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Optimization model for school transportation design based on economic and social efficiency

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ABSTRACT

The purpose of this paper is to design a model enabling recommendation for the new school transport planning proposals to achieve greater operational efficiency. It is a multi-objective optimization problem which includes minimization of the bus costs and total travel time for all students. The model is based on bus route planning according to changes in school starting times, so the buses can make more than one route.

The methodology is based on the School Bus Routing Problem, so that routes from different schools within a given time window are connected, and within the problem constraints so system costs are minimized. The proposed model has been programmed for application in any generic case.

This is a multi-objective problem, for which there are several possible solutions, depending on the weight assigned to each of the variables involved, economic versus social point of view. Therefore, the proposed model will be useful in school transport planning policy, given that it is a support model for making decisions, which seek efficiency in economic and social terms.

The model has been applied in some schools located in an area of Cantabria (Spain), resulting in 71 possible optimal options, which minimize school transport cost between 2.7% and 35.1% regarding current school transport routes, with different school start time and minimum travel time for students.

1. Introduction

School transport in Spain is a Special Regulated Public Transport Service financed by the Spanish regional governments through private sector contracts awarded via public tender each school year and represents a heavy financial burden for the regional government Departments of Education (Ibeas et al., 2006). This is partly because the companies find it difficult to use these vehicles for other purposes throughout the rest of the day, coupled with the existence of historically defined routes which have never been scrutinised for optimisation, not to mention the rigid school timetables.

The main goal of this study is to design an optimisation model enabling route planning proposals to be defined so they maximise efficiency from operational, economic and social standpoints. School opening and closing times will be modified by establishing time windows enabling the buses to cover more than one school route (see Fig. 1).

The aim of the research is to simultaneously optimise the group of school routes and connections between them. School route and connection optimisation is created so the differences among school opening times become dependent on the route planning to combine them and vice versa.

The analysis and research are aimed at finding a balance between profitability and quality of service, making it a multi-objective problem: economic (cost optimisation, regional government) and social objectives (journey time optimisation, users).

The initial hypotheses were: (1) bus capacity would be homogenous; and (2) the bus should arrive at the school 2–10 min before classes start. This would enable the students to arrive punctually but without having to wait too long. The input data required to resolve the model are: (1) location of the stops; (2) number of students per stop; and (3) destination school of each student.

The rest of the paper is organized as follows. Section 2 presents a brief review of existing literature. Section 3 describes the multi-objective optimization model proposed here to maximise school transport efficiency, and presents a brief description of the programme written to plan the bus fleet. Section 4 applies the proposed model in a specific area of Spain and presents the results obtained for this area. Finally, some conclusions have been made in section 5.

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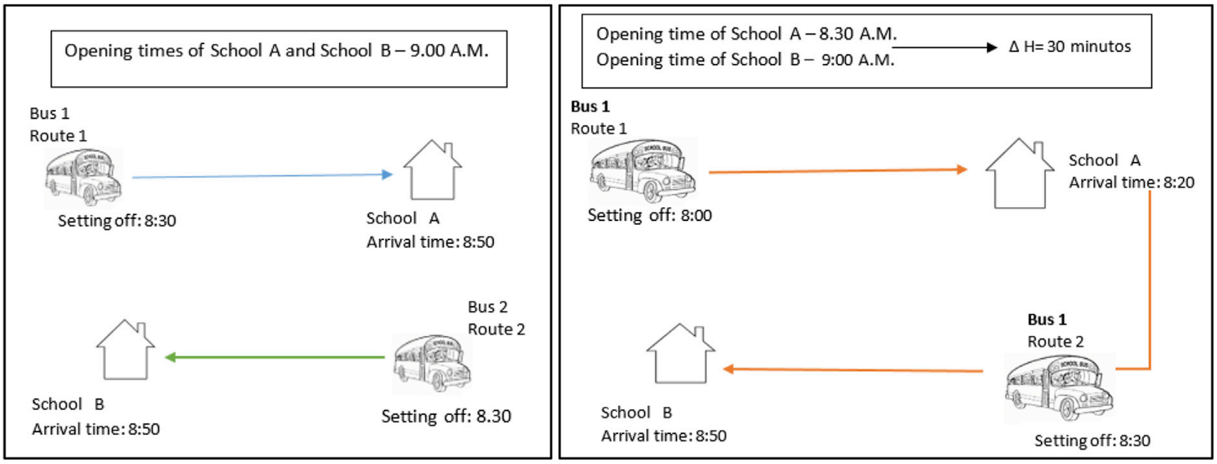

Fig. 1. The main idea (before-after).

2. State-of-the-art

Many papers are available describing research aimed at optimising school transport. Different viewpoints have been expressed in literature. Newton and Thomas (1974) presented a model to minimize both the total time required to complete all the routes (including time lapse) and the number of routes required to serve all the stops associated with the school. Spada et al. (2005) proposed a modelling approach focused on optimizing the bus service level and aims to minimize the children's time losses on the bus and at school before class starts. Another viewpoint is to minimize the economic cost, as in Thangiah et al. (2008) who presented heuristics to solve school routing problems that could lead to cost savings for governments. In Schittekat et al. (2013) the objective function is to minimize the total distance travelled by all buses, and by doing so, they had to determine (1) the stops to be covered; (2) which stop each student should use; and (3) the routes covering the selected stops.

Traditional VRP (Vehicle Routing Problem) seeks to generate efficient routes for a fleet of vehicles in order to deliver or collect products from depots for a set of customers (Laporte, 1992). Later, several extensions to the initial problem involving different constraints were developed. VRPTW (Vehicle Routing Problem Time Window) has vehicles with limited capacity and specific delivery time windows (Jean-Francois Cordeau et al., 2000) or SBRP (School Bus Routing Problem). SBRP seeks to plan an efficient schedule for a fleet of school buses where students are transported to and from school while satisfying various constraints (Park and Kim, 2010). According to Desrosiers et al. (1986), SBRP can be solved by five steps: data preparation, bus stop selection, bus route generation, school bell time adjustment, and route scheduling.

The school bus routing formulations focus on formulating extra constraints and/or objectives to take different factors into account: time window, bus stop selection, assigning students to buses, determining bus routes …. Desrosiers et al. (1986) added a maximum time constraint on each student's journey and/or time window, for their arrival at school. Fügenschuh (2009) considered the problem of programming the school bus by enabling the school opening times to be adapted to student transfer during the journey based on VRPTW, yet considered the routes to be basic input data. Ibeas et al. (2009) proposed the possibility of changing school entry and exit times, whereby the routes of each school in this case were input data, enabling a single bus serve multiple schools. On the other hand, Kim et al. (2012) propose a school bus scheduling problem where a bus can serve multiple trips for multiple schools but the school time window is fixed. Furthermore, Li and Fu (2002) presented an approach with multiple objectives where the number of buses, bus journey time and students' journey times were minimised. Bögl et al. (2015) take into account the possibility that pupils may change buses and analyse the impact of transfers on the service level in terms of user ride time and number of transfers.

As described below, this study differs in several ways from those mentioned above and others (Kontoravdis and Bard, 1995; Ho and Haugland, 2004; Park et al., 2012). In this study, the routing problem was solved at the same time as the vehicle planning problem to minimise journey times for the students and the number of buses being used within different time windows. This will enable future decision makers to give weight to these two economic and social criteria.

3. Methodology

The multi-objective optimisation model is a support tool for future decision makers. We will not be presenting a single solution, but rather various solutions to create a group of solutions which balance the economic and social factors. The objective function of the model is shown in expression 1 which will be decisive in planning school transport. The objective function is one of multi-objective optimization where 2 objectives are simultaneously minimized, i.e. operational costs and average time of routes (these variables will be explained later).

$$Min\ (\ operating\ costs;\ average\ travel\ time\ for\ routes) \quad (1)$$

The schematic shown in Fig. 2 was used in the development of the optimisation model and carried out in the following iterative manner:

- First phase, the routing problem per school is solved. Variables are the number of routes serving each school and maximum journey time allowed for them.
- Second phase, an optimisation model is used to solve the route combination problem; various routes are created for the same bus within the necessary time window, thereby providing multiple alternatives for the planning problem.
- Third phase, a pre-analysis is performed on all the alternatives obtained to find out which could minimise the objective function, and are, therefore, solutions to the model.

3.1. Routing problems per school

The routing problem per school was solved using SBRP (Bektaş and Elmastaş, 2007) (Sanhueza et al., 2012). The SBRP problem can be understood as the intersection of two well-known optimisation problems. The first, the problem of m travelling agents (m-TSP) (Salhi and Sari, 1997) is a generalisation of the TSP (Travelling Salesman Problem),

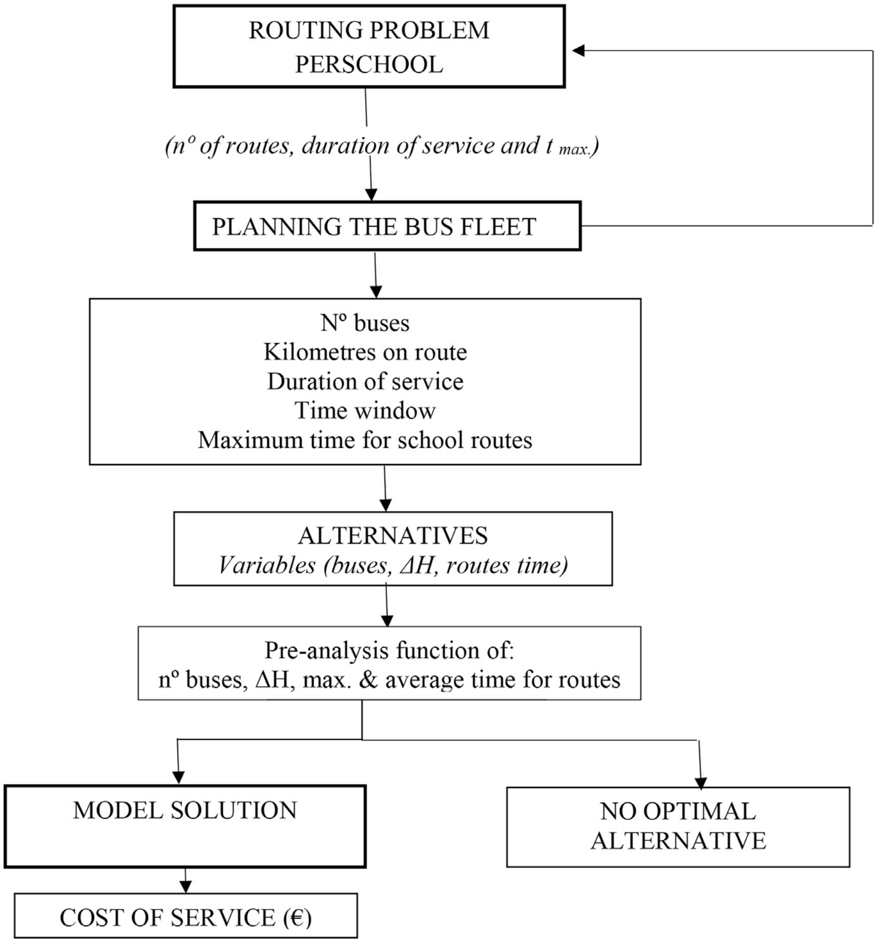

Fig. 2. Methodology.

whose purpose is to accurately create m routes, one per vehicle, so each stop is served only once. The second problem is that of packaging, i.e. picking up a group of students whose total number does not exceed bus capacity.

With the SBRP solution exact m routes need to be created. The number of routes is not fixed and varies between a maximum and minimum number of routes. The minimum number of routes will be that necessary to reach 50% of the bus capacity and the maximum will be necessary to reach bus capacity (equation (3)). The constraint of the maximum number of routes is applied to ensure said bus is not underused.

$$n^{o}\ routes\ min^{A} \leq n^{o}\ routes^{A} \leq n^{o}\ routes\ max^{A} \quad (2)$$

$$\frac{N^{o}\ pupils^{A}}{C} \leq n^{o}\ routes^{A} \leq \frac{N^{o}\ pupils^{A}}{0,5 \cdot C} \quad (3)$$

where:

- N^{o} pupils^{A}: the number of pupils going to school A.
- C: capacity of the buses.

A further variable has also been added to this problem, the maximum permitted route time. This represents a constraint on the routing problem which limits route duration and will change at 15 min intervals, up to maximum of 60 min as stipulated by school transport regulations.

The routing problem per school will not have a unique solution, so we will present various solutions, representing the sum of the combinations of these variables, number of routes and their maximum travelling time (nº routes, t_{max}) where a solution is possible. This knowledge provides: (a) the routes, which is the path followed by the bus from the first stop, and defined as header stop, until the last stop, which is the school; and (b) route time.

3.2. Planning the bus fleet

Once the routing problem has been solved per school, the following step is to plan the size of the bus fleet so that a bus is available to cover routes serving one or more schools within a time window (Jean-François Cordeau et al., 2001) (Garcia-Najera and Bullinaria, 2011), which cannot exceed 60 min. In other words, the aim is to minimise the number of buses required, which is the same as minimising the economic costs, taking into account journey times and time window of all possible combinations The objective function of this vehicle planning problem (equation (4)) addresses the need to find the minimum combination required to serve the group of schools (A). Each combination is obtained from the Cartesian product of schools and sets (equations (5) and (6)).

Equation (7) shows that the number of buses required is the difference between the number of routes per school and connections between them.

$$Min\,(Comb) \quad (4)$$

$$Comb = A^{j} \times C_{A^{j}} = \{A^{j}, C_{A^{j}} : \quad A^{j} \in A \quad and \quad C_{A^{j}} \in \ \mathrm{Set}_{A^{j}}\} \quad (5)$$

$$\mathrm{Set}_{A^{j}} = (n^{o}\ routes_{A^{j}}, start\ time_{A^{j}}, t.\ max_{A^{j}}),\ \forall A^{j} \in A,\ \forall\, n^{o}\ routes,\ \forall\, t.\ max, \forall\, start\ time \quad (6)$$

$$N^o buses_{Comb.} = \sum_{A^j \in Z} N^o routes^{A^j} - \sum_{A^j \in Z} N^o Connec.^{A^j} \quad \forall Comb, \forall A^j \in A \tag{7}$$

S.t.:

$$0 \leq t.connec^{comb}_{A^j, A^{j+1}, i} \leq \ldots\ldots\ldots \leq t.connec^{comb}_{A^{n-1}, A^n, i} \leq \Delta H_{max}, \quad \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \tag{8}$$

$$0 \leq t.connec^{comb}_{A^j, A^{j+1}, i} \leq t.connec^{comb}_{A^j, A^{j+1}, i} + t.connec^{comb}_{A^{j+1}, A^n, i} \leq \Delta H_{max}, \quad \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \tag{9}$$

$$t.connec^{comb}_{A^j, A^n, i} \geq t.connec^{comb}_{A^j, A^{j+1}, i} + t.connec^{comb}_{A^{j+1}, A^n, i}, \quad \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \tag{10}$$

$$\Delta H^{A^n}_{A^j} = Start\ time^{comb}_{A^n} - Start\ time^{comb}_{A^j} \leq 60\ min., \quad \forall Comb, \forall A^j \in A \tag{11}$$

$$N^o poss.connec.^{A^j} = \sum x^{comb}_{A^j,i} = \begin{cases} 0\ if\ \Delta H < t.connec^{comb}_{A^j, A^{j+1}}, \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \\ 1\ if\ \Delta H \geq t.connec^{comb}_{A^j, A^{j+1}}, \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \end{cases} \tag{12}$$

$$t.connec^{comb}_{A^j, A^{j+1}, i} = \begin{cases} 0 & if\ \Delta H < t.connec^{comb}_{A^j, A^{j+1}}, \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \\ t.connec^{comb}_{A^j, A^{j+1}, i} & if\ \Delta H \geq t.connec^{comb}_{A^j, A^{j+1}}, \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \end{cases} \tag{13}$$

$$\sum_{A^j \in Z} \sum_{A^{j+1} \in} t.connec^{comb}_{A^j, A^{j+1}, i} = \sum_{A \in Z} \sum_{i \in N} \left(t.move^{comb}_{A^j, i} + t.routes^{comb}_{i, A^{j+1}} \right), \quad \forall Comb, \forall A^j, A^{j+1} \in A, i \in N \tag{14}$$

$$N^o Connec.^{A^j} = \begin{cases} N^o\ poss.\ connec.^{A^j} & if\ N^o\ routes^{A^j} \geq n^o\ poss.connec.^{A^j}\ \forall Comb, \forall A^j \in A \\ N^o\ routes^{A^j} & if\ N^o\ routes^{A^j} < n^o\ poss.connec.^{A^j}\ \forall Comb, \forall A^j \in A \end{cases} \tag{15}$$

$$\sum_i x_{A^j, i} = N^o\ routes_{A^j} \quad \forall Comb, \forall A^j \in A \tag{16}$$

$$\sum_A x_{A^j, i} = 1 \quad \forall Comb, \forall i \in N \tag{17}$$

where:

- ΔH = time window necessary to connect the schools.
- T. max. = Maximum time allowed to cover the route.
- $N^o\ routes^{A^j}$ = Number of trips made to school A^j or, similarly, the number of buses.
- $N^o Connec.^{A^j}$ = the number of connections made from school A^j to the other schools.
- $N^o\ possible\ connec.^{A^j}$ = is the number of possible connections from school A^j to the other schools complying with the time window constraint.
- $x^{comb}_{A^j,i}$ = dummy variable which could take a value of 1 if there is enough time to use one of the buses that arrives to A^j to perform the service of route i.
- $t.connec^{comb}_{A^j, A^{j+1}, i}$ = journey time from school A^j to school A^{j+1} through the header i, for any combination.
- $t.move^{comb}_{A,i}$ = journey time from school A^j to header of route i, which is a route to destination school A^{j+1}.
- $t.routes^{comb}_{i,A^{j+1}}$ = journey time of the route from header i to school A^{j+1}.

Equations (8)–(11), represent how the time window is obtained, in other words, where each school is located within the time frame (A^j, A^{j+1} …, A^{n-1},A^n), knowing that the position of the schools along the timeline varies, thereby making all time combinations possible. Therefore, the time window will be the difference between the start time of the last school situated in the time frame minus the start time of the first school (eq. (11)).

Constraint (12) represents whether it is possible to make connections between schools and the route header stop of other schools. Constraints (13) and (14) represent their duration. (Fig. 3).

Constraint (15) represents the number of buses required to connect the schools, in other words, the number of connections will be (at the most) the number of routes (buses) which the connecting school has available. Furthermore, constraints (16) and (17) stop the headers being connected by more than one vehicle.

Once the vehicle planning problem has been solved for all the combinations of ΔH – nº routes – maximum time, multiple alternatives become available which have minimised the number of buses required. From these multiple alternatives (Alt) we know:

- Nº of buses required
- Time required to cover each route
- Time window required for that alternative
- Km driven by each bus

However, not all of these alternatives are going to be optimal from an economic or social aspect so a pre-analysis has been performed to find the solutions for the multi-objective optimisation model. The pre-analysis will be the function of the number of buses (Nº buses), time window (ΔH), average journey times ($T_{ave.}$) and average maximum time for schools routes ($T_{max.}$). This requires that the Nº buses are minimized and the other three variables fixed (18), and the same for the $T_{ave.}$ (19), knowing that $T_{ave.}$ And T_{max} are connected:

$$\min(N^o buses)_{Alt^o}, \quad \forall \Delta H^i, \Delta H^{i+1}, T^i_{ave}, T^{i+1}_{ave}, T^i_{max}, T^{i+1}_{max} \in Alt, \quad \forall \Delta H^i = \Delta H^{i+1}, \forall T^{i+1}_{ave} = T^{i+1}_{ave}, \forall T^{i+1}_{max} = T^{i+1}_{max} \tag{18}$$

$$\min(T_{ave})_{Alt}, \quad \forall N^o buses^i, N^o\ buses^{i+1}, \Delta H^i, \Delta H^{i+1}, T^i_{max}, T^{i+1}_{max} \in Alt, \quad \forall N^o buses^i = N^o\ buses^{i+1}, \forall \Delta H^i = \Delta H^{i+1}, \forall T^i_{max} = T^{i+1}_{max} \tag{19}$$

where:

$$T_{average} = \frac{\sum \frac{\sum t^C_{route}}{n^o routes^C} * n^o pupils^C}{\sum n^o pupils} \quad \forall Alt \tag{20}$$

$$T_{ave.max.} = \frac{\sum \frac{\sum t^C_{ave.max}}{n^o routes^C} * n^o pupils^C}{\sum n^o pupils} \quad \forall Alt \tag{21}$$

With this pre-analysis, those alternatives that minimize the economic and social cost are obtained.

3.2.1. Programming the planning bus fleet

A program written in Python 3.5 has been used to plan the bus fleet. It provides a fast way to efficiently resolve the issue, and the possibility of applying this methodology in any generic case (regardless of the number and size of schools). The program input data are the results obtained from the routing problem per school: nº routes, maximum time allowed to cover the route, journey time of the route and header of the route. Whenever the complexity of the tasks made it feasible, functional programming was used.

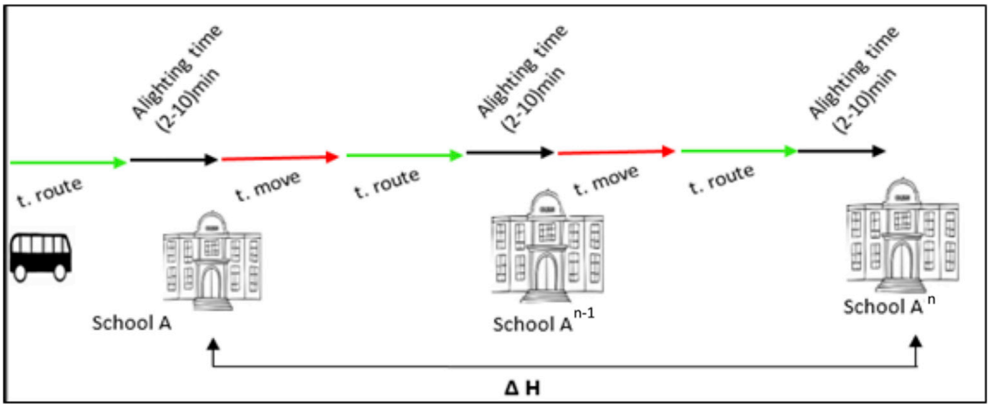

Fig. 3. Time band. Connection times between schools.

Once a routing choice (number of buses, and maximum time of the routes) and a bus arrival timetable have been defined, the adjacency matrix between schools and route headers can be built. An element a_{ij} of A will be 1 if it is possible for a bus to leave the children at school i, and drive to the starting point of route j in time; or 0 otherwise. This alternative will require a number of buses equal to the number of routes, minus how many times a bus can be re-utilized after finishing one. Calculating this value requires solving a linear programming optimization problem.

In order for the script to be able to compare all possible routing alternatives, the concrete schedule of every vehicle in each possibility is not needed, but just the number of buses required to materialize it. Thus, instead of directly solving the entire linear programming problem (4), the script evaluates a simpler one, and uses the result of the latter to calculate the solution of the former. This is achieved by manipulating the A_{ij} in the following manner:

a. Let $n\_available_i$ be the number of buses that can be re-used after arriving at school *i*. Each school can initially share as many buses as routes serve it.
b. Let *n_reuse* be the number of times that any bus, after finishing a route, is assigned to another one. Its initial value is zero.
c. Let S_j be number of the sum of the elements of column *j* (this means it is the number of schools from where a bus could depart after leaving the children there, and still be in time to cover route *j*). All columns where $S_j < 2$ will be removed, taking into account the following possibilities:
   o If $S_j = = 0$: no extra action is needed. It is impossible for the bus that covered route *j* to be in time for any other.
   o If $S_j = = 1$: there is a single cell a_{ij} with a value of 1. This means that there is only is one way to re-use a bus to cover the route *j*.
      • We increase the number of times a bus is re-used in the alternative, *n_reuse*, by 1.

We decrease the number of buses available for re-use at the school represented by row *j*, $n\_available_j$, by 1. If it reaches zero, we remove the row that represents that school from *A* and go back to c.

After this process, we would either have a zero-dimension matrix, in which case the number of buses needed for the current planning option would be the total number of routes minus "n_shared"; or a non-zero adjacency matrix, which would be solved using a liner programming solver (Coin-or LP). In this instance, the number of buses needed would be the total number of routes, minus "n_shared", minus the solution to the LP problem.

The program can present the information in three different manners:

• Raw output, which includes all cases analysed.
• Filtered output, described in (18), (19).
• Pareto frontier output: the set of possible choices that are Pareto-efficient.

3.3. Economic and social aspects

Once the solutions to the multiple objective optimisation model are available, the economic cost of each solution can be calculated. This process was supported by a previous study on school transport costs (Ibeas et al., 2006). Knowing the kilometres travelled per bus and the hours dedicated to the service, it can be possible to obtain the total direct costs ($Z_{TotalDC}$). The total direct costs of a transport company include the total running cost (RC), total direct labour force (bus drivers) cost (LC) and remaining fixed financial bus cost (FC), thus:

$$Z_{TotalDC} = RC + LC + FC \tag{22}$$

Direct costs must also be added indirect costs (computing, amortization …) and a profit to obtain the final economic cost school transport contract.

On the other hand, the social cost of school transport includes the duration of the routes and required time windows affecting the users, i.e. the pupils. The pupils wish to minimize the duration of the routes, because it is directly related to the time they spend on the bus. The route time has been calculated assigning an average speed to the links used in school transport network. Furthermore, according to the number of pupils in each stop, a different boarding time at the stops has been assigned. The average travel time of one solution is calculated applying the weighted average of the average route time of each school and the number of pupils who go to that school, in order to give more weight to the routes that drive more pupils. (eq. (20)).

The required time window is the school opening and closing times; changes in these hours affect students and their families, which is why it has been taken into account. These two variables are also known for each solution.

The objective of the proposed model is act as support tool for future decision-makers on issues of school transport, and as a result of which a range of solutions are obtained. The future decision depends on whoever decides to give more importance to the economic cost or the average route time. Furthermore, there will be a necessary time window per possible solution. These three variables will enable the future decision-maker to choose the optimum solution for his/her case study.

4. Application of the proposed model

The proposed model has been applied to an area in the Cantabria region (Spain) containing three primary schools (CP. Santa Juliana, CP. Manuel Liaño Beristaín and CP. Cantabria). The following information is known about these schools: the number of pupils, destination schools of those using school transport, location of stops and the number of pupils at

Table 1
Combinations for solving the routing problem.

	Nº pupils	Nº routes	Max. time	Nº solutions
CP. Santa Juliana	85	2-3-4	30-45-60	8
CP. Manuel Liaño Beristaín	55	1–2		4
CP. Cantabria	244	5-6-7		9

each.

The routing problem has been solved considering the number of routes and the journey time constraint as variables. The ArcGis geographic information system software was used to solve the problem and a total of 21 solutions were found for combinations of the number of routes and maximum time variables, as shown in Table 1. It is worth mentioning, that not all combinations (Nº routes - Max.Time) provide a solution to the problem. For example, Nº routes = 1 and max. time = 30 min, does not provide any solution to the problem due to the constraints of SBRP (Stop at all bus stops, road speed constraints …).

Fig. 4 represents an example of a routing problem solution, e.g. the case of Santa Juliana School with four routes (each one in different colour) and maximum time to cover the route is 30 min. The journey time of each route is shown, likewise bus stop location and school itself.

The second phase solves the planning problem using the solutions provided by the routing problem. The application created in Python for this step has been implemented obtaining 17,021 possible alternatives. However, which of these alternatives is optimal from an economic or social point of view is an unknown factor. To do this, a pre-analysis, as mentioned above, is performed (function of the number of buses, average journey time, maximum time for school routes and time window).

Fig. 5 represents the 71 results obtained from the pre-analysis of the multi-objective optimisation model, which all are possible. The Fig. 5 is a 3D graph in which the axes are represented as the mean children ride time, the average of the maximum time for school routes and the time window. Furthermore, the number of buses needed in each solution is represented with different coloured dots (changing from green to red as the number of buses decrease).

It shows that to attain the minimum average route time, minimum average of the maximum time for school routes and minimum time window, 12 buses are needed. However, if 5 buses are used, the average route time and time window increase significantly, and the average of the maximum time for school routes for some solutions increase too. Therefore, when the time window increases, the number of buses decreases and for the same number of buses (dots with the same colour) when the time window increases the average route time and the average of the maximum time for school routes tends to decrease. The maximum time of the route were taken into account, because it may be possible that there are solutions with one very long and the others very shorts and the average route time of these solutions could be similar to the solutions with all of the routes with similar route time. For the users and for the future decision-maker, these solutions are really different, this is a reason to take into account the maximum route time. It is important to remark that the maximum time for school routes and the time window are variables that are not in the objective function and therefore are not object of minimization, but are variables that have been taken into account in the model.

In addition, there are also cases without changes in school schedules (ΔH = 0), yet their routes vary compared to the current situation because they are historical ones which have not been updated.

Once the 71 solutions have been obtained, the number of buses, average journey time, average maximum time for school routes and time window of each solution are known. Furthermore, the route of each solution is known, in other words, the order in which each bus visit the stops and schools, and route time of each. Therefore, we have schools with earlier or later opening times in relation to others. To classify this,

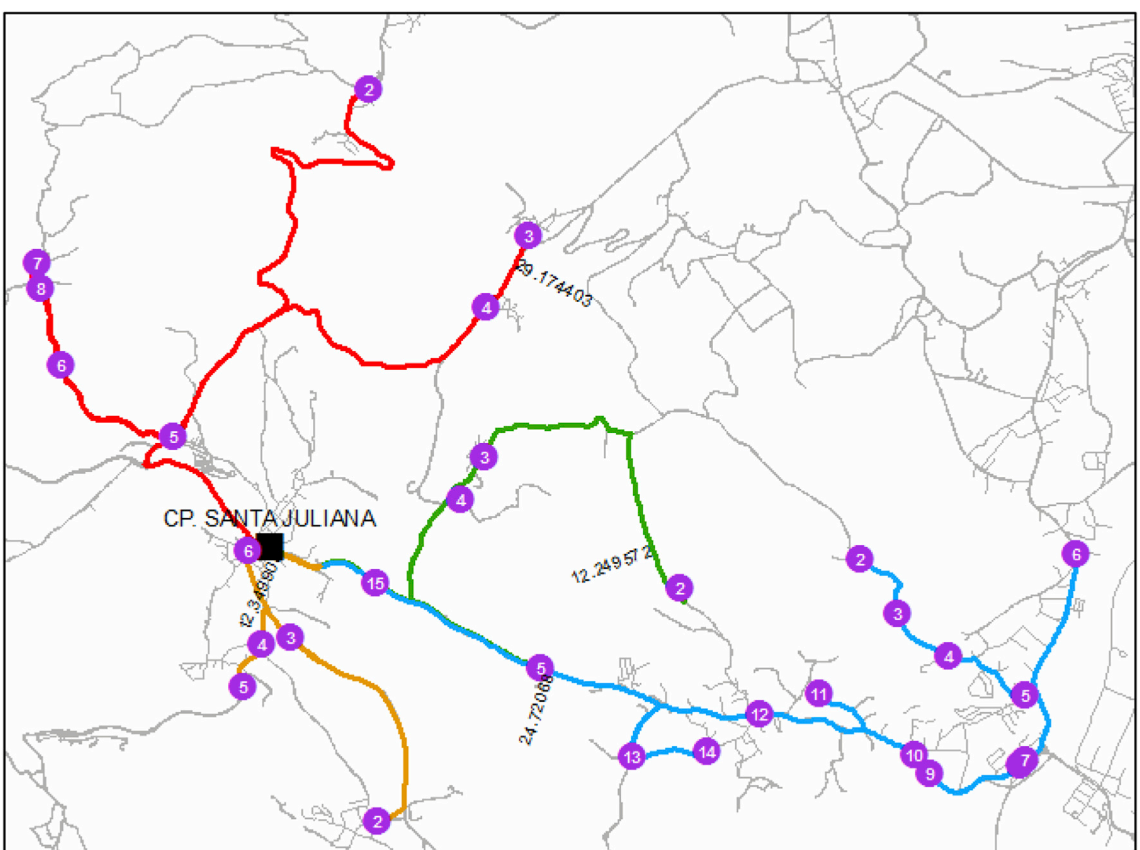

Fig. 4. Solution of routing problem: Santa Juliana school, nº routes = 4 and t.max = 30 min.

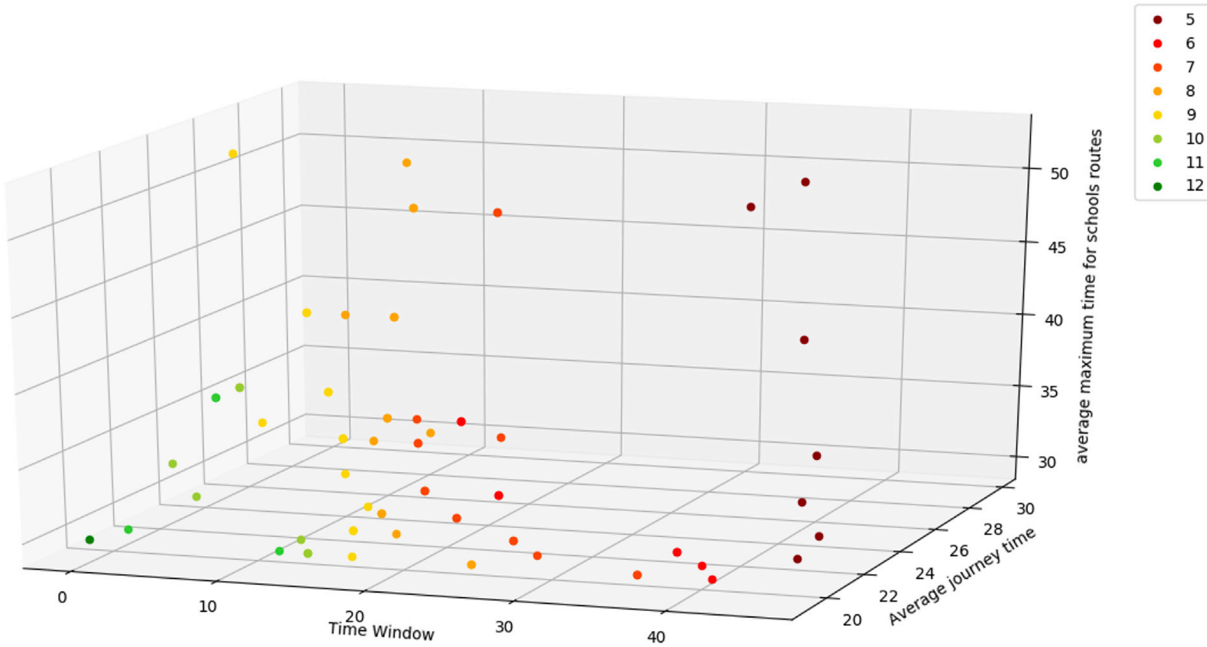

Fig. 5. Window time vs average route time vs nº of buses.

Table 2
Connecting cases between schools.

	From	To stop headers of
case A	CP. Cantabria	CP. Manuel Liaño Beristaín and/or CP. Santa Juliana
case B	CP. Santa Juliana	CP. Cantabria and/or CP. Manuel Liaño Beristaín
case C	CP. Manuel Liaño Beristaín	CP. Cantabria and/or CP. Santa Juliana
case D	CP. Cantabria and/or CP. Santa Juliana	CP. Manuel Liaño Beristaín
case E	CP. Cantabria and/or CP. Manuel Liaño Beristaín	CP. Santa Juliana
case F	CP. Santa Juliana and/or CP. Manuel Liaño Beristaín	CP. Cantabria
case G	No changes in school start time.	

Table 2 shows the different cases obtained from the 71 solutions. For example, Case A, the buses perform one or two routes, i.e. first a route that visits the stops whose destination is CP. Cantabria, the buses cover the route until arriving at the school where the students alight. Then the buses move to the header stop, to carry out the second route (if the bus has been assigned more than one route). In this case, it moves to the header stop of the CP. Manuel Liaño Beristaín and/or CP. Santa Juliana routes. As a result, we know that CP. Cantabria has an earlier opening time than the other two schools.

This is simply a support model for the future decision-maker, who will be responsible for deciding which of these alternatives is the final solution. To this end, the decision-maker will have to weight the economic costs and average route time, in addition to considering which time window is available for the different alternatives. Furthermore, the future decision-maker may come across additional constraints unrelated to the problem in hand, which must be considered when making the final decision. Said constraints, may for example, be related to school starting times or a student's maximum length of time on a bus.

To enable the future decision-maker to better comprehend the support model, Fig. 6 shows the solutions obtained from the multi-objective model (Eq. (1)) for one of the aforementioned cases, namely Case F (buses connecting Santa Juliana school and Manuel Liaño Beristaín school to the header stop of Cantabria school), which involves 13 of the 71 results, each represented in a different colour where the red one represents the current situation.

Fig. 6 shows three variables that characterize the 13 solutions: €/journey, average time for routes and ΔH. Of these three variables, the first two are the variables that the optimization model minimizes, and the last one (ΔH) is a variable that, although is not subject of minimization, provides information useful to the future decision maker. Therefore, through only one graph three variables that characterize the solutions could be seen (average time for routes, €/journey, ΔH).

Furthermore, the upper left graph quadrant (economic costs versus average route time) shows in black the Pareto boundary. We can minimize the average route time, the economic cost or chose an intermediate solution. Thus, if we want to minimize the average route time, will have to move along de Ox axis ($\alpha = 0°$) until the perpendicular from our position touches the Pareto boundary. In the same way, if we want to minimize economic cost, we will travel along the Oy axis ($\alpha = 90°$). If the future decision-maker wants an intermediate optimal solution, he should use: $\alpha \mid 0° < \alpha < 90°$.

For example, if there were no constraints of any kind unrelated to the problem, the future decision-maker for Case F, would choose one of these solutions: (1) only economic aspects count (yellow dot, €1039/day; 52 min. time window and average route time of 26 min), (2) only average route times count (lilac dot, €1188/day; 50 min. time window, and average route time of 19 min.). But, if it is not the case and the future decision-maker includes some restrictions, for example, (1) changes in start time of schools is 15 min at most, is to say that the time window is 30 min at most and (2) none of the pupils could travel more than 35 min (the maximum time a pupil currently travels on the bus). In addition, the future decision-maker wants the most economical solution that fulfils the two previous restrictions. In this case, the solution would be the navy blue dot of Fig. 6:

- ΔH = 30 min.
- €/journey = 1.288 €
- Nº buses = 9 buses. 3 buses serve CP. Santa Juliana school, 2 buses serve CP. Manuel Liaño school and 6 buses serve CP. Cantabria school, and from these 6 buses one bus come from CP. Santa Juliana school, other bus come from CP. Manuel Liaño school and the other 4 come from the depot.

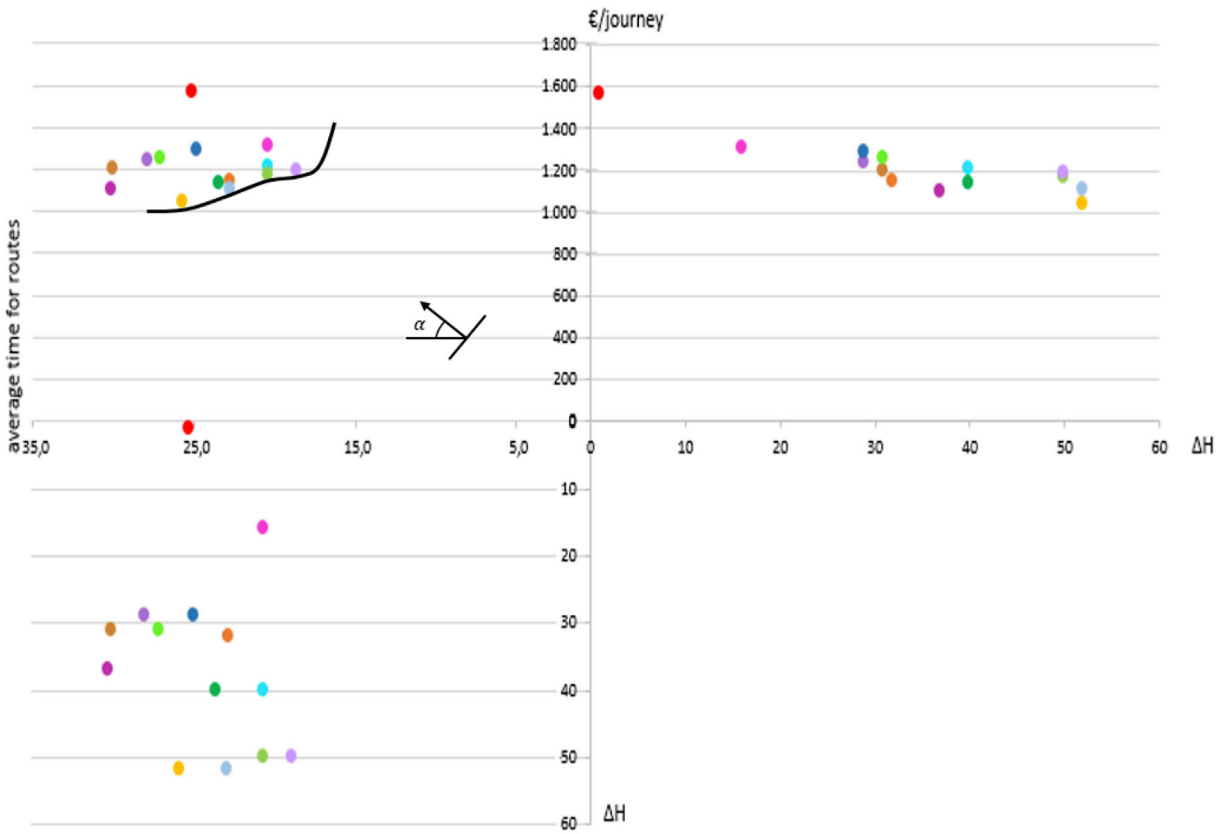

Fig. 6. Case F. Result of the objective function: Economic cost vs ΔH vs average route time.

- The average route time is 24 min
- The longer route is the one that finishes at CP. Cantabria school with a duration of 34 min.

In Fig. 6 we can see economic savings are obtained for any solution in relation to the current situation (without school timetable changes). These savings may vary between 16.5% and 33.7%. Furthermore, the average route time is lower for some of the solutions obtained; although at least a 15 min. time window is required for any of the solutions. Note that the greater the time window the greater economic savings obtained.

Only the results for Case F have been shown in this article; however, similar results were obtained for the other cases (A, B, C, D, E, and G). Economic savings ranging from 2.7% to 35.1% were obtained for the 71 results of the multi-objective optimization model while the average route time decreased for 56 of the 71 results. Furthermore, economic costs dropped as the time window increased.

5. Conclusions

This article presents a multi-objective optimization support model for the person responsible for school transport policy planning. The main contribution of this model principles that it enables modification of school entry and exit times, i.e. creating time windows between schools, thereby solving the route problem of each school as well as the bus fleet planning problem.

The methodology applied resolves two problems, i.e. (a) The route problem of each school, resolved using SBRP; and (b) the vehicle planning problem. To resolve the vehicle planning problem, a program was created in Python to enable application of this methodology to any generic case.

The multi-objective optimization model likewise minimizes: (1) economic costs affecting the regional public administration; and (2) the average route time affecting school transport users, i.e. students. In addition, to minimizing these 2 variables, it also considers the time window necessary for each of the optimization model solutions, which likewise affects users.

To sum up, the model enables school transport optimization from both economic and social viewpoints; moreover, it can be applied to any generic case and provide more than one solution. The aim is to aid the person responsible for planning, namely the future decision-maker to achieve an optimum solution per case study, which will depend on the weight assigned to the economic costs and average route time, as well as additional problem constraints in each case (time window or maximum route time or any other constraint).

The multi-objective optimization model presented herein has been applied to an area of Cantabria (Spain) where there are 3 schools with a total of 384 students using school transport. This model has provided 71 possible solutions, where 4 variables are known: (1) economic costs, (2) average route time of the 3 schools, (3) duration of each route; and (4) time window for said solution to be possible. All the 71 solutions obtained improved the regional public administration costs, achieving savings ranging from 2.7% to 35.1%. In 52 out of the 71 possible solutions the average route time is lower than the current. Therefore, this model proves it can aid the future decision-maker to obtain a solution different from the present, which improves economic and/or social aspects.

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