise comparison mechanism in which strategies that yield higher payoffs are more likely to be copied and spread through the population.

More specifically, let π_i denote the payoff of player i at a given time (we omit the time index for simplicity). The player then compares this payoff with that of other players, under the assumption that the focal agent i evaluates outcomes according to their own risk perception level. Accordingly, we denote by π_{ji} the payoff of player j computed using the focal agent's risk perception level r_i.

Agents can imitate only those peers who lie within their social network. Hence, player j is chosen at random from among the neighbors of player i. Player i then adopts the strategy of player j with a probability p that increases with the payoff difference—$(\pi_{ji} - \pi_i)$—and is given by Traulsen et al. [48] as:

$$p = \frac{1}{1 + e^{-\beta(\pi_{ji} - \pi_i)}}. \tag{8}$$

The free parameter β represents the selection of the evolutionary intensity and captures the probability of errors during the imitation process. Consequently, the player i can adopt the strategy of another player j even when the payoff of j is lower.

The evolutionary game dynamics of a CRD also incorporates a mutation function, which enables random changes in players' strategies at each time-step t. Specifically, the player i can switch to a random strategy with mutation probability p_{mut}; while, with probability $1 - p_{\text{mut}}$, the player can mimic the strategy of another randomly selected agent j, according to the Fermi function introduced above.

3. Methods details and experimental setup

We describe the main parameters and configuration of the scenarios used for our experiments to ensure reproducibility. First, Section 3.1 describes the agent-based computational environment. Later, Section 3.2 specifies the configuration of the collective dilemma. Finally, Sections 3.3 and 3.4 describe how we generate the initial opinions and synthetic networks, respectively.

3.1. Computational framework for the simulations

The experiments are based on Monte Carlo (MC) agent-based simulations [49,50], performed in computer clusters to obtain the stationary states of the model specifications. The simulation software is programmed in Python. Evolution proceeds in discrete steps involving the payoff calculation and the imitation update rule for all players in each group. All specifications of the model are run for 30 independent MC realizations and for a maximum number of 5×10^3 synchronous time-steps, reaching a stationary stable state and having a low MC realizations' deviation. The presented results are obtained by averaging the last 25% of the simulation time-steps in the independent MC realizations.

3.2. Specific configurations of the CRD

We employ a population of $Z = 10^3$ individuals in all experiments. The mutation probability p_{mut} is set to 0.01. In addition, the parameter β of the Fermi update function is fixed at 5 in all experiments. With respect to the CRD settings, the minimum coordination threshold within a group is set to $n_{pg} = 0.75$, and the size of the group is $N = 5$.[1] At the beginning of the simulation, the strategies are evenly distributed in the population (50% cooperators and 50% defectors). The rest of the parameters of the game are specified

[1] The same settings with larger group sizes were tested and resulted in full defection outcomes.

in the respective sections. For the HK model, the parameter ε is responsible for controlling the evolution of the consensus of opinions and will be described in the experiments.

Finally, we define the main performance indicator as the final fraction of cooperators in the population, f_C, to evaluate how cooperation evolves.

3.3. Opinion generation of the risk perceptions

In contrast to standard CRD models, we consider heterogeneous risk perceptions. Therefore, instead of a single scalar value, the input risk parameter is modeled as a probability distribution. Our experiments include both unimodal and bimodal input distributions. For the unimodal case, we consider the truncated normal distributions in $[0,1]$, parameterized as $N(\mu_0, \sigma_0^2)$. Importantly, we distinguish between the distribution parameters (μ_0, σ_0^2) and the resulting sample mean and variance ($\bar{r}$ and s_r^2), which may differ substantially due to truncation, particularly when μ_0 is close to the boundary and σ_0^2 is large. As a limiting case, when $\sigma_0^2 = 0$, the model reduces to homogeneous CRD with fixed perception $r = \mu_0$. The bimodal input distributions are designed to reflect a polarized scenario in which most agents belong to one of two clusters: one concerned and the other unconcerned. To this end, we initialize the population with U-shaped Beta distributions, $\mathrm{Beta}(\alpha, \beta)$, where $\alpha, \beta < 1$.[2]

In addition, we measure the final average perception of risk, $\bar{r}$, and its variance, s_r^2. The latter metric allows us to assess whether perceived risk in the population has converged to a consensus or remains fragmented/polarized.

3.4. Network generation for peer interaction

Both the selection of peers for imitation and the evolution of risk perception opinions are constrained by a social network connecting all agents in the population. In the OD model, this implies that $w_{ij}^t = 0$ if there is no link between agents i and j. If a link exists, the weight w_{ij}^t is computed according to Eq. (4).

The social network topology is scale-free (SF) with an average degree $\langle k \rangle = 4$ and density 4×10^{-3}, unless other specifications are given in the experiment. To avoid structural artifacts, we perform MC runs on multiple network realizations.

The network was generated using the Barabási-Albert algorithm [51] with $m = 3$. Specifically, the Barabási-Albert algorithm is a network growth model used to generate graphs exhibiting a SF degree distribution, characterized by the presence of hubs (nodes with an exceptionally high number of links). The procedure starts with a small initial network of m_0 nodes. The network then grows iteratively by adding new nodes one by one. Each new node connects to m existing nodes in the network, where $m \le m_0$. The key to the algorithm lies in the connection mechanism, which is based on the principle of preferential attachment: the probability that a new node connects to an existing node i is proportional to the degree (number of links) k_i of that node. This process ensures that nodes that are already highly connected become even more connected, leading to the formation of the SF network structure.

4. Results

In this section, we first compare cooperation levels for CRDs when having initial unimodal distributions of risk perception in Section 4.1. In Section 4.2, we further explore the main insights when starting in a polarized population of players about risk perception. Next, we evaluate the impact of the social network topology underlying the OD mechanism in Section 4.3. Finally, in Section 4.4, we observe the outcomes when considering stubbornness in the underlying OD mechanism, both linked and unlinked to the payoffs calculation.

4.1. Cooperation levels under initial unimodal risk perception distributions

We evaluate here the impact of the OD mechanism on cooperation with respect to a CRD with static risk perceptions, assuming initially unimodal distributions. For this purpose, we use the standard HK model described in Section 2.2.1 as the OD mechanism. Fig. 2 summarizes the results of cooperation across a wide range of truncated normal distributions (with varying mean and variance) and OD intensities, controlled by the confidence threshold ε.

In general, cooperation increases gradually as ε grows, reflecting that the stronger consensus achieved by the OD mechanism with interaction among individuals with highly divergent opinions enhances cooperation. Importantly, the average risk perception $\bar{r}$ is almost always unchanged, so a consensus is reached around the initial mean. To illustrate how OD modifies risk perceptions, Fig. 3 tracks the distribution dynamics for $\varepsilon = 0.1$ and $\varepsilon = 0.5$, starting from $N(0.2, 0.5^2)$. The corresponding cooperation outcomes are those in the middle column of the upper left plot in Fig. 2. Overall, there is no scenario configuration in which the OD mechanism leads to a noticeable reduction in cooperation. In the worst case, f_C decreases are always below 0.1.

The upper left plot in Fig. 2 corresponds to the most difficult scenario ($\mu_0 = 0.2$), where the population is initially biased toward low perceptions of risk. Here, the OD mechanism produces a notable gain: the cooperation increase reaches 0.4 percentage points when $\sigma_0 = 0.5$ and $\varepsilon = 0.4$, and remains high at 0.38 for $\varepsilon = 0.5$. A similar pattern is observed for larger initial distributions with a higher parameter σ_0 (right columns). In contrast, when the initial distribution is narrower (left columns), there is no increase in

[2] Please do not mislead the α parameter of the Beta distribution with the stubbornness level α in the OD process described in Section 2.2.2.

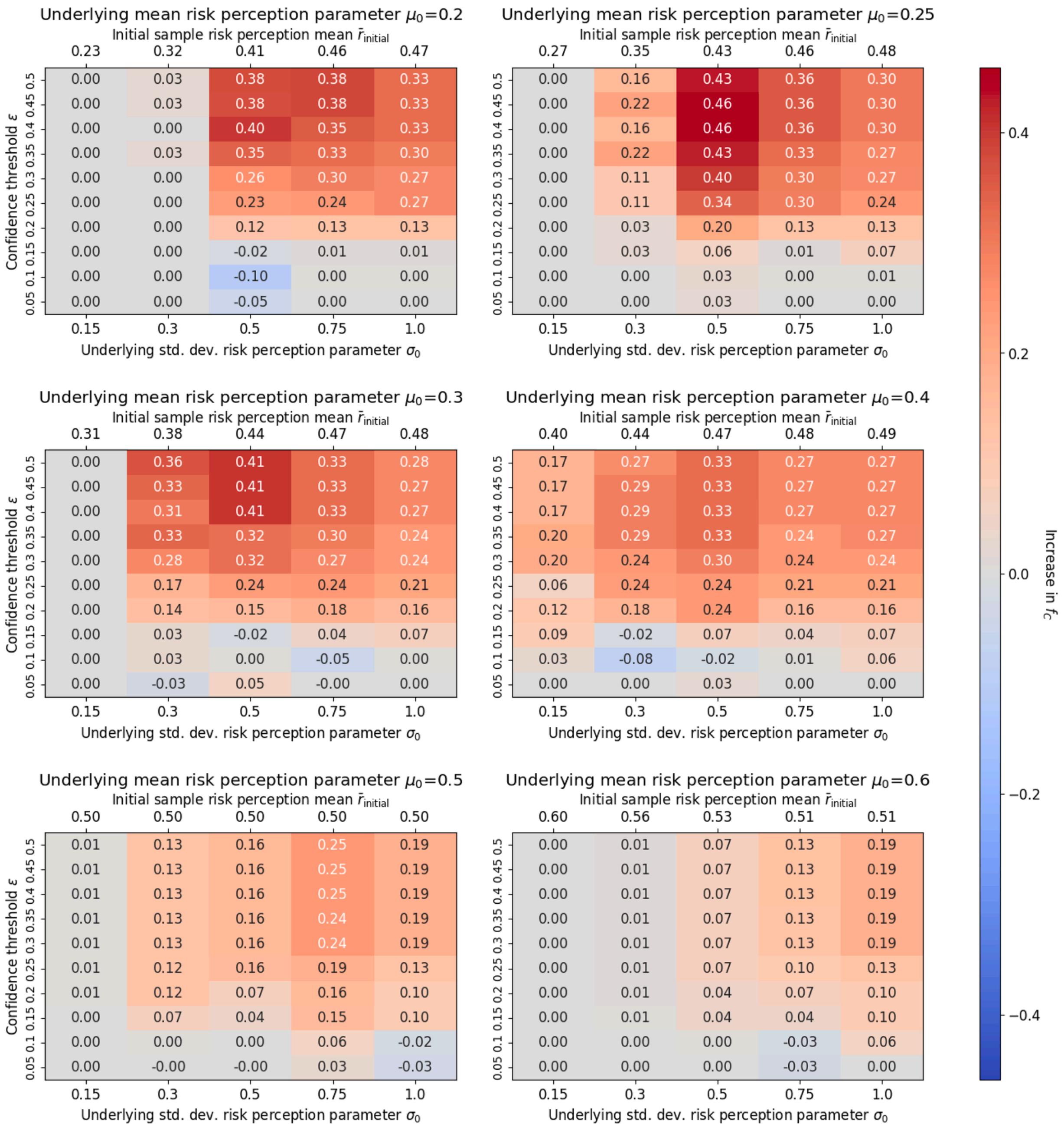

Fig. 2. Heatmaps showing the difference between the cooperation final frequency f_C for different parameter values of the used HK model and unimodal distribution around a mean risk perception $\bar{r}_{\mathrm{initial}}$ and the case of static risk perception in the same value. Cooperation generally increases with the OD strength (ε). The most remarkable increases are observed when the initial $\bar{r}$ is around 0.38 and 0.48, provided that the initial distribution is sufficiently sparse.

cooperation. The latter is unsurprising, given that perceptions are already clustered, leaving little room for the OD mechanism to alter the baseline scenario.

For distributions with a higher mean parameter μ_0, the same trend (greater cooperation with a stronger OD mechanism) holds although it becomes less pronounced for $\mu_0 \geq 0.5$. However, because truncation can make μ_0 misleading, attention should be paid to the mean of the real sample $\bar{r}$. If we look at Fig. 2, increases in cooperation exceed 0.3 when $\bar{r}$ lies between approximately 0.38 and 0.48, provided that the variance is sufficiently large. This suggests the existence of an approximate “consensus window” of risk perceptions: when opinions diverge around this range, cooperation increases significantly in the case of a fixed and invariant risk value.

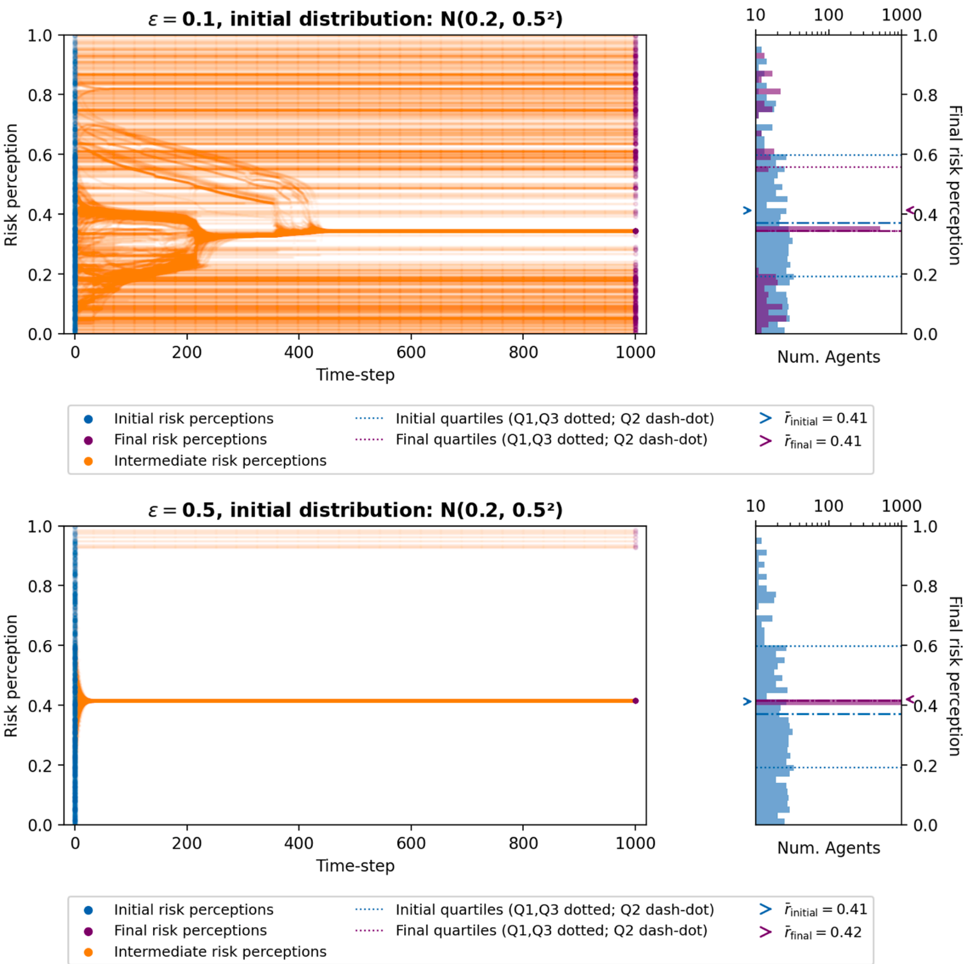

Fig. 3. Risk perceptions evolution over time for $\varepsilon = 0.1$ and $\varepsilon = 0.5$ using the OD model, with initial risk perceptions sampled from a truncated $N(0.2, 0.5^2)$. Initial and final distributions are shown as vertical blue and purple bars. Dotted lines indicate Q_1 and Q_3, the dash-dotted line indicates the median, and the mean is marked externally. Plots show that the mean perception remains almost unchanged. For $\varepsilon = 0.5$, an (almost) complete consensus is reached at the mean (0.41). In contrast, for $\varepsilon = 0.1$, a partial consensus emerges below the mean, collapsing Q_1 and Q_2 in such value. The remaining opinions are fragmented: lower values cluster within the bottom 20% of the interval, whereas higher values remain spread across the broader range above Q_3.

A closer look at the OD evolution in Fig. 3 shows that the mean remains stable in both cases, whereas the distribution evolves differently depending on ε. With weak OD ($\varepsilon = 0.1$), fragmentation persists: a partial consensus is formed at a value below the mean (with collapsing quartiles Q_1 and Q_2). However, this scenario produces almost the same cooperation outcome as static risk perceptions (see Fig. 2). In contrast, with strong OD ($\varepsilon = 0.5$), opinions converge to the mean, and cooperation increases notably.

4.2. Cooperation levels under risk polarization

We now study the effect of heterogeneous and evolving risk perceptions when the population is initially polarized. Fig. 4 shows the cooperation results under this assumption, considering a wide range of ε values and three different U-shaped Beta distributions around a mean value of $\bar{r}_{\text{initial}} = 0.5$ and with varying variances, along with the final variances associated with risk perception.

The overall pattern mirrors that observed with unimodal distributions: stronger OD driven by higher values of ε tends to increase cooperation. However, the magnitude of this increase depends on the specific polarization scenario. The greatest increase occurs with the U-shaped Beta distribution with medium variance ($s_r^2 = 0.16$), where the OD mechanism with $\varepsilon = 0.5$ increases cooperation by 0.45 percentage points relative to the static risk perception case. In this scenario, the baseline cooperation frequency (without OD) is $f_C = 0.34$.

A comparable increase is found in the less extreme scenario ($s_r^2 = 0.13$). Here, static risk perceptions already sustain higher cooperation ($f_C = 0.46$), since a more dispersed initial distribution produces more intermediate agents with sufficient risk perception to

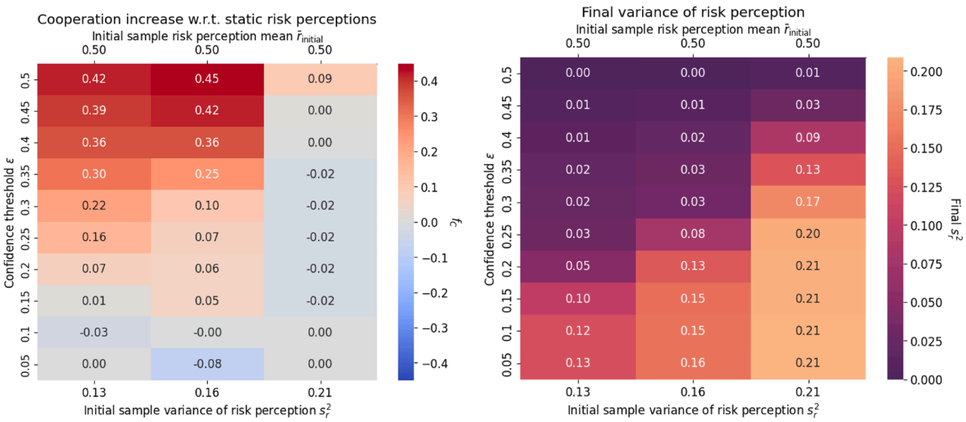

Fig. 4. Heatmaps showing the difference between the cooperation final frequency f_C (left) and final variance of risk perception (right) for different parameter values of the OD model and polarized initialization around a mean value of $\bar{r}_{\text{initial}} = 0.5$ and the case of static risk perception in the same mean value. The cooperation increases with the OD strength (ε). The most remarkable cooperation increases are observed in the polarized scenarios given by $s_r^2 = 0.16$ and $s_r^2 = 0.13$. Achieving the consensus (near zero variance) requires a higher ε when the initial distribution is more polarized.

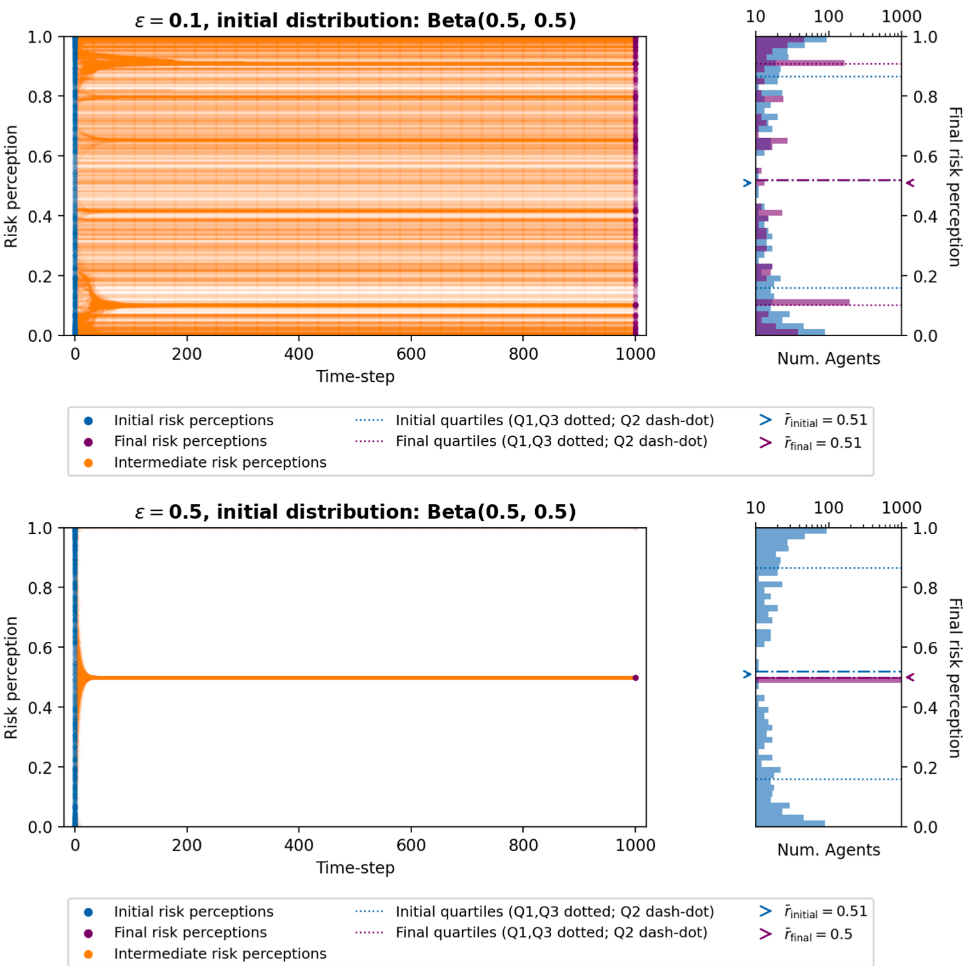

Fig. 5. Risk perceptions evolution over time for $\varepsilon = 0.1$ and $\varepsilon = 0.5$ using the OD model, with initial risk perceptions sampled from a bimodal Beta(0.5, 0.5) distribution with variance $s_r^2 = 0.13$. Initial and final distributions are shown as vertical blue and purple bars. Dotted lines indicate Q_1 and Q_3, the dash-dotted line indicates the median, and the mean is marked externally. Plots show that the mean perception remains almost unchanged. For $\varepsilon = 0.5$, a complete consensus is reached at the mean (0.5). In contrast, for $\varepsilon = 0.1$, risk perceptions remain bimodal, although the two modes move slightly closer together.

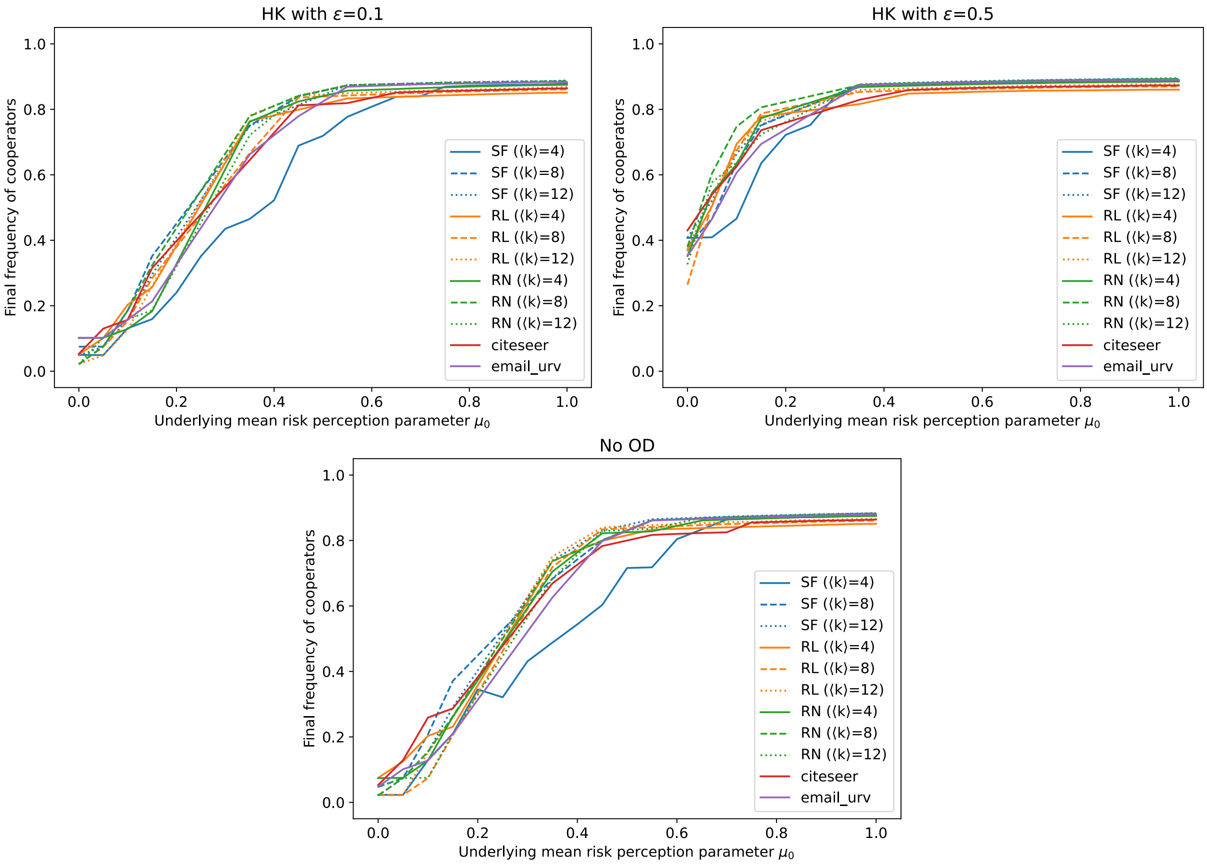

Fig. 6. Final frequency of cooperators f_C with and without OD for $\varepsilon = 0.1$ and $\varepsilon = 0.5$, across 11 social network topologies. Initial risk perceptions follow truncated normal distributions generated with $\sigma_0 = 0.5$ and varying μ_0. Results show that stronger OD mechanisms lead to higher cooperation. While the SF network with the lowest average degree shows slightly lower cooperation, this difference diminishes under the strongest OD intensity, confirming the overall robustness of the OD effect across topologies.

cooperate. In contrast, in the case of the most extreme polarization ($s_r^2 = 0.21$), the game simulation starts from a very low level of cooperation ($f_C = 0.05$) and still experiences an increase of up to 0.09 percentage points through the OD mechanism.

As shown in the right panel, in all three scenarios, the increase in cooperation is accompanied by consensus formation, reflected in the nearly zero final variance in risk perception. Therefore, the mechanism by which cooperation increases as consensus on risk perception emerges extends robustly from the unimodal to the polarized initial conditions.

Finally, Fig. 5 illustrates the evolution driven by the OD of risk perceptions for $\varepsilon = 0.1$ and $\varepsilon = 0.5$, with the initial U-shaped Beta distribution with variance $s_r^2 = 0.13$. When OD is stronger ($\varepsilon = 0.5$), consensus is reached around the mean perception of risk (0.5), similar to what is observed with unimodal distributions in Fig. 3. However, with $\varepsilon = 0.1$, polarization persists.

4.3. Cooperation levels under different social network topologies

Previous results were obtained using an SF network with $\langle k \rangle = 4$ to model OD interactions. We now continue our analysis by examining whether different network topologies, with varying degrees of heterogeneity, influence cooperation in the OD process. Specifically, we consider SF, Ring Lattice (RL) and Random Networks (RN). Each topology is analyzed with three different average degrees $\langle k \rangle \in \{4, 8, 12\}$, resulting in nine synthetic network topologies. RL and RN networks are generated with the small-world rewiring algorithm [52], using rewiring probabilities $p = 0$ and $p = 1$, respectively.

In addition, we include two real-world social networks. The first, *email-URV* [53], is based on email traffic at Rovira i Virgili University in Spain, with nodes representing email addresses and links representing email exchanges. It comprises 1133 nodes, has an average degree $\langle k \rangle = 9.61$, and a density of 9×10^{-3}. The second network, *CiteSeer* [54], is a citation network in which nodes represent scientific papers and links represent citations. It comprises 1681 nodes, with $\langle k \rangle = 3.45$ and a density of 2×10^{-3}.

Fig. 6 presents the cooperation results (f_C) for the 11 topologies. The analysis considers unimodal truncated normal input distributions with increasing mean parameter μ_0, two OD intensities ($\varepsilon \in \{0.1, 0.5\}$), and a baseline without OD. It should be noted that the social network also affects the strategy update process, so results differ even in the absence of OD. First, in all cases, we observe the

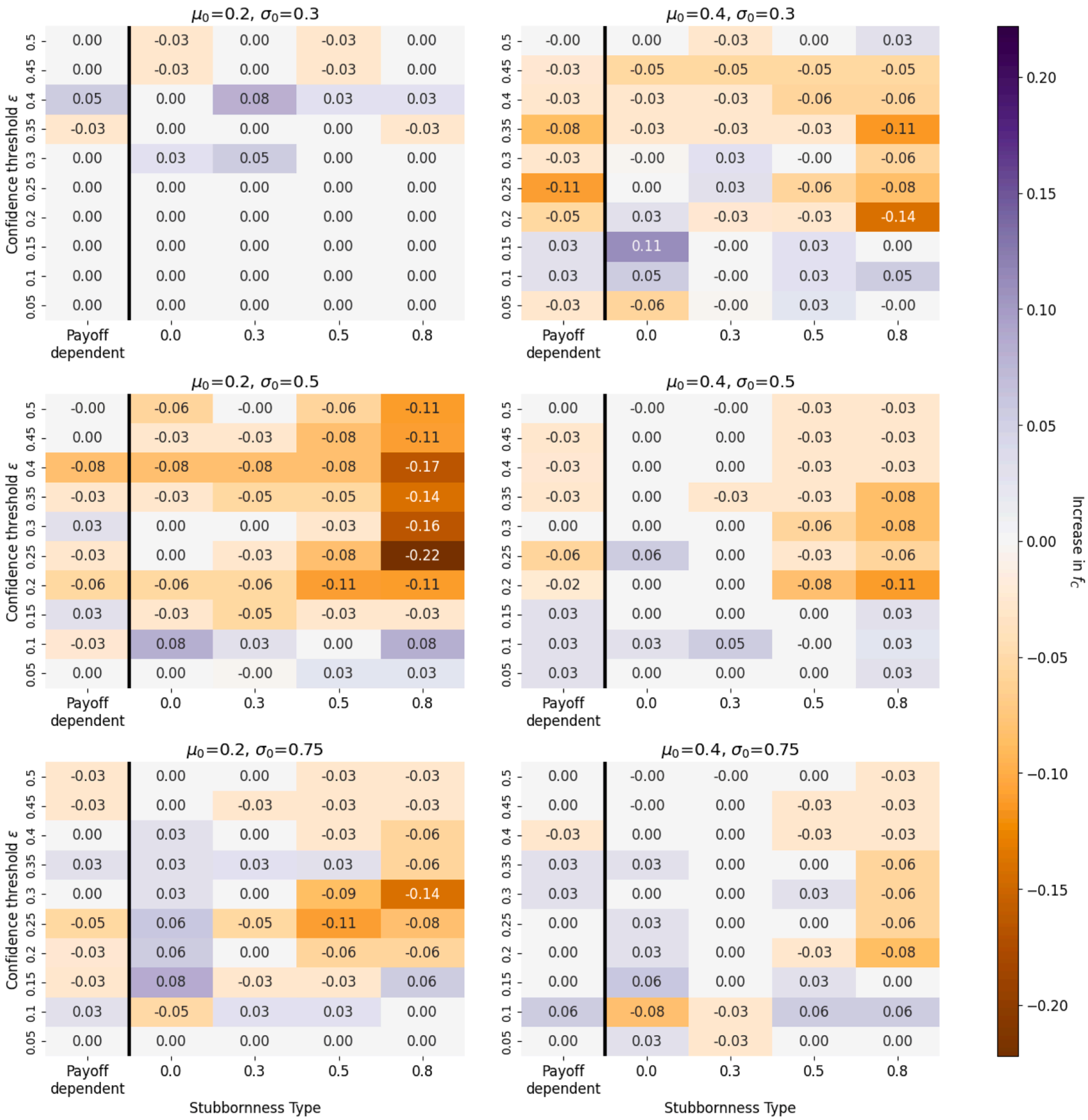

Fig. 7. Heatmaps showing the increase in cooperation for different configurations of the HK model with stubbornness relative to the standard HK model. The y-axis represents the OD intensity (ε), while the x-axis represents the stubbornness mechanism: the payoff-related (Eq. (7)) for the first column, and a static fixed α (Eq. (6)) for the remaining columns. Different heatmaps show different initial unimodal distributions with different μ_0 and σ_0. Overall, the OD mechanism shows no significant differences across configurations, with the most notable increase observed for $\alpha = 0.8$, particularly when $\mu_0 = 0.2$ and $\sigma_0 = 0.5$.

expected pattern: the higher the perceived risk (that is, the stronger the bias of the input distribution toward large values, controlled by μ_0), the easier the game becomes and, consequently, the higher the level of cooperation. Second, consistent with Sections 4.1 and 4.2, the OD mechanism leads to an increase in cooperation: a weak OD ($\varepsilon = 0.1$, top left plot), produces dynamics similar to the static perception baseline (blue). In contrast, a strong OD ($\varepsilon = 0.5$, top right plot) increases cooperation, particularly when initial perceptions are biased toward low values.

However, a key additional result is that these patterns are remarkably robust across all 11 topologies, without substantial differences. Only the SF network with the lowest average degree (solid blue line) exhibits slightly lower cooperation. However, this effect is specific to the strategy update process and diminishes under the strongest OD intensity ($\varepsilon = 0.5$). These results rule out

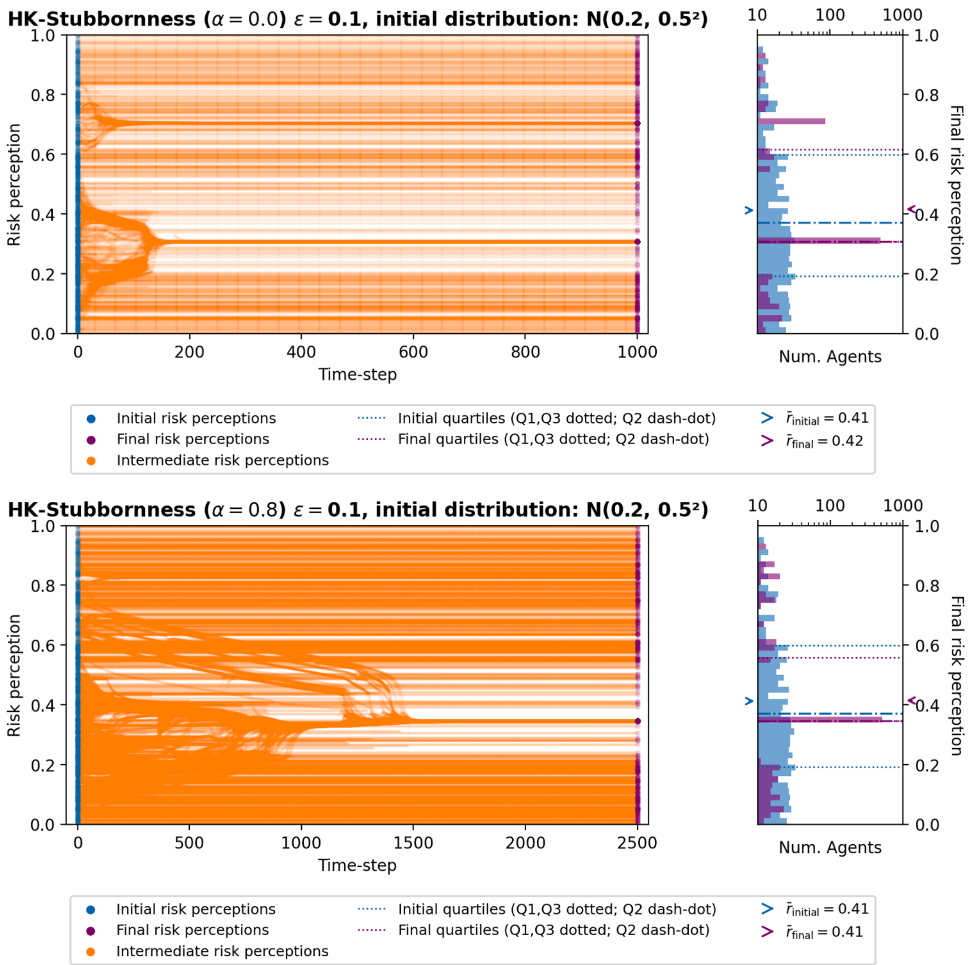

Fig. 8. Risk perceptions evolution over time for $\varepsilon = 0.1$ using the OD model with stubbornness level α of 0.0 and 0.8 (Eq. (6)). Initial risk perceptions are sampled from a truncated $N(0.2, 0.5^2)$. Initial and final distributions are shown as vertical blue and purple bars. Dotted lines indicate Q_1 and Q_3, the dash-dotted line indicates the median, and the mean is marked externally. Plots show that the mean perception remains almost unchanged. A partial consensus emerges below the mean, collapsing Q_1 and Q_2 in such value. When stubbornness is set to zero, a secondary cluster of risk perceptions emerges around 0.7. High stubbornness levels lead to slower convergence of risk perceptions.

topology-dependent effects and reinforce our main insight: populations with OD-driven consensus on risk perception achieve higher cooperation values than populations with static risk perceptions.

4.4. Impact of stubbornness in risk evolution on cooperation levels

In all experiments conducted so far, we consider the standard HK model as the OD mechanism. Finally, we explore an alternative OD mechanism to assess whether our results depend specifically on the standard HK model. In particular, we consider two extensions of the HK model with stubborn agents, in which agents exhibit an attachment (or stubbornness) level α to their previous risk perception. We analyze two variants: (i) a static version, where α is fixed and homogeneous throughout the population (see Section 2.2.2); and (ii) a payoff-dependent version, where the stubbornness of each agent at each time-step depends on its most recent payoff, so that more successful agents display greater stubbornness (see Section 2.2.3).

Fig. 7 reports the resulting differences in cooperation levels with respect to the standard HK model under unimodal initial risk perception distributions (see Fig. 2 for reference). We consider both the payoff-related stubbornness mechanism (left column of the heatmaps) and the static variant for four values of $\alpha \in \{0.0, 0.3, 0.5, 0.8\}$. Overall, we did not observe substantial deviations from the baseline HK results, particularly in the payoff-dependent extension, where the results are almost indistinguishable from the standard model.

Some minor effects emerge only in extreme cases. When $\alpha = 0.0$, differences tend to be slightly positive, indicating marginally higher cooperation than in the standard HK model, whereas for $\alpha = 0.8$ differences are generally negative, corresponding to slightly lower levels of cooperation. However, in general, these patterns are neither systematic nor quantitatively relevant.

The largest deviation is observed for an initial distribution with $\mu_0 = 0.2$, $\sigma_0 = 0.5$ and an intermediate confidence bound $\varepsilon = 0.25$, where cooperation decreases by 0.22 percentage points relative to the standard HK case. This outcome is equivalent to the one obtained with the absence of OD altogether (see the upper-left panel of Fig. 2). Even in this case, the general pattern remains unchanged: strong opinion convergence toward consensus enhances cooperation, but high stubbornness delays this effect.

To further examine how stubbornness affects the evolution of risk perceptions, we visualize opinion trajectories for a unimodal initial distribution with $\mu_0 = 0.2$ and $\sigma_0 = 0.5$. For $\varepsilon = 0.5$, the dynamics are essentially identical to those observed in the standard HK model. For illustrative purposes, Fig. 8 shows the evolution of $\varepsilon = 0.1$ and two extreme values of α (0.0 and 0.8). These cases correspond to the middle-left panel of Fig. 7, where both scenarios show a modest increase in cooperation of 0.08 percentage points. When $\alpha = 0.0$, a secondary opinion cluster emerges around 0.7. In contrast, for $\alpha = 0.8$, the final distribution of risk perceptions closely matches that of the standard HK model (see Fig. 3), with the main difference being a slower convergence toward equilibrium. However, these subtle variations in OD translate into negligible differences in cooperation results relative to those reported in Section 4.1.

Similar results are obtained under polarized initial conditions. In summary, apart from marginal effects at extreme stubbornness values (namely, a very slight increase in cooperation when agents are not stubborn and a mild reduction when stubbornness is high), the results obtained in this section closely align with those reported in Sections 4.1 and 4.2. This is particularly true for the payoff-dependent stubbornness mechanism, which yields outcomes nearly indistinguishable from the standard HK model. Consequently, these additional experiments reinforce the main conclusion of this work: in both sparsely unimodal and polarized heterogeneous risk perception scenarios, opinion exchange under OD mechanisms that promote convergence toward the mean perception tends to enhance overall cooperation.

5. Concluding remarks

We have studied the influence of subjective and evolving risk perceptions in CRDs. To this end, we extended a classical mathematical CRD model with homogeneous and static risk perceptions to a more practical and real model. Specifically, we integrated a social network (the same as the one used for the imitation rule, but independent of the CRD interaction groups) through which agents or players with heterogeneous risk perceptions update their opinions on risk of failure at each time-step via an OD mechanism. This extension also required redefining the evolutionary update: When an agent compares its payoff with others, both payoffs are calculated according to the risk perception of the focal agent.

The experiments were extensive in examining the implications of having evolving perceptions of risk with respect to the traditional model of invariant and exogenous risk value. We also compared 11 different social networks for the OD mechanism, covering various types of topology, average degrees, and densities. Additionally, we explore multiple intensities of OD (that is, varying confidence thresholds), OD extensions incorporating stubbornness, and alternative initial risk perception distributions, including unimodal populations with different means and variances, as well as bimodal polarized scenarios of varying intensities.

The key insight is that the evolution of population risk perception leads to a substantial increase in cooperation of up to 40%, compared to the case of static risk perception. This effect is obtained when the final consensus of the population (driven by the OD mechanism) is still around the mean of the initial distribution of risk values. The observed phenomenon emerges in unimodal scenarios with moderately low mean risk perceptions and sufficient dispersion for opinion change, as well as in polarized bimodal populations. Crucially, these results are valid regardless of the heterogeneity of the social network topology underlying the OD mechanism. Furthermore, the results remain consistent across different OD variants, with agent stubbornness acting as a mild regulator of opinion convergence, and thus of the resulting increase in cooperation.

These findings are useful in helping cooperation in the real dilemmas these models represent, such as climate change and other disasters. The players in these games (i.e., countries and institutions) have very different motivations and risk perceptions. The divergence of opinions on this matter is exacerbated by ideological positions, where misinformation can spread more widely. The main idea of this paper highlights the importance of communication and peer opinion exchange as a mechanism to foster cooperation and, in turn, to improve the effectiveness of the group in facing collective risks. The benefits of risk consensus (while preserving initial mean perception) are particularly relevant in polarized scenarios. Our findings for this scenario illustrate the global advantage of raising the risk awareness of the least-aware individuals, even if this comes at the cost of lowering the risk awareness of the most risk aware.

6. Limitations and future works

The risk perception in the HK model with stubbornness is linked to the payoff of the players, but this is a time-variant modification of the agent's parameter of the model. Then, one limitation of the study is the absence of a direct feedback mechanism between the success of the group and the future opinions or perceptions of the risk of the players, following the hypotheses proposed by recent contributions [31,32]. Also, other limitation is that the model assumes that players play in homogeneous groups of fixed size while other CRD variants, such as the model from [17], take into account the social network of the agents to create heterogeneous groups of interaction.

One can also study more sophisticated opinion models [55] and more realistic formation of players' opinions about risk, such as the inclusion of aging or the trend to maintain past opinions and the influence of institutional communication mechanisms to achieve cooperation. Calibrating the system with real opinions from digital platforms such as Reddit could also increase the validity of the outcomes. Human experiments also show that the mechanisms behind avoiding collective risks depend on the interaction between the type of behavior, communication, and timing [3].

In this context, a significant practical application is to observe how players can be exposed to the media influence of institutions or other stakeholders to bias their perception of the risk. Furthermore, and in a more general way, a framework in which an OD fusion process can arise from the trust of individuals in institutions, in peers, and in a context of cooperation, similar to the work of Zino et al. [18]. Another line of inquiry involves the use of AI-based autonomous agents to promote cooperation. Finally, other public good games can follow a similar perception-belief approach to the one proposed here to make them more realistic.

CRediT authorship contribution statement

Manuel Chica: Writing – original draft, Validation, Resources, Methodology, Funding acquisition, Conceptualization; **Víctor A. Vargas-Pérez:** Visualization, Software, Investigation, Writing – original draft; **Juan M. Hernández:** Writing – original draft, Validation, Methodology, Investigation, Conceptualization.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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