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Editorial

Symmetric and Asymmetric Distributions: Theoretical Developments and Applications III

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Abstract: A summary of the eleven papers published in this special issue is presented here. This volume was the last in a series of special issues dealing with symmetric and non-symmetric continuous probability distributions. The works presented in this issue propose new probabilistic models and extend the properties of other existing models in the statistical literature.

Keywords: censored data; continuous distribution; EM-algorithm; Lambert Transformation; logit; regression; simulation

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1. Introduction

This Special Issue is the last of three volumes published by *Symmetry* devoted to symmetric and asymmetric probability distributions, involving the proposal of new models and studying other properties of existing models. Based on the seminal work of [1], who introduced the skew-normal distribution, this aspect of distribution theory has been an inexhaustible source of ideas and information for the statistical community working in applied and theoretical fields. Other contributions in this area include [2–4]. Although this Special Issue was open to submissions relating to any type of asymmetric model—i.e., discrete or continuous and univariate or multivariate—only continuous models were to this issue.

The importance of the property of symmetry (and asymmetry) when modelling empirical data is widely known. For example, incorporating an asymmetry parameter into binary regression models with a logistic link can better explain the probability of the event being examined and improve the capacity for prediction. Something similar occurs if the link function is based on a normal distribution (probit model). Additionally, in the classic linear regression model, the error term assumes a normal distribution with zero mean and standard deviation $\sigma > 0$. That is, the error is considered to be symmetric about the zero value. In practice, however, this assumption is too strict, and the errors may contain a certain degree of asymmetry. Thus, the proposal of a normal distribution seems inappropriate in this situation. In this Special Issue, this is discussed in [5].

In this Special Issue on symmetric and skew distributions, statistical researchers, both theoretical and applied, were invited to submit original contributions that could have immediate application in disciplines where the importance of symmetry and asymmetry in modelling is essential. Of course, with the advancement of computational tools, the applications of these models have been greatly simplified, allowing the management of databases to provide solutions to real problems. These problems usually appear in the fields of economics, environmental sciences, biometrics, engineering, medicine, etc.

2. Contributions

The Gompertz distribution, a continuous probability distribution with positive support, can be considered as an alternative to the half-normal distribution. This distribution is often applied to describe the distribution of adult lifespans by demographers and actuaries for survival analysis. The contribution of [6] presents the slashed version of this distribution, with the result being a more flexible distribution with tails heavier than those of the Gompertz distribution. Furthermore, Ref. [7] proposes, related to the Gompertz distribution, a new four-parameter lifetime model denoted the exponentiated generalized inverted Gompertz distribution. This model can represent the lifetimes with upside-down bathtub-shaped hazard rates and is suitable for describing cases of negative and positive skewness.

Ref. [8] provides the extensions and properties of the multivariate skew-normal distribution, giving us a total theoretical contribution to this particular issue. This work discusses a valuable model for dealing with data with a normal distribution. However, in practice, the skewness of data and truncation can usually be detected. Thus, the normal assumption is not allowed. These authors study several properties of the truncated multivariate skew-normal distribution by obtaining distributional results through affine transformations, marginalization, and conditioning.

As it is well known, many lifetime distributions have been used as population models for use in risk analysis and reliability mechanisms. Ref. [9] propose a novel procedure for the estimation of stress-strength reliability in the case of two independent unit-half-normal distributions, which can fit asymmetrical data with either a positive or negative skew and different shape parameters.

It is known that specific estimation problems exist in some members of the family of logistic-type distributions. Strategies for dealing with this problem are examined in [10]. In this work, the authors focus their attention on the three-parameter type I generalized logistic distribution, which presents the problem that the parameter space must be restricted for the existence of maximum likelihood estimators. The authors of this contribution propose the use of an interesting Bayesian approach to solve this problem.

Related to the widely examined skew-normal distribution, Ref. [11] introduces an asymmetric regression model designed for censored non-negative data based on a mixture of centered exponentiated log-skew-normal and Bernoulli distributions. The logit link is considered by connecting the discrete part with the continuous distribution, showing in detail the score function and the information matrix, among other properties. An application pertaining to the study of the measles vaccine was used to illustrate the applicability of the proposed model.

In [12], the skew-elliptical sinh-alpha-power distribution is introduced, and as a natural development, the skew-elliptical log-linear Birnbaum–Saunders alpha-power regression model is studied. This new distribution contains two additional parameters that control skewness and kurtosis. The main characteristics of this novel distribution are studied, as well as the process used for the estimation of its parameters.

Ref. [13] propose a new distribution to model bounded data that exhibit asymmetry. The distribution is obtained directly from the Lambert-F distribution generator under a uniform baseline distribution. In this way, the Lambert-F generator alters the symmetry of the uniform distribution, which allows asymmetric shapes to be created in the proposed density.

In [14], the inverse gamma and power series distributions are mixed to obtain an inverse gamma power series class of distributions.

Finally, the classical logit model, which can explain a dichotomous dependent variable as a function of factors or covariates that can influence the response variable, is contemplated in [15]. The authors introduce a new skew-logit link for item response theory by considering the arctan transformation, which was first introduced in other settings by [16]. Due to this, they are able to obtain a very flexible link function from a new class of generalized distributions. This approach assumes an asymmetric model, which can be reduced to the standard logit model for a particular case of parameters that control the distribution's symmetry.

3. Futher Elements

Most of the works presented in this issue are devoted to the maximum likelihood estimation of the parameters of the proposed models, their asymptotic distribution, and exact and asymptotic confidence intervals. Different methods of estimation are discussed in [14], including a novel EM algorithm for computing the maximum likelihood estimates of its parameters. Additionally, in most of the works, the performance of the estimators based on Monte Carlo simulations, mean squared error, average bias and length, and coverage probabilities, are also provided, and a quantile regression framework is considered in [13]. Furthermore, these works are illustrated with numerous examples, including some of the real data used in their investigations. In this regard, Ref. [10] illustrates the potential of Bayesian estimation in their work by applying the proposed method to real-world data related to the copper metallurgical engineering area. Refs. [6–8,10,14] apply their results using real data pertaining different engineering settings.

Ref. [5] applies their methodology to the mortality of adult beetles subjected to five hours of exposure to gaseous carbon disulphide. This data set has been used widely in the statistical literature since appearing in [17] and has been discussed in many papers, such as [18]. Finally, Ref. [11] carries out an interesting regression model for censored data in the field of medicine.

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