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Sum of Products of Consecutive Gibonacci Numbers

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Gibonacci numbers are defined by the recurrence $u_{n+1} = u_n + u_{n-1}$ for $n \geq 1$, with general initial values $u_0 \geq 0$ and $u_1 \geq 0$, not both null.

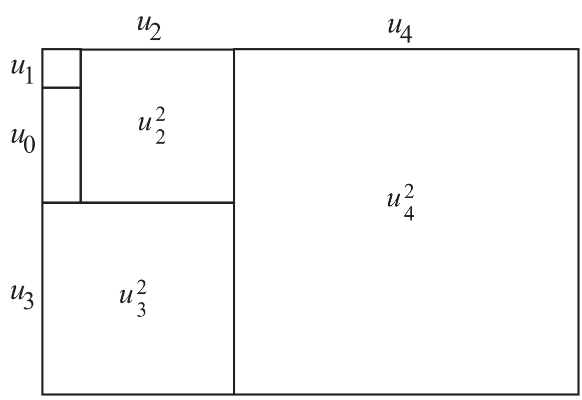
Here, we visually prove a formula for the sums of products of consecutive Gibonacci numbers by using rectangular tilings. Identities about sums of squares of any Fibonacci sequence were proved by Alfred Brousseau [1, p.147]. These results are also proved by a different combinatorial argument in [2, Identities 41 and 42].

Proposition. For $n \geq 1$, $\sum_{k=1}^{n} u_k u_{k+1} = \begin{cases} u_{n+1}^2 - u_0 u_2 & \text{if } n \text{ is odd,} \\ u_{n+1}^2 - u_1^2 & \text{if } n \text{ is even.} \end{cases}$

Corollary. For $n \geq 1$, $\sum_{k=1}^{n} F_k F_{k+1} = \begin{cases} F_{n+1}^2 & \text{if } n \text{ is odd,} \\ F_{n+1}^2 - 1 & \text{if } n \text{ is even.} \end{cases}$

Also, $\sum_{k=1}^{n} L_k L_{k+1} = \begin{cases} L_{n+1}^2 - 2 & \text{if } n \text{ is odd,} \\ L_{n+1}^2 - 1 & \text{if } n \text{ is even.} \end{cases}$

Proof. The figure shows the situation for $n = 3$ for the case of n odd in the proposition. The closely related visual argument for the even case is left to the reader.



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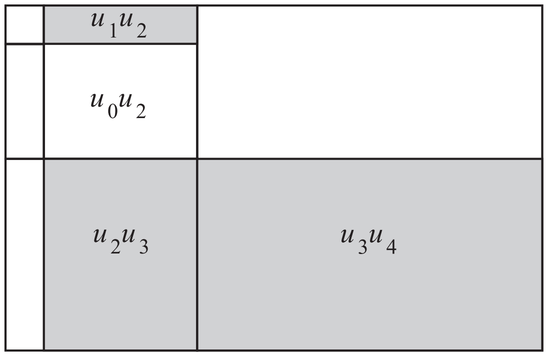

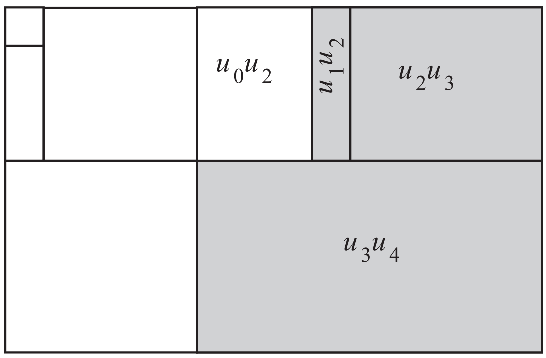

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