Additive problem posing with integers by students in 6th year of Primary Education and 1st year of Secondary Education

This work shows how 266 students in the 6th year of Primary Education and the 1st year of Secondary Education pose additive problems with integers based on two questions raised from the abstract and number line dimensions. Transfers from these two dimensions to the contextual dimension are studied, depending on the propositional structure of the problem, the type of model used, and its semantic structure. The problems posed reproduce the ones worked on in class and are, for the most part, problems of change that use displacement models with little contextual variety. The results in 1st year of Secondary Education do not exceed those obtained in the 6th year of Primary Education, which indicates that the formulation of problems does not develop spontaneously, and that greater knowledge does not imply a better understanding of the connections of contextual situations with operations and with the number line. From the results it is inferred that problem posing is a skill that improves understanding and problem solving, stimulates curiosity, motivation, and creativity, and, therefore, it is necessary to design a specific instruction to improve this.

Keywords: integers numbers; additive situations; contextual dimension; concrete models

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Introduction

Problem solving is a fundamental activity in mathematics education and constitutes a central axis of the mathematics curriculum of several countries, which places it as one of the specific competences to be developed in the different educational stages (NCTM, 2000; Real Decreto 157/2022; Real Decreto 217/2022). In addition, the formulation of problems by students has emerged for several decades as a skill that fosters the learning of mathematics, so it has relevance both at the curricular level (Real Decreto 157/2022; Real Decreto 217/2022) and in research (Cai et al., 2015).

Problem posing is the activity of creating, proposing, or inventing mathematical problems, which includes both the generation of new problems and the modification or reformulation of given problems (Cai et al., 2015). This activity is typical of teachers in the exercise of their profession, so problem posing studies have been carried out with teachers under training (Leavy & Hourigan, 2020) and with teachers in practice (Kapicoglu & Arikan, 2022). These studies suggest the incorporation of problem posing activities into the initial and continuous training programs of Primary and Secondary Education mathematics teachers. In addition, problem posing is an effective way of learning mathematics (Kilpatrick, 1987; Polya, 1965) and, although its educational potential is far from being fully exploited (Espinoza et al., 2016; Torregrosa et al., 2021), its incorporation into mathematics teaching-learning processes is increasingly proposed (Ayllón & Gómez, 2014; Ayllón et al., 2016; Irvine, 2017; Munayco & Solís, 2021; Zhang et al., 2022).

Research on problem posing has mainly focused on the characteristics of the creation process and, to a lesser extent, on how it influences the mathematical content addressed. In addition, the problems posed by students are a means of evaluating the knowledge they have acquired on a certain topic. In this sense, Fernández and Carrillo (2020) examined arithmetic and geometric problems invented from illustrations by fifth-year students in primary school; Espinoza et al. (2016) studied how third-year students from Secondary Education enunciated arithmetic problems from figures with numerical information and verbal situations with three numerical data; Torregrosa et al. (2021) asked sixth-year students from Primary Education to invent problems with geometric patterns; and Bruno (2000) analysed the additive problems posing of integer numbers carried out by Secondary Education students. This work is based on a similar approach, analysing the formulation of problems by students in the 6^{th} year of Primary Education (aged 11-12) and 1^{st} year of Secondary Education (aged 12-13) based on two initial situations with integer numbers, which were previously provided.

Therefore, this study is situated in the transition from Primary to Secondary Education in Spain. While students have an initial approach to integers and additive operations in the last year of Primary Education, in the first year of Secondary Education the in-depth study of integers takes place. From this perspective, this research delves into the analysis of the characteristics of students' productions when they pose additive problems in the domain of integers. Moreover, it studies whether greater knowledge about integers and additive operations might imply a greater ability to pose additive problems with integers by comparing the results obtained from students belonging to the two different courses.

Theoretical framework

Problem posing in mathematics

Problem posing is a complex mathematical process that requires a personal, original, and creative contribution to contents (Ayllón et al., 2016; Espinoza et al. 2016; Munayco & Solís, 2021).

When formulating a problem, it is necessary to think about a situation in a certain context, critically analyse the statement that is going to be proposed, examine the data, and observe the different resolution strategies that it could have (Ayllón et al., 2016). From this perspective, creating problems is a mental activity that requires establishing relationships between mathematical concepts and using different cognitive processes, such as selecting, understanding, organizing, editing, and translating information from one representation to another (Christou et al., 2005).

Research in the field of problem solving considers that there is a high correlation between problem solving and problem posing skills (Ayllón & Gómez, 2014; Silver & Cai, 2005). This implies that, generally, students with greater problem-solving skills pose more problems and with a greater degree of complexity (English, 1998), and also that students instructed in problem posing tend to obtain better results in problem-solving than those who have received traditional instruction (Ayllón & Gómez, 2014; Rudnitsky et al., 1995; Bruno, 2000). That is, problem posing is a tool that improves problem-solving skills (Ayllón et al., 2016; Munayco & Solís, 2021).

Problem posing requires establishing relationships between mathematical concepts and structures, which is a useful skill in problem solving (Ayllón et al., 2016; Cankoy, 2014; Espinoza et al., 2016). In addition, to formulate problems, it is necessary to properly choose the information and data in the statement, which helps to understand the problem in depth and reduce resolution errors (Ayllón et al., 2016; Brown & Walter, 1993; Espinoza et al., 2016; Munayco &Solís, 2021). Different studies have focused on the creative capacity that problem posing can develop in students, since it is an activity that requires flexibility to change the approach to problems and develop different solutions to them (Ayllón et al., 2016; Espinoza et al., 2016; Porras & Castro, 2019; Silver & Cai, 2005).

The formulation of problems promotes the effective participation of students in their learning process, stimulating their curiosity, motivation, and increasing their school performance and success (Munayco & Solís, 2021; Porras & Castro, 2019). On the other hand, different studies have observed that learning to pose problems fosters in students a favourable disposition towards mathematics, reducing fear, frustration and anxiety towards the subject and increasing their commitment and responsibility (Brown & Walter, 1993; Fernández & Carrillo 2020; Munayco & Solís, 2021; Porras & Castro, 2019). In addition, using problem posing as a teaching strategy helps teachers evaluate students' understanding of mathematical contents and processes, as well as their perceptions and attitudes towards the subject (Ayllón et al., 2016; Silver & Cai, 2005).

Problem posing can be done before, during, or after problem solving (Ayllón et al., 2016; Espinoza et al. 2016; Fernández & Carrillo, 2020). It is done before resolution, if the problem is formulated from a given situation or condition, and its goal is to create a problem. Problem posing can be carried out during the resolution of a problem when it is reformulated, by proposing a simpler one or another similar one, facilitating its understanding. It can be done after the resolution, when previously solved problems are reformulated, by modifying the initial one; and in the latter case, a strategy to follow is to ask yourself: "What if...?" (Brown & Walter, 1993), and it is carried out in the "look back" stage shown in Polya's method (1965).

Stoyanova and Ellerton (1996) identified three situations from which problems can be formulated: free situation, semi-structured situation, and structured situation. A problem-posing task will be referred to as free "when students are asked to generate a problem from a given, contrived or naturalistic situation. Some directions may be given to prompt certain specific actions" (p. 519); as semi-structured "when students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences"; and as structured "when problem-posing activities are based on a specific problem" (p. 520). In this research, the problems formulated by the students will be analysed from two semi-structured situations.

Teaching integer numbers

The formalization of the integer numbers was a time-consuming process due to attempts by mathematicians to connect them with reality. This process lasted until the 19th century when integers became considered a mathematical creation that allowed the operations and properties of natural numbers to be extended (Glaeser, 1981). The historical evolution and moment of acceptance of integers are similar to the teaching sequence of the different number sets in compulsory schooling (Bishop et al., 2010). Thus, the initial teaching of numbers in the primary stage begins with the natural numbers, continues with the positive rational numbers, and, at the end of primary or beginning of secondary school, integers are introduced, culminating in compulsory schooling with the study of real numbers.

In this sequence of successive numerical extensions, a didactic effort is made to connect and make sense of numbers and their operations with reality. In the case of integers, as in the historical process, the learning effort involves didactic difficulties, widely collected in educational research for several decades (Bofferding, 2014; Bruno, 2000; Bruno and Martinón, 1997; Janvier, 1983). In this way, when using integers, students start from specific previous knowledge about positive numbers that may cause difficulties and errors in learning them (Bishop et al., 2010; Cid, 2003; Herrera and Zapatera, 2019; Quevedo et al., 2023; Zapatera, 2021; Zapatera et al., 2024).

Integers are usually introduced through concrete models, through close contextual situations, visual representations, or even manipulative materials, so that they can be used to justify their meaning and operational rules. Cid (2003) distinguishes between neutralization and displacement models. On the one hand, in neutralization models, integers express quantities of a certain magnitude with opposite meanings that are neutralized, such as assets and debts, profits and losses, people entering and leaving, positive and negative charges, etc. On the other hand, in displacement models, the idea of numbers as positions or movements in the number line is used, which can be contextualized with situations of temperatures, elevator movements, sea level, or the imaginary chronological axis.

Since both neutralization and displacement models admit different situations in real contexts, other authors propose the teaching of integers by observing three dimensions of their knowledge: contextual, number line, and abstract (Bruno, 1997; Bruno and Martinón, 1997; Peled, 1991). Thus, numbers and operations in the contextual dimension refer to concrete models or real-life situations; in the number line dimension, numbers are represented with points and operations with vectors; and in the abstract dimension, numbers are used only through their symbology and operational rules.

In this way, following a teaching strategy that encompasses the three dimensions of knowledge, students must be able to express situations with integers in the three dimensions and transfer or translate them from one to another (Bruno and Cabrera,

2005) (Figure 1). This research focuses on how students in the 6th year of Primary Education and 1st year of Secondary Education formulate additive problems (contextual dimension) based on information given in the abstract dimension and the number line dimension.

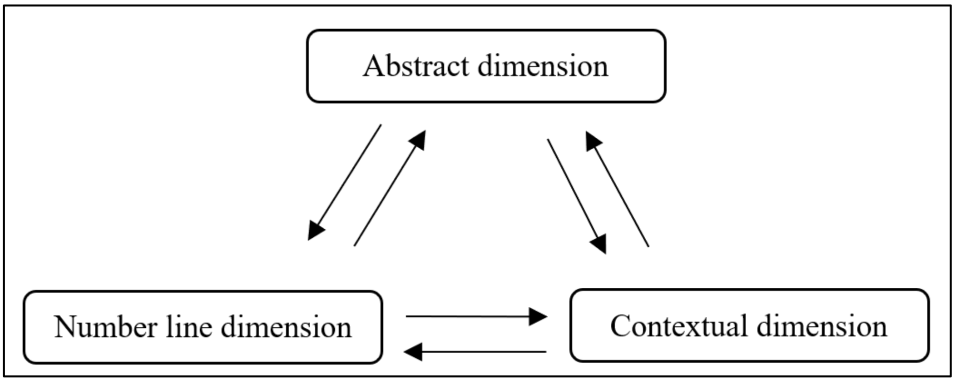
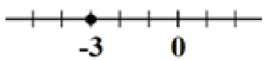


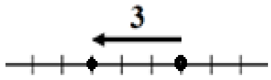
Figure 1. Three dimensions of numerical knowledge and their transfers

The contextual dimension involves the use of real situations and, therefore, the resolution of additive and multiplicative problems. This research focuses only on additive situations, so Vergnaud's classification of problems (1982) and Nesher's semantic categories (1982) have been adopted.

In this way, we distinguish three uses of numerical situations that can be represented in the three dimensions: (1) state, if the measure of a quantity of magnitude is expressed at a certain instant, (2) variation, if the change that occurs in a state is expressed over time and (3) comparison, if the comparison between two states is expressed at a certain instant (Table 1).

Table 1. Uses of numerical situations and representation on the number line

State		*The temperature is 3 degrees below zero* *I owe €3*
Variation		*The temperature rose 3 degrees* *I got €3*

Comparison		*In Tenerife there are 3 degrees less than in Mexico* *I have €3 less than you*

From these numerical situations, four types of additive problems are determined, according to their semantic structure, which admits different unknowns: (1) change, if one state varies, (2) comparison if two states are compared, (3) combination, if two states are added and (4) two changes, if two variations are added (Bruno and Martinón, 1997).

Research objectives

The general objective of this research is to analyse the problems formulated by students in the 6th year of Primary Education (aged 11-12) and the 1st year of Secondary Education (aged 12-13), based on the information of two additive situations initially given in the number line and the abstract dimensions.

The specific aims are defined as follows:

(1) To analyse the success in the approach to mathematical problems of students in the 6th year of Primary Education and 1st year of Secondary Education based on the propositional structure (or syntactic structure), the concrete model, and the semantic structure.

(2) To compare the differences observed in the transfers between the abstract (additive operation) and the number line (representation of points and vector) dimensions towards the contextual dimension (problem posing).

(3) To analyse the differences in the problems posed by 6th year Primary Education students and 1st year Secondary Education students.

Methodology

Participants

This research involved 266 students from the same school: 127 students from five groups in the 6th year of Primary Education (aged 11-12) and 139 students from five other groups in the 1st year of Secondary Education (aged 12-13). We will refer to them from now on as Primary and Secondary students, respectively.

The students had received instruction on the integers according to the approach of their teachers, following the curricular indications and completely independent of the research. In the 6th year of Primary Education, integers, their representation, and addition were introduced through contextualized problems; and in the 1st year of Secondary Education, the concepts previously studied in 6th year Primary Education had been extended to the multiplicative field, and the training had been completed with greater formalization in the operations.

Data collection and analysis

A questionnaire with two questions was used for data collection (Figure 2).

1. Write the statement of a problem from operation 2 – 3 = -1
2. Write the statement of a problem from the points and the vector indicated on the number line:

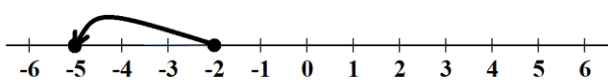


Figure 2. Questions discussed in this study

These tasks involve making transfers of additive situations from the abstract dimension (2 – 3= -1) and number line dimension (representing the operation 2 – 3= -1 through two points and a vector) to the contextual dimension (posing a problem).

The different categories of analysis are described in the following subsection, according to the scheme shown in Figure 3.

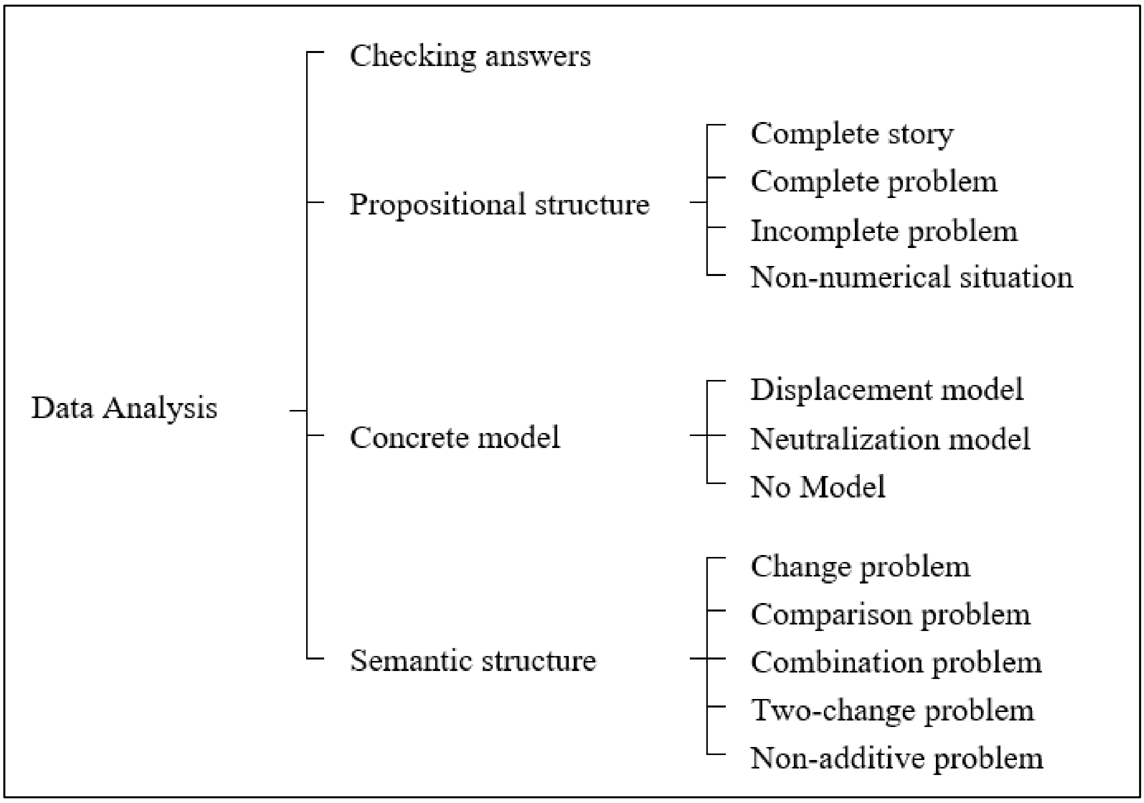


Figure 3. Mathematical analysis scheme

Checking answers

An answer is considered correct (C) if it is a complete story or problem, and the propositions used fit the conditions of the numbers given in the operation or the vector. In other cases, the answer will be categorised as incorrect (I).

Analysis of the propositional structure

The answers are classified, depending on the numerical situations, into four syntactic structure: (1) complete story, if it describes one numerical situation with three pieces of data and without any question, (2) complete problem, if it describes one situation with two pieces of data and a question is posed, or an unknown is declared, (3) incomplete problem, or situation, if it describes one situation with one or two pieces of data, without any question, or unknown, and (4) non-numerical situation, if it does not describe a numerical situation. Some examples are presented in Table 2.

Table 2. Examples of answers according to the propositional structure for operation 2 – 3 = -1

Incomplete situation	*Maria was on floor 2 and went down 3 floors to pick up her car that was on floor -1*
Complete problem	*Maria was on the 2nd floor and went down 3 floors to pick up her car. What floor was the car on?*
Incomplete problem	*Maria was on the 2nd floor and went down 3 floors to pick up her car.*
Non-numerical situation	*Maria is looking for her car*

Analysis of the concrete model

Responses are categorized based on the concrete model used in the story or problem: (1) displacement, if it is using a displacement model, (2) neutralization, if it is using a neutralization model, and (3) no concrete model, if it is not using any concrete model. Some examples are presented in Table 3.

Table 3. Examples of answers according to the model used for operation 2 – 3 = -1

Displacement model	*María was on floor 2 and went down 3 floors to pick up her car that was on floor -1*
Neutralization model	*María had €2, If she lost €3, what is Maria's final balance?*
Non-concrete model	*María is looking for her car*

Analysis of the semantic structure

Responses are classified based on the typologies of additive problems: (1) change, if it describes a change problem, (2) comparison, if it describes a comparison problem, (3) combination, if it describes a combination problem, (4) two-change, if it describes a two-change problem, and (5) non-additive problem, if it does not describe an additive problem. Some examples are presented in Table 4.

Table 4. Examples of answers according to the type of problems for operation 2 – 3 = -1

Change	*The temperature was 2 degrees above zero and went down 3 degrees. What's the temperature now?*
Comparison	*Juan has 2 euros and María has 3 euros less than Juan. How much does Maria owe?*
Combination	*Paula had 2 euros and owed 3 euros. What is her financial situation?*
Two changes	*Juan earned 2 euros in the morning and lost 3 euros in the afternoon. How much did he earn or lose throughout the day?*
Non-additive problem	*Write the operation*

Stages

The data analysis has been carried out in four stages:

(1) Transfer from the abstract dimension to the contextual dimension.

(2) Transfer from the number line dimension to the contextual dimension.

(3) Comparison between transfers from abstract and number line dimensions to contextual dimension.

(4) Comparison between the transfers of ~~6th year~~ Primary and Secondary students.

Results

Transfer from the abstract dimension

Through the transfer from the abstract to the contextual dimension, students posed problems according to the initial numerical situation provided: 2 - 3 = -1. Table 5 shows the results from all the participants, according to the propositional structure developed, the concrete model presented (if any), and the semantic structure proposed.

Table 5 Results of the transfer from the abstract to the contextual dimension (2 – 3 = -1)

		Primary			Secondary		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Propositional structure	Complete story	1	2	3	1	1	2
	Complete problem	48	15	63	44	15	59
	Incomplete problem	0	12	12	0	30	30
	Non-numerical situation	0	2	2	0	1	1
	No answer	0	20	20	0	8	8
Concrete model	Displacement	35	10	45	28	22	50
	Neutralisation	14	16	30	17	23	40
	Non-concrete model	0	5	5	0	2	2
	No answer	0	20	20	0	8	8
Semantic structure	Change	34	14	48	28	29	57
	Comparison	6	1	7	6	6	12
	Combination	9	6	15	8	9	17
	Two changes	0	2	2	2	1	3
	Non-additive problem	0	8	8	1	2	3
	No answer	0	20	20	0	8	8
	Total	49	51	100	45	55	100

Checking answers

When posing problems from the operation 2 – 3 = -1, almost half of the participants were able to successfully create complete stories and problems. Particularly, 49% of Primary students and 45% of Secondary students have stated additive problems correctly from the given numerical operation.

Analysis of the propositional structure

In the two courses, the participants have formulated similar percentages of stories, complete problems, and non-numeric situations. However, Secondary students have formulated more incomplete problems, 30% compared to the 12% presented by Primary students. However, Primary students have left more questions blank, 20% compared to the 8% from Secondary students.

Analysis of the concrete model

45% of Primary students and 50% of Secondary students ~~Education~~ have raised problems using the displacement model, while 30% and 40% of students, respectively, have raised problems using neutralization models.

The contexts used in displacement models are mainly temperature variations and, to a lesser extent, the floors of a building; and the contexts used in neutralization models are predominantly profit and loss and, to a lesser extent, earn-lose, give-take, etc.

Analysis of the semantic structure

48% of Primary students have described problems of change, 7% of comparison, 15% of combination, and 2% of two changes. When analysing the problems posed by Secondary students, these percentages are, respectively, 57%, 12%, 17%, and 3%. Table 6 shows representative examples of student responses to question 1. In the coding, students are designated with a "P" or an "S" depending on their grade level, where P stands for Primary and S for Secondary, followed by the student number. Therefore, P.67 is the 67th student in Primary.

Table 6. Examples of student responses to question 1

		C / I	Answers	Student
Propositional structure	Complete story	C	*In an exam I got a score of 2 points but they took off 3 points for making mistakes, and in the end, I got -1*	P.67
		I	*I was on floor -1 and I went down 3 floors. I am now on floor -4*	P.01
	Complete problem	C	*In Tejeda the temperature was 2ºC and at night it dropped to 3ºC. How many degrees was it at night?*	S.07
		I	*I had €3 and I spent €2 to buy a bag of sweets. How much do I have left now?*	P.101
	Incomplete problem	C	*Isabel has 2 candies, but she owes 3.*	P.15
	Non-numerical situation	I	*Why are you taking my candies?*	S.38
Concrete models	Displacement	C	*In a building, my mother went down 3 floors. If she was on the second floor, which floor is she on now?*	P.78
		I	*The water temperature in the morning is at 2 degrees and at night at -3. What is the temperature at night?*	P.46
	Neutralisation	C	*I have €2 and I owe €3*	P.26
		I	*If I take two candies from Paco and he has 3, how many candies does Paco have now?*	S.132
	Non-concrete model	C	*Find the operation*	P.32
Semantic structure	Change	C	*Sergio had 2 lollipops, and they took 3 from him. How many does he have left?*	S.28
		I	*Pepe had two cases stolen*	P.112
	Comparison	C	*In Madrid, it is 2 degrees, and in Valencia, it is 3 degrees less*	P.98
		I	*John is on a mountain 1000 m above the sea, and Nerea is 1001 m below Juan*	P.40
	Combination	C	*Federico has 2 candies, but he owes his sister 5*	S.11
		I	*I have €2 and I owe -3, what is my account balance?*	P.11
	Two changes	C	*If Carlos gets 2 and has 3 stolen, how many does he have left?*	S.03
		I	*Javier gave his brother €2, two days later his brother gave him €3, how much money does Javier have now?*	S.15
	Non-additive problem	C	*In Dolomites the temperature is 2ºC, but it drops by 1 degree every minute. In 3 minutes, what is the temperature?*	S.90
		I	*Find the operation*	P.32

Transfer from the number line dimension

Through the transfer from the number line to the contextual dimension, students posed problems that were developed from the vector (-2, -5) of the number line. Table 7 shows the results from all the participants, according to the propositional structure developed, the concrete model presented (if any), and the semantic structure proposed.

Table 7. Results of the mathematical analysis of the transfer from the number line dimension (v (-2, -5))

		Primary Students			Secondary Students		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Propositional structure	Complete story	3	0	3	1	1	2
	Complete problem	52	21	73	44	12	56
	Incomplete problem	0	16	16	0	37	37
	Non-numerical situation	0	3	3	0	1	1
	No answer	0	5	5	0	4	4
Concrete model	Displacement	42	24	66	32	30	62
	Neutralisation	13	12	25	13	20	33
	Non-concrete model	0	4	4	0	1	1
	No answer	0	5	5	0	4	4
Semantic structure	Change	41	24	65	28	28	56
	Comparison	4	4	8	6	6	12
	Combination	10	4	14	10	5	15
	Two changes	0	2	2	1	6	7
	Non-additive problem	0	6	6	0	6	6
	No answer	0	5	5	0	4	4
	Total	55	45	100	45	55	100

Checking answers

When posing problems from the vector (-2, -3), half of the participants were able to successfully create complete stories and problems. Particularly, 55% of Primary students and 45% of Secondary students have formulated additive problems correctly from the provided vector.

Analysis of the propositional structure

~~Of~~ Primary students have described more complete problems than Secondary students, 73% compared to 56%. However, the percentage of incomplete problems is much higher for Secondary students, 37% compared to 16%. 3% of Primary students have described a non-numeric situation and 5% have left the answer blank; these percentages fall to 1% and 4%, respectively, in Secondary students.

Analysis of the concrete model

66% of Primary students and 62% of Secondary students have used displacement models, and 25% and 33% have used neutralization models, respectively.

As in the formulations of problems from the abstract dimension, the displacement models used are, mostly, those of temperature and building floors and in those of neutralization, those of have-owe and earn-lose, give-take, etc.

Analysis of the semantic structure

65% of Primary students have described problems of change, 8% of comparison, 14% of combination, and 2% of two changes. When analysing the problems posed by Secondary students, the percentages are, respectively, 56%, 12%, 15%, and 7%. Table 8 shows representative examples of student responses to question 2.

Table 8 Examples of student responses to question 2

		C/I	Answers	Student
Propositional structure	Complete story	C	*It was very cold this morning. Tonight, it was -2ºC and in the morning the temperature dropped 3º, so we are at -5ºC*	S.92
		I	*In the match, my team was -2 points down, and the other team scored a triple, and now we are -35 points down*	S.80
	Complete problem	C	*I was on floor -2 and I had to go down 3 more floors. What floor am I on?*	P.03
		I	*Carlos had 2 cookies, and they stole 5 from him. How many cookies does he have left?*	S.24
	Incomplete problem	I	*Laura owed Juan 2 sweets, but now she also owes Pedro 3 sweets*	S.119
	Non-numerical situation	I	*The number line represents the solution of a problem*	P.41
Concrete model	Displacement	C	*In Helsinki, it was -2ºC at night, and it dropped 3ºC in the morning. What was the temperature in the morning?*	S.39
		I	*I'm on the -5 floor and I'm going up 3*	P.26
	Neutralisation	C	*I owe my friend €2, and I owe Lucas €3. How much money do I have to pay?*	P.17
		I	*5 gifts have been taken from you*	S.03
	Non-concrete model	I	*Negative numbers*	P.47
Semantic structure	Change	C	*Marta went down to the parking lot from floor -2 to -5*	P.22
		I	*The current temperature is 2º if it drops 3º, what's the temperature?*	S.09
	Comparison	C	*In my house it is -2 degrees at night and in my grandmother's house -5, what is the temperature difference?*	S.113
		I	*Today in Las Palmas it is 17ºC and yesterday it was 2ºC lower. How many degrees was the temperature yesterday?*	P.127
	Combination	C	*I owe Claudio 2 sweets and Iñaki 3*	S.31
		I	*I have 2€ but I owe Pedro 5. How much money do I have?*	P.103
	Two changes	C	*Mr Paco had 2 pencils stolen and now 3 more stolen*	P.112
		I	*Juan gives €2 to his sister and €3 to his cousin*	S.35
	Non-additive problem	I	*-2 – 3*	S.110

Comparison between transfers from abstract and number line dimensions

This section presents a comparison between the results of the students' productions of the two courses in each of the transfers (abstract-context and number line-context). Results are summarized in Table 9.

Table 9. Results of transfers between abstract-context and number line-context dimensions (data from primary and secondary school students together)

		Abstract-Context			Number line-Context		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Propositional structure	Complete story	1	1	2	2	1	3
	Complete problem	46	15	61	48	16	64
	Incomplete problem	0	21	21	0	27	27
	Non-numerical situation	0	2	2	0	2	2
	No answer	0	14	14	0	4	4
Concrete model	Displacement	31	16	47	37	27	64
	Neutralisation	16	20	36	13	16	29
	Non-concrete model	0	3	3	0	3	3
	No answer	0	14	14	0	4	4
Semantic structure	Change	31	21	52	34	26	60
	Comparison	6	3	9	5	5	10
	Combination	8	8	16	10	5	15
	Two changes	1	2	3	1	4	5
	Non-additive problem	1	5	6	0	6	6
	No answer	0	14	14	0	4	4
	Total	47	53	100	50	50	100

The results of transfers from the abstract and number line dimensions are similar, although slightly higher than the number line dimension, 50% versus 47%. In the transfer from the number line, more problems have been described, both complete and incomplete, while in the transfer from the operation, there are more blank answers (14%).

For both types of transfers, the students tend to use more frequently the displacement mode, although the difference with the neutralization models is more significant in the transfers from the number line dimension. The order from highest to lowest use of the different types of problems has been the following for both transfer types: change, combination, comparison, and two changes.

Moreover, in both questions, we can find responses that describe unfeasible situations, although some of them are adjusted to the vector operation. Some examples are: “I was 2 years old, and they took 3 years off” (A1.02) or “In a tree there are 2 leaves and in autumn 5 leaves fall” (A6.40).

Additionally, several answers contain expressions that simultaneously use the binary and unary meanings of the minus sign. Some examples are: “... I owe -3” (A6.11) and “... the temperature fell -3º” (A1.12), which would be equivalent to “I have 3”, “... the temperature increased 3ºC”.

Comparison of results between students in the 6th year of Primary Education and the 1st year of Secondary Education

With the objective of comparing the problems posed by students from Primary Education and Secondary Education, Table 10 presents the results obtained for both courses, by averaging the results of the two tasks.

Table 10. Comparison of the transfer results from Primary students and Secondary students

		Primary Students			Secondary Students		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Propositional structure	Complete story	2	1	3	1	1	2
	Complete problem	50	18	68	44	13	57
	Incomplete problem	0	14	14	0	34	34
	Non-numerical situation	0	2	2	0	1	1
	No answer	0	13	13	0	6	6
Concrete model	Displacement	38	17	55	30	26	56
	Neutralisation	14	14	28	15	21	36
	Non-concrete model	0	4	4	0	2	2
	No answer	0	13	13	0	6	6
Semantic structure	Change	37	19	56	28	28	56
	Comparison	5	2	7	6	6	12
	Combination	10	5	15	9	7	16
	Two changes	0	2	2	1	4	5
	Non-additive problem	0	7	7	1	4	5
	No answer	0	13	13	0	6	6
	Total	52	48	100	45	55	100

Primary students have formulated more correct problems than Secondary students, specifically, 52% compared to 45%. Perhaps it is due to the tendency of Secondary Education participants to “economize” resources, obviating the question. As some pointed out in the subsequent discussion of results, this might be due to the consideration that the question was not necessary to describe the situation. 34% of the answers in Secondary students and 14% of the answers in Primary students are incomplete problems with two data that fit the conditions imposed.

Primary students leave more blank answers than Secondary students, 13% compared to 6%; this circumstance is especially striking in the abstract dimension, in which the percentage of blank answers of Primary students reaches 20%.

The use of displacement models is similar in both years, 55% and 56%; but Secondary students use more neutralization models than those in the Primary students, 36% compared to 28%.

The frequency of use in the different types of semantic structures of additive situations maintains the same order in both years: change, combination, comparison, and two changes.

In summary, Figure 4 shows the overall results of the transfers from the abstract and number line dimensions to the contextual dimension, for both years and concerning the total number of participants. These data are taken from tables 5 and 7.

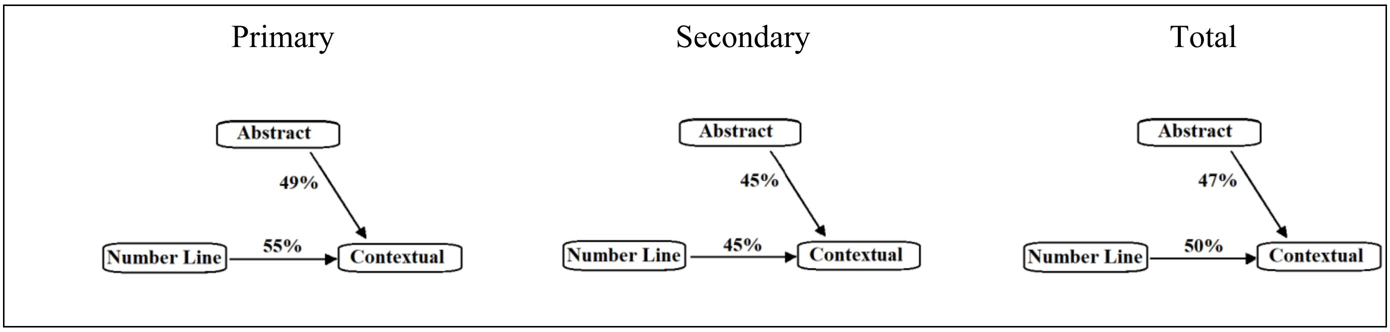
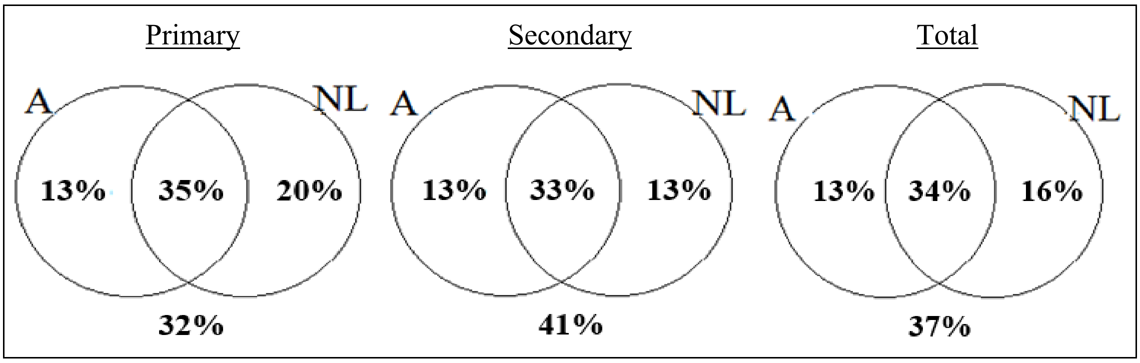


Figure 4. Overall results of transfers

Moreover, Figure 5 summarises the percentage of students who have successfully transferred from the abstract dimension (A), from the number line dimension (NL), and globally to the contextual dimension. 34% of students have completed both transfers, 35% in Primary Education compared to 33% in Secondary Education. 37% of all students have not made either of the two transfers, which is composed of the 32% of Primary students compared to 41% of Secondary students.



A: Abstract dimension NL: Number line dimension

Figure 5. Students in each level who correctly solved 2, 1, and 0 transfers

13% of the students in both courses have correctly carried out only the transfer from the abstract dimension, while 20% of the Primary students and 13% of the Secondary students have correctly carried out only the transfer from the number line.

Discussion

Without specific instruction, almost half of the students in this study have been able to formulate simple additive problems, based on initial information given abstractly or on the number line.

Contrary to expectations, the results of transferring situations to the contextual dimension are slightly higher when starting from the number line dimension than when starting from the abstract dimension, especially for Primary students. Some research states that the number line can cause difficulties in subtracting negative numbers (Janvier, 1983); however, these difficulties did not arise clearly in this study because students could pose situations in which positive numbers were subtracted, avoiding the most complex situations about subtracting a negative number, that is, a-(-b). In addition, an approach to teaching integers based on contextualized situations must be linked to representations in the number line, as this induces the use of semantic structures of different types of problems (Bruno and Cabrera, 2005).

In the question that begins in the abstract dimension, there were more blank answers than in the question that begins in the dimension of the number line. This circumstance was relevant in elementary students, where one out of five students did not answer the question. This observation confirms that, for students with less initial knowledge of integers, the graphical representation of an additive situation in the form of a vector on the number line is more understandable and intuitive than the symbolic representation. Along these lines, although focused on problem solving rather than invention, Bruno and Martinón (1997) states that students have "greater ease and confidence in solving problems using the number line than with operations" (p. 257).

Moreover, almost a third of the participants have only been able to correctly perform one of the two transfers. This observation is confirmed by Bruno and Martinón (1997), who considers that "there is a certain disconnection between problem solving using operations and the number line" (p. 257).

Students show their preference for problems of the semantic structure of change in the two types of questions analysed. This preference is manifested more in the transfer from the number line, since the image of the vector favours giving an example of variation. In addition, change problems are usually those that are solved more frequently in class and those that appear in greater proportion in textbooks (Ayllón et al., 2011). However, although there are more displacement situations and more change problems, it is striking that there is a significant percentage of situations that respond to neutralization models and with a combination semantic structure.

Very little variety has also been observed concerning the contexts used. In the displacement models, their contextualization is reduced, exclusively, to situations with temperatures and, to a lesser extent, to floors of a building and heights above sea level; the situations in the neutralization models are reduced to have-owe and get-lose situations contextualized in euros, candies, school supplies, fruits... This is because students do not use negative numbers extensively in their daily lives. Therefore, they mainly use contexts learned in class. These results coincide with the findings of other research by Ayllón et al (2011) and Irvine (2017), which show that students, when posing problems, usually limit themselves to reproducing the problems worked on in class. This is confirmed in the thesis by English (1998) which indicates that the type of problems formulated by students depends on the instruction received.

The propositional structure most used in both grades is that of complete problems, that is, situations consisting of two pieces of information and a question. However, in the two questions analysed, the Primary students described more complete problems than the Secondary students. However, if the answers that fit the situation but did not pose the question had been considered correct, the results of the Secondary students would have been slightly higher than those of the Primary students. In any case, the fact that the differences in the results of both grades are not relevant shows that problem posing is not a task that is developed exclusively based on knowledge of integers; that is, greater knowledge does not imply a better understanding of how contextual situations or problems are connected with operations and the number line.

Although the rest of the negative numbers are not raised in the questions, in some formulation’s expressions are used in which the binary and unary meanings of the minus sign overlap, which shows the difficulty of differentiating both meanings (Bofferding, 2014) and causes an artificial language. Other formulations analysed, although sometimes adjusted to the conditions imposed by the situations expressed in the operation or the vector, describe unreal or impossible situations. These formulations confirm the results obtained by Ayllón et al. (2016) when analysing the coherence of the problems invented by Primary students, since the description of impossible, unreal, or absurd situations could be due to teaching that is far removed from the context and reality of the student.

Conclusions

The general objective of this research is to analyse the problems formulated by 266 students in the Primary and Secondary students (11 to 13 years old). This research provides information on the knowledge about integer numbers in students of two educational levels in transition from primary to secondary school.

Analysing the problems posed by the students helps, in a way, to observe how they understand the additive situations. The analysis carried out on the problems formulated by the students has been profound, since it is approached from different educational research interests: two educational levels (last year of primary and first year of secondary school), relating dimensions of numerical knowledge (abstract-context and number line-context) and observing the characteristics of the problems (propositional structure, concrete model and semantic structure).

The action of posing problems involves a high level of abstraction and requires a significant mastery of the content studied, such as the appropriate use of language, concepts, procedures, and mathematical properties (Silver, 2013). From the in-depth analysis of the problems written by the students in this study, and observing their characteristics, it can be inferred that problem posing is a skill strongly influenced by the training received and that it is not developed simply by expanding mathematical knowledge.

The problems formulated by students in this study exhibited simple structures and were often poorly written and lacked originality, showing limited variation from classroom contexts. These weaknesses have also been observed in research on future teachers in writing problems of this topic (Kapıcıoğlu & Arıkan, 2022). The ability to pose problems does not develop spontaneously in learners; it requires explicit and consistent instruction. Problem formulation is not a common practice in the learning of mathematics, but its relevance has been increasing in recent years. In the case of integer numbers, it requires that students observe the different semantic structures based on stories associated with certain operations (Bruno, 2000; Rudnitsky et al., 1995). Furthermore, mathematics is often taught in isolation from other curricular areas, especially language. However, it is known that posing problems can help improve problem solving, allow progress in understanding environmental situations, stimulate curiosity, motivation, and creativity, and promote a cross-curricular approach in which other curricular subjects participate.

Problem posing is a latent ability in students that can be improved through practice. Therefore, based on the results of this research, we believe that problem posing requires further research to fully develop its didactic potential and prove that it is an effective tool for learning mathematics and solving problems. Thus, the developing of the potential offered by problem posing in mathematics teaching requires the active participation of teachers, which is why we support its incorporation into initial and continuing education programs.

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Data available on request from the authors

The data that support the findings of this study are available from the corresponding author, [author initials], upon reasonable request.

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