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Abstract

The aim of this work is to accurately estimate from an image the parameters of some ellipses and their relative positions with respect to a given pattern. The process is characterized because it is fully automated and is robust against image noise and occlusions. We have built a calibrator pattern with two planes each containing several ordered circles in known 3D positions. Our method is able to estimate the position of every ellipse and to put them in correspondence with the original calibrator circles.

1 Introduction

In this paper we tackle the problem of automatically detecting a set of ellipses on an image and ordering them according to an original pattern. The method that we propose here deals with images like in Fig. 1 in where there is an image of a *calibrator* which is often used to calibrate a camera [7, 14]. This is the original pattern and, in this case, it is composed of two orthogonal planes each one containing twenty circles in known 3D positions. When we take a photo of a circle in 3D it becomes an ellipse in the projection plane. We are interested in detecting the center of every ellipse – in the image plane – and put all of them in correspondence with the original 3D position of each center of circle without the need for human interaction.

In this paper we deal with two different problems. The first aims for detecting the parameters of the ellipses. In fact we are only interested in the center of the ellipses which will match the projection of the circle centers. For the problem of camera calibration a good accuracy is also expected although it is not necessary for the following step. Given the center of the ellipses the second task aims for determining which are the relative positions of every center by respect to the original pattern – we will refer to this task as the ordering of centers of ellipses. In Fig. 3 we show the relative positions for the circles of the calibrator planes.

The ellipse parameters are detected by means of a Hough transform (see [10]) with parameter space reduction. This method is often used to detect shapes like lines, circles and ellipses. With this transform we express the ellipse equation in the parameter space. Every contour pixel contributes –in a voting system– to some ellipse parameters. We use an accumulator with two dimensions to maintain the information of the votes of every ellipse parameter configuration. The most voted parameters represent the ellipses which have been detected. An ellipse has 5 parameters so it requires a 5-dimensional parameter space but in our case we reduce the parameter space to only compute the center – 2 parameters – of the ellipses. Some techniques (like [1, 9, 16, 13, 8] have appeared to overcome the excessive computational cost and space requirements needed. They normally decompose the 5 dimensional parameter space into several sub spaces of fewer dimensions. At the end of this process we obtain the center of the ellipses in pixel precision. One of the advantages of this transform is that it is very robust against noise and occlusions. We have also improved this process by limiting the radius of the ellipses.

The following step is to select a fixed number of similar ellipses – 40 in the case of Fig. 1 –. We get the ellipses which have a similar area so we apply a simple recursive seed region growing algorithm from the centers of the ellipses to obtain the area of every ellipse. From the median sized ellipse we may select the fixed number of ellipses and remove those which have a very big or small area. We also remove the centers which do not lay inside a black region.

The last step in ellipse detection is to refine their parameters from the image contours. We have used a least-squares minimization method (see [2]) to accurately obtain the center of the ellipses – we also obtain the other three parameters, but they are not utilized in the rest of the process–. As we mentioned above, this step is not necessary if we are only interested in ordering the ellipses, but it is useful if we want to use this information for other purposes like in camera calibration where we need a good accuracy. This method is similar to other related ellipse or general conic fitting methods like [4, 5, 11, 12].

Once we have detected the ellipse centers we are concerned with the problem of putting in correspondence the 2D center points on the image with their original 3D points on the scene. We previously know which are the positions of the circle centers in 3D but no a priori information about the 2D points is supplied. If the 3D center points are ordered, this task is the same as ordering the ellipse centers on the image (like in Fig. 3). The difficult is to locate some reference ellipses, for example, those situated at the contours of the calibrator or at the corners. In some human assisted applications, like the "Camera Calibration Toolbox for Matlab", or in works about camera calibration like [17], normally the user is asked for selecting the four points at the corners by clicking with the mouse and then the application automatically recover the positions of the rest of points. Our goal is to avoid this in order to make it fully automatic the selection of corner points.

Previously we determine the neighbourhood relation between the ellipse centers. For this we build a Delaunay triangulation from the ellipse centers. This gives us a regular triangulated mesh in the image domain that allows us to search for the closest neighbours of every point. We will see that if the triangulation is regular enough – this is normally the case if the calibrator is not very leaning with respect to the image plane – then the center of the corner ellipses are included in one or two triangles. In this way we may select the corner points.

With these four points we compute the homografies – linear transformation between 2D points – between these corners and the corners of the original pattern. We obtain the best homografy with less error for the two planes. Finally we estimate the position and orientation of every plane in the image.

With this idea it is only necessary to obtain two corners with their 3 closest neighbours – one on each plane –. In fact we obtain the four corners and then select the best ones. The information we use for ordering the ellipses is composed of eight ellipses. If this minimum information is correctly recovered then we may order the entire calibrator and, from the inverse of the homografies, we may also recover the coordinates of the occluded ellipses.

In Sect. 2 we explain the method and its steps. In Sect. 3 we show some experiments for a variety of configurations. Also we show some results in where there are some occluded regions on the calibrator. Finally, in Sect. 4 we explain the most important conclusions and contributions of this work.

2 The Method

In Fig. 1 there is a calibrator which is a pattern commonly used to calibrate projective cameras. It is composed of two orthogonal planes each one containing a set of circles. We also note that the two planes are symmetrical and the position of the circles is regularly distributed on the planes. Many of such calibrators are design in this way in order to easily know the 3D position of the centers of every circle. We will see later that a so symmetrical distribution makes it more difficult to estimate the correct position and orientation of the planes. There has been some approaches in where the shapes to detect are not simple circles but some set of special shapes that facilitates the process of detecting which shape is in which position. Others have proposed methods for calibrators which are not so regular in the distribution of the circles, so it is easier to detect the orientation of the planes by simply detecting the position of some circles. In our case we can not make these assumptions and we have to automatically detect the orientation of the planes not only with the information of the circles but also with their relative positions on the image.

In the process of strong camera calibration we need a set of 3D points and their correspondent 2D points on the image. The set of 3D points are already known and in the case of Fig. 1 corresponds to the position of the center of the circles. The set of 2D points have to be detected on the projection image. To calibrate the camera it is very important that the estimation of the 2D points is accurate in order to obtain a good projection matrix, $\mathbf{P}$. The 2D points on the image corresponds to the center of the ellipses that result from the projection of every circle, so the first task is to accurately detect the center of all ellipses. Secondly we have to put in correspondence the 2D points on the image with their original 3D points on the real calibrator.

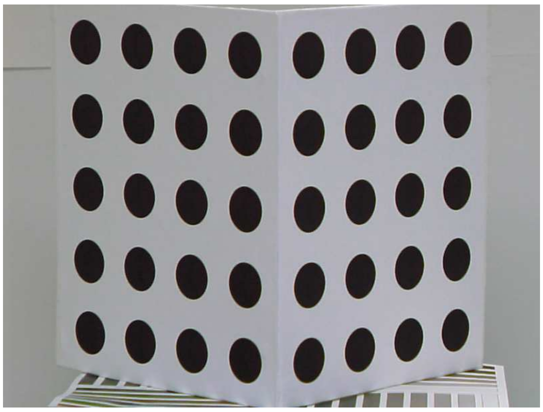


Figure 1: Image of a calibrator. It is composed of two orthogonal planes each one containing twenty circles in known 3D position

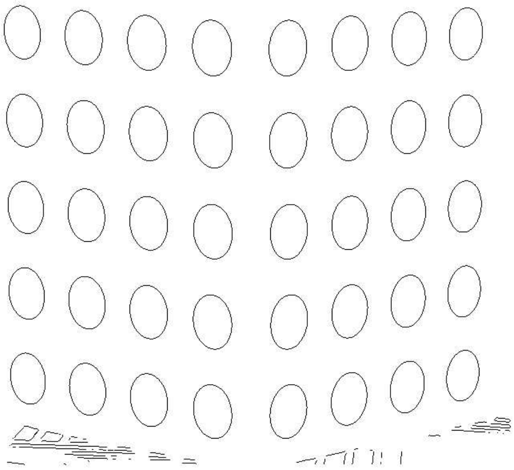


Figure 2: Image contours computed with the technique explained in [3]

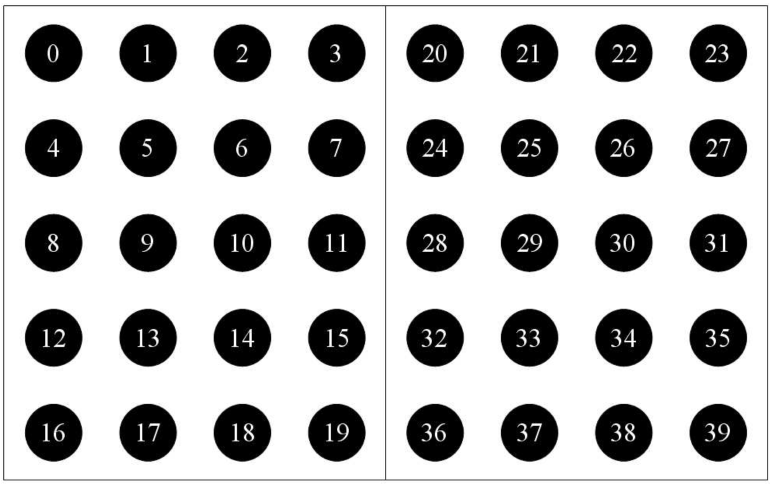


Figure 3: Ordering of circles in the original pattern. The relative position is fixed from left to right and from top to bottom. The second plane is also numbered in the same way from 21 to 39

If the 3D points are initially ordered like in Fig. 3 then we have to order the ellipses on the image in the same way, taking into account that the calibrator may be horizontal, vertical or in any other orientation.

This method also aims for being fully automatic. Given the input image, we have to detect the center of the ellipses and then put in correpondence these centers with the original centers of the circles.

The steps of the algorithm are as follows:

Algorithm 1 *Ellipse detection and ordering*

Ellipse detection

Hough transform for ellipse detection

Select n similar ellipses

Refine ellipse parameters

Ellipse ordering

Find corner ellipses

Compute the best homografy for each plane

Estimate orientation and position of the plames

Recover the position of occluded ellipses

The first step is to detect the center of the ellipses. Here we apply a Hough transform for ellipse detection with parameter space reduction. We only recover the ellipse centers which are only two parameters, (x, y) – ellipses are defined by means of five parameters –. It is possible that in this process we may obtain more ellipses than expected, so it is necessary to select an exact number of ellipses – the calibrator in Fig. 1 has 40 – according to their area size and their colour. Finally we make a better estimation on the parameters of the ellipses by means of a minimum least squares process. The Hough transform gives us the center of ellipses in pixel precision and we are searching for a better accuracy.

The second main step is the ordering of the selected ellipses. We select the corner ellispes. We create a Delaunay triangulation to form a regular mesh which allows us to easily find the corners. Then we estimate the homografies with the original 3D planes of the calibrator, given the 3 closest neighbours of the corners. The homografies are useful because they put in correspondence the centers of ellipses in the image plane with the centers of the circles in 3D. We select the two homografies – one for each plane – that have greater number of matchings and minimum error. Thanks to the homografies we may estimate the relative position of every ellipse in every plane. The following step is to estimate the orientation of the calibrator – whether it is vertical or horizontal – and thus determine which plane goes first and maybe to invert the initial ordering of the ellipses. Finally it is possible that if there are some ellipses that have not been detected by the Hough transform or were initially occluded by another object, their center and position in the plane could be recovered from the inverse homografies.

2.1 Detecting ellipses

2.1.1 Hough transform for ellipse detection

Given an image, the first step is to detect the center of the ellipses. We use the Hough transform to do so [ref]. The Hough transform is a technique that locates shapes in images. It is useful to extract shapes like lines, circles and ellipses. It is based on a voting system in where every contour pixel votes for the possible ellipse parameters that it could belong to. The parameters that have been more voted are the detected ellipses. The Hough transform is very robust against occlusions and noise.

We have implemented a Hough transform with parameter space reduction for ellipse detection. The idea behind this is illustrated in Fig. 4. Instead of computing the five parameters of an ellipse we only calculate its center. This is to avoid the computational cost and memory requirements of the original transform. We estimate from every two points and their tangent lines, the points (x_t, y_t) and (x_m, y_m) which gives a line where the ellipse center is included.

We extract the contours of the image (see [3]) and for every pair of contour points in a neighbourhood we compute these lines and accumulates the voting in one array. The point in the accumulator with maximum number of votes will give us the center of the ellipses.

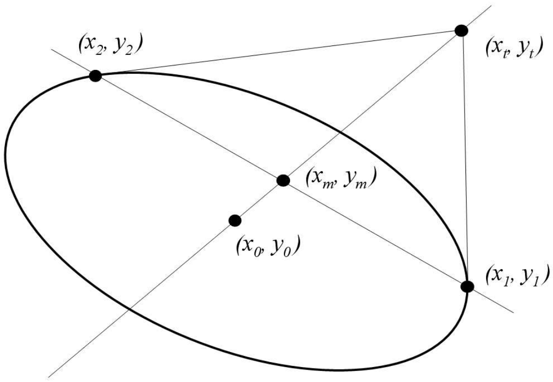


Figure 4: The straight line $[(x_t, y_t) - (x_m, y_m)]$ may be computed from two points on the ellipse, (x_1, y_1) and (x_2, y_2), and the tangents to the gradient in the points. The center of the ellipse is included in this line

Figure 5: Gray level image representation of the accumulator. White regions represent less voted pixels and black regions are the most voted ones

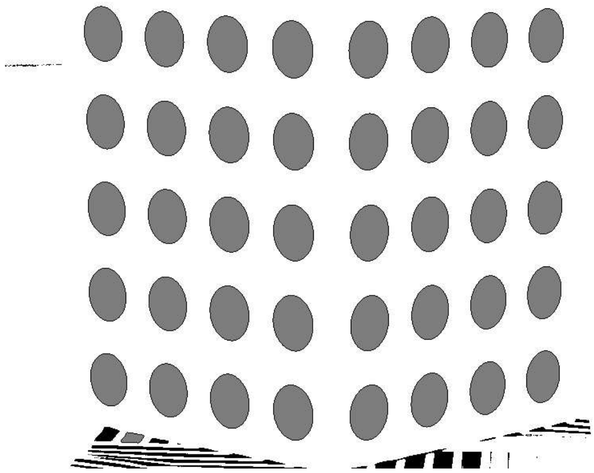


Figure 6: Threshold image. In grey the selected ellipses through the seed region growing method

In Fig. 5 we may see the accumulator that results from applying the Hough transform to the image in Fig. 1.

We have forced the algorithm to obtain a fixed number of best candidates from the accumulator which are the most voted coordinates. Normally the number of candidates will be greater than the number of ellipses in the calibrator pattern. This allows us to avoid leaving good candidates over. In a posteriori step we will remove those selected points which are not ellipses.

We have also improved the Houg transform by reducing the size of the window to look for pair of points – (x_1, y_1) and (x_2, y_2) in Fig. 4– and the radius or length of the voting lines. This is possible because the size of the ellipses we are looking for are not very big and all of them are similar. The effect of these two improvements is that the time necessary to find the centers reduce to a few seconds.

After this step the center of the ellipses are obtained in pixel precision. We are interested in accurately computing this centers. In a following section we will propose a method to refine these paramaters.

Normally and depending on the complexity of the images, the Hough transform may detect other points which are not ellipse centers. We will need a process to remove those candidates.

2.1.2 Select n similar ellipses

Our calibrator has a fixed number of circles so we are interested in detecting only the same number of ellipses in the image plane. It is usual that the Hough transform detect more ellipses than expected and we have designed the previous step to do so. Thus it is necessary to discard some of the ellipses.

The first step is to apply a threshold to the image. We obtain a black and white image in where the ellipses are set to black and the background to white. It is then easy to remove the center of ellipses that does not lay on a black region. It is also possible to identify some centers that lay on a black region – corresponding to additional ellipses or similar shapes – or that several centers are associated to the same ellipse.

For the last two cases what we do is to compare the area of the ellipses. To get their area we have implemented a recursive seeded region growing method. We use the centers as seeds.

In our case we are searching for black regions on a white background. In this case one pixel belongs to the region if its colour is black. We test that the center is in a black region and that there are no more than one center by region.

Using the information of the area of every ellipse we estimate the median value and then look for the n ellipses – n the number of circles on the calibrator – whose area are closer to this one.We also discard the regions which are very big or very small in comparison with the median ellipse.

Another approach to find the candidates ellipses is to compute the histogram of the area of the ellipses. The maximum of the histogram will represent that several ellipses have the same size, so it would be a good

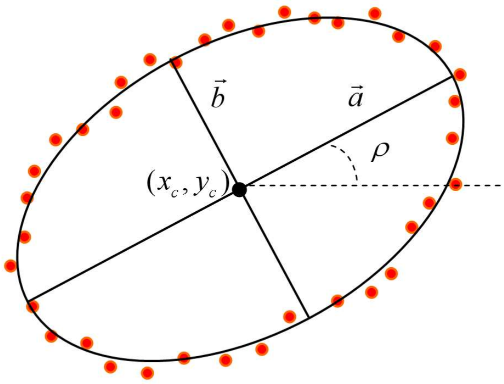


Figure 7: Least-square minimization method to fit a set of points to an ellipse

discriminant size. We may also consider that if our calibrator has two planes the area of the ellipses are similar by planes. We could find two maximums – one for each plane – and then find the ellipses which better aproximates to both maximums.

One of the drawbacks of the histogram is that we do not have many points, so it is possible that we do not get a unique maximum.

2.1.3 Refine ellipse parameters

Previous to accurately get the parameters of the ellipses we initialize these from the ellipse centers computed with the Hough transform and the points estimated with the seed region growing method.

$$\mathbf{x} = \mathbf{x}_0 + \vec{a}\cos(\theta) + \vec{b}\sin(\theta) \tag{1}$$

An ellipse may be expressed in polar coordinates as in (1), where $\mathbf{x}_0 = (x_c, y_c)$, $\vec{a} = (a_x, a_y)^T$ and $\vec{b} = (b_x, b_y)^T$.

If $a = \|\vec{a}\|$ is the magnitud of the greater axis and $b = \left\|\vec{b}\right\|$ the magnitud of the smaller one and knowing that both axis are orthogonal then an ellipse may be expressed by 5 parameters, (x_c, y_c, a, b, ρ). Given a set of 2D points, $\{\mathbf{x}_i\}_{i=1,\dots,N}$, on the image we are interested in finding the ellipse parameters which best fits the set of points as in Fig. 7. We implement a least square method (see [2]) in the following way:

$$E(x_c, y_c, a, b, \rho) = \sum_{i=1}^{N} dist(\mathbf{x}_i, \mathbf{x})$$

where $dist(\mathbf{x}_i, \mathbf{x})$ is the distance from a point to the ellipse. To reach the solution this equation is minimized and the solution is obtained by means of a gradient descend method. This method needs initial values for the parameters: The initial center of the ellipse is the one obtained by the Hough transform and the axis are computed by finding the farthest and nearest points on the contour of the ellipse.

2.2 Ordering ellipses

That calibrator we are considering has two planes which are normally orthogonal. The problem of ordering the ellipses here is the same as given the 3D positions of the center of the circles to look for the correspondent 2D projections on the image.

It is important to note that our calibrator is highly symmetrical: The size of all the circles are the same and they are regularly distributed in the planes. The two planes are identically design. This makes things much more complex because it is difficult to determine the position and orientation of a plane by respect to the other.

The idea behind the process of ordering the ellipses is that of selecting the four corner ellipses – those situated in the corners of the calibrator – and put them in correspondence with the corners on the original

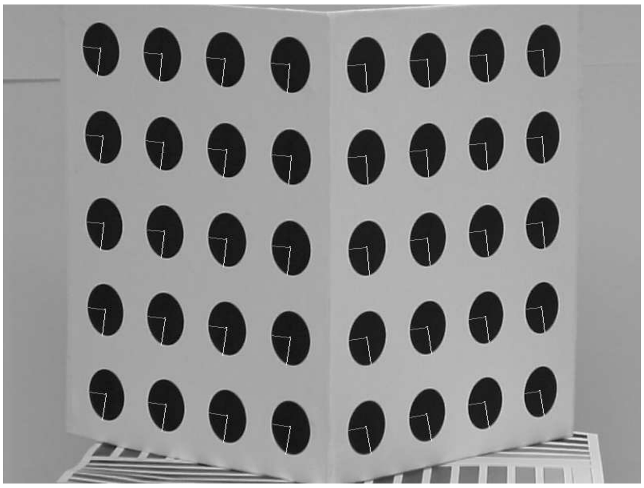


Figure 8: Parameters of the ellipses. The axis of every ellipse is drawn on the calibrator image. The center is at the intersection of both axis

pattern. If we may estimate this transformation then we can easily estimate the correspondences for the rest of points. This is carried out by means of a linear transformation, or a homografy between 2D planes.

This idea is usually applied in human assisted or semi-automated enviroment in where a user select the four corners and the algorithm would manage to do the rest. The goal is that our algorithm would be fully-automatic, also for selecting the corners.

The outline of the process is as follows: First we find the corner ellipses and their three closest neighbours. Then we compute the homografies between the image calibrator planes and the original calibrator planes and select the best two ones, one for each plane. The last step is to estimate the position and orientation of the two planes on the image.

In fact we only need eight center points to do this process because it is the only information needed to compute the homografies. When we find the homografies we know the projective transformation between the original planes and the image planes, so we could reproject the 3D circle centers, by means of the inverse homografies, on to the image and find the center of the ellipses on the image. This allows us to estimate the position of occluded ellipses on the image.

2.2.1 Find ellipse corners

The first step is to know the distribution of the center of ellipses on the image. We tackle the problem of determining which are the neighbours of every center point. It is difficult to estimate which are the neighbours in a projective plane since distances between ellipses are dependent on the position of the calibrator and may vary inside both planes of the calibrator. It is also difficult to estimate how many neigbours have a single point. And even more, the distribution is dependent on which points we consider first.

In order to tackle this problem we have made used of Delaunay triangulations, that is, we create a regular triangulation, which is a simple way of defining a neighbourhood relation. With this approach we assure that all points belong to some triangles – normally more than one – and that the triangle will be composed of the nearest points. The Delaunay triangulation of a point set is a collection of edges satisfying an "empty circle" property: for each edge we can find a circle containing the edge's endpoints but not containing any other points.

To create the triangulation we proceed as follows: Given the set of ellipse centers we calculate four points which are the envelope for all points. Then we create an initial triangulation with these four points. The rest of the process consist of randomly inserting one point at a time. Once we have inserted all the points, we remove the initial envelope.

In Fig. 9 we can see a typical triangulation for the calibrator.

We select the four corners of the calibrator from the triangulation given in the previous step. We will assume that the calibrator is not very leaning in the sense that it stays in front of the camera – we are

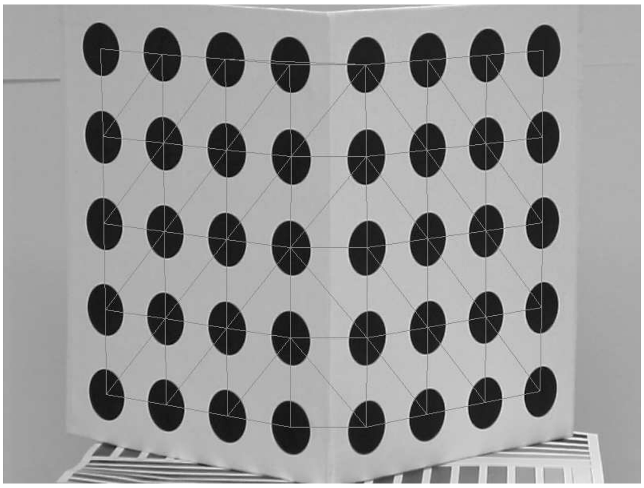


Figure 9: Delaunay triangulation

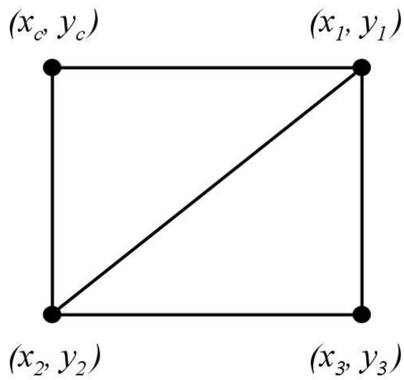


Figure 10: The ellipse corner, (x_c, y_c), belongs to one triangle, like the upper corners of Fig. 9

able to see better the two planes rather than the bottom or upside of the calibrator. If this happens the triangulation is regular and we will obtain a triangulation like in Fig. 9.

If we also assume that all the ellipse centers have been detected and are close to their correct position, one point is a corner if it belongs to one or two triangles of the triangulation. We may guess this if we look at Figs. 10 and 11. The inverse is not always true, that is, if one point belongs to one or two triangles then it is a corner.

What is always true is that if one point belongs to two triangles in the triangulation then it is a contour point – an ellipse center positioned in the border of the calibrator –. If it only belongs to one triangle then it is necessary a corner. All the interior points will always belong to more than three triangles.

For these reasons we search for the center points that belong to one or two triangles. In this set we will have the corner ellipses and other ellipses that belong to the contour of the calibrator.

We also recover from the information of the triangles and for every point the other four neighbours – see Figs. 10 and 11 – which will be used later to compute the homografies.

In the case we have a configuration like in Fig. 11, we look for the triangles that contain the corner ellipse center, (x_c, y_c), and extract the other points of the two triangles.

In the case of a configuraction like in Fig. 10, we have to search for the triangle that share the opposite edge to the corner ellipse center and obtain the fourth point, (x_3, y_3).

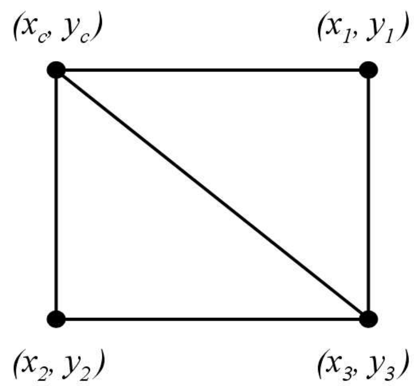


Figure 11: The ellipse corner, (x_c, y_c), belongs to two triangles, like the lower corners of Fig. 9

2.2.2 Compute the best homografy for each plane

The purpose in this step is to find the transformation that it exists between the calibrator planes on the image and the original planes of the calibrator. If we find these transformations, then it is easy to put in correspondence the 2D points with the 3D points of the pattern. We have to consider here that in our example we have two planes, what means that we have to look for two differtent transformations. A homografy is a linear transformation between two planes . In our case these planes are in 2D. If we have a set of corresponding points $\mathbf{x}'_i \leftrightarrow \mathbf{x}_i$ in two planes that have been obtained through a projective transformation, then there exist a linear transformation, $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$, that relates them. Here we use homogeneous vectors, $\mathbf{x}_i = (x_i, y_i, w_i)$ and $\mathbf{x}'_i = (x'_i, y'_i, w'_i)$. The homografy is a 3×3 matrix

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \quad (2)$$

Because we work with homogeneous coordinates the relation is true up to a non-zero scale factor so we may use the cross product, $\mathbf{x}'_i \times \mathbf{H}\mathbf{x}_i = \mathbf{0}$, to solve the system. Our unknowns are the h_{ij} coeficients so we may express the previous system as $\mathbf{A}_i\mathbf{h} = 0$. We look for a non-zero solution so we may force the norm of $\mathbf{h}$ to be equal 1, so the number of unknowns are 8. We need at least four points to compute the homografy. The solution is the unit singular vector corresponding to the smallest singular value of $\mathbf{A}$. In order to avoid solving a not well conditioned system it is usual to make a previous normalization of $\mathbf{x}$ and $\mathbf{x}'$ – $\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x}$, $\tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$– such that the centroid of the points is at the origin and the average distance is $\sqrt{2}$ (see [6]). We have a new system, $\tilde{\mathbf{x}}'_i \times \tilde{\mathbf{H}}\tilde{\mathbf{x}}_i = 0$, and the original homografy is obtained as $\mathbf{H} = \mathbf{T}'^{-1}\tilde{\mathbf{H}}\mathbf{T}$.

For every corner point detected in the previous step and their closest three neighbours we compute the homografies to the original calibrator planes. We apply the homografies to all the center points detected on the image and compute the difference between the original center of circles and the newly transformed center of ellipses – if the difference is smaller than a given threshold, then it is considered that the two points are correspondent –. We select the homografies that have greater number of corresponding points in 2D and 3D and smaller error. At the end of this process we obtain two homografies – one for each plane of the calibrator–. We study the two homografies in order to avoid that they share the same set of center points.

$$\cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} \quad (3)$$

It is interesting to note that if the calibrator is not symmetrical then the homografies would determine directly which are the associated corners on the original pattern, so we could compute the four homografies and take the one that has a greater number of guessed. Due to the symmetry of the pattern we only need to compute the homografies with two different corners. These corners are determined by means of the cosine (3).Using the vectors on the image and the ones in Fig 12 we select those which are better aligned (bigger values for the cosine).

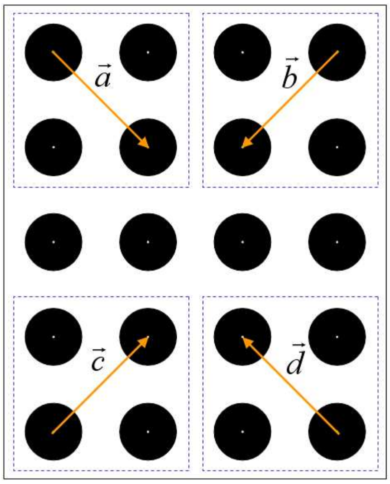


Figure 12: Vectors used to compute the homografies

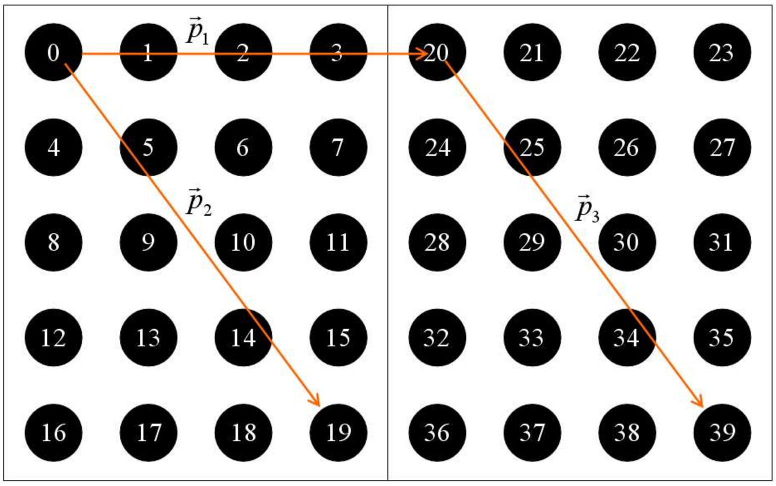


Figure 13: Correct position of the ellipses and orientation of the planes

When we have selected the two homografies, we refine them by taking into account all the center points detected on the plane. This gives us a better transformation for the planes. This will be useful in the last step to accurately recover the information of occluded ellipses.

The information of the homografies allows us to put in correspondence the points on the image and the points on the original calibrator, but we do not know the relative position of the planes on the image. If the calibrator would only have one plane, then the algorithm would finish in this step, but due to presence of two planes there still remains a final process to determine which plane is situated before the other.

2.2.3 Estimate orientation and position of the plames

The last step is to determine the relative position and orientation of the planes.

The original 3D point circle centers of the calibrator are ordered like in Fig. 3.

When we find the homografies of the two planes we do not know which plane is on the left – goes first – and which on the right – goes second –. Thanks to the homografies we know the ellipses that belong to each plane but we do not have information of how the planes are situated. For this we make use of three vectors, $\vec{p}_1, \vec{p}_2$ and $\vec{p}_3$, like in Fig. 13, and the cosine equation (3) to estimate the orientation and position. The first vector relates both planes and allows us to determine their position . The second and third ones allows us

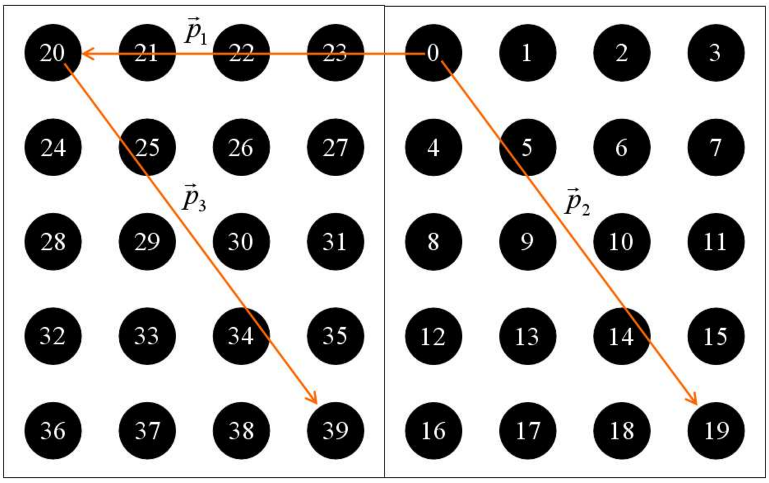


Figure 14: The planes are wrongly positioned

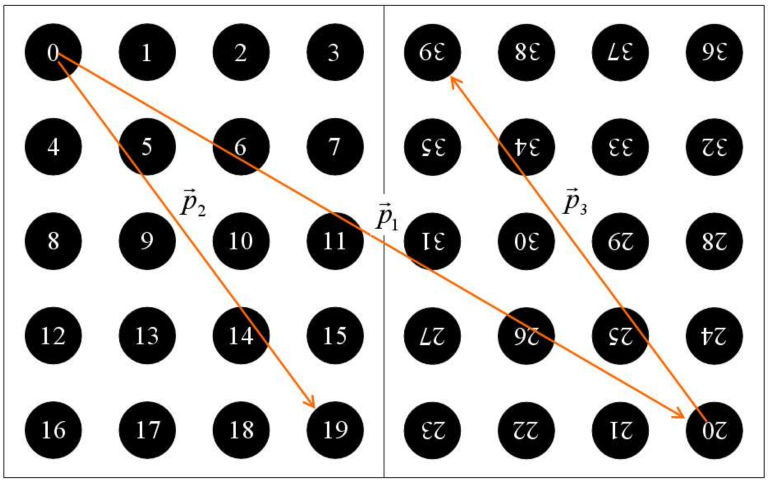


Figure 15: The second plane is rotated

to determine the orientation of the planes.

In Figs 14 and 15 we have two images with wrong configurations of the planes. In the first one the planes are not well situated. We can identify this by testing $\cos(\vec{p}_1, \vec{p}_2) < 0$. In the second one, one of the planes is rotated with respect to the other. This occurs when $\cos(\vec{p}_2, \vec{p}_3) < 0$.

We could have estimated the orientation and position directly from the homografies, but in our case this is not enough because of the symmetry of the planes. If we would have designed the calibrator less symmetrical, then this step could have been omitted.

2.2.4 Recover the position of occluded ellipses

Thanks to the information of the homografies and their inverses we may estimate the position of occluded ellipses. The homografies are calculated from 4 different points – the corner and their 3 closest neighbours –, so we only need 8 points for the two homografies. If they are correctly estimated then we could obtain the rest of the ellipses from the inverse homografies and the original 3D center points. Given the homografy, $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$, we can calculate the image points as $\mathbf{x}_i = \mathbf{H}^{-1}\mathbf{x}'_i$. Remember that we use homogeneous notation so we have to normalize the vector in order to put it in image coordinates (third coordinate must be equal 1, $\mathbf{x}_i = (x_i, y_i, 1)$).

Ideally if we only detect the 8 correct ellipses we may estimate the position of the rest. However it is more robust to compute the homografies from a bigger set of center points.

At the end of the process we select the center of ellipses that have been obtained from the least square method and that have a correspondent 3D point. It is possible that an ellipse from outside the calibrator has been detected, but thanks to the homografy we may determine if it is an ellipse correspondent to a circle on the calibrator and decide to include it or not. We know that an ellipse is occluded if a 3D center point does not have a correspondent 2D image center point. In this case we recover it by means of the inverse homografy.

In the experimental results we show several examples of occluding ellipses.

3 Experimental results

In this section we show several representative configurations that we have used to test our method. Apart from these tests we have applied our method to a large number of images. Most of them have been succesfully detected and ordered by only varying a little the parameters of the application. We show results for typical positions of the calibrator, such as vertical or horizontal front views of the pattern, and more complex configurations in where the calibrator is in a middle position between vertical and horizontal, is far from the camera, is very leaning with respect to the image plane or is situated close to another similar square calibrator pattern.

In another subsection we show several results for images with occluding ellipses. In some cases we have that the number of occluded ellipses is high and we test for different positions and orientations of the calibrator.

We have taken the images with different types of cameras and with different image resolutions. We have converted the colour images to grey level ones. The intensity values for the images are between 0 and 255.

The application has four parameters: The first parameter is the number of ellipses, *nel*, to be detected in the Hough transform process. This parameter is fixed to 60 and is used at the end of the Hough transform step to select a fixed number of ellipse centers from the accumulator. This number is usually greater than the actual number of circles on the calibrator, 40. In the following step the application only selects 40 or less number of ellipses as explained in Sect. 2.1.2. The second parameter is a threshold for computing the edges of the image, *th*. This parameter is specified by means of a percentage and is used in the Hough transform process to compute the edges of the image and to compute the thresholded image in the seed region growing step. This parameter is normally fixed to 0.5. The third parameter is the radius of ellipses, *radius*, and is used to accelerate the Hough transform process. Its default value is 50. This value is valid for a large number of images and is reduced only if the size of the calibrator is very small.

For every trial we show the original image, the image with the Delaunay tringulation, the detected parameters for the ellipses and the final ordering of the planes.

In Figs. 16, 17 and 18 we have several trials of the calibrator with different orientations.

In these images we show the original image, the Delauny triangulation obtained for the 2D points, the parameters computed for every ellipse on the image and finally the positions of every ellipse that are obtained automatically from our method.

In Fig. 19 the calibrator is far from the image plane. In this case it is enough to reduce the radio size for the Hough detection process.

In Fig. 21 the circle calibrator is put beside a square calibrator and occupies less than the half the image.

3.1 Experimental results with occlusions

From Fig. 22 to 25 there are some experiments with occluded ellipses. In these cases we may appreciate that the information of the occlusions is not taken into account in the triangulations. The only information necessary to find the homografies are the ellipse corners.

This is not a problem if at least two ellipse corners – one on each plane – may be recovered. The last test was configured in order to hide two corners – the two at the bottom – and regard that at the top of the calibrator the other two corners are correctly detected. We see that they are correctly ordered and the position of the occluded ellipses are also recovered.

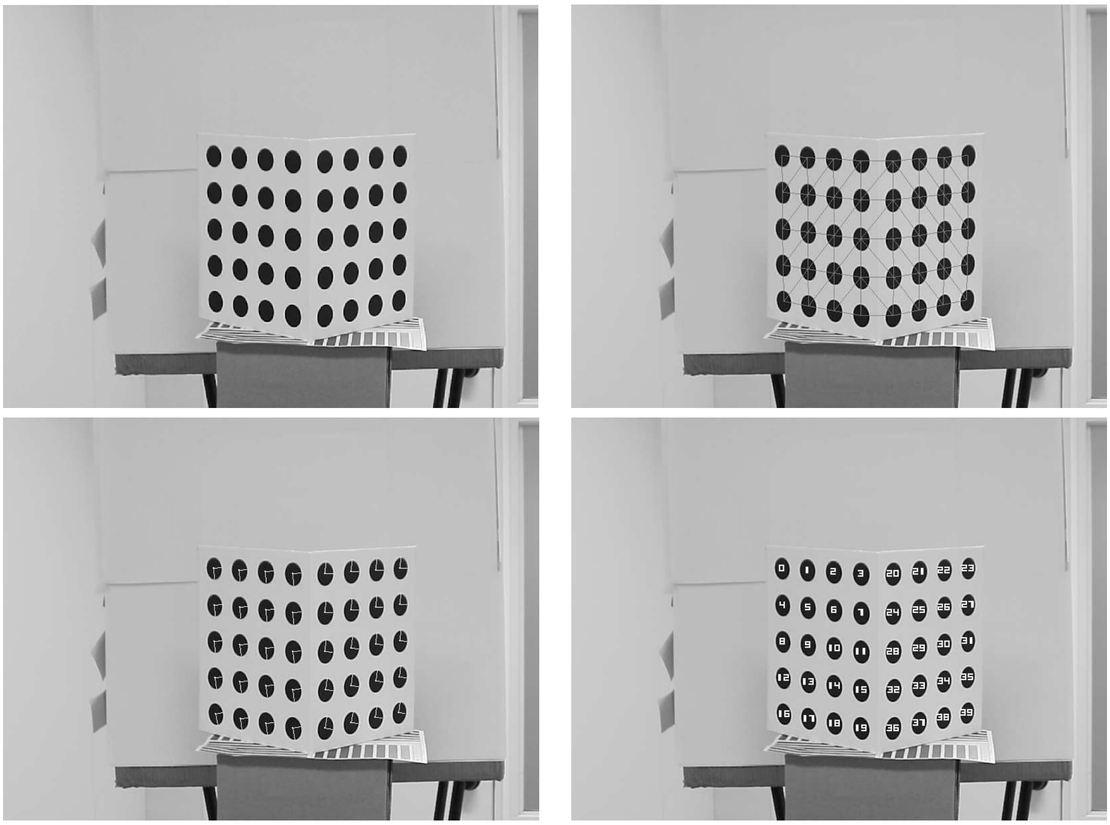

0 1 2 3 20 21 22 23
4 5 6 7 24 25 26 27
8 9 10 11 28 29 30 31
12 13 14 15 32 33 34 35
16 17 18 19 36 37 38 39

Figure 16:

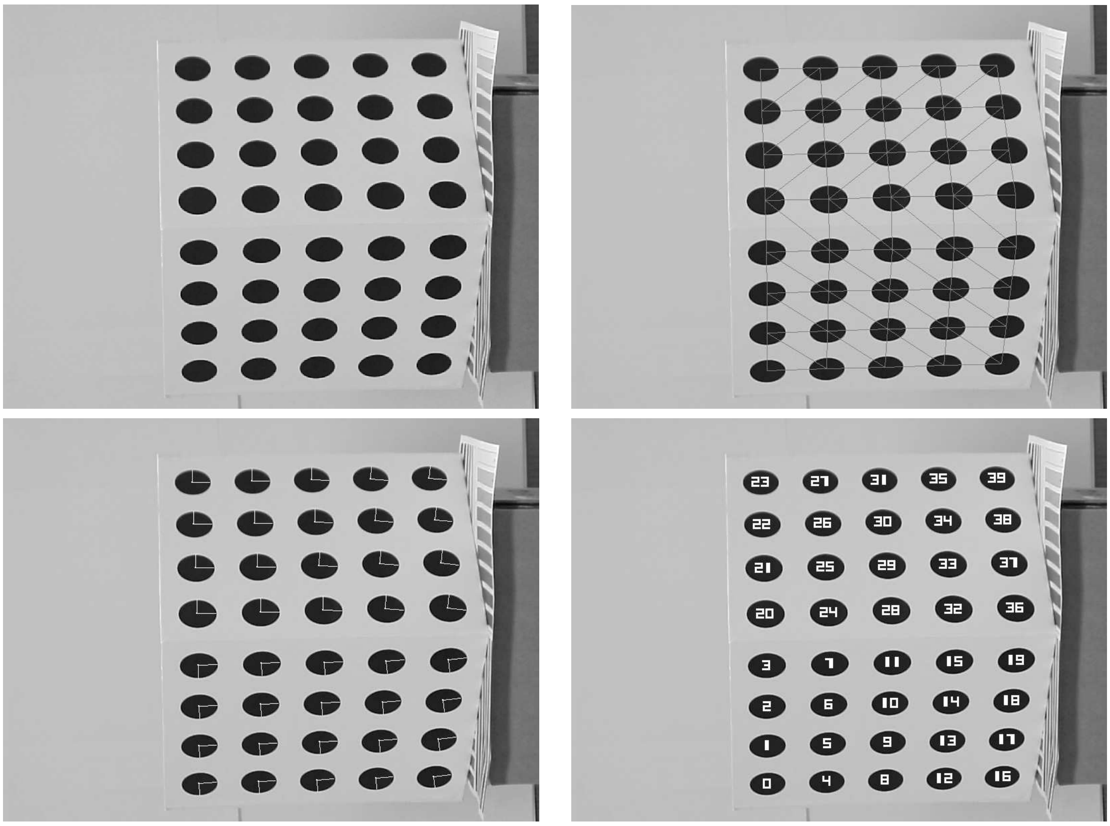

23 27 31 35 39
22 26 30 34 38
21 25 29 33 37
20 24 28 32 36
3 7 11 15 19
2 6 10 14 18
1 5 9 13 17
0 4 8 12 16

Figure 17:

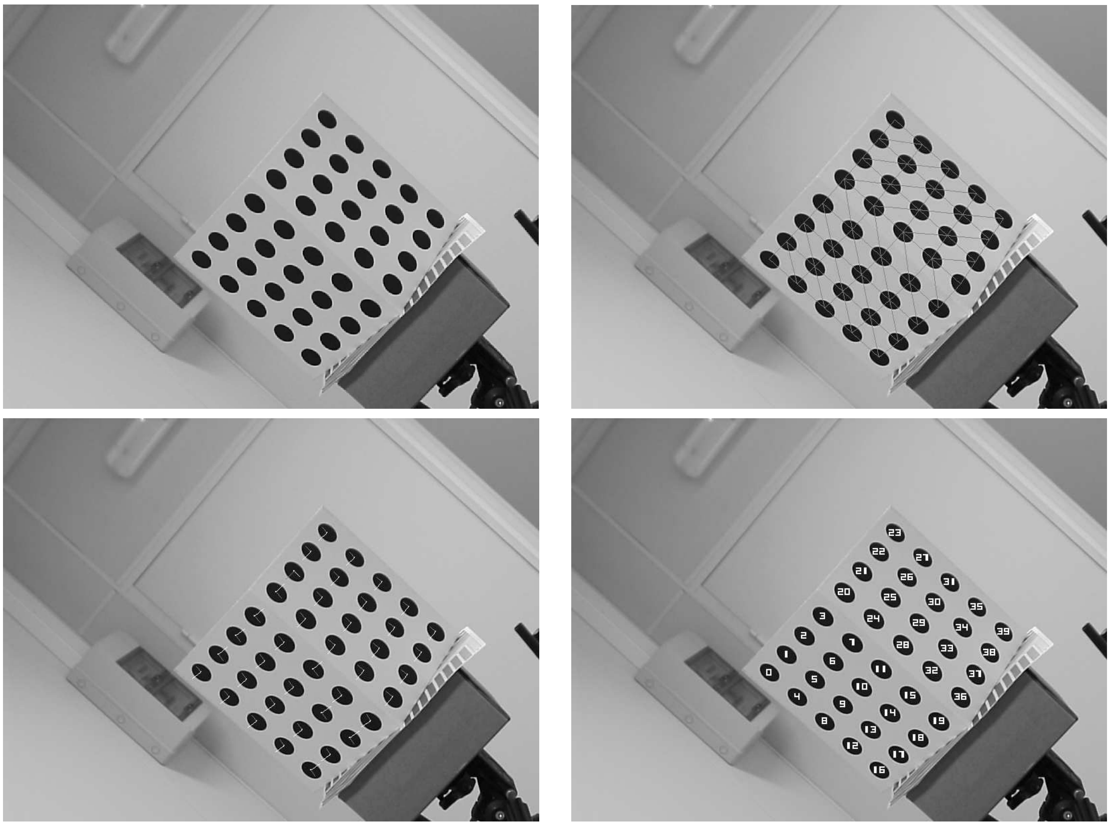
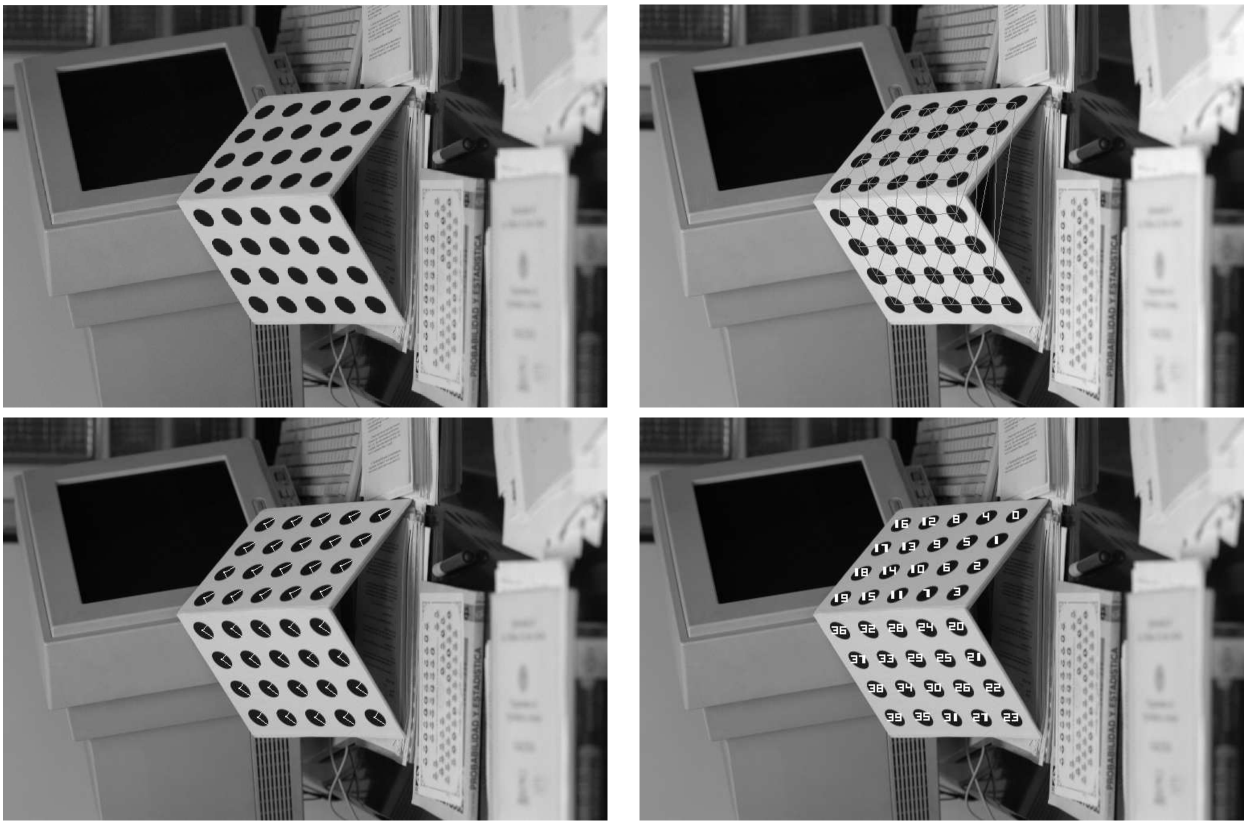


Figure 18:

Figure 19:


PROBABILIDAD Y ESTADISTICA
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17 13 9 5 1
18 14 10 6 2
19 15 11 7 3
36 32 28 24 20
37 33 29 25 21
38 34 30 26 22
39 35 31 27 23

Figure 20:

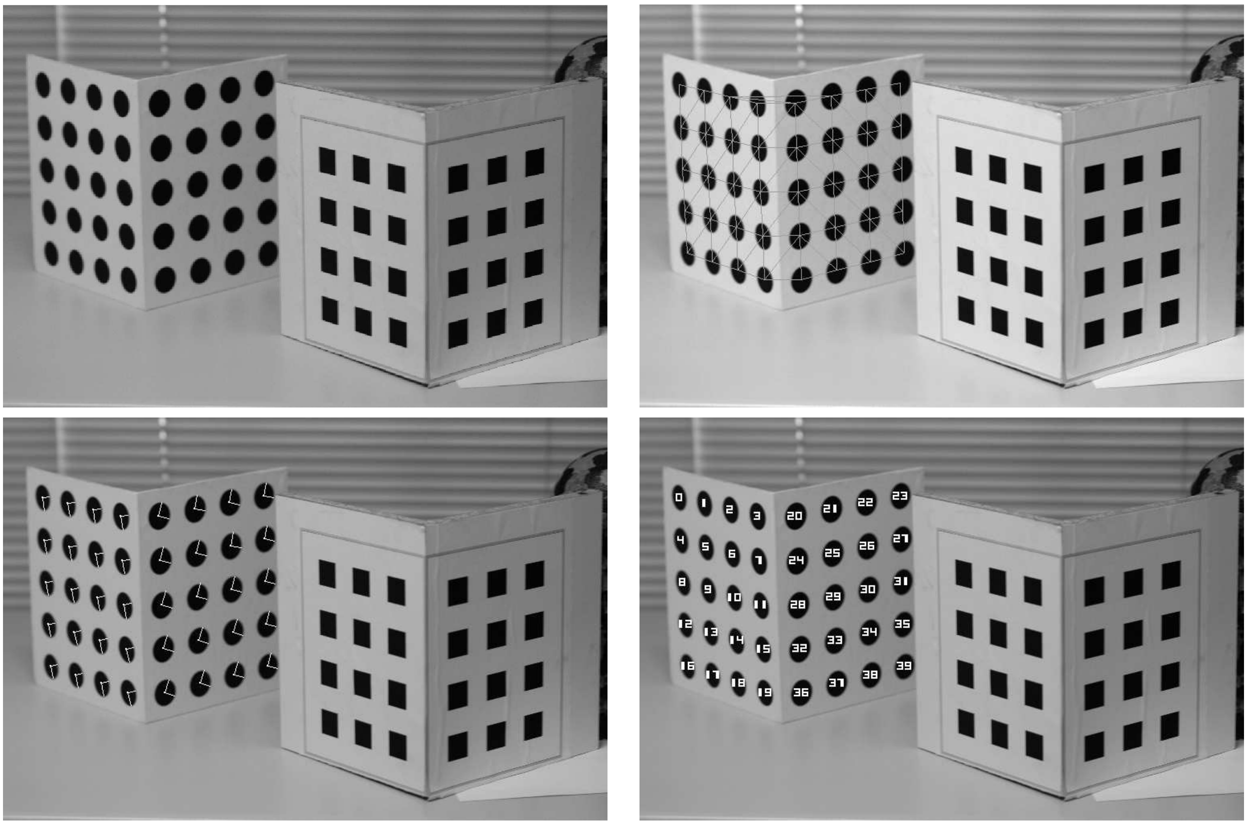

Figure 21:

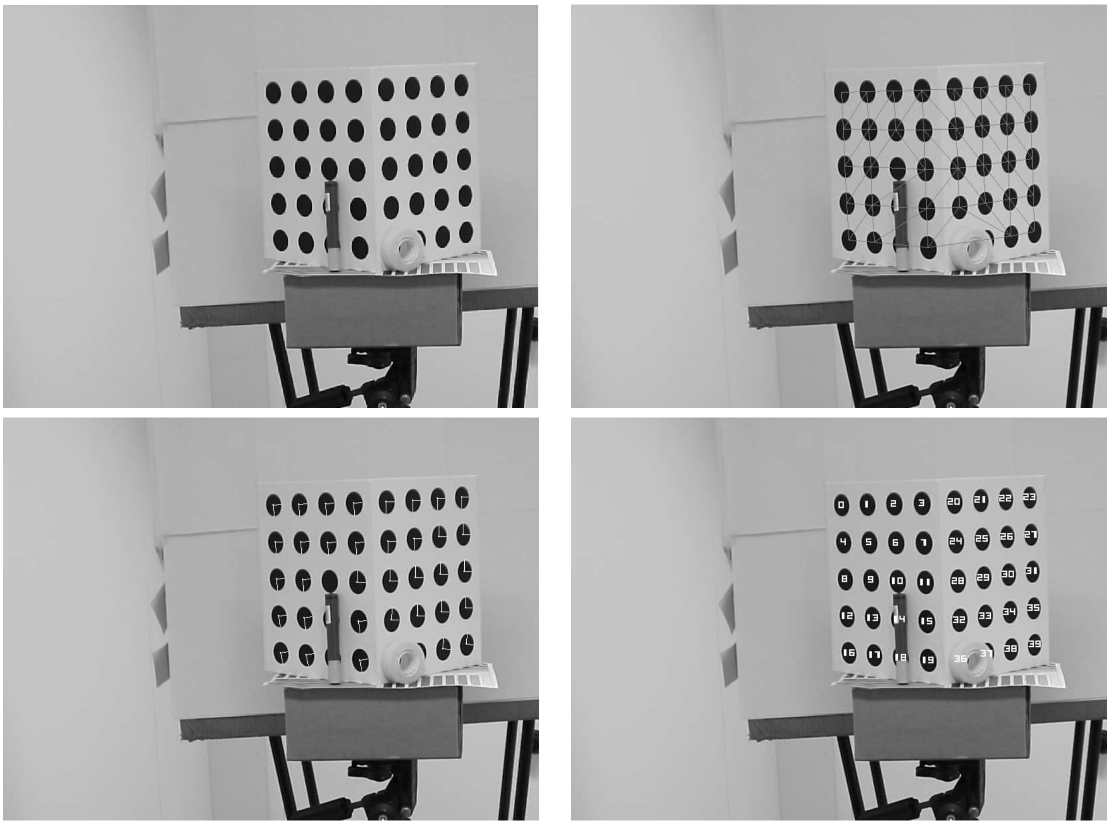

Figure 22:

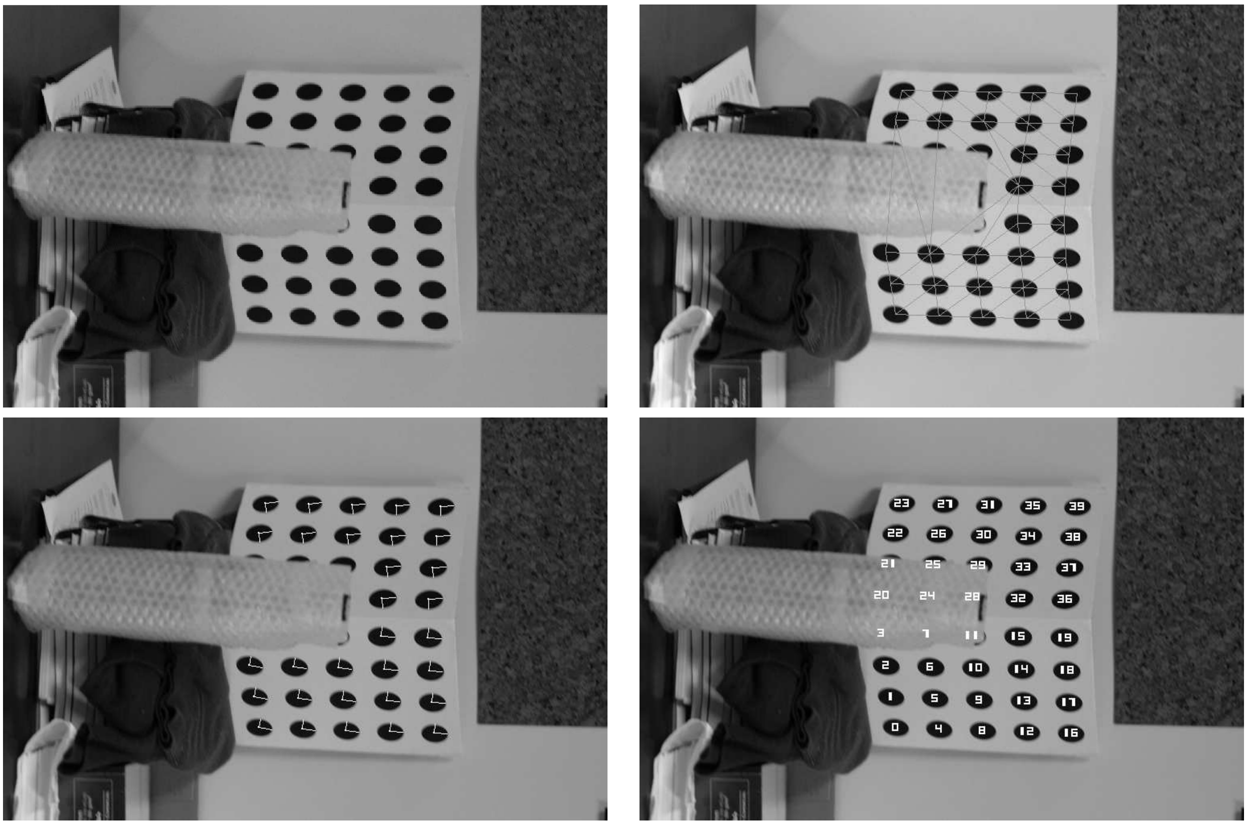

Figure 23:

In the second example (Fig. 23) there is a bottle in front of the calibrator. There are 9 occluded ellipses that are correctly recovered.

Finally in Fig. 25 there is a chair that is occluding two corners of the calibrator. Regard that the ellipses are also well ordered.

In this case our method has ordered the pattern correctly because it has been able to detect two corners, one on each plane.

4 Conclusions

In this work we have presented a very robust technique for automatically recovering the ellipse centers and ordering them according to their 3D positions. The method is fully automatic with no need for human interaction and it is very robust against a great variety of positions and orientations of the calibrator. It is also very robust against occlusions on the calibrator planes and, furthermore, we can easily recover the 2D position of the occluded ellipses with a good accuracy.

We have separated the process of detecting the ellipses and the process of putting in correspondence the 2D and 3D points. This is very useful since we can easily use the second process for other kinds of calibrators, for example calibrators with squares instead of circles which are also very common for camera calibration. In this case the corners of the squares are detected by means of a technique like Harris corner detection and the ordering process is exactly the same but instead of using the center of ellipses we would use the corner of the projected squares.

In the ellipse detection process we have used the Houg transform with parameter reduction and we have improved the computation speed by reducing the radius of the ellipses and the size of the neighbouring window. This method is very robust against noise and occlusions. A good accuracy for the ellipse parameters

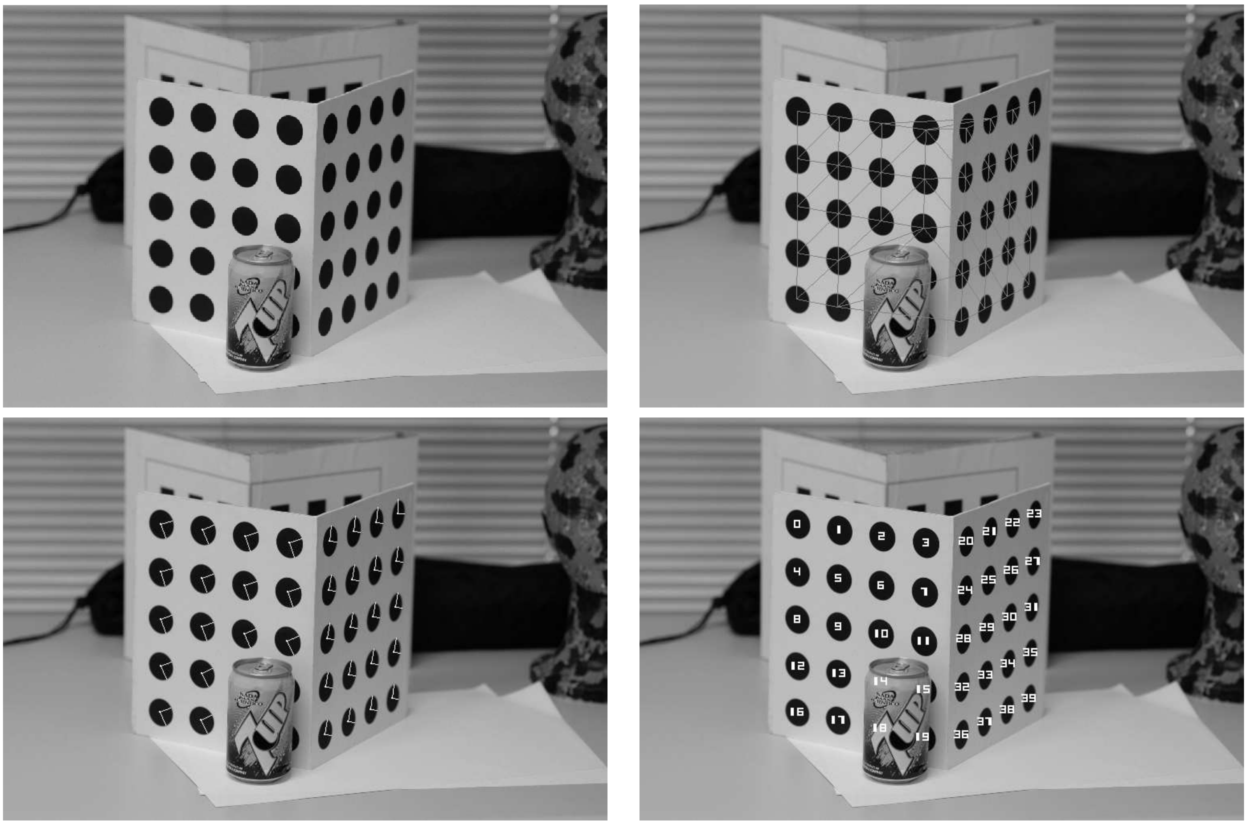

Figure 24:

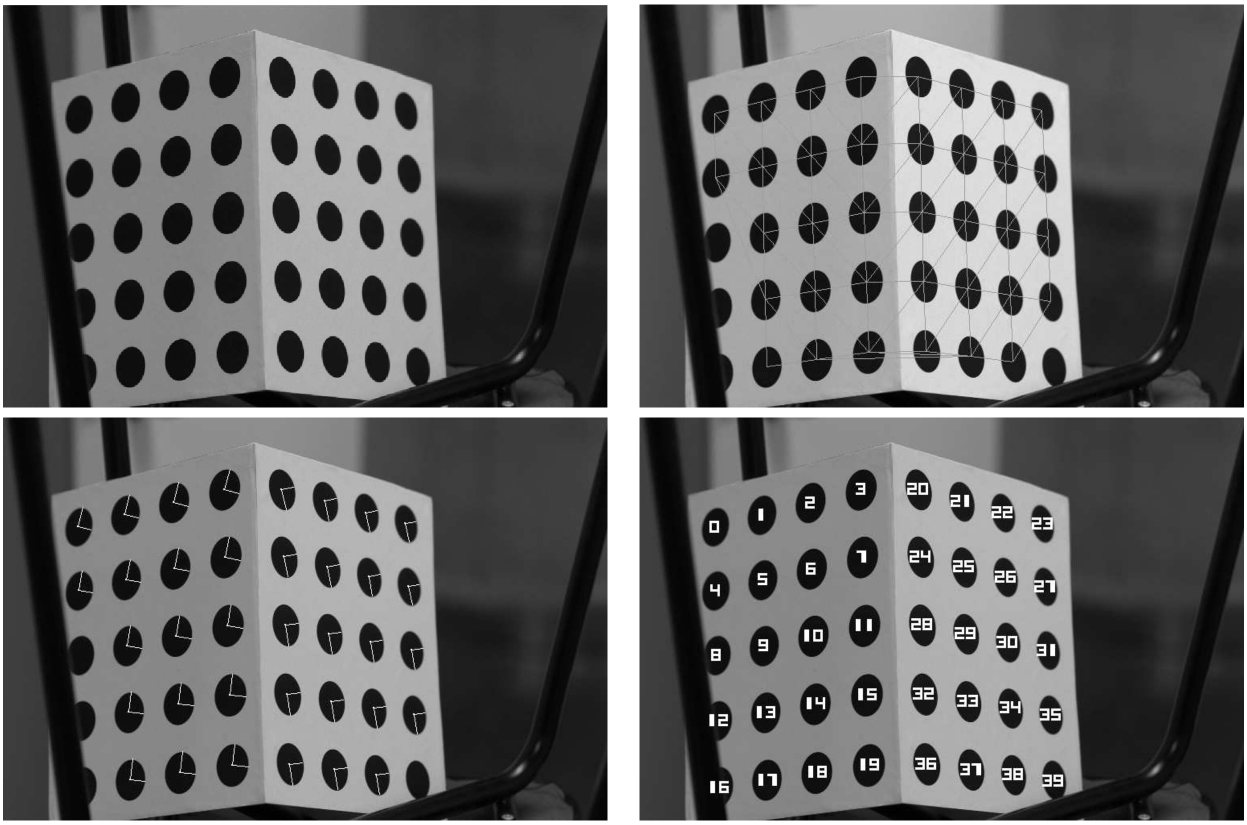
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Figure 25:

is obtained by means of a least square method. This accuracy is not necessary for the ordering of ellipses but it is normally needed for other purposes like camera calibration.

In the process of ordering the ellipses we only need the information of two ellipse corners, one in every plane. For this, we only need to find eight ellipses (the four ellipses of every corner). In the process we find the four corners and select the best one for each plane. The use of homografies to find the transformation between the original and the projected planes is a powerful tool that makes the method very robust against occluding ellipses or bad detected centers of ellipses. Thanks to the estimation of the homografies we may estimate the positions of the rest of the ellipses only knowing a few of them. This allows us to recover the position of occluded ellipses.

The kind of pattern we have considered is a symmetrical calibrator pattern which is in some sense more complicated than a non symmetrical one. When the calibrator is so symmetrical we need to estimate the position and orientation of the planes once we have computed the homografies. If the calibrator would not be so symmetrical then the homografies would determine directly the position and orientation of the planes, without the need for a posteriori step.

In the experimental results we have shown through a large number of tests that the method is quite robust and it normally works for a great variety of different position and orientations of the calibrator without changing the initial parameters. Then we have shown more complex experiments that have been succeesfully solved by changing the parameters. Finally we showed several experiments with occluded ellipses.

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