



# International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: [www.tandfonline.com/journals/tmes20](http://www.tandfonline.com/journals/tmes20)

## Additive problem posing with integers by students in 6th year of Primary Education and 1st year of Secondary Education

Alberto Zapatera Llinares, Eduardo Quevedo Gutiérrez, Rubén Lijó Sánchez & Alicia Bruno Castañeda

To cite this article: Alberto Zapatera Llinares, Eduardo Quevedo Gutiérrez, Rubén Lijó Sánchez & Alicia Bruno Castañeda (06 Oct 2025): Additive problem posing with integers by students in 6th year of Primary Education and 1st year of Secondary Education, International Journal of Mathematical Education in Science and Technology, DOI: [10.1080/0020739X.2025.2562282](https://doi.org/10.1080/0020739X.2025.2562282)

To link to this article: <https://doi.org/10.1080/0020739X.2025.2562282>



Published online: 06 Oct 2025.



Submit your article to this journal [↗](#)



View related articles [↗](#)



View Crossmark data [↗](#)



# Additive problem posing with integers by students in 6th year of Primary Education and 1st year of Secondary Education

Alberto Zapatera Llinares<sup>a</sup>, Eduardo Quevedo Gutiérrez<sup>b</sup>, Rubén Lijó Sánchez<sup>b</sup> and Alicia Bruno Castañeda<sup>c</sup>

<sup>a</sup>Department of Innovation and Didactic Training, University of Alicante, Alicante, Spain; <sup>b</sup>Department of Mathematics, University of Las Palmas de Gran Canaria, Las Palmas de Gran Canaria, Spain; <sup>c</sup>Department of Mathematical Analysis, University of La Laguna, San Cristóbal de La Laguna, Spain

## ABSTRACT

This work shows how 266 students in the 6th year of Primary Education and the 1st year of Secondary Education pose additive problems with integers based on two questions raised from the abstract and number line dimensions. Transfers from these two dimensions to the contextual dimension are studied, depending on the propositional structure of the problem, the type of model used, and its semantic structure. The problems posed reproduce the ones worked on in class and are, for the most part, problems of change that use displacement models with little contextual variety. The results in 1st year of Secondary Education do not exceed those obtained in the 6th year of Primary Education, which indicates that the formulation of problems does not develop spontaneously, and that greater knowledge does not imply a better understanding of the connections of contextual situations with operations and with the number line. From the results it is inferred that problem posing is a skill that improves understanding and problem solving, stimulates curiosity, motivation and creativity, and, therefore, it is necessary to design a specific instruction to improve this.

## ARTICLE HISTORY

Received 12 June 2024

## KEYWORDS

Integers numbers; additive situations; contextual dimension; concrete models

## MSC

97F40; 97B10; 97B50

## 1. Introduction

Problem solving is a fundamental activity in mathematics education and constitutes a central axis of the mathematics curriculum of several countries, which places it as one of the specific competences to be developed in the different educational stages (NCTM, 2000; Real Decreto 157/2022; Real Decreto 217/2022). In addition, the formulation of problems by students has emerged for several decades as a skill that fosters the learning of mathematics, so it has relevance both at the curricular level (Real Decreto 157/2022; Real Decreto 217/2022) and in research (Cai et al., 2015).

Problem posing is the activity of creating, proposing or inventing mathematical problems, which includes both the generation of new problems and the modification or reformulation of given problems (Cai et al., 2015). This activity is typical of teachers in the

exercise of their profession, so problem posing studies have been carried out with teachers under training (Leavy & Hourigan, 2020) and with teachers in practice (Kapıcıoğlu & Arıkan, 2022). These studies suggest the incorporation of problem posing activities into the initial and continuous training programmes of Primary and Secondary Education mathematics teachers. In addition, problem posing is an effective way of learning mathematics (Kilpatrick, 1987; Polya, 1965) and, although its educational potential is far from being fully exploited (Espinoza et al., 2016; Torregrosa et al., 2021), its incorporation into mathematics teaching-learning processes is increasingly proposed (Ayllón et al., 2016; Ayllón & Gómez, 2014; Irvine, 2017; Munayco & Solís, 2021; Zhang et al., 2022).

Research on problem posing has mainly focused on the characteristics of the creation process and, to a lesser extent, on how it influences the mathematical content addressed. In addition, the problems posed by students are a means of evaluating the knowledge they have acquired on a certain topic. In this sense, Fernández and Carrillo (2020) examined arithmetic and geometric problems invented from illustrations by fifth-year students in primary school; Espinoza et al. (2016) studied how third-year students from Secondary Education enunciated arithmetic problems from figures with numerical information and verbal situations with three numerical data; Torregrosa et al. (2021) asked sixth-year students from Primary Education to invent problems with geometric patterns; and Bruno (2000) analysed the additive problems posing of integer numbers carried out by Secondary Education students. This work is based on a similar approach, analysing the formulation of problems by students in the 6th year of Primary Education (aged 11–12) and 1st year of Secondary Education (aged 12–13) based on two initial situations with integer numbers, which were previously provided.

Therefore, this study is situated in the transition from Primary to Secondary Education in Spain. While students have an initial approach to integers and additive operations in the last year of Primary Education, in the first year of Secondary Education the in-depth study of integers takes place. From this perspective, this research delves into the analysis of the characteristics of students' productions when they pose additive problems in the domain of integers. Moreover, it studies whether greater knowledge about integers and additive operations might imply a greater ability to pose additive problems with integers by comparing the results obtained from students belonging to the two different courses.

## 2. Theoretical framework

### 2.1. Problem posing in mathematics

Problem posing is a complex mathematical process that requires a personal, original and creative contribution to contents (Ayllón et al., 2016; Espinoza et al., 2016; Munayco & Solís, 2021).

When formulating a problem, it is necessary to think about a situation in a certain context, critically analyse the statement that is going to be proposed, examine the data and observe the different resolution strategies that it could have (Ayllón et al., 2016). From this perspective, creating problems is a mental activity that requires establishing relationships between mathematical concepts and using different cognitive processes, such as selecting, understanding, organising, editing and translating information from one representation to another (Christou et al., 2005).

Research in the field of problem solving considers that there is a high correlation between problem solving and problem posing skills (Ayllón & Gómez, 2014; Silver & Cai, 2005). This implies that, generally, students with greater problem-solving skills pose more problems and with a greater degree of complexity (English, 1998), and also that students instructed in problem posing tend to obtain better results in problem-solving than those who have received traditional instruction (Ayllón & Gómez, 2014; Bruno, 2000; Rudnitsky et al., 1995). That is, problem posing is a tool that improves problem-solving skills (Ayllón et al., 2016; Munayco & Solís, 2021).

Problem posing requires establishing relationships between mathematical concepts and structures, which is a useful skill in problem solving (Ayllón et al., 2016; Cankoy, 2014; Espinoza et al., 2016). In addition, to formulate problems, it is necessary to properly choose the information and data in the statement, which helps to understand the problem in depth and reduce resolution errors (Ayllón et al., 2016; Brown & Walter, 1993; Espinoza et al., 2016; Munayco & Solís, 2021). Different studies have focused on the creative capacity that problem posing can develop in students, since it is an activity that requires flexibility to change the approach to problems and develop different solutions to them (Ayllón et al., 2016; Espinoza et al., 2016; Porras & Castro, 2019; Silver & Cai, 2005).

The formulation of problems promotes the effective participation of students in their learning process, stimulating their curiosity, motivation and increasing their school performance and success (Munayco & Solís, 2021; Porras & Castro, 2019). On the other hand, different studies have observed that learning to pose problems fosters in students a favourable disposition towards mathematics, reducing fear, frustration and anxiety towards the subject and increasing their commitment and responsibility (Brown & Walter, 1993; Fernández & Carrillo, 2020; Munayco & Solís, 2021; Porras & Castro, 2019). In addition, using problem posing as a teaching strategy helps teachers evaluate students' understanding of mathematical contents and processes, as well as their perceptions and attitudes towards the subject (Ayllón et al., 2016; Silver & Cai, 2005).

Problem posing can be done before, during or after problem solving (Ayllón et al., 2016; Espinoza et al., 2016; Fernández & Carrillo, 2020). It is done before resolution, if the problem is formulated from a given situation or condition, and its goal is to create a problem. Problem posing can be carried out during the resolution of a problem when it is reformulated, by proposing a simpler one or another similar one, facilitating its understanding. It can be done after the resolution, when previously solved problems are reformulated, by modifying the initial one; and in the latter case, a strategy to follow is to ask yourself: 'What if ... ?' (Brown & Walter, 1993), and it is carried out in the 'look back' stage shown in Polya's method (1965).

Stoyanova and Ellerton (1996) identified three situations from which problems can be formulated: free situation, semi-structured situation and structured situation. A problem-posing task will be referred to as free 'when students are asked to generate a problem from a given, contrived or naturalistic situation. Some directions may be given to prompt certain specific actions' (p. 519); as semi-structured 'when students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences'; and as structured 'when problem-posing activities are based on a specific problem' (p. 520). In this research, the problems formulated by the students will be analysed from two semi-structured situations.

## 2.2. Teaching integer numbers

The formalisation of the integer numbers was a time-consuming process due to attempts by mathematicians to connect them with reality. This process lasted until the nineteenth century when integers became considered a mathematical creation that allowed the operations and properties of natural numbers to be extended (Glaeser, 1981). The historical evolution and moment of acceptance of integers are similar to the teaching sequence of the different number sets in compulsory schooling (Bishop et al., 2010). Thus, the initial teaching of numbers in the primary stage begins with the natural numbers, continues with the positive rational numbers, and, at the end of primary or beginning of secondary school, integers are introduced, culminating in compulsory schooling with the study of real numbers.

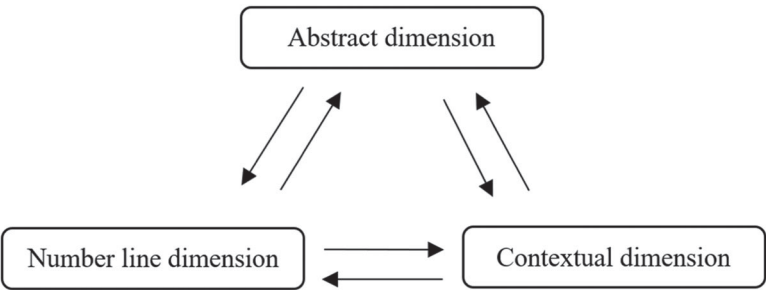
In this sequence of successive numerical extensions, a didactic effort is made to connect and make sense of numbers and their operations with reality. In the case of integers, as in the historical process, the learning effort involves didactic difficulties, widely collected in educational research for several decades (Bofferding, 2014; Bruno, 2000; Bruno & Martínón, 1997; Janvier, 1983). In this way, when using integers, students start from specific previous knowledge about positive numbers that may cause difficulties and errors in learning them (Bishop et al., 2010; Cid, 2003; Herrera & Zapatera, 2019; Quevedo et al., 2023; Zapatera, 2021; Zapatera et al., 2024).

Integers are usually introduced through concrete models, through close contextual situations, visual representations or even manipulative materials, so that they can be used to justify their meaning and operational rules. Cid (2003) distinguishes between neutralisation and displacement models. On the one hand, in neutralisation models, integers express quantities of a certain magnitude with opposite meanings that are neutralised, such as assets and debts, profits and losses, people entering and leaving, positive and negative charges, etc. On the other hand, in displacement models, the idea of numbers as positions or movements in the number line is used, which can be contextualised with situations of temperatures, elevator movements, sea level or the imaginary chronological axis.

Since both neutralisation and displacement models admit different situations in real contexts, other authors propose the teaching of integers by observing three dimensions of their knowledge: contextual, number line and abstract (Bruno, 1997; Bruno & Martínón, 1997; Peled, 1991). Thus, numbers and operations in the contextual dimension refer to concrete models or real-life situations; in the number line dimension, numbers are represented with points and operations with vectors; and in the abstract dimension, numbers are used only through their symbology and operational rules.

In this way, following a teaching strategy that encompasses the three dimensions of knowledge, students must be able to express situations with integers in the three dimensions and transfer or translate them from one to another (Bruno & Cabrera, 2005) (Figure 1). This research focuses on how students in the 6th year of Primary Education and 1st year of Secondary Education formulate additive problems (contextual dimension) based on information given in the abstract dimension and the number line dimension.

The contextual dimension involves the use of real situations and, therefore, the resolution of additive and multiplicative problems. This research focuses only on additive situations, so Vergnaud's classification of problems (1982) and Nesher's semantic categories (1982) have been adopted.



**Figure 1.** Three dimensions of numerical knowledge and their transfers.

**Table 1.** Uses of numerical situations and representation on the number line.

|            |  |                                                                             |
|------------|--|-----------------------------------------------------------------------------|
| State      |  | The temperature is 3 degrees below zero I owe €3                            |
| Variation  |  | The temperature rose 3 degrees I got €3                                     |
| Comparison |  | In Tenerife there are 3 degrees less than in Mexico I have €3 less than you |

In this way, we distinguish three uses of numerical situations that can be represented in the three dimensions: (1) state, if the measure of a quantity of magnitude is expressed at a certain instant, (2) variation, if the change that occurs in a state is expressed over time and (3) comparison, if the comparison between two states is expressed at a certain instant (Table 1).

From these numerical situations, four types of additive problems are determined, according to their semantic structure, which admits different unknowns: (1) change, if one state varies, (2) comparison if two states are compared, (3) combination, if two states are added and (4) two changes, if two variations are added (Bruno & Martínón, 1997).

2.3. Research objectives

The general objective of this research is to analyse the problems formulated by students in the 6th year of Primary Education (aged 11–12) and the 1st year of Secondary Education (aged 12–13), based on the information of two additive situations initially given in the number line and the abstract dimensions.

The specific aims are defined as follows:

- (1) To analyse the success in the approach to mathematical problems of students in the 6th year of Primary Education and 1st year of Secondary Education based on the propositional structure (or syntactic structure), the concrete model and the semantic structure.
- (2) To compare the differences observed in the transfers between the abstract (additive operation) and the number line (representation of points and vector) dimensions towards the contextual dimension (problem posing).

- (3) To analyse the differences in the problems posed by 6th year Primary Education students and 1<sup>st</sup> year Secondary Education students.

### 3. Methodology

#### 3.1. Participants

This research involved 266 students from the same school: 127 students from five groups in the 6th year of Primary Education (aged 11–12) and 139 students from five other groups in the 1st year of Secondary Education (aged 12–13). We will refer to them from now on as Primary and Secondary students, respectively.

The students had received instruction on the integers according to the approach of their teachers, following the curricular indications and completely independent of the research. In the 6th year of Primary Education, integers, their representation and addition were introduced through contextualised problems; and in the 1st year of Secondary Education, the concepts previously studied in 6th year Primary Education had been extended to the multiplicative field, and the training had been completed with greater formalisation in the operations.

#### 3.2. Data collection and analysis

A questionnaire with two questions was used for data collection (Figure 2).

These tasks involve making transfers of additive situations from the abstract dimension ( $2 - 3 = -1$ ) and number line dimension (representing the operation  $2 - 3 = -1$  through two points and a vector) to the contextual dimension (posing a problem).

The different categories of analysis are described in the following subsection, according to the scheme shown in Figure 3.

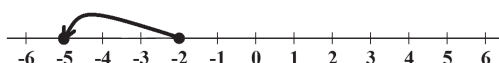
##### 3.2.1. Checking answers

An answer is considered correct (C) if it is a complete story or problem, and the propositions used fit the conditions of the numbers given in the operation or the vector. In other cases, the answer will be categorised as incorrect (I).

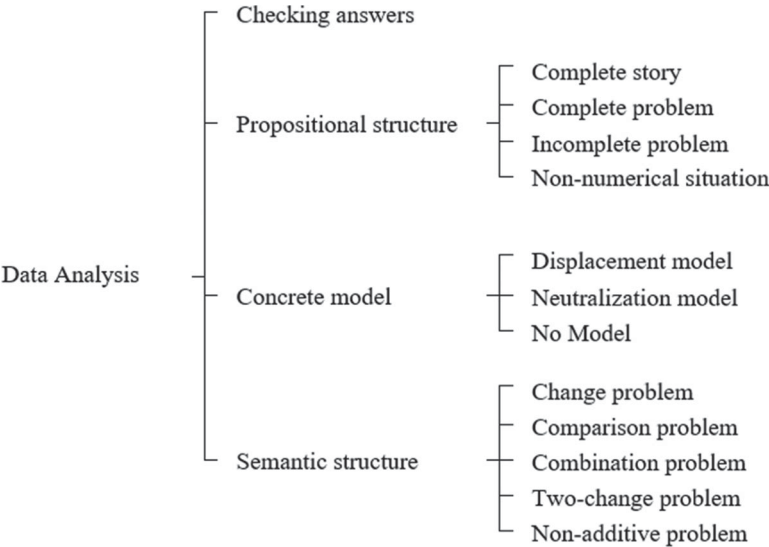
##### 3.2.2. Analysis of the propositional structure

The answers are classified, depending on the numerical situations, into four syntactic structure: (1) complete story, if it describes one numerical situation with three pieces of data and without any question, (2) complete problem, if it describes one situation with two pieces of data and a question is posed, or an unknown is declared, (3) incomplete problem, or

1. Write the statement of a problem from operation  $2 - 3 = -1$
2. Write the statement of a problem from the points and the vector indicated on the number line:



**Figure 2.** Questions discussed in this study.



**Figure 3.** Mathematical analysis scheme.

**Table 2.** Examples of answers according to the propositional structure for operation  $2-3 = -1$ .

|                         |                                                                                                  |
|-------------------------|--------------------------------------------------------------------------------------------------|
| Incomplete situation    | Maria was on floor 2 and went down 3 floors to pick up her car that was on floor $-1$            |
| Complete problem        | Maria was on the 2nd floor and went down 3 floors to pick up her car. What floor was the car on? |
| Incomplete problem      | Maria was on the 2nd floor and went down 3 floors to pick up her car.                            |
| Non-numerical situation | Maria is looking for her car                                                                     |

**Table 3.** Examples of answers according to the model used for operation  $2-3 = -1$ .

|                      |                                                                                       |
|----------------------|---------------------------------------------------------------------------------------|
| Displacement model   | Maria was on floor 2 and went down 3 floors to pick up her car that was on floor $-1$ |
| Neutralisation model | Maria had €2, If she lost €3, what is Maria's final balance?                          |
| Non-concrete model   | Maria is looking for her car                                                          |

situation, if it describes one situation with one or two pieces of data, without any question, or unknown and (4) non-numerical situation, if it does not describe a numerical situation. Some examples are presented in Table 2.

**3.2.3. Analysis of the concrete model**

Responses are categorised based on the concrete model used in the story or problem: (1) displacement, if it is using a displacement model, (2) neutralisation, if it is using a neutralisation model and (3) no concrete model, if it is not using any concrete model. Some examples are presented in Table 3.

**3.2.4. Analysis of the semantic structure**

Responses are classified based on the typologies of additive problems: (1) change, if it describes a change problem, (2) comparison, if it describes a comparison problem, (3) combination, if it describes a combination problem, (4) two-change, if it describes a two-change

**Table 4.** Examples of answers according to the type of problems for operation  $2-3 = -1$ .

|                      |                                                                                                                        |
|----------------------|------------------------------------------------------------------------------------------------------------------------|
| Change               | The temperature was 2 degrees above zero and went down 3 degrees. What's the temperature now?                          |
| Comparison           | Juan has 2 euros and María has 3 euros less than Juan. How much does Maria owe?                                        |
| Combination          | Paula had 2 euros and owed 3 euros. What is her financial situation?                                                   |
| Two changes          | Juan earned 2 euros in the morning and lost 3 euros in the afternoon. How much did he earn or lose throughout the day? |
| Non-additive problem | Write the operation                                                                                                    |

problem and (5) non-additive problem, if it does not describe an additive problem. Some examples are presented in Table 4.

### 3.3. Stages

The data analysis has been carried out in four stages:

- (1) Transfer from the abstract dimension to the contextual dimension.
- (2) Transfer from the number line dimension to the contextual dimension.
- (3) Comparison between transfers from abstract and number line dimensions to contextual dimension.
- (4) Comparison between the transfers of Primary and Secondary students.

## 4. Results

### 4.1. Transfer from the abstract dimension

Through the transfer from the abstract to the contextual dimension, students posed problems according to the initial numerical situation provided:  $2-3 = -1$ . Table 5 shows the results from all the participants, according to the propositional structure developed, the concrete model presented (if any) and the semantic structure proposed.

**Table 5.** Results of the transfer from the abstract to the contextual dimension ( $2-3 = -1$ ).

|                         |                         | Primary |           |       | Secondary |           |       |
|-------------------------|-------------------------|---------|-----------|-------|-----------|-----------|-------|
|                         |                         | Correct | Incorrect | Total | Correct   | Incorrect | Total |
| Propositional structure | Complete story          | 1       | 2         | 3     | 1         | 1         | 2     |
|                         | Complete problem        | 48      | 15        | 63    | 44        | 15        | 59    |
|                         | Incomplete problem      | 0       | 12        | 12    | 0         | 30        | 30    |
|                         | Non-numerical situation | 0       | 2         | 2     | 0         | 1         | 1     |
|                         | No answer               | 0       | 20        | 20    | 0         | 8         | 8     |
| Concrete model          | Displacement            | 35      | 10        | 45    | 28        | 22        | 50    |
|                         | Neutralisation          | 14      | 16        | 30    | 17        | 23        | 40    |
|                         | Non-concrete model      | 0       | 5         | 5     | 0         | 2         | 2     |
|                         | No answer               | 0       | 20        | 20    | 0         | 8         | 8     |
| Semantic structure      | Change                  | 34      | 14        | 48    | 28        | 29        | 57    |
|                         | Comparison              | 6       | 1         | 7     | 6         | 6         | 12    |
|                         | Combination             | 9       | 6         | 15    | 8         | 9         | 17    |
|                         | Two changes             | 0       | 2         | 2     | 2         | 1         | 3     |
|                         | Non-additive problem    | 0       | 8         | 8     | 1         | 2         | 3     |
|                         | No answer               | 0       | 20        | 20    | 0         | 8         | 8     |
|                         | Total                   | 49      | 51        | 100   | 45        | 55        | 100   |

#### **4.1.1. Checking answers**

When posing problems from the operation  $2-3 = -1$ , almost half of the participants were able to successfully create complete stories and problems. Particularly, 49% of Primary students and 45% of Secondary students have stated additive problems correctly from the given numerical operation.

#### **4.1.2. Analysis of the propositional structure**

In the two courses, the participants have formulated similar percentages of stories, complete problems and non-numeric situations. However, Secondary students have formulated more incomplete problems, 30% compared to the 12% presented by Primary students. However, Primary students have left more questions blank, 20% compared to the 8% from Secondary students.

#### **4.1.3. Analysis of the concrete model**

45% of Primary students and 50% of Secondary students have raised problems using the displacement model, while 30% and 40% of students, respectively, have raised problems using neutralisation models.

The contexts used in displacement models are mainly temperature variations and, to a lesser extent, the floors of a building; and the contexts used in neutralisation models are predominantly profit and loss and, to a lesser extent, earn-lose, give-take, etc.

#### **4.1.4. Analysis of the semantic structure**

48% of Primary students have described problems of change, 7% of comparison, 15% of combination and 2% of two changes. When analysing the problems posed by Secondary students, these percentages are, respectively, 57%, 12%, 17% and 3%. Table 6 shows representative examples of student responses to question 1. In the coding, students are designated with a 'P' or an 'S' depending on their grade level, where P stands for Primary and S for Secondary, followed by the student number. Therefore, P.67 is the 67th student in Primary.

### **4.2. Transfer from the number line dimension**

Through the transfer from the number line to the contextual dimension, students posed problems that were developed from the vector  $(-2, -5)$  of the number line. Table 7 shows the results from all the participants, according to the propositional structure developed, the concrete model presented (if any) and the semantic structure proposed.

#### **4.2.1. Checking answers**

When posing problems from the vector  $(-2, -3)$ , half of the participants were able to successfully create complete stories and problems. Particularly, 55% of Primary students and 45% of Secondary students have formulated additive problems correctly from the provided vector.

#### **4.2.2. Analysis of the propositional structure**

Primary students have described more complete problems than Secondary students, 73% compared to 56%. However, the percentage of incomplete problems is much higher for

**Table 6.** Examples of student responses to question 1.

|                         |                         | C / I | Answers                                                                                                                              | Student |
|-------------------------|-------------------------|-------|--------------------------------------------------------------------------------------------------------------------------------------|---------|
| Propositional structure | Complete story          | C     | In an examination I got a score of 2 points but they took off 3 points for making mistakes, and in the end, I got $-1$               | P.67    |
|                         |                         | I     | I was on floor $-1$ and I went down 3 floors. I am now on floor $-4$                                                                 | P.01    |
|                         | Complete problem        | C     | In Tejada the temperature was $2^{\circ}\text{C}$ and at night it dropped to $3^{\circ}\text{C}$ . How many degrees was it at night? | S.07    |
|                         |                         | I     | I had $\text{€}3$ and I spent $\text{€}2$ to buy a bag of sweets. How much do I have left now?                                       | P.101   |
| Concrete models         | Incomplete problem      | C     | Isabel has 2 candies, but she owes 3.                                                                                                | P.15    |
|                         | Non-numerical situation | I     | Why are you taking my candies?                                                                                                       | S.38    |
|                         | Displacement            | C     | In a building, my mother went down 3 floors. If she was on the second floor, which floor is she on now?                              | P.78    |
|                         |                         | I     | The water temperature in the morning is at 2 degrees and at night at $-3$ . What is the temperature at night?                        | P.46    |
| Semantic structure      | Neutralisation          | C     | I have $\text{€}2$ and I owe $\text{€}3$                                                                                             | P.26    |
|                         |                         | I     | If I take two candies from Paco and he has 3, how many candies does Paco have now?                                                   | S.132   |
|                         | Non-concrete model      | C     | Find the operation                                                                                                                   | P.32    |
|                         |                         | C     | Sergio had 2 lollipops, and they took 3 from him. How many does he have left?                                                        | S.28    |
|                         | Change                  | I     | Pepe had two cases stolen                                                                                                            | P.112   |
|                         |                         | C     | In Madrid, it is 2 degrees, and in Valencia, it is 3 degrees less                                                                    | P.98    |
|                         |                         | I     | John is on a mountain 1000 m above the sea, and Nerea is 1001 m below Juan                                                           | P.40    |
|                         | Combination             | C     | Federico has 2 candies, but he owes his sister 5                                                                                     | S.11    |
|                         |                         | I     | I have $\text{€}2$ and I owe $-3$ , what is my account balance?                                                                      | P.11    |
|                         | Two changes             | C     | If Carlos gets 2 and has 3 stolen, how many does he have left?                                                                       | S.03    |
|                         |                         | I     | Javier gave his brother $\text{€}2$ , two days later his brother gave him $\text{€}3$ , how much money does Javier have now?         | S.15    |
|                         | Non-additive problem    | C     | In Dolomites the temperature is $2^{\circ}\text{C}$ , but it drops by 1 degree every minute. In 3 min, what is the temperature?      | S.90    |
|                         |                         | I     | Find the operation                                                                                                                   | P.32    |

Secondary students, 37% compared to 16%. 3% of Primary students have described a non-numeric situation and 5% have left the answer blank; these percentages fall to 1% and 4%, respectively, in Secondary students.

#### 4.2.3. Analysis of the concrete model

66% of Primary students and 62% of Secondary students have used displacement models, and 25% and 33% have used neutralisation models, respectively.

As in the formulations of problems from the abstract dimension, the displacement models used are, mostly, those of temperature and building floors and in those of neutralisation, those of have-owe and earn-lose, give-take, etc.

**Table 7.** Results of the mathematical analysis of the transfer from the number line dimension ( $v(-2, -5)$ ).

|                         |                         | Primary Students |           |       | Secondary Students |           |       |
|-------------------------|-------------------------|------------------|-----------|-------|--------------------|-----------|-------|
|                         |                         | Correct          | Incorrect | Total | Correct            | Incorrect | Total |
| Propositional structure | Complete story          | 3                | 0         | 3     | 1                  | 1         | 2     |
|                         | Complete problem        | 52               | 21        | 73    | 44                 | 12        | 56    |
|                         | Incomplete problem      | 0                | 16        | 16    | 0                  | 37        | 37    |
|                         | Non-numerical situation | 0                | 3         | 3     | 0                  | 1         | 1     |
|                         | No answer               | 0                | 5         | 5     | 0                  | 4         | 4     |
| Concrete model          | Displacement            | 42               | 24        | 66    | 32                 | 30        | 62    |
|                         | Neutralisation          | 13               | 12        | 25    | 13                 | 20        | 33    |
|                         | Non-concrete model      | 0                | 4         | 4     | 0                  | 1         | 1     |
|                         | No answer               | 0                | 5         | 5     | 0                  | 4         | 4     |
| Semantic structure      | Change                  | 41               | 24        | 65    | 28                 | 28        | 56    |
|                         | Comparison              | 4                | 4         | 8     | 6                  | 6         | 12    |
|                         | Combination             | 10               | 4         | 14    | 10                 | 5         | 15    |
|                         | Two changes             | 0                | 2         | 2     | 1                  | 6         | 7     |
|                         | Non-additive problem    | 0                | 6         | 6     | 0                  | 6         | 6     |
|                         | No answer               | 0                | 5         | 5     | 0                  | 4         | 4     |
|                         | Total                   | 55               | 45        | 100   | 45                 | 55        | 100   |

#### 4.2.4. Analysis of the semantic structure

65% of Primary students have described problems of change, 8% of comparison, 14% of combination and 2% of two changes. When analysing the problems posed by Secondary students, the percentages are, respectively, 56%, 12%, 15% and 7%. Table 8 shows representative examples of student responses to question 2.

### 4.3. Comparison between transfers from abstract and number line dimensions

This section presents a comparison between the results of the students' productions of the two courses in each of the transfers (abstract-context and number line-context). Results are summarised in Table 9.

The results of transfers from the abstract and number line dimensions are similar, although slightly higher than the number line dimension, 50% versus 47%. In the transfer from the number line, more problems have been described, both complete and incomplete, while in the transfer from the operation, there are more blank answers (14%).

For both types of transfers, the students tend to use more frequently the displacement mode, although the difference with the neutralisation models is more significant in the transfers from the number line dimension. The order from highest to lowest use of the different types of problems has been the following for both transfer types: change, combination, comparison and two changes.

Moreover, in both questions, we can find responses that describe unfeasible situations, although some of them are adjusted to the vector operation. Some examples are: 'I was 2 years old, and they took 3 years off' (A1.02) or 'In a tree there are 2 leaves and in autumn 5 leaves fall' (A6.40).

Additionally, several answers contain expressions that simultaneously use the binary and unary meanings of the minus sign. Some examples are: '... I owe  $-3$ ' (A6.11) and '... the temperature fell  $-3^\circ$ ' (A1.12), which would be equivalent to 'I have 3', '... the temperature increased  $3^\circ\text{C}$ '.

**Table 8.** Examples of student responses to question 2.

|                         |                    | C/I | Answers                                                                                                                                                        | Student |
|-------------------------|--------------------|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| Propositional structure | Complete story     | C   | It was very cold this morning. Tonight, it was $-2^{\circ}\text{C}$ and in the morning the temperature dropped $3^{\circ}$ , so we are at $-5^{\circ}\text{C}$ | S.92    |
|                         |                    | I   | In the match, my team was $-2$ points down, and the other team scored a triple, and now we are $-35$ points down                                               | S.80    |
|                         | Complete problem   | C   | I was on floor $-2$ and I had to go down 3 more floors. What floor am I on?                                                                                    | P.03    |
|                         |                    | I   | Carlos had 2 cookies, and they stole 5 from him. How many cookies does he have left?                                                                           | S.24    |
|                         | Incomplete problem | I   | Laura owed Juan 2 sweets, but now she also owes Pedro 3 sweets                                                                                                 | S.119   |
| Concrete model          | Displacement       | I   | The number line represents the solution of a problem                                                                                                           | P.41    |
|                         |                    | C   | In Helsinki, it was $-2^{\circ}\text{C}$ at night, and it dropped $3^{\circ}\text{C}$ in the morning. What was the temperature in the morning?                 | S.39    |
|                         | Neutralisation     | I   | I'm on the $-5$ floor and I'm going up 3                                                                                                                       | P.26    |
|                         |                    | C   | I owe my friend $\text{€}2$ , and I owe Lucas $\text{€}3$ . How much money do I have to pay?                                                                   | P.17    |
|                         | Non-concrete model | I   | 5 gifts have been taken from you                                                                                                                               | S.03    |
| Semantic structure      | Change             | I   | Negative numbers                                                                                                                                               | P.47    |
|                         |                    | C   | Marta went down to the parking lot from floor $-2$ to $-5$                                                                                                     | P.22    |
|                         | Comparison         | I   | The current temperature is $2^{\circ}$ if it drops $3^{\circ}$ , what's the temperature?                                                                       | S.09    |
|                         |                    | C   | In my house it is $-2$ degrees at night and in my grandmother's house $-5$ , what is the temperature difference?                                               | S.113   |
|                         |                    | I   | Today in Las Palmas it is $17^{\circ}\text{C}$ and yesterday it was $2^{\circ}\text{C}$ lower. How many degrees was the temperature yesterday?                 | P.127   |
|                         |                    | C   | I owe Claudio 2 sweets and Iñaki 3                                                                                                                             | S.31    |
|                         | Combination        | I   | I have $2\text{€}$ but I owe Pedro 5. How much money do I have?                                                                                                | P.103   |
|                         |                    | C   | Mr Paco had 2 pencils stolen and now 3 more stolen                                                                                                             | P.112   |
|                         | Two changes        | I   | Juan gives $\text{€}2$ to his sister and $\text{€}3$ to his cousin                                                                                             | S.35    |
|                         |                    | I   | $-2-3$                                                                                                                                                         | S.110   |

#### 4.4. Comparison of results between students in the 6th year of Primary Education and the 1st year of Secondary Education

With the objective of comparing the problems posed by students from Primary Education and Secondary Education, Table 10 presents the results obtained for both courses, by averaging the results of the two tasks.

Primary students have formulated more correct problems than Secondary students, specifically, 52% compared to 45%. Perhaps it is due to the tendency of Secondary Education participants to 'economize' resources, obviating the question. As some pointed out in the subsequent discussion of results, this might be due to the consideration that the question was not necessary to describe the situation. 34% of the answers in Secondary students and 14% of the answers in Primary students are incomplete problems with two data that fit the conditions imposed.

**Table 9.** Results of transfers between abstract-context and number line-context dimensions (data from primary and secondary school students together).

|                         |                         | Abstract-Context |           |       | Number line-Context |           |       |
|-------------------------|-------------------------|------------------|-----------|-------|---------------------|-----------|-------|
|                         |                         | Correct          | Incorrect | Total | Correct             | Incorrect | Total |
| Propositional structure | Complete story          | 1                | 1         | 2     | 2                   | 1         | 3     |
|                         | Complete problem        | 46               | 15        | 61    | 48                  | 16        | 64    |
|                         | Incomplete problem      | 0                | 21        | 21    | 0                   | 27        | 27    |
|                         | Non-numerical situation | 0                | 2         | 2     | 0                   | 2         | 2     |
|                         | No answer               | 0                | 14        | 14    | 0                   | 4         | 4     |
| Concrete model          | Displacement            | 31               | 16        | 47    | 37                  | 27        | 64    |
|                         | Neutralisation          | 16               | 20        | 36    | 13                  | 16        | 29    |
|                         | Non-concrete model      | 0                | 3         | 3     | 0                   | 3         | 3     |
|                         | No answer               | 0                | 14        | 14    | 0                   | 4         | 4     |
| Semantic structure      | Change                  | 31               | 21        | 52    | 34                  | 26        | 60    |
|                         | Comparison              | 6                | 3         | 9     | 5                   | 5         | 10    |
|                         | Combination             | 8                | 8         | 16    | 10                  | 5         | 15    |
|                         | Two changes             | 1                | 2         | 3     | 1                   | 4         | 5     |
|                         | Non-additive problem    | 1                | 5         | 6     | 0                   | 6         | 6     |
|                         | No answer               | 0                | 14        | 14    | 0                   | 4         | 4     |
|                         | Total                   | 47               | 53        | 100   | 50                  | 50        | 100   |

**Table 10.** Comparison of the transfer results from primary students and secondary students.

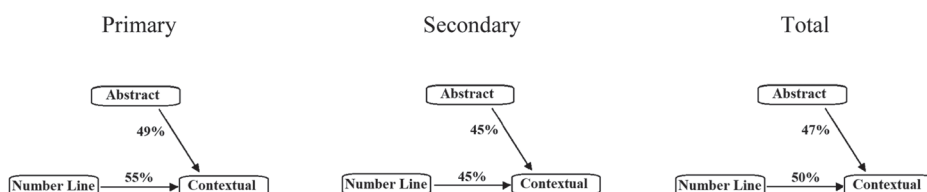
|                         |                         | Primary Students |           |       | Secondary Students |           |       |
|-------------------------|-------------------------|------------------|-----------|-------|--------------------|-----------|-------|
|                         |                         | Correct          | Incorrect | Total | Correct            | Incorrect | Total |
| Propositional structure | Complete story          | 2                | 1         | 3     | 1                  | 1         | 2     |
|                         | Complete problem        | 50               | 18        | 68    | 44                 | 13        | 57    |
|                         | Incomplete problem      | 0                | 14        | 14    | 0                  | 34        | 34    |
|                         | Non-numerical situation | 0                | 2         | 2     | 0                  | 1         | 1     |
|                         | No answer               | 0                | 13        | 13    | 0                  | 6         | 6     |
| Concrete model          | Displacement            | 38               | 17        | 55    | 30                 | 26        | 56    |
|                         | Neutralisation          | 14               | 14        | 28    | 15                 | 21        | 36    |
|                         | Non-concrete model      | 0                | 4         | 4     | 0                  | 2         | 2     |
|                         | No answer               | 0                | 13        | 13    | 0                  | 6         | 6     |
| Semantic structure      | Change                  | 37               | 19        | 56    | 28                 | 28        | 56    |
|                         | Comparison              | 5                | 2         | 7     | 6                  | 6         | 12    |
|                         | Combination             | 10               | 5         | 15    | 9                  | 7         | 16    |
|                         | Two changes             | 0                | 2         | 2     | 1                  | 4         | 5     |
|                         | Non-additive problem    | 0                | 7         | 7     | 1                  | 4         | 5     |
|                         | No answer               | 0                | 13        | 13    | 0                  | 6         | 6     |
|                         | Total                   | 52               | 48        | 100   | 45                 | 55        | 100   |

Primary students leave more blank answers than Secondary students, 13% compared to 6%; this circumstance is especially striking in the abstract dimension, in which the percentage of blank answers of Primary students reaches 20%.

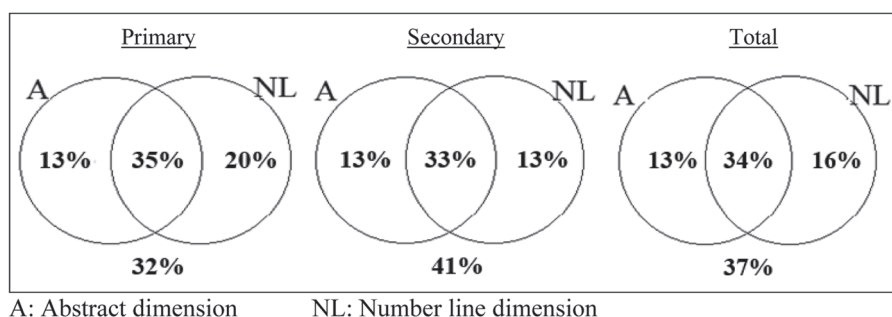
The use of displacement models is similar in both years, 55% and 56%; but Secondary students use more neutralisation models than those in the Primary students, 36% compared to 28%.

The frequency of use in the different types of semantic structures of additive situations maintains the same order in both years: change, combination, comparison and two changes.

In summary, Figure 4 shows the overall results of the transfers from the abstract and number line dimensions to the contextual dimension, for both years and concerning the total number of participants. These data are taken from Tables 5 and 7.



**Figure 4.** Overall results of transfers.



**Figure 5.** Students in each level who correctly solved 2, 1, and 0 transfers.

Moreover, Figure 5 summarises the percentage of students who have successfully transferred from the abstract dimension (A), from the number line dimension (NL) and globally to the contextual dimension. 34% of students have completed both transfers, 35% in Primary Education compared to 33% in Secondary Education. 37% of all students have not made either of the two transfers, which is composed of the 32% of Primary students compared to 41% of Secondary students.

13% of the students in both courses have correctly carried out only the transfer from the abstract dimension, while 20% of the Primary students and 13% of the Secondary students have correctly carried out only the transfer from the number line.

## 5. Discussion

Without specific instruction, almost half of the students in this study have been able to formulate simple additive problems, based on initial information given abstractly or on the number line.

Contrary to expectations, the results of transferring situations to the contextual dimension are slightly higher when starting from the number line dimension than when starting from the abstract dimension, especially for Primary students. Some research states that the number line can cause difficulties in subtracting negative numbers (Janvier, 1983); however, these difficulties did not arise clearly in this study because students could pose situations in which positive numbers were subtracted, avoiding the most complex situations about subtracting a negative number, that is,  $a - (-b)$ . In addition, an approach to teaching integers based on contextualised situations must be linked to representations in the number line, as this induces the use of semantic structures of different types of problems (Bruno & Cabrera, 2005).

In the question that begins in the abstract dimension, there were more blank answers than in the question that begins in the dimension of the number line. This circumstance was relevant in elementary students, where one out of five students did not answer the question. This observation confirms that, for students with less initial knowledge of integers, the graphical representation of an additive situation in the form of a vector on the number line is more understandable and intuitive than the symbolic representation. Along these lines, although focused on problem solving rather than invention, Bruno and Martínón (1997) states that students have ‘greater ease and confidence in solving problems using the number line than with operations’ (p. 257).

Moreover, almost a third of the participants have only been able to correctly perform one of the two transfers. This observation is confirmed by Bruno and Martínón (1997), who considers that ‘there is a certain disconnection between problem solving using operations and the number line’ (p. 257).

Students show their preference for problems of the semantic structure of change in the two types of questions analysed. This preference is manifested more in the transfer from the number line, since the image of the vector favours giving an example of variation. In addition, change problems are usually those that are solved more frequently in class and those that appear in greater proportion in textbooks (Ayllón et al., 2011). However, although there are more displacement situations and more change problems, it is striking that there is a significant percentage of situations that respond to neutralisation models and with a combination semantic structure.

Very little variety has also been observed concerning the contexts used. In the displacement models, their contextualisation is reduced, exclusively, to situations with temperatures and, to a lesser extent, to floors of a building and heights above sea level; the situations in the neutralisation models are reduced to have-owe and get-lose situations contextualised in euros, candies, school supplies, fruits... This is because students do not use negative numbers extensively in their daily lives. Therefore, they mainly use contexts learned in class. These results coincide with the findings of other research by Ayllón et al. (2011) and Irvine (2017), which show that students, when posing problems, usually limit themselves to reproducing the problems worked on in class. This is confirmed in the thesis by English (1998) which indicates that the type of problems formulated by students depends on the instruction received.

The propositional structure most used in both grades is that of complete problems, that is, situations consisting of two pieces of information and a question. However, in the two questions analysed, the Primary students described more complete problems than the Secondary students. However, if the answers that fit the situation but did not pose the question had been considered correct, the results of the Secondary students would have been slightly higher than those of the Primary students. In any case, the fact that the differences in the results of both grades are not relevant shows that problem posing is not a task that is developed exclusively based on knowledge of integers; that is, greater knowledge does not imply a better understanding of how contextual situations or problems are connected with operations and the number line.

Although the rest of the negative numbers are not raised in the questions, in some formulation’s expressions are used in which the binary and unary meanings of the minus sign overlap, which shows the difficulty of differentiating both meanings (Bofferding, 2014) and causes an artificial language. Other formulations analysed, although sometimes adjusted to

the conditions imposed by the situations expressed in the operation or the vector, describe unreal or impossible situations. These formulations confirm the results obtained by Ayllón et al. (2016) when analysing the coherence of the problems invented by Primary students, since the description of impossible, unreal or absurd situations could be due to teaching that is far removed from the context and reality of the student.

## 6. Conclusions

The general objective of this research is to analyse the problems formulated by 266 students in the Primary and Secondary students (11–13 years old). This research provides information on the knowledge about integer numbers in students of two educational levels in transition from primary to secondary school.

Analysing the problems posed by the students helps, in a way, to observe how they understand the additive situations. The analysis carried out on the problems formulated by the students has been profound, since it is approached from different educational research interests: two educational levels (last year of primary and first year of secondary school), relating dimensions of numerical knowledge (abstract-context and number line-context) and observing the characteristics of the problems (propositional structure, concrete model and semantic structure).

The action of posing problems involves a high level of abstraction and requires a significant mastery of the content studied, such as the appropriate use of language, concepts, procedures and mathematical properties (Silver, 2013). From the in-depth analysis of the problems written by the students in this study, and observing their characteristics, it can be inferred that problem posing is a skill strongly influenced by the training received and that it is not developed simply by expanding mathematical knowledge.

The problems formulated by students in this study exhibited simple structures and were often poorly written and lacked originality, showing limited variation from classroom contexts. These weaknesses have also been observed in research on future teachers in writing problems of this topic (Kapicioğlu & Arıkan, 2022). The ability to pose problems does not develop spontaneously in learners; it requires explicit and consistent instruction. Problem formulation is not a common practice in the learning of mathematics, but its relevance has been increasing in recent years. In the case of integer numbers, it requires that students observe the different semantic structures based on stories associated with certain operations (Bruno, 2000; Rudnitsky et al., 1995). Furthermore, mathematics is often taught in isolation from other curricular areas, especially language. However, it is known that posing problems can help improve problem solving, allow progress in understanding environmental situations, stimulate curiosity, motivation and creativity, and promote a cross-curricular approach in which other curricular subjects participate.

Problem posing is a latent ability in students that can be improved through practice. Therefore, based on the results of this research, we believe that problem posing requires further research to fully develop its didactic potential and prove that it is an effective tool for learning mathematics and solving problems. Thus, the developing of the potential offered by problem posing in mathematics teaching requires the active participation of teachers, which is why we support its incorporation into initial and continuing education programmes.

## Acknowledgements

We acknowledge the project PID2022–139007NB-I00 (MINECO-Spain) of the Spanish Government.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This work was supported by Project PID2022–139007NB-I00 (MINECO-Spain) of the Spanish Government.

## Data availability statement

The data associated to this article uploaded to a permanent repository, and the link to this repository is <http://doi.org/10.17632/nzdz6vwmgn.1>

## References

- Ayllón, M. F., Castro, E., & Molina, M. (2011). Invención de problemas y tipificación de problema ‘difícil’ por alumnos de Educación Primaria. In E. M. Marín, G. Fernández, L. J. Blanco, & M. Palarea (Eds.), *Investigación en educación matemática XV*, 1 (pp. 277–286). SEIEM.
- Ayllón, M. F., Gallego, J. L., & Gómez, I. A. (2016). La actuación de estudiantes de educación primaria en un proceso de invención de problemas. *Perfiles Educativos*, 38(152), 51–67. <https://doi.org/10.22201/iisue.24486167e.2016.152.57588>
- Ayllón, M. F., & Gómez, I. A. (2014). La invención de problemas como tarea escolar. *EA, Escuela Abierta*, 17(1), 29–40. <https://doi.org/10.29257/EA17.2014.03>
- Bishop, J. P., Lamb, L., Philipp, R., Schappelle, B., & Whitacre, I. (2010). A developing framework for children’s reasoning about integers. In P. Brosnan, D. Erchick, & L. Flevaris (Eds.), *Proceedings of the 32nd annual conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. VI, pp. 695–702). The Ohio State University.
- Bofferding, L. (2014). Negative integer understanding: Characterizing first graders’ mental models. *Journal for Research in Mathematics Education*, 45(2), 194–245. <https://doi.org/10.5951/jresmetheduc.45.2.0194>
- Brown, S. I., & Walter, M. I. (1993). Problem posing in mathematics education. In Stephen I. Brown & Marion I. Walter (Eds.), *Problem posing: Reflection and applications* (pp. 16–27). Lawrence Erlbaum Associates.
- Bruno, A. (1997). La enseñanza de los números negativos: aportaciones de una investigación. *Números*, 29, 5–18.
- Bruno, A. (2000). El alumnado redacta problemas aditivos de números negativos. *Revista EMA*, 5(3), 236–251.
- Bruno, A., & Cabrera, N. (2005). Una experiencia sobre la representación en la recta de números negativos. *Cuadrante*, 14(2), 25–41. <https://doi.org/10.48489/cuadrante.22797>
- Bruno, A., & Martín, A. (1997). Procedimientos de resolución de problemas aditivos con números negativos. *Enseñanza de las ciencias: revista de investigación y experiencias didácticas*, 15(2), 249–258. <https://doi.org/10.5565/rev/ensciencias.4180>
- Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In F. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical Problem Posing: From Research to Effective Practice*, (pp. 3–34). Springer. [https://doi.org/10.1007/978-1-4614-6258-3\\_1](https://doi.org/10.1007/978-1-4614-6258-3_1)

- Cankoy, O. (2014). Interlocked problem posing and children's problem posing performance in free structured situations. *International Journal of Science and Mathematics Education*, 12(1), 219–238. <https://doi.org/10.1007/s10763-013-9433-9>
- Christou, C., Mousoulides, N., Pittalis, M., & Pitta-Pantazi, D. (2005). Resolución de problemas y planteamiento de problemas en un entorno de geometría dinámica. *El entusiasta de las matemáticas*, 2(2), 125–143. <https://doi.org/10.54870/1551-3440.1029>
- Cid, E. (2003). La investigación didáctica sobre los números negativos: estado de la cuestión. *Publicaciones del seminario matemático García de Goleano*, 25, 1–40. Universidad de Zaragoza.
- English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, 29(1), 83–106. <https://doi.org/10.2307/749719>
- Espinoza, J., Lupiáñez, J., & Segovia, I. (2016). Un estudio de los problemas inventados por estudiantes de secundaria en España. *Revista de Educación de la Universidad de Granada*, 23, 85–101.
- Fernández, M. E., & Carrillo, J. C. (2020). Un acercamiento a la forma en que los estudiantes de primaria formulan problemas. *Revista de Educação Matemática*, 17, 1–19. <https://doi.org/10.37001/remat25269062v17id257>
- Glaeser, G. (1981). Epistémologie des nombres relatifs. *Recherches en Didactique des Mathématiques*, 2(3), 303–346.
- Herrera, J. L., & Zapatera, A. (2019). El número como cantidad física y concreta, un obstáculo en el aprendizaje de los números enteros. *PNA: Revista de Investigación en Didáctica de la Matemática*, 13(4), 197–220. <https://doi.org/10.30827/pna.v13i4.8226>
- Irvine, J. (2017). Planteamiento de problemas en clases de matemáticas de consumo: no solo para futuros matemáticos. *El entusiasta de las matemáticas*, 14(1), 387–412. <https://doi.org/10.54870/1551-3440.1404>
- Janvier, C. (1983). The understanding of directed numbers. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the 15th Annual Conference of the North American Chapter of PME* (Vol. 2, pp. 295–300). International Group for the Psychology of Mathematics Education, North American Chapter, Montréal.
- Kapicioğlu, G., & Arıkan, E. E. (2022). The analysis of problem posing skills about integers of prospective primary school mathematics teachers who have experienced in problem posing. *International Journal of Psychology and Educational Studies*, 9(1), 211–229. <https://doi.org/10.52380/ijpes.2022.9.1.555>
- Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 123–147). Lawrence Erlbaum Associates.
- Leavy, A., & Hourigan, M. (2020). Posing mathematically worthwhile problems: Developing the problem-posing skills of prospective teachers. *Journal of Mathematics Teacher Education*, 23(4), 341–361. <https://doi.org/10.1007/s10857-018-09425-w>
- Munayco, E., & Solís, B. P. (2021). Comprensión, invención y resolución de problemas. *Polo del conocimiento*, 6(2), 46–63. <https://doi.org/10.23857/pc.v6i2.2236>
- National Council of Teachers of Mathematics. (2000). [NCTM]. *Principles and standards for school mathematics*.
- Nesher, P. (1982). Levels of description in the analysis of addition and subtraction word problems. In E. T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 25–38). Lawrence Erlbaum.
- Peled, I. (1991). Levels of knowledge about signed numbers: Effects of age and ability. In F. Furinghetti (Ed.), *Proceedings of the 15th Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 145–152). International Group for the Psychology of Mathematics Education.
- Polya, G. (1965). *Cómo plantear y resolver problemas*. Trillas.
- Porras, K., & Castro, E. (2019). Representaciones matemáticas y la invención de problemas desde la modelización. In Y. Morales-López, Y. & A. Ruíz (Eds.), *Comité Interamericano de Educación Matemática 2020* (Vol. 16, pp. 2604–2611). Comité Interamericano de Educación Matemática (CIAEM).

- Quevedo, E., Zapatera, A., & Lijó, R. (2023). Invención de situaciones aditivas con números enteros. In E. C. Jiménez-Gestal, ÁA Magreñán, E. Badillo, & E. y. P. Ivars (Eds.), *Investigación en Educación Matemática XXVI* (pp. 459–466). SEIEM.
- Real Decreto 157/2022. de 1 de marzo, por el que se establecen la ordenación y las enseñanzas mínimas de la Educación Primaria. Boletín Oficial del Estado, 52, de 2 de marzo de 2022, 24386-24504.
- Real Decreto 217/2022. de 29 de marzo, por el que se establece la ordenación y las enseñanzas mínimas de la Educación Secundaria Obligatoria. Boletín Oficial del Estado, 76, de 30 de marzo de 2022, 41571-41789.
- Rudnitsky, A., Etheredge, S., Freeman, S. J., & Gilbert, T. (1995). Learning to solve addition and subtraction word problems through a structure-plus-writing approach. *Journal for Research in Mathematics Education, JRME*, 26(5), 467–486. <https://doi.org/10.2307/749433>
- Silver, E. A. (2013). Problem-posing research in mathematics education: Looking back, looking around, and looking ahead. *Educational Studies in Mathematics*, 83(1), 157–162. <https://doi.org/10.1007/s10649-013-9477-3>
- Silver, E. A., & Cai, J. (2005). Evaluación del planteamiento de problemas matemáticos del alumnado. *Enseñanza de las matemáticas para niños*, 12(3), 129–135. <https://doi.org/10.5951/TCM.12.3.0129>
- Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. C. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Mathematics Education Research Group of Australasia.
- Torregrosa, A., Albarracín, L., & Deulofeu, J. (2021). Resolución e invención de problemas: la estrategia de resolución con relación al problema inventado. In P. D. En Diago, D. F. Yáñez, M. T. González-Astudillo, & D. Carrillo (Eds.), *Investigación en Educación Matemática XXIV* (pp. 611–618). SEIEM.
- Vergnaud, G. (1982). A classification of cognitive tasks and operation of thought involved in additions and subtraction problems. En T.P. Carpenter, J.M. Moser, & T.A. Romberg (eds.) *Addition and subtraction: A cognitive perspective*, (pp. 39–59). Lawrence Erlbaum.
- Zapatera, A. (2021). Obstáculos epistemológicos en el aprendizaje de los números enteros. En A. Rojas (Ed.), *Avances en Matemática Educativa. Teorías diversas* (8, pp. 121–135). Editorial Lectorium.
- Zapatera, A., Quevedo, E., González, S.-, Santana, A., & y Álamo, J. (2024). Obstáculos y dificultades de los alumnos en la incorporación de los números enteros. *AIEM – Avances de investigación en educación matemática*, 24, 41–63 (in press).
- Zhang, L., Cai, J., Song, N., Zhang, H., Chen, T., Zhang, Z., & Guo, F. (2022). Mathematical problem posing of elementary school students: The impact of task format and its relationship to problem solving. *ZDM – Mathematics Education*, 54(3), 497–512. <https://doi.org/10.1007/s11858-021-01324-4>