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How are patent decisions affected by environmental regulation?☆

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ABSTRACT

This paper examines how the presence of environmental regulation may induce shorter or longer patents. In the absence of environmental regulation, the patent office faces a well-known tradeoff: a longer patent yields a welfare benefit from inducing more R&D investment, but generates a welfare loss from allowing a longer monopoly during the patent period. When environmental policy is present, we show that the welfare loss is emphasized (ameliorated) when the environmental agency is less (more) flexible than the patent office, thus inducing shorter (longer) patents. We also consider green innovations, showing that environmental policy becomes less stringent, and patent decisions approach those in the absence of regulation.

1. Introduction

The optimal setting of patents has been extensively examined since Nordhaus (1969) and Takalo (2001). These studies consider the trade-off between the social loss from market monopolization during the patent period and the social benefit of inducing more investment in R&D, lowering production costs for the innovator during the patent and for all firms in the industry after the patent expires. More recently, a growing number of scholars have proposed the shortening of patent lengths, or their complete elimination for certain goods; see Boldrin and Levine (2013).[1] These studies, however, consider that firms do not generate environmental damages and, as a consequence, assume that firms do not face environmental regulation. Many patentees compete in polluting industries, such as the Saudi Arabian Oil Company (holder of over 1030 patents in 2023), BASF (333 patents), Walmart (266 patents), the Haier Group (253 patents), Exxon Mobil (238 patents), or Nippon Steel Corporation (211 patents); among others. In addition, several patentees operate in the energy industry, such as BP (deepwater drilling technology), Chevron (hydraulic fracturing), ConocoPhillips (Arctic drilling), and TotalEnergies (oil sands extraction).[2]

In this paper, we analyze the interaction between patents and environmental policy, showing that patent lengths are shortened by the presence of this policy, thus providing a new argument to reconsider patent decisions. This result is robust to changes in the environmental regulator's ability to revise emission fees in each period, or the green features of the innovation. Patents in the energy industry often exhibit

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[1] In many countries, a utility patent is granted to a product, process, machine, manufacture, chemical compound, or asexually reproducible plant for 20 years, while a design patent varies from 15 years in the US (The United States Patent and Trademark Office, 2017) and China (Zhou, 2021) or 20 years in Japan (Japan Patent Office, n.d.) to a maximum of 25 years in Europe (European Union Intellectual Property Office, n.d).

[2] Other examples include EOG Resources (horizontal drilling technology), Occidental Petroleum (using CO_2 injection to facilitate oil extraction), Devon Energy (steam-assisted gravity drainage of oil sands), and Halliburton (hydraulic fracturing).

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green characteristics, such as General Electric (low NO_x combustion systems), Vestas Wind Systems and NextEra Energy (wind turbines and blades), Siemens and BP (carbon capture and storage technology), ExxonMobil and Shell (biofuels), and First Solar, SunPower, and Panasonic (ultra-thin photovoltaic technology).

For comparison purposes, we consider a setting similar to Takalo's (2001) where, in the first stage, the patent office (PO) sets a patent length; in the second stage, the innovator responds investing in R&D to reduce its marginal production cost (process innovation)[3]; and, in the third stage, firms compete. During the patent period, the innovator uses its technology to operate as a monopolist, and once the patent expires, the technology is publicly available.

As a benchmark, we first examine an industry where environmental regulation is absent, with our results confirming Nordhaus' tradeoff. We then study how environmental policy affects this tradeoff. For completeness, we allow for this policy to occur in three different stages: in the first, second, or third stage.

When emission fees are set in the first stage, before patent and investment decisions are made, our model represents regulatory settings where environmental agencies cannot easily revise future policies. While many countries adjust emission fees according to inflation or pollutant intensity, few update their policies according to changes in the market structure (e.g., whether the industry becomes more competitive, if large R&D investments are made, or if cost-reducing innovations occur), thus closely fitting our model. Examples include the U.S. Environmental Protection Agency (EPA) permit annual fees (Title V Operating Permits), initially set at $32/ton in 1996, and only adjusted for inflation every year in September. Similarly, the sanctions for firms exceeding CO_2 emissions in the EU-ETS, originally set at 100 Euro/ton in 2003, have remained unchanged, only being adjusted for inflation since 2013; and a similar argument applies to the criminal provisions of the US Clean Air Act (42 U.S.C. 7413) which have not been revised since its enactment in 1990. Likewise, Finland carbon tax law was not amended for 14 years, between 1997 and 2011, although it has been more recently adjusted in 2018.[4] In addition, emission fees need to be approved by the legislature, whereas patent lengths are directly decided by PO officials.

In this context, we show that the presence of environmental regulation induces higher costs (net of taxes), less R&D investments, and a lower expected social return of the innovation, which ultimately induces the PO to respond with shorter patents than when regulation is absent. This "patent differential" grows when pollution is more severe and the innovation does not bring large cost-reduction effects, indicating that it is in these contexts when the PO should be more vigilant about environmental policy being active.

If, instead, the environmental agency (EA) acts in the second stage, after the PO has set patent lengths, our model still captures settings where the EA cannot easily revise future environmental policies. Yet, the EA is, comparatively, more flexible than the PO in this setting. We adopt a standard approach in the industrial organization literature, assuming less flexibility to strategies chosen close to the first stage of the game. In this context, we demonstrate that the emission fee imposes the same cost increase in all scenarios: before the patent expires, after it expires, and when no innovation occurs. Therefore, the welfare loss that the PO considers when deciding to prolong the patentee's legal monopoly for one more period is unaffected by environmental regulation. Welfare gains from a longer patent are, however, larger, since firms invest less with than without regulation, yielding a larger marginal effect from these lower investments. As a result, the PO has incentives to set longer patents than in the absence of environmental policy.

A similar argument applies when the EA becomes more flexible, acting in the third stage after patents and investments are made.[5] In this case, the environmental regulator observes whether the patent is still in force and whether the innovation was successful, thus being able to set emission fees that induce the socially optimal output in every scenario. Thus, aggregate output coincides before and after the patent expires, eliminating the welfare loss arising from allowing for a longer monopoly (i.e., "breaking" the Nordhaus' tradeoff), and inducing the PO to set longer patents than under no regulation.

Overall, our results suggest that, as the EA's ability to revise emission fees increases, the regulatory agency induces aggregate output that is closer to the first best in each industry setting (i.e., before the patent expires, after it expires, and if there is no innovation). This ameliorates the welfare loss from extending the patent for more periods (that is, allowing a legal monopoly), ultimately reducing the Nordhaus' tradeoff. As a consequence, the PO has incentives to set longer patents than when the EA is absent. In contrast, when this agency cannot easily revise environmental policy, exhibiting more rigidity than the PO, our results indicate that this policy emphasizes the welfare loss from allowing for longer patents, strengthening the Nordhaus' tradeoff, ultimately inducing the PO to set shorter patents than in the absence of regulation.

Our policy recommendations entail that POs should take into account the environmental effects of new technologies, not necessarily in their patent decisions, but because these effects directly impact emission fees. While accurate estimations of pollution damages are, admittedly, difficult for the PO, our findings indicate that the consideration of average environmental damages across industries could be welfare-improving, relative to settings where the PO completely ignores the environmental effects of innovations or regulation.

Finally, we test how our findings are affected by considering two extensions. First, we allow for innovations to not only reduce firms' production costs but also lower their pollution intensity.[6] Because the EA anticipates less pollution and sets less stringent emission fees, firms' investment decisions and the PO's patent policy become more similar to the setting without regulation, thus generating smaller changes in their behavior. In summary, our results indicate that POs can essentially ignore the presence of environmental regulation when innovations are extremely green, but must consider this regulation when setting patent lengths otherwise.

In contrast, when we allow for concave demand, the EA anticipates more aggregate output and pollution, requiring a more stringent emission fee. As a consequence, patents become shorter than under linear demand, and firms have less incentives to invest in R&D. Comparing our results with and without environmental policy, we show that the introduction of emission fees leads to shorter patents and less R&D investment, with both effects intensifying as demand becomes more concave.

[3] Examples of process innovations with cost-reducing benefits include: (i) methods for making higher value products from sulfur containing crude oil, assigned to Saudi Arabian Oil Company in 2010 (patent number 7790018); (ii) method for making molecular sieves comprising silicoaluminophosphate 44, assigned to Exxon Mobil in 2001 (patent number 6319487); and (iii) method for supplying hydrogen-containing reducing gas to shaft part of blast furnace, assigned to Nippon Steel Corporation in 2021 (patent number 10961596).

[4] Globally, carbon tax revenues have been relatively constant from 2006 to 2016, as reported by World Bank (2023).

[5] The sanctions in the US Clean Water Act (33 U.S.C. 1319), for instance, have been frequently revised, with their last modifications happening in 2018 and 2019.

[6] According to Globaldata.com, in 2022, for instance, there were over 43,800 new green patents in the automotive sector, more than 5800 in the chemical industry, over 6800 in the industrial goods and machinery industry, and 2900 in agriculture and forestry. In the energy sector, there were about 6000 patents filed globally; as reported by the International Energy Agency (2021).

1.1. Related literature

Theoretical studies. Our model follows the seminal work by Nordhaus (1969) about optimal patent lengths, which identified the tradeoff between the marginal static loss of allowing a monopoly market that induces the innovator to recover its R&D costs, and the marginal dynamic gain that the innovation produces on society once the patent expires. More specifically, we use Takalo's (2001) game-theoretic analysis of these two effects.[7] This literature extended along different dimensions, such as: (1) the optimal scope (or breadth) of patents, as in Merges and Nelson (1990), Gilbert and Shapiro (1990), Klemperer (1990), and Gallini (1992); (2) licensing of the patent to other firms, as in Denicolò (1996), Gallini and Scotchmer (2002), and Hattori (2017); (3) the protection that the patent gives to the innovator against other products that could be infringing the patent (often referred to as the "height" of the novelty requirement), as in van Dijk (1996), La Manna (1992), Matutes et al. (1996), and O'Donoghue et al. (1998); or (4) the development of complementary innovations, as in Heller and Eisenberg (1998) and Shapiro (2001). For a survey of the literature, see Eckert and Langinier (2014) to show the effect of agency interactions.

Our paper examines how optimal patents are affected with and without environmental regulation. One of the few articles exploring the connection between the EA and the PO's decisions is Gerlagh et al. (2014), which also considers patents in clean energy R&D, but assumes that firms invest in abatement R&D to reduce their pollution intensity. In contrast, we study an innovation that reduces the firms' production costs, thus being more similar to the standard patent model in Takalo (2001), helping us identify how the EA's presence affects patent policy.

Langinier and Chaudhuri (2020) analyze the effects of knowledge appropriability on patentability requirements and emission fees in the presence of environmentally-conscious consumers. The authors show that green R&D is maximized when emission fees are neither too high to make the investment unprofitable nor too low to yield insufficient abatement. However, they consider fixed patent lengths and exogenous fees, and we endogenize both of them.

Empirical studies. A substantial body of empirical research has also examined the role of patents as indicators of technological progress. Mansfield (1986), for instance, investigates the effectiveness of patents in promoting innovation, while Lerner (1994) analyzes the impact of patent scope on firm value, specifically, in the context of biotechnology firms. Similarly, Harhoff et al. (1999) and Hall et al. (2005) demonstrate that firms holding frequently cited patents possess higher market valuations.[8]

Another area of research utilizes patent data to assess technological advancements specifically in renewable energy. Popp (2002) and Popp et al. (2011), for example, analyze the relationship between technological progress, as measured by patents, and investment in renewable energy technologies. In a similar context, Huenteler et al. (2016) use patents to identify technological life-cycles within solar and wind power energy sectors.

Another line of the literature empirically studies how government policies – such as subsidies, tax incentives, climate policies, and feed-in tariffs – promote more patent innovation in renewable energy. Examples include Johnstone et al. (2010) for several types of renewables, Dechezleprêtre et al. (2011) for different climate-mitigation technologies, Noailly (2012) for energy efficiency in buildings, Lindman and Söderholm (2016) for the wind energy sector, Costantini et al. (2017) for energy efficiency in the residential and technological sectors, and Wurlod and Noailly (2018) for energy intensity across different industries.

Finally, a strand of recent empirical studies examines the relationship between patents development and CO_2 emissions in different regions. Wang and Wei (2020), for instance, examine how patent development interacts with environmental regulation to influence emissions across OECD countries. Cheng et al. (2021) further analyze how technological innovation, measured by the development of patents on CO_2 emissions, affect pollution in OECD countries, finding asymmetric results across regions. Similarly, Li et al. (2021) focusing on China, find evidence of an inverted U-shape relation between patent generation and CO_2 emissions. For other recent OECD studies, see Agnelli et al. (2023) about green innovations in the automotive sector, Dechezleprêtre et al. (2024) for renewable energy adoption, and Peñalosa and Kleine-Rueschkamp (2024) for the spatial distribution of green patenting activities.

The remainder of this article is organized as follows. Section 2 describes the model. As a benchmark, Section 3 identifies equilibrium behavior when the EA is absent and Section 4 examines how our results are affected when the EA is present. Section 5 discusses our main results and their policy implications, and Section 6 concludes.

2. Model

Following Takalo (2001), consider an innovator with R&D cost function $C(x) = \frac{1}{2}\gamma x^2$, where $\gamma > 0$ denotes its R&D efficiency, that is, a lower γ represents a greater efficiency. As in Belleflamme and Peitz (2015), we assume that γ is sufficiently large to yield $x \in [0, 1]$ in equilibrium, thus allowing for x to be interpreted as the innovator's probability of success. The inverse demand function is $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.

Without the innovation, every firm faces marginal cost of production c, where $0 \leq c \leq 1$, and we assume that they interact in a perfectly competitive market, yielding zero profit, $\pi^N = 0$, where superscript N denotes "no innovation." When the innovator is successful, its marginal cost decreases to $c - \alpha$, where $0 \leq \alpha \leq c$ denotes the cost-reduction effect of the innovation (process innovation). When $\alpha = 0$, the innovation is inconsequential but when $\alpha = c$, the firm's marginal cost is reduced to zero.

The innovator receives a patent during $T \geq 0$ periods, which lets the firm use the technology that decreases its marginal cost of production to $c - \alpha$, earning monopoly profit π^P, where superscript P represents a "patent" period. Once the patent expires, the innovation becomes public, allowing other firms to enjoy this technology as well, and every firm earns competitive profits $\pi^C = 0$, where superscript C denotes "competition."[9]

In the absence of patents, aggregate output occurs at the point where the demand $p(Q)$ and the marginal cost c intersect with one another, that is, Q_0 solves $p(Q_0) = c$, or $Q_0 = 1 - c > 0$.

We consider the following time structure of the game:

1. In the first stage, the environmental agency (EA) sets an emission fee, τ.
2. In the second stage, the patent office (PO) observes τ, and responds choosing a patent length T.
3. In the third stage, the innovator observes the fee τ and patent length T, and responds with its R&D investment, $x(T, \tau)$.
4. In the fourth stage, firms compete in output.

[7] For an introduction, see the presentation in Belleflamme and Peitz (2015), p. 541–542).

[8] Jaffe and Trajtenberg (2005) and Sampat and Ziedonis (2005) also study patent citation, but focusing on how they serve as proxies for technological diffusion and innovation spillovers.

[9] Marginal cost c is measured in dollars and so is the cost-reduction effect of the innovation, α, and per-unit emission fee, τ. The R&D cost $C(x) = \frac{1}{2}\gamma x^2$ is measured in dollars, with investment x being measured in dollars too. Parameter γ indicates the firm's inefficiency at investing in R&D (i.e., a low value of γ entails that the firm is efficient, but a high value of γ implies the firm is inefficient) and it is measured in dollars per unit. Finally, patent length T is measured in years.

Table 1
Regulatory scenarios.

No regulation	No EA	Baseline (Section 3)
Regulation	EA acting first	Inflexible regulation (Section 4.1)
	EA acting second	Flexible regulation (Section 4.2)
	EA acting third	More flexible regulation (Section 4.3)
Extensions	Green innovations	Extension (Section 5.1)
	Non-linear demand	Extension (Section 5.2)

This time structure indicates that emission fees are administratively difficult to revise after the PO sets a patent length or after firms invest in R&D.[10] Hence, it characterizes the policy constraints that several environmental regulatory agencies face, where fees have been rarely revised in the last decades, as reported in the Introduction. (We separately examine other time structures in Sections 4.2–4.3.)

The PO's welfare function is $W = CS + PS$, which includes consumer and producer surplus, since POs in most countries ignore pollution when setting patent lengths. In contrast, the EA's welfare function is

$$W_{EA} = CS + PS + Tax - Env, \tag{1}$$

where $Tax = \tau Q$ denotes total tax collection, which guarantees that emission fees are revenue neutral; and $Env \equiv dQ^2$ represents environmental damages, which are increasing and convex in aggregate output, as in Poyago-Theotoky (2007), Lambertini et al. (2017), and Bárcena-Ruiz et al. (2023), among others. Parameter d captures pollution severity and satisfies $d > 1/2$ to guarantee positive emission fees in all settings.

We first analyze equilibrium behavior in a setting where pollution is not addressed by the EA, becoming a three-stage game that we solve by backward induction. For comparison purposes, Section 3 closely follows Takalo (2001), while Section 4 focuses on the role of environmental policy. Section 5 examines two extensions, as robustness checks of our modeling assumptions: allowing for innovations to not only reduce production costs but also pollution intensity (green innovations) and non-linear demand. Table 1 summarizes the cases we consider and the section where each is studied.

3. Equilibrium analysis when the EA is absent

3.1. Third stage, output decisions

After the patent expires (in periods $t \geq T$), every firm i enjoys production cost $c - \alpha$. In this context, firms interact in a perfectly competitive market, with aggregate output $Q^C = 1 - (c - \alpha)$, which solves $p(Q^C) = c - \alpha$, yielding zero profits for every firm, that is, $\pi^C = 0$.

Before the patent expires ($t < T$), the innovator enjoys production cost $c - \alpha$, choosing a monopoly output $q^m = \frac{1-(c-\alpha)}{2}$ that yields monopoly price $p^m \equiv \frac{1+(c-\alpha)}{2}$. We define the cost-reduction effect of the innovation, as captured by α, to be "non-radical" as in Belleflamme and Peitz (2015), if the innovator's monopoly price during the patent exceeds its rivals' common marginal cost, $p^m = \frac{1+(c-\alpha)}{2} > c$, or $\alpha < 1 - c$. Otherwise, patent lengths would be undefined, even in the absence of environmental regulation. The innovator, then, becomes the only seller, setting a price below its rivals' common marginal cost, c, that is, $p^P = c - \varepsilon$, where $\varepsilon \to 0$; and patent output is $q^P = Q_0 = 1 - c$.

In this context, the innovator earns monopoly profits in every patent period, as follows,

$$\pi^P = \left[p^P - (c - \alpha)\right] q^P = \alpha (1 - c)$$

which are positive during the patent but zero afterwards, that is, $\pi^P > \pi^C = 0$. Patent profits increase with the cost-reduction effect of innovation α but decrease in the initial cost c.

3.2. Second stage, R&D investment

Anticipating output decisions in the last stage, the innovator chooses its R&D investment, x, to solve

$$\max_{x \geq 0} \; x \Pi_0(T) + (1 - x) \int_0^{+\infty} e^{-rt} \pi^N dt - \frac{1}{2} \gamma x^2 \tag{2}$$

where the first two terms represent the innovator's expected return from its investment, which is successful with probability x earning $\Pi_0(T)$, but unsuccessful with probability $(1 - x)$ earning profits π^N in every period, and

$$\Pi_0(T) = \int_0^T e^{-rt} \pi^P dt + \int_T^{+\infty} e^{-rt} \pi^C dt = \frac{\alpha (1 - c) \left(1 - e^{-rT}\right)}{r}$$

where r denotes the discount factor satisfying $0 < r \leq 1$, and the second equality considers $\pi^C = 0$ and $\pi^P = \alpha(1 - c)$. Note that $\Pi_0(T)$ is increasing in the patent length T, that is, $\Pi_0'(T) = \pi^P e^{-rT} \geq 0$, meaning that the innovator can earn monopoly profits π^P during more periods.

Lemma 1. *The innovator's equilibrium investment in R&D is* $x_0(T) = \frac{\Pi_0(T)}{\gamma} = \frac{\alpha(1-c)\left(1-e^{-rT}\right)}{\gamma r}$, *which is increasing and concave in* T, *increasing in* α, *and decreasing in* c *and* γ.

As expected, the innovator has stronger incentives to invest in R&D with longer patents, thus providing this firm with a longer monopoly, although these incentives increase at a decreasing rate. In contrast, investment decreases when net costs increase (higher $c - \alpha$) or it becomes more costly (higher γ).

3.3. First stage, patent length

The PO anticipates the firms' decisions in the subsequent stages of the game, choosing the patent length, T, that solves the following problem,

$$\max_{T \geq 0} \; x_0(T) \left(\int_0^T e^{-rt} W_0^P dt + \int_T^{+\infty} e^{-rt} W_0^C dt \right) + \left[1 - x_0(T)\right] \int_0^{+\infty} e^{-rt} W_0^N dt - \frac{1}{2} \gamma \left[x_0(T)\right]^2$$

where $x_0(T)$ originates from Lemma 1, and the first (second) term denotes the welfare with (without) innovation. In addition, $W_0^P = \frac{(1-c)^2}{2} + \alpha (1 - c)$ denotes welfare in each patent period, $W_0^C = \frac{[1-(c-\alpha)]^2}{2}$ measures welfare after the patent expires, and $W_0^N = \frac{(1-c)^2}{2}$ represents welfare without the innovation. Simplifying, the above problem becomes

$$\max_{T \geq 0} \; x_0(T) S_0(T) + \int_0^{+\infty} e^{-rt} W_0^N dt - \frac{1}{2} \gamma \left[x_0(T)\right]^2 \tag{3}$$

where $S_0(T)$ represents the social return of the innovation, relative to that without the innovation, defined as

$$S_0(T) = \left(\int_0^T e^{-rt} W_0^P dt + \int_T^{+\infty} e^{-rt} W_0^C dt \right) - \int_0^{+\infty} e^{-rt} W_0^N dt.$$

While profits satisfy $\pi^P > \pi^C = \pi^N = 0$, the welfare ranking is $W_0^N < W_0^P < W_0^C$ for all parameter values, indicating that firms prefer longer patents than the PO. Next, differentiating the PO's problem in expression (3) with respect to T, yields

$$\underbrace{\frac{\partial x_0(T)}{\partial T} S_0(T)}_{MDG_0(T)} = \underbrace{x_0(T) \left(\gamma \frac{\partial x_0(T)}{\partial T} - \frac{\partial S_0(T)}{\partial T} \right)}_{MSL_0(T)}. \tag{4}$$

[10] The PO corresponds to the U.S. Patent and Trademark Office (USPTO), the Canadian Intellectual Property Office (CIPO), or the European Patent Office (EPO), among others. For generality, however, we label this agency as the PO.

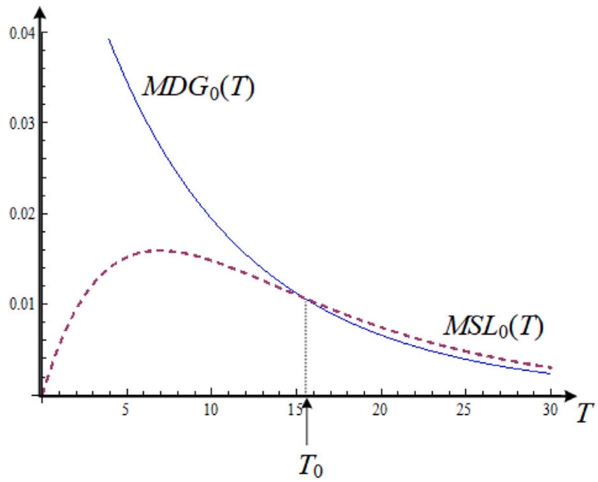


Fig. 1. Patent length when the EA is absent.

The left-hand side of expression (4) measures the marginal dynamic gain, $MDG_0(T)$, from extending the patent for one more period. Intuitively, the firm increases its R&D intensity $x_0(T)$, which increases the expected social welfare. The right-hand side, in contrast, captures the marginal static loss, $MSL_0(T)$, from a longer patent, in the form of a larger R&D cost (first term), and a lower consumer surplus due to extending the monopoly for more periods (second term). The next lemma examines how these effects are separately affected by a longer patent.

Lemma 2. *$MSL_0(T)$ increases in T at a decreasing rate when $T < \frac{\log 2}{r}$, decreases in T at an increasing rate when $\frac{\log 2}{r} \le T < \frac{\log 4}{r}$, and decreases in T at a decreasing rate otherwise. In contrast, $MDG_0(T)$ decreases in T at a decreasing rate, and more significantly than $MSL_0(T)$ does if and only if $\frac{\log 2}{r} < T < \overline{T}_0$, where cutoff $\overline{T}_0 \equiv \frac{1}{r}\log\left[\frac{4(1-(c-\alpha))}{\alpha}\right]$.*

Fig. 1 illustrates the results in Lemma 2, showing that $MDG_0(T)$ is unambiguously decreasing in T, while $MSL_0(T)$ originates at zero, first increases in T until reaching a maximum at $\frac{\log 2}{r}$, and then decreases in T.[11] Furthermore, $MDG_0(T)$ decreases faster than $MSL_0(T)$ for all $T < \overline{T}_0$, but slower afterwards. The optimal patent length, T_0, then occurs at the point where $MDG_0(T) = MSL_0(T)$, which we identify in Proposition 1.

Lemma 3. *$MSL_0(T)$ and $MDG_0(T)$ exhibit the following comparative statics:*

1. *Cost-reduction effect, α: Both $MDG_0(T)$ and $MSL_0(T)$ increase in α, but $MSL_0(T)$ increases more substantially than $MDG_0(T)$ does if and only if $T > \underline{T}_0$.*
2. *Initial cost, c: Both $MDG_0(T)$ and $MSL_0(T)$ decrease in c, but $MDG_0(T)$ decreases more significantly than $MSL_0(T)$ does if and only if $T < \widehat{T}_0$,*

where $\underline{T}_0 \equiv \frac{1}{r}\log\left[\frac{2[3\alpha+2(1-c)]}{3\alpha}\right]$, $\widehat{T}_0 \equiv \frac{1}{r}\log\left[\frac{2[\alpha+2(1-c)]}{\alpha}\right]$, and cutoffs satisfy $\frac{\log 2}{r} < \underline{T}_0 < \widehat{T}_0 < \overline{T}_0$.

Thus, a larger cost-reduction effect (higher α) produces a more significant pivot in $MSL_0(T)$ than in $MDG_0(T)$ if the patent is long enough ($T > \underline{T}_0$, which is confirmed by Proposition 1 below), implying that the optimal patent becomes shorter. The opposite argument applies when initial cost, c, increases, where $MDG_0(T)$ shifts downwards more than $MSL_0(T)$ does, thus yielding a shorter patent.

Proposition 1. *When the EA is absent, the optimal patent length is $T_0 = \frac{1}{r}\log\left[\frac{2[1-(c-\alpha)]}{\alpha}\right]$, where $T_0 \in (\underline{T}_0, \widehat{T}_0)$. In addition, T_0 decreases in both α and c. Finally, the innovator invests $x_0 = \frac{\alpha(1-c)[\alpha+2(1-c)]}{2\gamma r[1-(c-\alpha)]}$, which increases in the cost-reduction effect α but decreases in cost c.*

Therefore, when firms benefit from larger cost-reduction effects (higher α), they experience stronger incentives to invest in R&D. Anticipating these incentives, the PO can set shorter patents. The above proposition also shows that, when firms' initial cost is low or the cost-reduction effect of R&D is high, the net cost, $c-\alpha$, decreases, providing firms with more incentives to invest.

4. Equilibrium analysis when the EA is present

We now examine how our above equilibrium results are affected when firms face environmental regulation. We first study the case in which the EA sets emission fees in the first stage (Section 4.1), when the EA chooses fees in the second stage (Section 4.2), and finally when it acts in the third stage (Section 4.3).

4.1. The EA acts in the first stage

In this context, equilibrium behavior in stages 2–4 is analogous to that in stages 1–3 in the model without environmental regulation, but increasing firms' costs in all scenarios: from c to $c+\tau$ in the absence of innovation; and from $c-\alpha$ to $c-\alpha+\tau$ when innovation takes place.

In the third stage, every firm chooses its R&D investment, x, to solve a problem analogous to (2), yielding an equilibrium investment of $x(T,\tau) = \frac{\alpha(1-c-\tau)(1-e^{-rT})}{\gamma r}$, which is decreasing in the emission fee τ. In addition, this investment satisfies

$$0 < x(T,\tau) < x_0(T) \text{ and } 0 < x'(T,\tau) < x_0'(T),$$

implying that firms invest less with than without regulation and that, when facing a longer patent, they increase their investments less significantly when facing regulation than otherwise.

The PO's problem is analogous to (3), yielding similar expressions of $MDG(\tau)$ and $MSL(\tau)$, which are now a function of fee τ. Fig. 2 depicts these curves: first, evaluated at $\tau = 0$ which coincide with MDG_0 and MSL_0 in Fig. 1, denoted as $MDG(0)$ and $MSL(0)$; second, evaluated at $\tau = 0.1$, where both curves shift downwards relative to $\tau = 0$; and, finally, evaluated at $\tau = 0.2$, further shifting both curves downwards. As shown above, regulation lowers investment incentives, decreasing both $MDG(\tau)$ and $MSL(\tau)$. As illustrated by the figure, the downward shift in $MDG(\tau)$ dominates that in $MSL(\tau)$, implying that the crossing point between $MDG(\tau)$ and $MSL(\tau)$ moves leftward. Therefore, the equilibrium patent length decreases in the stringency of the emission fee, τ; as confirmed by the patent T that solves the PO's problem, which is

$$T_{EA} = \frac{1}{r}\log\left[\frac{2\left[1-(c-\alpha+\tau)\right]}{\alpha}\right]. \quad (5)$$

Hence, the PO sets shorter patents when firms' initial costs are higher (c or τ are higher) because the patent gives rise to a smaller cost-reduction effect (for a given value of α), ultimately reducing the social benefit of the patent.

The EA anticipates the welfare in each case, W^P_{EA}, W^C_{EA}, and W^N_{EA}, as defined in Eq. (1), setting fee τ to solve

$$\max_{\tau\ge 0}\ x(T,\tau)\left(\int_0^T e^{-rt}W^P_{EA}dt + \int_T^{+\infty} e^{-rt}W^C_{EA}dt\right) \quad (6)$$
$$+\left[1-x(T,\tau)\right]\int_0^{+\infty} e^{-rt}W^N_{EA}dt - \frac{\gamma}{2}\left[x(T,\tau)\right]^2$$

The EA, then, faces a tradeoff: a more stringent fee reduces expected pollution if an innovation occurs, yielding a welfare gain; but it also

[11] For illustration purposes, the figure considers $\alpha = 1/4$, $c = 2/3$, and $r = 1/10$. Other parameter values yield similar results and can be provided by the authors upon request.

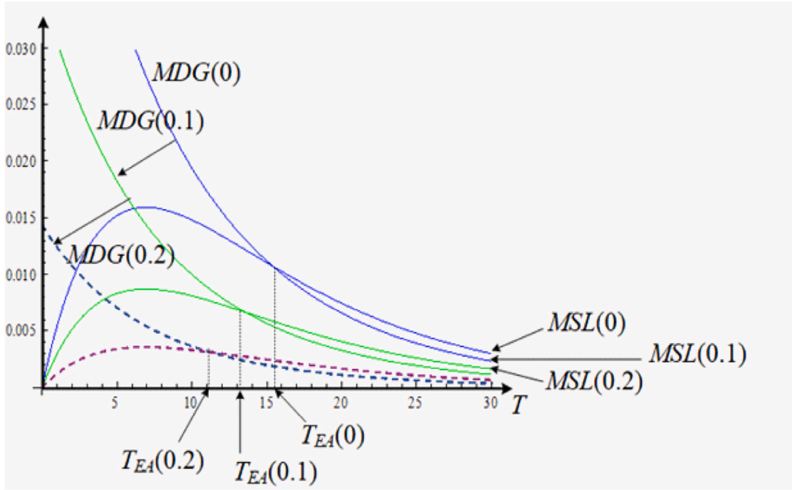
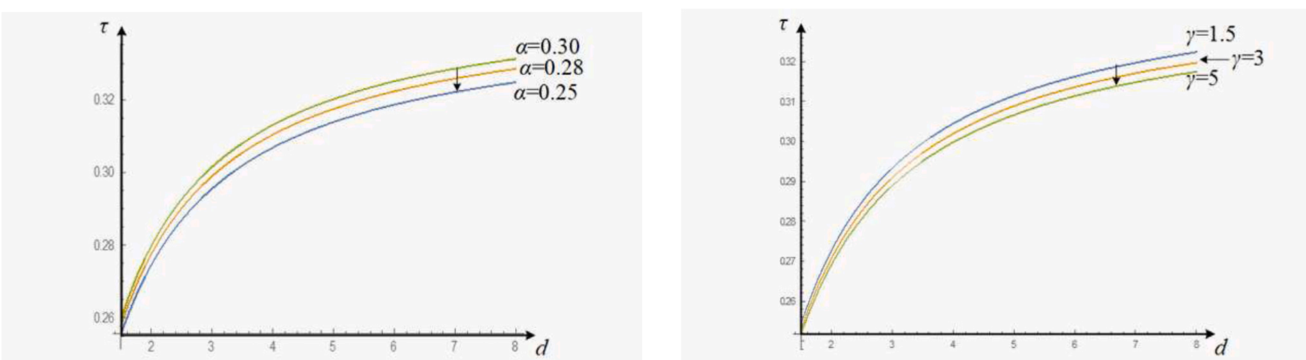


Fig. 2. Patent lengths with environmental regulation.



(a) Fee τ^*, changes in α

(b) Fee τ^*, changes in γ

Fig. 3.

reduces firms' investment, $x(T,\tau)$, making the innovation less likely to occur, lowering expected welfare. Alternatively, in the absence of innovation, the EA would set a fee $\tau^N = \frac{2d(1-c)}{1+2d}$ that induces a first-best output and pollution. Similarly, with an innovation, the agency would set a linear combination between fees $\tau^P = \frac{2d(1-c)-\alpha}{1+2d}$ and $\tau^C = \frac{2d(1-c)+2d\alpha}{1+2d}$, where τ^P is the fee in the monopoly market in each patent period and τ^C represents the fee in the perfectly competitive market after the patent expires, and $\tau^C > \tau^P > \tau^N$ to account for the market monopolization during patent periods. When the patent length is nil, the more stringent fee τ^C would induce a first best; and when the patent is infinite, the less stringent τ^P would be socially optimal. Therefore, the EA sets an emission fee that is less stringent than τ^C, but more than τ^P; becoming an average of these fees, weighted by the relative probability of an innovation occurring.

Differentiating with respect to τ in problem (6) yields intractable results, and we rely on numerical simulations. Figs. 3a–3b depict emission fee τ^* as a function of pollution severity, d, in the horizontal axis; and evaluated at different values of α (Fig. 3(a) and γ (Fig. 3(b). As a benchmark, we consider $\gamma = \frac{3}{2}$, $\alpha = \frac{1}{4}$, $c = \frac{2}{3}$, and $r = \frac{1}{10}$. As expected, emission fees are more stringent as pollution becomes more damaging (higher d).

When the innovation generates smaller cost-reduction effects (lower α), production is more costly, and the EA anticipates less output and pollution, setting a less stringent emission fee. A similar argument applies when the cost of investing in R&D, γ, increases, where the agency anticipates less innovation, higher expected production costs and, as a consequence, less pollution.

Table 2 reports patent lengths at similar parameter values as in Fig. 3, with the benchmark (top row) considering $d = \frac{3}{2}$. Patents become shorter when the innovation is more cost-reducing (higher α). Intuitively, this indicates the presence of a direct effect of α in T_{EA} since, as shown above, T_{EA} decreases in α; but also an indirect effect, since α produces an increase in emission fee τ^*, which in turn decreases T. Both effects, then, move in the same direction, ultimately reducing the patent length; as shown in Table 3.

In contrast, more costly production (higher c) produces a negative direct effect on T_{EA}; but induces a less stringent fee τ^* and, thus, a longer patent (positive indirect effect). Our results, however, identify that the negative direct effect dominates, yielding a shorter patent. This argument also applies when future periods are more heavily discounted (higher r), since the patent length becomes shorter (negative direct effect) but the emission fee is less stringent thus increasing the patent length (positive indirect effect). Overall, Table 2 shows that the direct effect dominates, shortening the patent length. As a robustness check, tables A2–A.6 in the appendix evaluate Table 2 at different parameter values, changing one parameter at a time, and confirm that our qualitative results and comparative statics are unaffected.

Changes in parameters d and γ, however, only give rise to an indirect effect on T_{EA} by affecting the stringency of the emission fee, since these parameters are not arguments of T_{EA}. To see this point, note that more severe pollution (higher d) entails a more stringent emission fee, τ^*, which lowers the patent length, T_{EA}. While the PO ignores pollution in its decisions (i.e., T_{EA} is unaffected by d), this agency anticipates that a more stringent emission fee increases firms' net production cost,

Table 2
Equilibrium outcomes with and without regulation — The EA acts first.

		Regulation			No regulation		Comparison	
		τ^*	T_{EA}	x_{EA}	T_0	x_0	ΔT	Δx
	Benchmark	0.2555	9.6409	0.0802	15.4045	0.4365	5.7636	0.3563
Higher α	$\alpha = 0.18$	0.2517	10.6703	0.0642	17.4112	0.3299	6.7409	0.2657
	$\alpha = 0.20$	0.2526	10.3235	0.0693	16.7398	0.3611	6.4163	0.2918
	$\alpha = 0.22$	0.2536	10.024	0.0740	16.1548	0.3917	6.1308	0.3177
	$\alpha = 0.24$	0.2548	9.7612	0.0783	15.6398	0.4217	5.8786	0.3434
	$\alpha = 0.26$	0.2563	9.5271	0.0820	15.1822	0.4512	5.6551	0.3692
	$\alpha = 0.28$	0.2579	9.3161	0.0853	14.7727	0.4802	5.4566	0.3949
	$\alpha = 0.30$	0.2598	9.1239	0.0880	14.4036	0.5088	5.2797	0.4208
	$\alpha = 0.32$	0.2619	8.9473	0.0901	14.0691	0.5370	5.1218	0.4469
Higher c	$c = 0.35$	0.4900	11.8774	0.1853	19.7408	0.9329	7.8634	0.7476
	$c = 0.40$	0.4531	11.5529	0.1677	19.1692	0.8529	7.6163	0.6852
	$c = 0.45$	0.4162	11.2186	0.1504	18.5630	0.7734	7.3444	0.6230
	$c = 0.50$	0.3792	10.8738	0.1335	17.9176	0.6944	7.0438	0.5609
	$c = 0.55$	0.3422	10.5180	0.1170	17.2277	0.6161	6.7097	0.4991
	$c = 0.60$	0.3051	10.1504	0.1009	16.4866	0.5385	6.3362	0.4376
	$c = 0.65$	0.2679	9.7705	0.0853	15.6862	0.4618	5.9157	0.3765
	$c = 0.70$	0.2307	9.3774	0.0703	14.8160	0.3864	5.4386	5.4386
Higher r	$r = 0.15$	0.2538	6.4632	0.0549	10.2696	0.2910	3.8064	0.2361
	$r = 0.20$	0.2528	4.8612	0.0417	7.7022	0.2183	2.8410	0.1766
	$r = 0.25$	0.2523	3.8957	0.0336	6.1618	0.1746	2.2661	0.1410
	$r = 0.30$	0.2519	3.2502	0.0282	5.1348	0.1455	1.8846	0.1173
	$r = 0.35$	0.2516	2.7882	0.0242	4.4013	0.1247	1.6131	0.1005
	$r = 0.40$	0.2514	2.4412	0.0213	3.8511	0.1091	1.4099	0.0878
	$r = 0.45$	0.2513	2.1710	0.0189	3.4232	0.0970	1.2522	0.0781
	$r = 0.50$	0.2512	1.9547	0.0171	3.0809	0.0873	1.1262	0.0702
Higher d	$d = 2$	0.2746	9.0404	0.0582	15.4045	0.4365	6.3641	0.3783
	$d = 3$	0.2956	8.3378	0.0356	15.4045	0.4365	7.0667	0.4009
	$d = 4$	0.3068	7.9391	0.0242	15.4045	0.4365	7.4654	0.4123
	$d = 5$	0.3138	7.6822	0.0174	15.4045	0.4365	7.7223	0.4191
	$d = 6$	0.3186	7.5027	0.0129	15.4045	0.4365	7.9018	0.4236
	$d = 7$	0.3221	7.3703	0.0097	15.4045	0.4365	8.0342	0.4268
	$d = 8$	0.3248	7.2686	0.0074	15.4045	0.4365	8.1359	0.4291
	$d = 9$	0.3268	7.1880	0.0056	15.4045	0.4365	8.2165	0.4309
Higher γ	$\gamma = 2$	0.2542	9.6812	0.0613	15.4045	0.3274	5.7233	0.2661
	$\gamma = 3$	0.2529	9.7225	0.0417	15.4045	0.2183	5.6820	0.1766
	$\gamma = 4$	0.2522	9.7435	0.0316	15.4045	0.1637	5.6610	0.1321
	$\gamma = 5$	0.2517	9.7563	0.0254	15.4045	0.1310	5.6482	0.1056
	$\gamma = 6$	0.2514	9.7648	0.0213	15.4045	0.1091	5.6397	0.0878
	$\gamma = 7$	0.2512	9.7710	0.0183	15.4045	0.0935	5.6335	0.0752
	$\gamma = 8$	0.2511	9.7756	0.0160	15.4045	0.0818	5.6289	0.0658
	$\gamma = 9$	0.2510	9.7792	0.0143	15.4045	0.0728	5.6253	0.0585

thus making innovation less valuable for society, ultimately leading to shorter patents in equilibrium. The opposite argument applies when R&D becomes more costly (higher γ) since T_{EA} is not directly affected by γ, but it is indirectly via a less stringent emission fee (recall that the EA anticipates less investment in R&D, followed by less output and pollution). Overall, a lower fee τ^* yields a longer patent. Table 3 summarizes the direct and indirect effects from changes in each parameter, as well as their overall effect on T_{EA}.

Table 3
Direct and indirect effects on T_{EA}.

	Direct effect	Indirect effect	Overall effect on T_{EA}
Higher α	–	–	–
Higher c	–	+	–
Higher r	–	+	–
Higher d	None	–	–
Higher γ	None	+	+

Table 4 summarizes our results about how patent lengths, emission fees, and R&D investments are affected by changes in parameters.

Regulation vs. No regulation. The last two columns in Table 2 examine the effect of regulation on the patent length, $\Delta T \equiv T_0 - T_{EA}$, and on equilibrium investment, $\Delta x \equiv x_0 - x_{EA}$. Both ΔT and Δx are unambiguously positive, meaning that the presence of environmental policy induces less investment and shorter patents, as discussed above. As expected, this effect is particularly intense when the innovation does not bring large cost-reduction effects (lower α), pollution becomes more severe (higher d), and R&D is less costly (lower γ). It is in these contexts

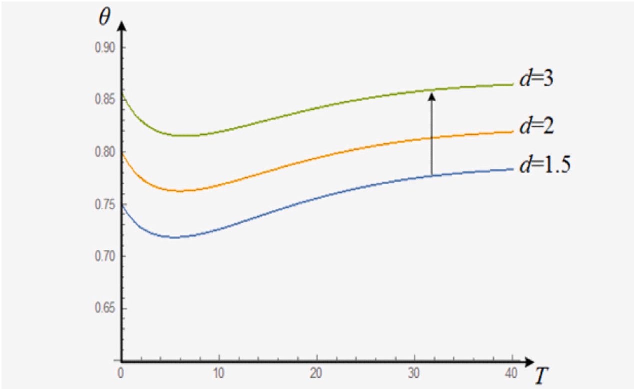


Fig. 4. Weight θ on fee τ^P.

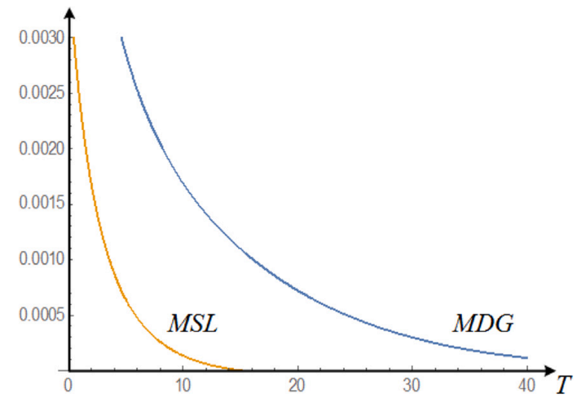


Fig. 5. MDG and MSL with EA acting second.

that a PO should be more aware of environmental policies, anticipating firms' responses to regulation, thus shortening patent lengths more significantly.

4.2. The EA acts in the second stage

Still considering the role of environmental regulation, assume that the EA acts in the second stage. This entails that, in the first stage, the PO sets the patent length T; in the second stage, the EA responds setting the emission fee τ; in the third stage, firms choose their investment in R&D, x; and in the fourth stage, firms compete.

In this context, equilibrium behavior in stages 3–4 is analogous to those in Section 4.1. In the second stage, the EA anticipates per-period welfare before the patent expires, W^P_{EA}, after the patent expires, W^C_{EA}, and in the absence of innovation, W^N_{EA}; as defined in (1). (For compactness, these expressions are presented in Appendix A.) Therefore, the agency sets the emission fee τ that solves

$$\max_{\tau\geq 0}\ W = x(T,\tau)\left(\int_0^T W^P_{EA}dt + \int_T^{+\infty} W^C_{EA}dt\right) + [1-x(T,\tau)]\int_0^{+\infty} W^N_{EA}dt - \frac{\gamma}{2}[x(T,\tau)]^2 \tag{7}$$

which, as shown in Appendix A, yields emission fee $\tau(T)$ that can be presented as the following linear combination

$$\tau(T) = \theta\tau^P + (1-\theta)\tau^C$$

where $\tau^P = \frac{2d(1-c)-\alpha}{1+2d}$ and $\tau^C = \frac{2d(1-c)+2d\alpha}{1+2d}$ represent the fee under monopoly and perfect competition, respectively, and satisfy $\tau^C > \tau^P$, indicating that pollution is larger after the patent expires, requiring a more stringent fee.

For compactness, weight $\theta \in [0,1]$ is presented in Appendix A, and Fig. 4 depicts this weight as a function of the patent length T.[12] When T is relatively short, the EA anticipates a short monopoly but a long period of perfect competition with low costs, yielding an overall increase in pollution. In this context, it seeks a more stringent fee, thus assigning a lower weight on τ^P, i.e., θ decreases in T. However, when patents become longer, the EA anticipates a monopoly during more periods, and thus less pollution. This agency sets, then, a less stringent fee, assigning a higher weight on τ^P, i.e., θ increases in T.

Fig. 4 also shows that weight θ increases in the severity of pollution (higher d), thus assigning more weight to the monopoly fee τ^P. However, this result does not imply that overall tax burden decreases when d increases, since both τ^P and τ^C are increasing in d. As shown in Table A.1 in Appendix A, the overall fee $\tau(T)$ is also increasing in d. This table reports comparative statics of θ, τ^P, τ^C, and $\tau(T)$ with respect to γ, c, α, and r.

In the first stage, the PO anticipates emission fee $\tau(T)$, which induces investment level $x(T,\tau(T))$, and yields welfare levels $W^P = \frac{(1-c-\tau(T))^2}{2} + \alpha(1-c-\tau(T))$, $W^C = \frac{[1-(c-\alpha+\tau(T))]^2}{2}$, and $W^N = \frac{[1-(c+\tau(T))]^2}{2}$. Unlike the EA, the PO does not directly consider environmental damages, but these welfare levels are still indirectly affected by pollution severity, d, since they are a function of the emission fee $\tau(T)$. The PO, then, chooses the patent length T to solve a problem analogous to (3), that is,

$$\max_{T\geq 0}\ W = x(T,\tau(T))\left(\int_0^T e^{-rt}W^P dt + \int_T^{+\infty} e^{-rt}W^C dt\right) + [1-x(T,\tau(T))]\int_0^{+\infty} e^{-rt}W^N dt - \frac{1}{2}\gamma[x(T,\tau(T))]^2 \tag{8}$$

Before solving (8), let us compare the PO's problem with and without regulation. In the absence of environmental regulation, the PO anticipates that a longer patent gives rise to a monopoly for one more period, yielding a welfare loss of $WL_0 \equiv W^C_0 - W^P_0 = \frac{\alpha^2}{2}$; and with regulation this welfare loss remains unaffected at $WL \equiv W^C - W^P = \frac{\alpha^2}{2}$. Intuitively, production costs are increased by the same fee τ before and after the patent expires, thus yielding the same output change.

While the marginal static loss from a longer patent is composed of two terms: (i) a longer monopoly, and (ii) higher investment costs; our results show that (i) is unchanged as a result of regulation but (ii) decreases. This occurs because investments are lower with than without regulation ($x_1(T) < x_0(T)$) and, due to the convexity of the cost of R&D function, marginal costs are also lower. In contrast, the marginal dynamic gain is larger since R&D gains are concave. Overall, both effects induce the PO to set longer patents.

Differentiating (8) with respect to T yields a highly non-linear first-order condition, but the marginal dynamic gain from a longer patent exceeds its marginal static loss for all values of T, as suggested above, and depicted in Fig. 5 (this figure considers the same parameter values as the benchmark of Table 1). Table A.2 in Appendix A shows that this ranking is robust to different parameter combinations.

4.3. The EA acts in the third stage

Consider that the EA has the ability to revise emission fees after patent lengths and R&D investments are chosen. For compactness, Appendix B identifies equilibrium investments, emission fees, and welfare in this context, while here we focus on examining the PO's decision.

In the current setting, the EA sets emission fee τ^P before the patent expires, τ^C after the patent expires, and τ^N when no innovation occurs, which satisfy $\tau^C > \tau^P > \tau^N$, to account for the market monopolization during patent periods. Therefore, these fees induce the same output

[12] For consistency, Fig. 4 considers the same parameter values as Figs. 2–3.

before and after the patent expires, yielding a PO's welfare of $W^P = W^C = \frac{(1+4d)[1-(c-\alpha)]^2}{2(1+2d)^2}$; whereas when innovation does not occur this welfare becomes $W^N = \frac{(1+4d)(1-c)^2}{2(1+2d)^2}$.

As a consequence, the social return of innovation, relative to no innovation, becomes $S_1 = \frac{\alpha[\alpha+2(1-c)](1+4d)}{2r(1+2d)^2}$, which is unaffected by the patent length, T; as opposed to $S_0(T)$. Hence, the PO's problem simplifies to

$$\max_{T\geq 0} \; x_1(T)S_1 - \int_0^{+\infty} e^{-rt}W^N dt - \frac{1}{2}\gamma\left[x_1(T)\right]^2 \tag{9}$$

where $\int_0^{+\infty} e^{-rt}W^N dt = \frac{(1-c)^2(1+4d)}{2r(1+2d)^2}$ is also constant in T. This maximization problem yields first-order condition $S_1 - \gamma x_1(T) \geq 0$, simplifying to $e^{-rT} \geq -\frac{(1-c)(1+6d)+2d\alpha}{2[1-(c-\alpha)](1+2d)}$. This condition holds with strict inequality for all parameter values, entailing that $S_1 > \gamma x_1(T)$. Therefore, the marginal dynamic gain from a longer patent exceeds its marginal static loss, as the following proposition shows.

Proposition 2. *Both $MDG_1(T)$ and $MSL_1(T)$ are unambiguously decreasing in d ; $MDG_1(T)$ decreases in T, $MSL_1(T)$ decreases in T if and only if $T > \frac{\log 2}{r}$; and $MDG_1(T) > MSL_1(T)$ for all T.*

The Nordhaus' trade-off, which emerges under no regulation, breaks down when the EA sets policy in the third stage. This occurs because the PO anticipates that a longer patent will *not* alter output, since it coincides before and after the patent expires.

5. Extensions

5.1. Allowing for green innovations

Consider that investments not only reduce production costs but also environmental damages (a "green" innovation). In particular, pollution severity decreases from d to $d - \lambda$, where $d \geq \lambda \geq 0$; but remains unaffected if the innovation is not successful. When $\lambda = 0$, the innovation does not reduce pollution severity, as in our main model; whereas when $\lambda = d$, the innovation makes output completely clean. (For compactness, we study the effect of green innovations in the setting of Section 4.1.)

In the fourth stage, equilibrium output levels are unaffected by the decrease in parameter d. In the third stage, the innovator chooses investment level, $x(T,\tau) = \frac{\alpha(1-c-\tau)(1-e^{-rT})}{\gamma r}$, which coincides with that in Section 4.1.

In the second stage, the PO chooses the optimal patent length similarly as in Section 4.1, and is unaffected by the environmental damages or green innovations (i.e., it is not a function of d or $d-\lambda$), that is, $T_{EA} = \frac{1}{r}\log\left[\frac{2[1-(c-\alpha+\tau)]}{\alpha}\right]$.

Finally, in the first stage, the EA anticipates patent length T_{EA} and chooses emission fee τ to solve

$$\max_{\tau\geq 0} \; x(T,\tau)\left(\int_0^T e^{-rt}W_{EA}^P(\lambda)dt + \int_T^{+\infty} e^{-rt}W_{EA}^C(\lambda)dt\right) \tag{6'}$$

$$+\left[1-x(T,\tau)\right]\int_0^{+\infty} e^{-rt}W_{EA}^N dt - \frac{\gamma}{2}\left[x(T,\tau)\right]^2$$

which is analogous to (6), but with welfare levels being a function of the green innovation parameter, λ.[13]

Differentiating with respect to τ yields a highly nonlinear first-order condition, not allowing for an explicit solution for $\tau(\lambda)$. Table 5 numerically evaluates the emission fee, patent length, and investment in equilibrium, at the same parameters as in Table 1. For comparison purposes, the top row considers $\lambda = 0$, obtaining the same results as in Table 1; while lower rows allow more intense green innovations (higher λ).

As expected, when innovations are greener (higher λ), the EA anticipates less pollution, and thus sets a less stringent emission fee $\tau(\lambda)$. Facing a lower fee, the PO responds with a longer patent T_{EA} in the second stage. Intuitively, the PO anticipates firms will face lower costs (net of taxes), implying that the cost-reduction effect of the patent is more intense. This makes the innovation more socially attractive and the PO sets longer patents. Finally, firms respond to longer patents and less stringent fees investing more in R&D. Therefore, while firms ignore the pollution-reduction effects of their innovation, they benefit from this innovation in the form of lower taxes and longer patents, ultimately providing them with incentives to invest more in R&D than when innovations do not reduce pollution.

The previous-to-last column in Table 5 evaluates the patent decrease due to environmental regulation, $\Delta T \equiv T_0 - T_{EA}$, which is positive for all parameter values, but decreases in λ. As the innovation becomes greener, environmental policy is less necessary (lower emission fees), entailing that the setting with regulation approaches that without fees, yielding more similar patent lengths. Similarly, the last column reports the investment reduction as a result of regulation, $\Delta x \equiv x_0 - x_{EA}$, which is also positive for all parameters and decreases in λ. In this case, firms face less stringent fees, and patents approach those without regulation, ultimately inducing similar R&D investments with and without environmental policy.

5.2. Allowing for non-linear demand

Consider an inverse demand function $p(Q,b) = 1 - Q^{(1+b)}$, where parameter $b \geq 0$. When $b = 0$, this inverse demand function simplifies to $p(Q,0) = 1-Q$, as in the main body of the paper. Otherwise, the function is decreasing at an increasing rate since $p'(Q,b) = -(1+b)Q^b < 0$ and $p''(Q) = -b(1+b)Q^{-(1-b)} < 0$. For illustration purposes, Fig. 6 depicts $p(Q,b) = 1 - Q^{(1+b)}$ evaluated at different values of parameter b ($b = 0$, $b = 1/2$, and $b = 1$), showing that a more concave inverse demand function becomes more bowed-out from the origin. As we examine in this section, this implies that, for a given marginal cost c, aggregate output increases when demand becomes more concave (higher b).

Table 4
Comparative statics.

	Patent length, T_{EA}	Fee stringency, τ^*	R&D investment, x_{EA}
Higher α	Shorter	More stringent	More investment
Higher c	Shorter	Less stringent	Less investment
Higher r	Shorter	Less stringent	Less investment
Higher d	Shorter	More stringent	Less investment
Higher γ	Longer	Less stringent	Less investment

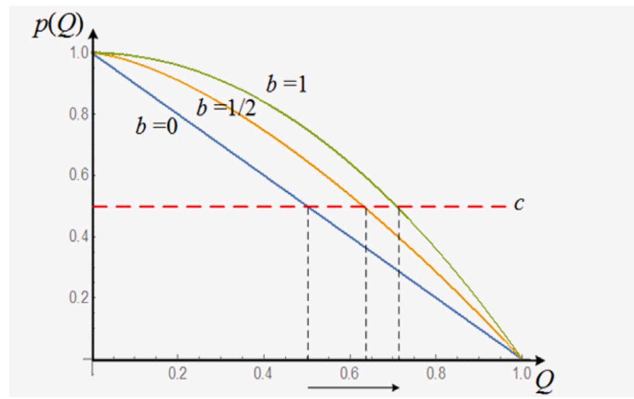


Fig. 6. Inverse demand $p(Q,b) = 1 - Q^{1+b}$.

[13] Welfare W_{EA}^N is unaffected by λ since the innovation does not occur in this setting, thus not bringing pollution-reduction effects. Welfare $W_{EA}^P(\lambda)$ and $W_{EA}^C(\lambda)$ are both unambiguously increasing in λ.

5.2.1. Equilibrium analysis when the EA is absent

Third stage, output decisions. After the patent expires (in periods $t \geq T$), every firm i enjoys production cost $c-\alpha$ in a perfectly competitive market. Then, aggregate output solves $p(Q^C, b) = c-\alpha$, which in this case entails $1-Q^{(1+b)} = c-\alpha$, yielding $Q^C(b) = (1-c+\alpha)^{\frac{1}{1+b}}$. When $b=0$, aggregate output simplifies to $Q^C(0) = 1-c+\alpha$, as in Section 3.1; but otherwise it decreases in b. Since the market is perfectly competitive, every firm earns zero profit, $\pi^C = 0$, which is unaffected by parameter b.

Before the patent expires ($t < T$), the innovator enjoys production cost $c-\alpha$, choosing a monopoly output $q^m(b) = \left(\frac{1-c+\alpha}{2+b}\right)^{\frac{1}{1+b}}$, which simplifies to $q^m(0) = \frac{1+c-\alpha}{2}$ in the special case that $b=0$. In every patent period, the innovator's monopoly price is $p^m(b) = 1 - \left[\left(\frac{1-c+\alpha}{2+b}\right)^{\frac{1}{1+b}}\right]^{1+b} = \frac{1+c-\alpha+b}{2+b}$, which simplifies to $p^m(0) = \frac{1+c-\alpha}{2}$ when evaluated at $b=0$. In this context, for the innovation to be non-radical, $p^m(b) > c$, we need that

$$\alpha < \overline{\alpha}(b) \equiv (1-c)(1+b),$$

where cutoff $\overline{\alpha}(b)$ simplifies to $\overline{\alpha}(0) = 1-c$ when evaluated at $b=0$. The innovator, then, becomes the only seller, setting a price below its rivals' common marginal cost, c, that is, $p^P = c-\varepsilon$, where $\varepsilon \to 0$; and patent output is $q^P(b) = Q_0 = (1-c)^{\frac{1}{1+b}}$, which collapses to $q^P(0) = 1-c$ when evaluated at $b=0$, as in Section 3.1.

In this context, the innovator earns monopoly profits in every patent period, as follows,

$$\pi^P(b) = \left[p^P - (c-\alpha)\right] q^P = \alpha(1-c)^{\frac{1}{1+b}}$$

which are positive during the patent but zero afterwards, that is, $\pi^P(b) > \pi^C(b) = 0$. When demand is linear, $b=0$, these profits simplify to $\pi^P(0) = \alpha(1-c)$, as in Section 3.1; but otherwise profit $\pi^P(b)$ increases in b. Intuitively, the patentee charges a price of c, which is unaffected by the demand curvature (as captured by parameter b), but sells more units, thus earning more profits per period.

Second stage, R&D investment. Anticipating output decisions in the last stage, the innovator chooses its R&D investment, x, to solve a problem analogous to (2),

$$\max_{x\geq 0}\ x\Pi_0(T,b) + (1-x)\int_0^{+\infty} e^{-rt}\pi^N(b)dt - \frac{1}{2}\gamma x^2 \tag{2'}$$

where

$$\Pi_0(T,b) = \int_0^T e^{-rt}\pi^P(b)dt + \int_T^{+\infty} e^{-rt}\pi^C(b)dt = \frac{\alpha(1-c)^{\frac{1}{1+b}}\left(1-e^{-rT}\right)}{r}$$

where the second equality considers $\pi^C(b) = 0$ and $\pi^P(b) = \alpha(1-c)^{\frac{1}{1+b}}$. Differentiating with respect to investment x in problem (2'), yields

$$x_0(T,b) = \frac{\Pi_0(T,b)}{\gamma} = \frac{\alpha(1-c)^{\frac{1}{1+b}}\left(1-e^{-rT}\right)}{\gamma r},$$

which exhibits the same comparative statics as $x_0(T)$ in Lemma 1, and increases in b since $\frac{\partial x_0(T,b)}{\partial b} = -\frac{\alpha(1-c)^{\frac{1}{1+b}}\left(1-e^{-rT}\right)\log(1-c)}{\gamma r(1+b)^2} > 0$ because $\log(1-c) < 0$ for all admissible values. Intuitively, this occurs because, as shown above, a more concave demand function helps the innovator earn higher per-period profits, inducing this firm to invest more in R&D than under linear demand. In addition, when $b=0$, this investment simplifies to $x_0(T,0) = x_0(T)$, as in Lemma 1.

First stage, patent length. The PO anticipates the firms' decisions in the subsequent stages of the game, choosing the patent length, T, that solves the following problem,

$$\max_{T\geq 0}\ x_0(T,b)\left(\int_0^T e^{-rt}W_0^P(b)dt + \int_T^{+\infty} e^{-rt}W_0^C(b)dt\right)$$

$$+\left[1-x_0(T,b)\right]\int_0^{+\infty} e^{-rt}W_0^N(b)dt - \frac{1}{2}\gamma\left[x_0(T,b)\right]^2$$

where $x_0(T,b)$ was identified above, and the term in the first (second) line denotes the welfare with (without) innovation. In addition, $W_0^P(b) = \frac{(1-c)(1+b)-c(1-c^{1+b})}{2+b} + \alpha(1-c)^{\frac{1}{1+b}}$ denotes welfare in each patent period, where consumer surplus is defined as the area under the inverse demand curve, $p(Q) = 1-Q^{1+b}$, that is, $CS_0^N(b) = \int_c^1\left(1-y^{1+b}\right)dy = \frac{(1-c)(1+b)-c(1-c^{1+b})}{2+b}$, which is unambiguously increasing in the demand curvature, b, since $\frac{\partial CS_0^N(b)}{\partial b} = \frac{1+c^{2+b}[(2+b)\log(c)-1]}{(2+b)^2} > 0$ for all admissible parameters. Profit $\alpha(1-c)^{\frac{1}{1+b}}$ is also increasing in b, implying that welfare $W_0^P(b)$ is increasing in b.

Similarly, $W_0^C(b) = (1-c+\alpha) - \frac{1-(c-\alpha)^{2+b}}{2+b}$ measures welfare after the patent expires, and $W_0^N(b) = \frac{(1-c)(1+b)-c(1-c^{1+b})}{2+b}$ represents welfare without the innovation. When demand is linear, $b=0$, these welfares coincide with those in section 3.3, i.e., $W_0^P(0) = \frac{(1-c)^2}{2} + \alpha(1-c)$, $W_0^C(0) = \frac{[1-(c-\alpha)]^2}{2}$, and $W_0^N(0) = \frac{(1-c)^2}{2}$. Otherwise, all welfare expressions increase in b.

Simplifying, the above problem, we find that

$$\max_{T\geq 0}\ x_0(T,b)S_0(T,b) + \int_0^{+\infty} e^{-rt}W_0^N(b)dt - \frac{1}{2}\gamma\left[x_0(T,b)\right]^2 \tag{3'}$$

where $S_0(T,b)$ represents the social return of the innovation, relative to that without the innovation, defined as

$$S_0(T,b) = \left(\int_0^T e^{-rt}W_0^P(b)dt + \int_T^{+\infty} e^{-rt}W_0^C(b)dt\right) - \int_0^{+\infty} e^{-rt}W_0^N(b)dt\ .$$

Next, differentiating the PO's problem in expression (3') with respect to T, yields

$$\underbrace{\frac{\partial x_0(T,b)}{\partial T}S_0(T,b)}_{MDG_0(b)} = \underbrace{x_0(T,b)\left(\gamma\frac{\partial x_0(T,b)}{\partial T} - \frac{\partial S_0(T,b)}{\partial T}\right)}_{MSL_0(b)}.$$

$MDG_0(b)$ and $MSL_0(b)$ are both highly non-linear in b, and Fig. 7 depicts these curves considering, for comparison purposes, the same parameter values as in Fig. 1 in the main body of paper, and evaluated at different values of b: (i) when $b=0$, yielding $MDG_0(0)$ and $MSL_0(0)$, which coincide with linear demand in Section 3.3; (ii) when $b=\frac{1}{2}$, resulting in $MDG_0(1/2)$ and $MSL_0(1/2)$; and (iii) when $b=1$, entailing $MDG_0(1)$ and $MSL_0(1)$.

First, a more concave demand function (higher b) produces an upward shift in $MDG_0(b)$, indicating that a longer patent provides firms with more incentives to invest in R&D, $\frac{\partial x_0(T,b)}{\partial T}$, than under linear demand. Second, a higher b also produces an upward shift in $MSL_0(b)$,

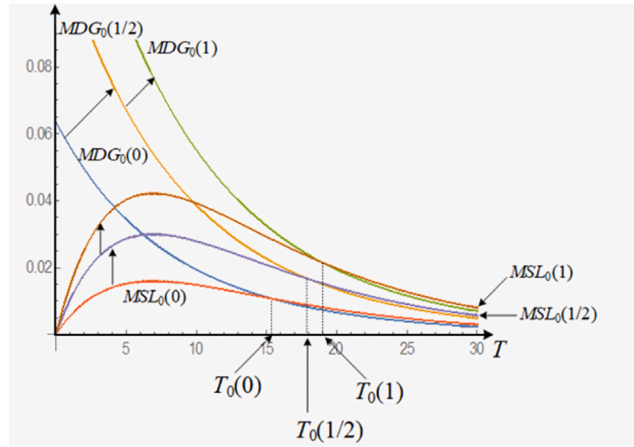


Fig. 7. $MDG_0(b)$ and $MSL_0(b)$ as a function of b.

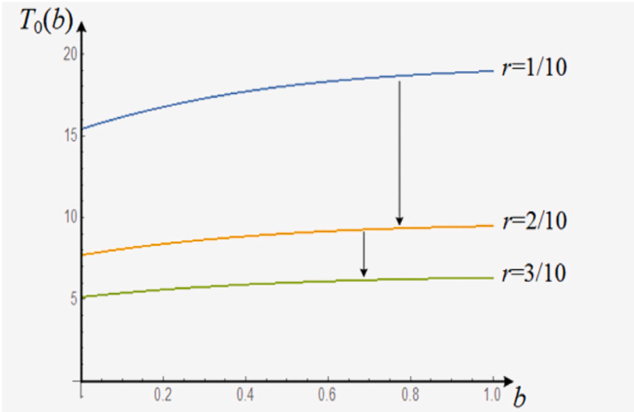


Fig. 8. Patent length without regulation, $T_0(b)$.

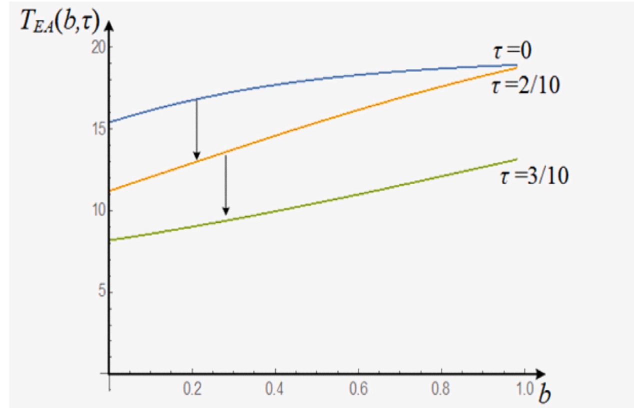


Fig. 9. Patent length $T_{EA}(b,\tau)$ at different values of τ.

implying that, because of the higher willingness-to-pay under a concave demand function, the legal monopoly of a patent reduces welfare more intensively than under linear demand. In other words, the gain in consumer surplus that arises after the patent expires increases as demand becomes more concave, making a longer patent relatively worse than under linear demand, $b = 0$. Overall, the former effect dominates, as illustrated in Fig. 7, leading the PO to set longer patents in equilibrium, i.e., $T_0(b) > T_0(0)$. In particular, the optimal patent length without environmental regulation is

$$T_0(b) = \frac{1}{r}\log\left[\frac{\alpha(2+b)\left[2-(1-c)^{\frac{1}{1+b}}\right]-2\left[c^{2+b}-(c-\alpha)^{2+b}\right]}{\alpha(2+b)\left[1-(1-c)^{\frac{1}{1+b}}\right]-\left[c^{2+b}-(c-\alpha)^{2+b}\right]}\right],$$

which in the case of linear demand, $b = 0$, simplifies to $T_0(0) = \frac{1}{r}\log\left[\frac{2[1-(c-\alpha)]}{\alpha}\right]$, coinciding that in Proposition 1. Fig. 8 depicts patent length $T_0(b)$ as a function of the demand curvature, b, considering the same parameter values as previous figures. Patent $T_0(b)$ is increasing in b, and shifts downwards as r increases; as in the main model.

5.2.2. Equilibrium analysis when the EA is present

Third stage, R&D decisions. In this context, equilibrium behavior in stages 2–4 is analogous to that in Section 4.1. In the third stage, every firm chooses its R&D investment, x, to solve a problem analogous to (2), yielding an equilibrium investment of $x(T,b,\tau) = \frac{\alpha(1-c-\tau)^{\frac{1}{1+b}}\left(1-e^{-rT}\right)}{\gamma r}$, which is decreasing in the emission fee τ. In addition, this investment satisfies

$$0 < x(T,b,\tau) < x_0(T,b) \;\text{ and }\; 0 < x'(T,b,\tau) < x_0'(T,b),$$

implying that firms invest less with than without regulation and that, when facing a longer patent, they increase their investments less significantly when facing regulation than otherwise.

Second stage, patent decisions. The PO's problem is analogous to (3), yielding similar expressions of $MDG(\tau)$ and $MSL(\tau)$, which are now a function of fee τ. A more stringent emission fee drives firms to invest less in R&D, as shown above, shifting $MDG(\tau)$ and $MSL(\tau)$ downwards, as analyzed in Section 4.1 (see Fig. 2 in the main body of the paper). When demand is linear $b = 0$ and, more generally, when b is relatively low, a more stringent regulation produces a larger downward shift in $MDG(\tau)$ than in $MSL(\tau)$, implying that the crossing point between $MDG(\tau)$ and $MSL(\tau)$ moves leftward, and the equilibrium patent length becomes shorter. In contrast, when demand is sufficiently concave (high b), a more stringent emission fee τ produces a larger downward shift in $MSL(\tau)$ than in $MDG(\tau)$, entailing that their crossing point moves rightward, and the equilibrium patent becomes longer.

In particular, the PO chooses an equilibrium patent that is a function of fee τ, as follows

$$T_{EA}(b,\tau) = \frac{1}{r}\log\left[\frac{\alpha(2+b)\left[2-(1-c-\tau)^{\frac{1}{1+b}}\right]-2\left[(c+\tau)^{2+b}-(c+\tau-\alpha)^{2+b}\right]}{\alpha(2+b)\left[1-(1-c-\tau)^{\frac{1}{1+b}}\right]-\left[(c+\tau)^{2+b}-(c+\tau-\alpha)^{2+b}\right]}\right], \tag{5'}$$

which simplifies to $T_{EA}(0,\tau) = \frac{1}{r}\log\left[\frac{2[1-(c-\alpha+\tau)]}{\alpha}\right]$ when demand is linear, $b = 0$; see Eq. (5) in Section 4.1. While the comparative statics of patent length $T_{EA}(b,\tau)$ with respect to b and τ are highly non-linear, Fig. 9 illustrates $T_{EA}(b,\tau)$ as a function of b, and evaluates it at different fee stringencies ($\tau = 0$, $\tau = 0.2$, and $\tau = 0.3$) considering, for comparison purposes, the same parameter values as Fig. 8 and $r = 1/10$.[14] When $\tau = 0$, environmental policy is absent, and the patent length becomes $T_{EA}(b,0)$, coinciding with that in $T_0(b)$, as depicted in Fig. 8.

As in the absence of regulation, the patent length increases when demand becomes more concave, i.e., $T_0(b)$ increases in b in Fig. 8 and, similarly, $T_{EA}(b,\tau)$ increases in b in Fig. 9. A more stringent emission fee produces an unambiguous shortening of the patent when demand is linear, as depicted by the downward shift in the vertical intercept of $T_{EA}(b,\tau)$ where $b = 0$. This result coincides with that in section 4.1, where the EA's presence leads to shorter patents under linear demands. A similar argument applies when demand is concave, but b is relatively low, where $T_{EA}(b,\tau)$ still shifts downwards when the emission fee becomes more stringent (higher τ), although less significantly than when $b = 0$; see the left side of Fig. 9.

First stage, emission fee. The EA anticipates the welfare in each case, $W^P_{EA}(b)$, $W^C_{EA}(b)$, and $W^N_{EA}(b)$, and solves a problem identical to (6),

$$\max_{\tau\geq 0}\; x(T,b,\tau)\left(\int_0^T e^{-rt}W^P_{EA}(b)dt + \int_T^{+\infty} e^{-rt}W^C_{EA}(b)dt\right) \tag{6'}$$
$$+\,[1-x(T,b,\tau)]\int_0^{+\infty} e^{-rt}W^N_{EA}(b)dt - \frac{\gamma}{2}\,[x(T,b,\tau)]^2$$

As in Section 4.1, differentiating with respect to fee τ yields intractable results, and we rely on numerical simulations. Table 6 considers, for comparison purposes, the same parameter values as in Table 2. The first row in the table assumes a linear demand, $b = 0$, obtaining the

[14] The initial condition for non-radical innovations, $\alpha < \overline{\alpha}(b)$, holds in this context since cutoff $\overline{\alpha}(b)$ originates at $\overline{\alpha}(0) = \frac{1}{3}$ when demand is linear, and increases in b, reaching $\overline{\alpha}(1) = \frac{2}{3}$ at the upper bound of b in Fig. 8. Therefore, condition $\alpha < \overline{\alpha}(b)$ is satisfied for all values of b.

Table 5
Equilibrium outcomes with green innovations.

	Regulation			No regulation		Comparison	
	$\tau(\lambda)$	T_{EA}	x_{EA}	T_0	x_0	ΔT	Δx
Benchmark, $\lambda = 0$	0.2555	9.6409	0.0802	15.4045	0.4365	5.7636	0.3563
$\lambda = 0.05$	0.2547	9.6667	0.0812	15.4045	0.4365	5.7378	0.3553
$\lambda = 0.10$	0.2538	9.6929	0.0822	15.4045	0.4365	5.7116	0.3543
$\lambda = 0.15$	0.2529	9.7195	0.0832	15.4045	0.4365	5.6850	0.3533
$\lambda = 0.20$	0.2520	9.7467	0.0843	15.4045	0.2183	5.6578	0.1340
$\lambda = 0.25$	0.2511	9.7743	0.0854	15.4045	0.1309	5.6302	0.0455
$\lambda = 0.30$	0.2502	9.8026	0.0866	15.4045	0.0935	5.6019	0.0069

same results as in the top row of Table 2. Subsequent rows, however, allow for demand to become more concave (higher b), identifying how equilibrium results are affected, including the optimal emission fee, τ^*; the patent length, $T_{EA} = T_{EA}(b, \tau^*)$; and the R&D investment, $x_{EA} = x_{EA}(T_{EA}, \tau^*)$.

Therefore, an increase in b induces a more stringent emission fee, τ^*. This occurs because, when demand becomes more concave, the EA anticipates more aggregate output and pollution in equilibrium, requiring a more stringent fee.

An increase in b, however, gives rise to direct and indirect effects on patent lengths, T_{EA}. In particular, as demand becomes more concave (higher b), the PO sets longer patents, as shown in equation (5') and Fig. 7 (positive direct effect). The EA responds to more concave demand setting a more stringent fee, which shortens the patent (i.e., negative indirect effect). Table 6 indicates that, when b is low, the former effect dominates, yielding an overall increase in patent lengths, T_{EA}. However, when concavity is further increased, the latter effect dominates, and patents shortened. A similar argument applies to R&D investments. More concave demand induces firms to anticipate larger profits, investing more in R&D (positive direct effect), but the EA responds to this demand change by setting more stringent fees, which reduce investment incentives (i.e., negative indirect effect). As reported in Table 6, the former (latter) effect dominates, yielding an overall increase (decrease) in R&D investments when b is relatively low (high, respectively).

In the absence of environmental regulation, a more concave demand (higher b) induces the PO to set longer patents. This result was already shown in Fig. 7 and confirmed in the column reporting T_0 in Table 6. A more concave demand, however, gives rise to positive direct and indirect effects, namely, firms anticipate larger per-period profits for a given patent length, and invest more in R&D; and the PO provides a longer patent, further reinforcing firms' incentives to invest.

The last two columns identify the role of environmental policy. First, introducing emission fees induces shorter patents, i.e., $\Delta T \equiv T_0 - T_{EA} > 0$ under all parameter conditions. This shortening effect is, in addition, augmented when demand becomes more concave, i.e., ΔT grows in b. A similar argument applies to R&D investments, which are lower with than without environmental regulation, $\Delta x \equiv x_0 - x_{EA} > 0$ for all parameters, with the differential expanding as demand becomes more concave.

6. Discussion

Our results help understand how environmental regulation affects patent policy. The presence of emission fees induces firms to invest less in R&D, which reduce the expected social return of the innovation, thus inducing the PO to set shorter patents.

A negative direct effect arises both with and without regulation, with patents shortening as firms' initial costs increase. With regulation, a positive indirect effect emerges, since emission fees become less stringent as costs increase, ultimately leading the PO to set longer patents. Overall, the negative direct effect dominates, yielding shorter patents. When the innovation becomes more cost-effective, however, the negative direct effect still arises, but the emission fee becomes more stringent, thus inducing a negative indirect effect, which reinforces the direct effect, implying an unambiguous patent reduction. A more severe pollution or less costly R&D do not affect patent lengths under no regulation, but they do with regulation. In particular, these changes induce a more stringent emission fee, thus producing a negative indirect effect on patents, lowering their lengths. These effects do not arise in the literature on patents that assume firms are not subject to environmental policy, such as Nordhaus (1969), Takalo (2001), or Denicolò (1996), among others; or in the literature that considers exogenous emission fees, see Langinier and Chaudhuri (2020).

When the EA acts in the second stage, firms' costs are uniformly affected by regulation in all scenarios (before and after the patent expires, and in the absence of innovation). This implies that the PO's decision to allow for longer patents, thus letting the monopolist persist for more periods, does not affect the severity of the welfare loss from the patent, relative to that without environmental regulation. Nonetheless, environmental policy lowers investment costs, thus reducing the overall welfare loss from an extended monopoly, so the PO can set longer patents to stimulate investments.

When the EA acts in the third stage, it can induce the same output level in all three scenarios, yielding the same welfare. In this setting, a longer patent does not affect social welfare at all, implying that the traditional welfare loss from a longer monopoly does not arise in the presence of regulation. Since, in addition, firms' R&D investment is lower with regulation, investment costs are lower as well, entailing that patent lengths become longer with regulation.

Overall, patents are shorter with than without environmental policy when the EA is relatively less flexible than the PO, but become longer otherwise. Hence, considering the EA's administrative ability to revise emission fees is important, since this flexibility can produce different effects in the PO's patent lengths decisions. Environmental agencies often face bureaucratic hurdles, not allowing for rapid adjustments of emission fees based on industry conditions, thus suggesting that POs should set shorter patents with than without regulation. Ignoring the presence of environmental policy when designing patent lengths will induce unnecessarily long monopolies, ultimately reducing social welfare.

Finally, we show that our results are robust to green innovations, when R&D investments not only reduce firms' production costs but also lower their pollution intensity. In particular, we demonstrate that green innovations help the EA lower the stringency of emission fees, which induces firms to increase their R&D investment, ultimately allowing the PO to set longer patents. In other words, as innovations become greener, environmental policy becomes less necessary, and patents approach those under no regulation. In these contexts, considering environmental regulation before setting patent lengths is less critical, but when innovations do not bring large reductions in pollution intensity, this consideration becomes more important for the PO. In the energy sector, our findings indicate that those innovations with low environmental impact should be granted patents that can ignore the interaction between the PO and EA. In contrast, energy innovations with severe environmental impacts should receive more attention when setting patent lengths.

Table 6
Equilibrium outcomes with and without regulation — The role of b.

		Regulation			No regulation		Comparison	
		τ^*	T_{EA}	x_{EA}	T_0	x_0	ΔT	Δx
	Benchmark, $b = 0$	0.2555	9.6409	0.0802	15.4045	0.4365	5.7636	0.3563
Higher b	$b = 0.25$	0.2897	9.7849	0.0849	17.0405	0.5661	7.2556	0.4813
	$b = 0.50$	0.3087	9.7694	0.0880	18.0441	0.6694	8.2747	0.5814
	$b = 0.75$	0.3196	9.6131	0.0888	18.6246	0.7515	9.0115	0.6627
	$b = 1.00$	0.3260	9.3587	0.0869	18.9531	0.8177	9.5944	0.7308
	$b = 1.25$	0.3296	9.0582	0.0829	19.1431	0.8720	10.0849	0.7891
	$b = 1.50$	0.3315	8.7581	0.0775	19.2628	0.9175	10.5047	0.8400
	$b = 1.75$	0.3325	8.4882	0.0715	19.3511	0.9564	10.8629	0.8849

Table 7
Summary of scenarios.

Regulatory setting	*Section*	*Patent length*	*R&D investment*
No EA	Section 3	Baseline	Baseline
EA acting first	Section 4.1	Shorter than baseline	Less than baseline
EA acting second	Section 4.2	Longer than baseline	Less than baseline
EA acting third	Section 4.3	Longer than baseline	Less than baseline
Green innovations	Section 5.1	Longer than Section 4.1	More than Section 4.1
Non-linear demand	Section 5.2	Longer/shorter than Section 4.1	More/less than Section 4.1

Table 7 compares patent lengths and R&D investments across different regulatory settings, summarizing our main findings.

While the PO does not consider environmental damages, we allow this agency to anticipate the strategic implications of environmental regulation. In contrast, much of the existing literature implicitly assumes that patent decisions operate in a regulatory vacuum, thereby neglecting how environmental policy influences firms' R&D and production decisions. Our analysis shows that ignoring regulatory feedback can lead to suboptimal patent policies. Although POs and EAs typically operate independently and pursue distinct objectives,[15] our results demonstrate that optimal patent policy must account for the regulatory environment in which firms operate. This interdependence is particularly important when market demand is highly concave, while it is mitigated in the case of green innovations, which face less stringent fees.

7. Conclusion

We examine the optimal setting of patent lengths in polluting industries under the influence of environmental regulation. The model considers a scenario where the PO sets patent lengths and firms invest in R&D to reduce production costs. We first assume that emission fees are administratively difficult to revise, reflecting real-world regulatory constraints; but then allow for the EA to revise emission fees in subsequent periods. Additionally, we consider green innovations that reduce costs and pollution intensity, showing how these innovations affect the interaction between patent policy and environmental regulation.

The results indicate that environmental policy generally leads to shorter patent lengths due to the increased costs and reduced R&D investments induced by emission fees. When the EA sets emission fees before the PO's decision, the PO responds with shorter patents to account for the higher costs and lower social return of innovations. Conversely, if the EA sets fees after the PO's decision, the PO may set longer patents, as the marginal dynamic gains from extended patents outweigh the marginal static losses. The study also finds that green innovations, which reduce pollution intensity, lead to less stringent emission fees and longer patents, as the social benefits of these innovations are higher. Overall, the findings suggest that patent offices should consider the environmental effects of new technologies when setting patent lengths, especially in the presence of stringent environmental regulations.

Further research. For tractability, our model considers assumptions that could be relaxed along several dimensions. First, the PO in our model considers consumer and producer surplus, but ignores environmental damages. While POs in most countries do not explicitly consider pollution effects in their patent assessments, it would be interesting to study if changing the PO's guidelines could lead to welfare-improving effects. Second, the market is perfectly competitive after the patent expires, which could be relaxed by allowing for oligopolistic competition. Third, production costs are linear, but a natural question is how the results are affected if, instead, these costs are convex, entailing that firms suffer from diseconomies of scale. Fourth, we assume that the PO can perfectly observe the severity of environmental damages used by the EA (parameter d). However, one could assume that the PO does not accurately observe this severity, potentially leading to further distortions in the patent length. Fifth, the cost-reducing effect of the innovation may be a function of the firm's investment (i.e., α being increasing in x).

Finally, our theoretical results can also be empirically tested, using patent length as the dependent variable, and emission fees, R&D investment, or number of green patents as independent variables. The regression results could help identify whether the presence of emission fees (or more stringent fees) has an effect on patent decisions. Alternatively, researchers can consider industries not being regulated by EA that suddenly become regulated by this agency, using difference-in-differences models to assess the impact of this policy change on patent lengths.

CRediT authorship contribution statement

Pak-Sing Choi: Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Ana Espínola-Arredondo:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Félix Muñoz-García:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Eugenio Diaz-Farina:** Writing – review & editing, Validation, Formal analysis, Conceptualization.

[15] A recent example is the cooperation between the National Oceanic and Atmospheric Administration (NOAA) and USPTO on climate-change related patents, including staff training and data exchanges, as reported by National Oceanic and Atmospheric Administration (2024).

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2025.108968.

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