A GIS based model for solving the planar Huff problem considering different demand distributions and forbidden regions

Rafael Suárez-Vega[1,2], Dolores R. Santos-Peñate[1,2], Pablo Dorta-González[1,2]

[1]Departamento de Métodos Cuantitativos en Economía y Gestión
[2]Instituto Universitario de Turismo y Desarrollo Económico Sostenible (Tides)
Universidad de Las Palmas de Gran Canaria
rafael.suarez@ulpgc.es

ABSTRACT

In this paper, we have used Geographical Information Systems (GIS) to solve the planar Huff problem considering different demand distributions and forbidden regions. Most of the papers connected with the competitive location problems consider that the demand is aggregated in a finite set of points. In other few cases, the models suppose that the demand is distributed along the feasible region according to a functional form, mainly a uniform distribution. In this case, in addition to the discrete and uniform demand distributions we have considered that the demand is represented by a population surface model, that is, a raster map where each pixel has associated a value corresponding to the population living in the area that it covers. Taking into account the demand distribution and the location and size of the existing facilities, we have obtained a raster map where each pixel has associated the estimated capture for a new competing firm if it decides to locate on it. Finally, a real example is solved where the solution for the three scenarios is compared.

Key-words: Competitive location, planar Huff problem, demand distribution, surface population model, forbidden region, GIS

1 INTRODUCTION

In this paper we investigate the follower competitive location problem in the leader-follower model. A new firm wants to establish a facility in a market where other competing firms already exist. We suppose that each facility is characterized by its quality level that may combine aspects such as size, prices, parking areas, opening hours, accessibility, and so on. Customers make their choice according to the attraction or utility that they perceive from the facilities. The attraction perceived by the customers from the facilities has been represented mathematically by an attraction function which increases with respect to the quality level and decreases with respect to the distance. Attraction functions are used to define the customer choice rule, which represents the customer behaviour and the customer flow in the market.

Assuming certain customer preferences, the firms, whose natural objective is the maximization of the market share or profit, take their location decisions, influencing with their actions the results and strategies of their competitor. This movement of individuals defines the basis of the location-spatial interaction models.

The first spatial interaction models were gravity models, which assumed analogies between human behaviour and Newtonian gravity laws. The basic gravity formulation, in which the movement of individuals between two points is inversely proportional to the distance separating them, was applied by Reilly (1931) and Converse (1949) to analyze retail market areas. Later, Huff (1964) proposed an alternative model to overcome certain limitations of the Reilly-type approach. According to this new model, the probability that a customer at *i* buys at a facility *j* is given by

$$P_{ij} = \frac{\dfrac{a_j}{d_{ij}^{\lambda}}}{\displaystyle\sum_{k=1}^{n} \frac{a_k}{d_{ik}^{\lambda}}}$$

where a_j represents the size of a facility *j*, d_{ij} is the distance (or travel time) from demand point *i* to facility *j*, and λ is a parameter which reflects the effect of the distance on the consumer's behaviour and whose value is estimated empirically. The quotient $\frac{a_j}{d_{ij}^{\lambda}}$ can be interpreted as an attraction function (where the attraction felt by a customer at point *i* towards facility *j* is directly proportional to the size of the facility and inversely proportional to a power of the distance between them).

The planar Huff based competitive location model, that is, the case where each point in the plane is a potential location for the new facility, has previously been studied. Drezner (1994) solved the planar problem considering Euclidean distances. In this case, the author considered that the demand was aggregated in a finite set of points and used the Weiszfeld algorithm to obtain the best location. This algorithm is a gradient based method that yields a local optimum, so the procedure must be run several times and the best solution obtained is then selected. Later, Drezner and Drezner (1997) proposed a problem resolution for the continuous demand case. This resolution is based on the assumption that the demand density function *f*(*x*,*y*) is known, that is, that there exists a known functional relationship between the coordinates (*x*,*y*) of a point and its population density. Due to the complexity of the procedure for the resolution of this problem, the authors proposed an approximation approach based on the aggregation of the demand.

In this paper, the planar Huff problem has been solved using GIS based tools. GIS makes the study of spatial interrelations easier, and the combination of GIS and location theory will help the development and application of location models. A review of the connections between GIS and Location Science can be found in Church (2002), and more recently, in Murray (2010), where an overview and discussion about how GIS has contributed to location science is provided. Examples of location model integration into a GIS framework can be found in Spaulding and Cromley (2007), and Suárez et al. (2011), as well as references cited in Church (2002) and Murray (2010).

Most of the papers connected with the competitive location problems consider that the demand is aggregated in a finite set of points. In other few cases, the models suppose that the demand is distributed along the feasible region according to a functional form, mainly a uniform distribution. In this paper, we have considered three scenarios for the demand distribution. In the first scenario we have considered that the demand of each administrative unit is aggregated at a point; in the second the demand of each administrative unit is uniformly distributed along its surface; and finally, a surface population map is considered in the third. A surface population map is a raster map where each pixel has associated a value corresponding to the population living in the area that it covers and that is calculated based on the population of the administrative unit and the land use associated to this point. This model has the advantage of being able to discriminate between populated and non-populated areas. The surface population models have been developed, among other considerations, to reduce the effect that the aggregation level of statistical information has over demographic studies.

In the location literature some works focused on analysing the error due to the demand aggregation which can be found. See, for example, Francis et al. (2008) for an interesting review about the aggregation measures for a large class of location models. For the particular case of competitive location models, the problem has been analyzed by Drezner and Drezner (1997). Of course, when an aggregation process is done, a trade off between computational time and accuracy occurs. For certain problems, time factor is vital, for example, for the choice of the shortest path for an ambulance to an emergency, but in competitive location, accuracy is more important than computational time. The location of a new facility is a decision for a long range temporary horizon and the firms need an efficient solution rather than an instant one.

The inclusion of forbidden regions, i.e. areas where the location of new facilities are not allowed, means an important increase in the complexity of the resolution for the traditional Operation Research methods (see for example McGarvey and Cavalier 2005). The use of the GIS procedure we propose in this paper allows us to exclude these regions easily without considering any analytical assumption.

The result of the proposed method is a raster map where each pixel of the feasible region has associated the estimated Huff capture for a new facility located on it. This solution format permits its use within a multi-criteria decision environment in combination with other criteria (minimization of the distance to the distribution centres or to the main roads, minimization of the cannibalization, etc.) as Suárez-Vega et al. (2011, 2012, 2014) have proposed.

The rest of the paper is organized as follows. The model and the resolution method are explained in Section 2. In Section 3 the problem of finding the best location for a new hypermarket on the island of Gran Canaria (Spain) is solved and the solution obtained using the different demand distributions are compared. Finally, we present some conclusions in Section 4.

2. SOLVING THE HUFF-BASED COMPETITIVE LOCATION PROBLEM ON THE PLANE

In this paper, we are going to solve a competitive location problem using GIS based tools. The geographical information can be presented in two formats: vector and raster. In a vector layer the objects are represented by means of points, lines or polygons. Each layer has associated a table where, for each element, the information for different attributes is stored. A raster layer is a matrix of cells (named pixels) which contain certain value and can be represented by giving each pixel a colour with respect to its value. Each raster map is determined by five parameters: number of columns (*ncol*) and rows (*nrow*), coordinates x and y of the lower left corner (*xllcorner*, *yllcorner*) and the size of the pixel (*cellsize*). Knowing these values, it is easy to associate a pixel and its coordinates in the map. If a pixel P is located at row r and column c, then $(x, y) = (xllcorner + (c-1) * cellsize, yllcorner + (r-1) * cellsize)$ are the coordinates of the lower left corner of P.

In this paper, the planar Huff problem has been solved within a raster environment. A new firm A wants to determine the best location for a new facility that must compete with the facilities that already exist in a market. Let x_0 and a_0 be the location and the quality level for the new facility and $X_p = \{x_1, x_2, ..., x_p\}$ and $A_p = \{a_1, a_2, ..., a_p\}$ the locations and the quality levels of the existing facilities. The attraction felt by customers at pixel P towards a facility j at x_j with quality level a_j is given by

$$a_{Pj} = \frac{a_j}{f\left(d\left(x_j, P\right)\right)},$$

where $f : \Re_0^+ \to \Re^+$ is a positive increasing function and $d\left(x_j, P\right)$ is the Euclidean distance between pixel P and x_j.

Let PM be the population map, that is, the raster map containing the demand distribution and let $w(P) \geq 0$ be the population at pixel $P \in PM$. Then, following the Huff model, the market share captured by a new facility with quality level a_0 located at point x_0 is given by

$$M(x_0) = \sum_{P \in PM} w(P) \frac{\dfrac{a_0}{f\left(d\left(x_0, P\right)\right)}}{\dfrac{a_0}{f\left(d\left(x_0, P\right)\right)} + \sum_{j=1}^{p} \dfrac{a_j}{f\left(d\left(x_j, P\right)\right)}}. \tag{1}$$

Therefore, the following problem must be solved to obtain the solution for our competitive location problem:

$$\underset{x_0 \in F}{Max}\, M(x_0), \tag{2}$$

where F is a raster map representing the feasible region containing all the potential locations for establishing the new facility. Note that under the assumption of a constant marginal cost, maximizing profit is equivalent to maximizing captured demand.

To solve the problem we have combined a C coded program with some ArcGis 9.3® tools. First, the feasible region and the population maps are exported to an ASCII file. Then the C program calculates $M(x_0)$ for each $x_0 \in F$ taking into account the coordinates of the existing facilities and their quality levels. The program stores this information in an ASCII file that can be imported to ArcGis in raster format. This final map reflects the estimated Huff capture for each pixel in the feasible region. Additionally, not only the best locations can be pinpointed but we can also compare the suitability of the different zones.

The C program solves problem (1) for each pixel in F. The complexity of solving problem (1) is o(*nrowPM***ncolPM*) where *nrowPM* and *ncolPM* are the number of rows and columns of the population map. To solve (2) it is necessary to solve problem (1) *nrowF***ncolF* times (*nrowF, ncolF* are number of rows and columns of the feasible region). The total complexity is o($nrowPM*ncolPM*nrowF*ncolF$). That means that the computational effort to solve our planar Huff model depends on the extent of the region analyzed and on the size of the pixels of both F and PM. Note that for the aggregate demand case, the population map PM can be substituted by a finite set of points resulting in a significant decrease in the complexity of problem (1).

The pixel size for the feasible region may depend on the surface needed for locating the new service. A traditional assumption is to suppose a pixel size such that at least four pixels are needed to cover the area required for the new service. For example, for locating a hypermarket with 5000 m^2 of sales surface with another 5000 m^2 of parking area usually it is not necessary to use a pixel size smaller than 50 m.

In order to reduce the computational effort it would be very interesting to consider the existence of forbidden regions. For example, water bodies, natural protected areas or zones with high slope, can be discarded from the feasible region. The incorporation of forbidden regions using traditional OR methods introduces an important complexity in the resolution methods whereas by employing GIS tools this process may be quite easy if the information is available in digital maps.

3. AN APPLICATION

The previous tools have been used to find the more promising locations for a new hypermarket on the island of Gran Canaria (Spain). According to the Law of Commercial Activity Regulation in the Canaries (B.O.C., 1994) a hypermarket is a large store which is a combination of a food store and a department store with a minimum sales area of 2500 m^2. In this application, the quality level is measured by the hypermarkets sales surface (m^2).

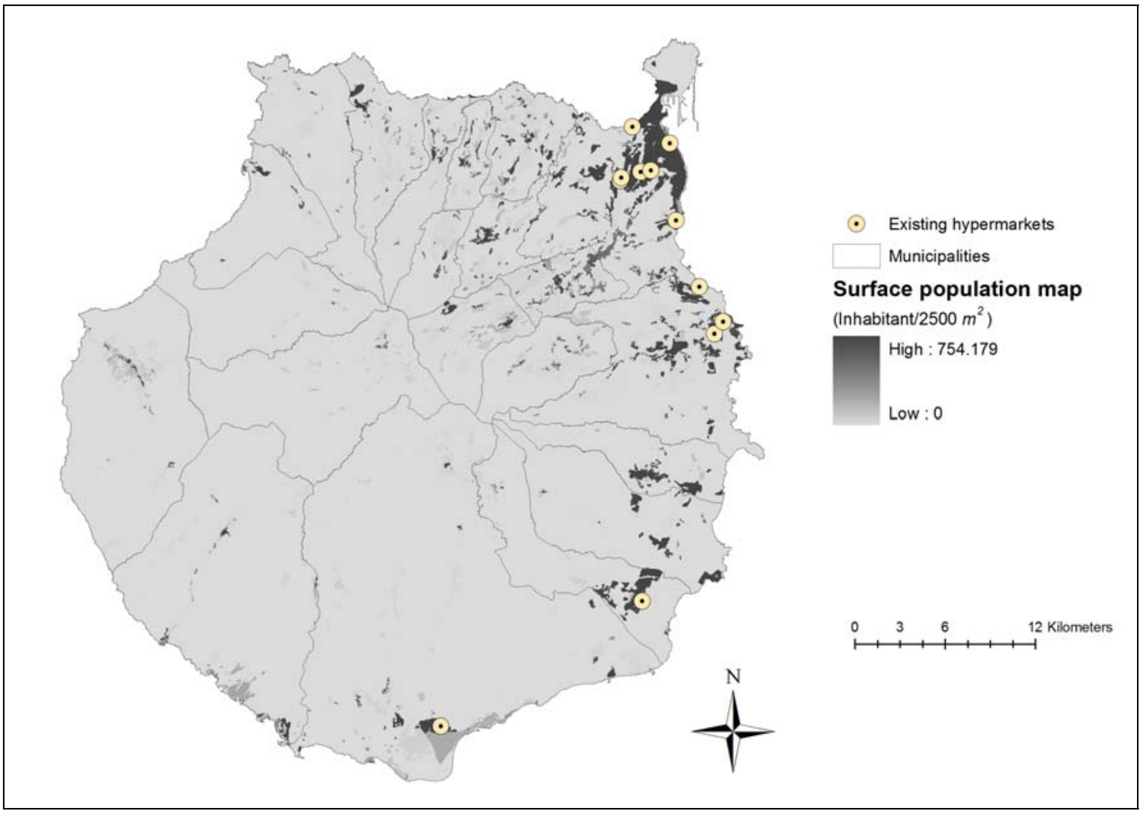

Figure 1. Population surface model and existing hypermarkets

Figure 1 shows the scenario where our problem has arisen. Gran Canaria has an almost circular shape with a surface of 155824.43 Ha and a population of 757821 inhabitants (2006 Census Data). As the Figure shows, in 2007 there already existed in Gran Canaria twelve hypermarkets. Note that, 51.7 % of the population and 67.9 % of the sales surface of the island is concentrated in the capital (the north-eastern part of the island). The rest of the hypermarkets, and an important part of the demand, are distributed on the eastern and southern sides of the island. The centre and western parts of the island are mountainous areas with a low density of population.

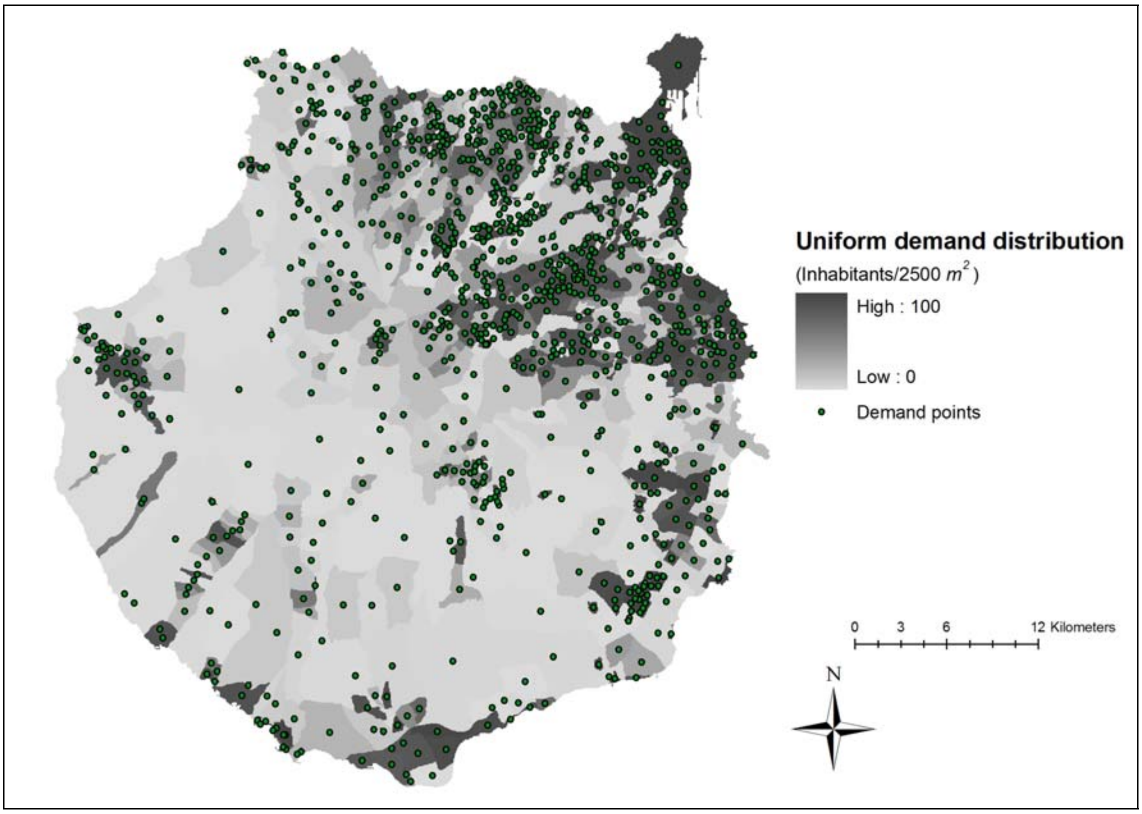

Figure 2. Uniform population distribution and aggregated demand points

The population map used was elaborated by Suárez-Vega et al. (2008). In short, a surface model of population is a raster map that stores the population of an area. Each pixel on the map has associated the population of the area that it covers. A Land Uses map in combination with a Constructions layer was used in order to generate the ancillary map for the dasymetric method. The population densities of the polygons in the ancillary layer have been calculated taking the aggregated density of an influence area (independently of the administrative limits). In this case, two values for the pixel size of the population maps have been considered in order to analyse the effect on the results of this parameter. In particular, we have considered population maps with 50 and 100 meters of pixel size, and therefore, a pixel represents a surface of 2500 and 10000 m^2, respectively.

The island of Gran Canaria is distributed into 1029 population entities (the less aggregated administrative unit considered by the Spanish National Institute of Statistics (INE)). Considering these administrative units, we have considered two demand distributions. First we have aggregate the demand of each population entity to its centroid to obtain a discrete demand distribution. Second, we have uniformly distributed the demand of each entity along its surface. Both demand distributions are shown in Figure 2. To analyse the effect of the pixel size of the population map on the problem results, sizes of 50 and 100 *m*. have been used for both the surface population and the uniform demand distribution maps.

To refine the feasible region Land Uses and Elevation maps were used. The Land Uses map (1/25000) from year 2002 provided by GRAFCAN (the official supplier of geographical information in the Canaries) was employed to discard some areas such as water bodies, beaches or protected areas. The elevation data were obtained from the

Shuttle Radar Topography Mission (SRTM) and areas with slopes higher than 30 % were discarded from the analysis for reducing the building problems.

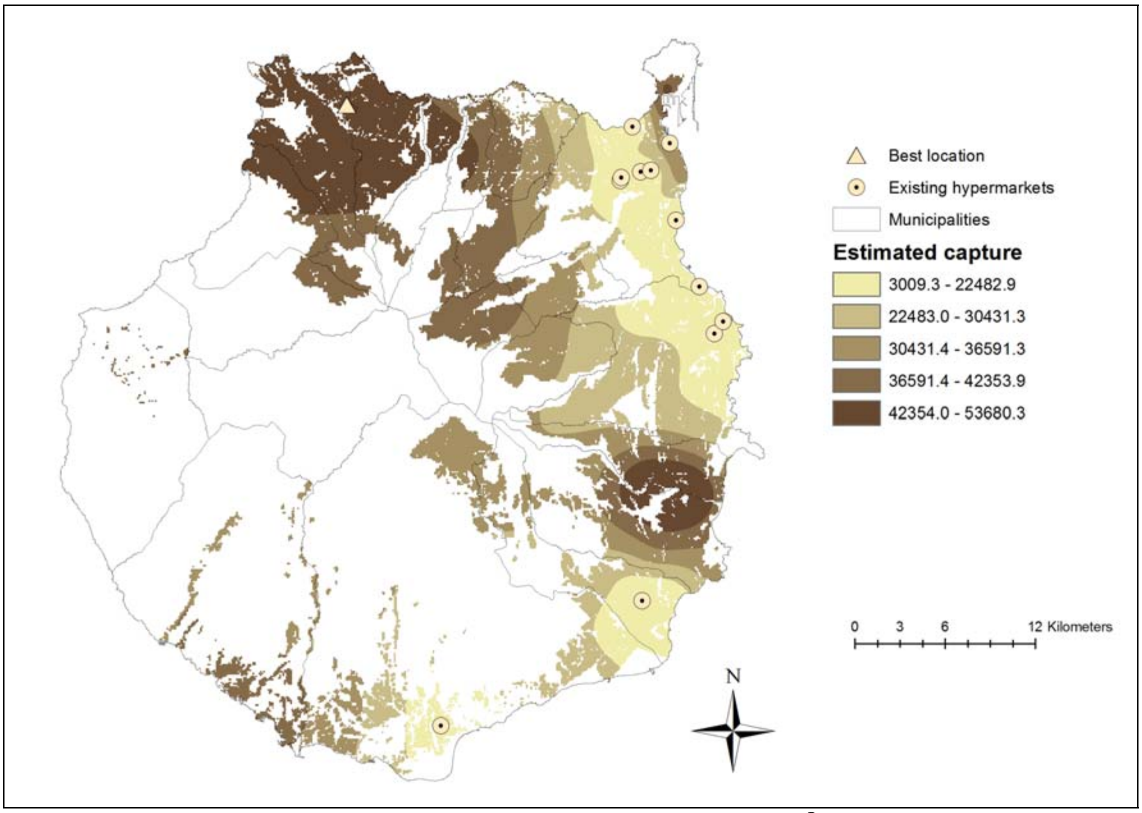

Figure 3. Capture maps for a new hypermarket of 2500 m^2 and λ = 2 using the population surface model.

To calculate the customers' attraction, function $f(d) = 1 + d^{\lambda}$ is considered. To reflect different customers' distance perceptions, three values of λ (0.5, 1, 2) have been used. Note that, an increase of λ means that customers present less disposition to travel. The problem has been solved considering three possible sizes for the new hypermarket, 2500, 5000, and 7500 m^2, taking a pixel size for the feasible region of 30, 50, and 60 meters, respectively.

The results obtained in each case can be plotted into a raster map where each pixel has associated the estimated capture for the new hypermarket if it were to be located on it. Note that the use of this map provides the decision maker with a broader vision of the problem and allows for comparisons between different alternatives in contrast to the traditional OR methods which only give a global optimum or, when it is not possible, a set of local optima.

For instance, Figure 3 shows the capture map when the population surface model is used to locate a 2500 m^2 hypermarket and the distance decay parameter is λ = 2. The map is symbolized using the quantile method, i.e., ordering the pixels in increasing order with respect to the capture, and classifying them into five classes with the same number of elements within each one. The point with the highest capture for the new store is marked with a triangle. The white zones are the areas that do not belong to the feasible region. Note that the most promising areas for locating the new hypermarket are mainly in the north-west of the island, a zone where no hypermarkets exist and the population is dispersed.

Table 1 shows the optimum values obtained by the new facility in the different scenarios analysed in this paper (columns four to six). Note that the optimal value is higher the greater the customers' resistance to travel for shopping, and of course, the greater the size of the new store. We assume that the best results are the obtained by the surface model because it is the most realistic population distribution. So, we will consider the solution to this model as the best solution among the three solutions obtained. From this point of view, columns seven and eight show the percentage error achieved for the optimal values obtained by the uniform and discrete models, respectively, with respect to the best solution. Note that when the pixel size is 50 *m*, the error is around double that obtained for size 100 *m*. This means that an increase of the precision in the population map implies an important improvement in the solution for the surface population model, but not in the uniform distribution model. There is not a significant difference in the error obtained by the uniform model (an average error of 2.47 %) and by the discrete model (an average error of 2.40 %).

Size (m^2)	λ	Pixel size (m^2)	Population surface	Uniform distribution	Discrete demand	Uniform error (%)	Discrete error (%)
2500	0.5	100	26513.8	26264.3	26431.6	1.13	1.17
		50	27406.9	26217.6	26431.6	2.44	2.30
	1	100	30998.3	30769.1	31035.6	1.14	1.25
		50	33869.8	30714.2	31035.6	2.34	2.27
	2	100	50682.7	45452.1	46300.5	1.15	1.34
		50	53680.4	45385.2	46300.5	2.80	2.82
5000	0.5	100	51148.2	50664.6	50849.3	1.12	1.16
		50	52785.7	50563.7	50849.3	2.43	2.29
	1	100	58036.1	57597.9	57845.3	1.11	1.24
		50	62413.4	57491.1	57845.3	2.35	2.27
	2	100	76817.8	75117.1	75002.3	1.18	1.39
		50	83800.5	74002.0	75002.3	2.64	2.65
7500	0.5	100	74111.6	73385.7	73714.6	1.12	1.16
		50	76389.5	73311.7	73714.6	2.41	2.29
	1	100	82462.8	81789.2	82058.0	1.14	1.23
		50	87787.4	81585.8	82058.0	2.34	2.26
	2	100	101550.1	101047.7	100847.2	1.14	1.34
		50	109726.1	100795.7	100847.23	2.47	2.48

Table 1. Optimum values and errors for the different scenarios solved

As we have previously commented, the result of this GIS tools is a map that reflects the estimated Huff capture for each pixel in the feasible region. These maps can be compared in order to obtain error maps. Figure 4 shows a map representing the error resulting from the use of the discrete demand model instead of the surface population model (surface population capture minus discrete capture). Using this map, the high underestimated zones (in dark tones) and overestimated areas (in clear tones) of the discrete model can be clearly detected. Note that, in this case, the zones with the highest captures in the population surface model are underestimated by the discrete model and usually, the areas close to the existing facilities are overestimated.

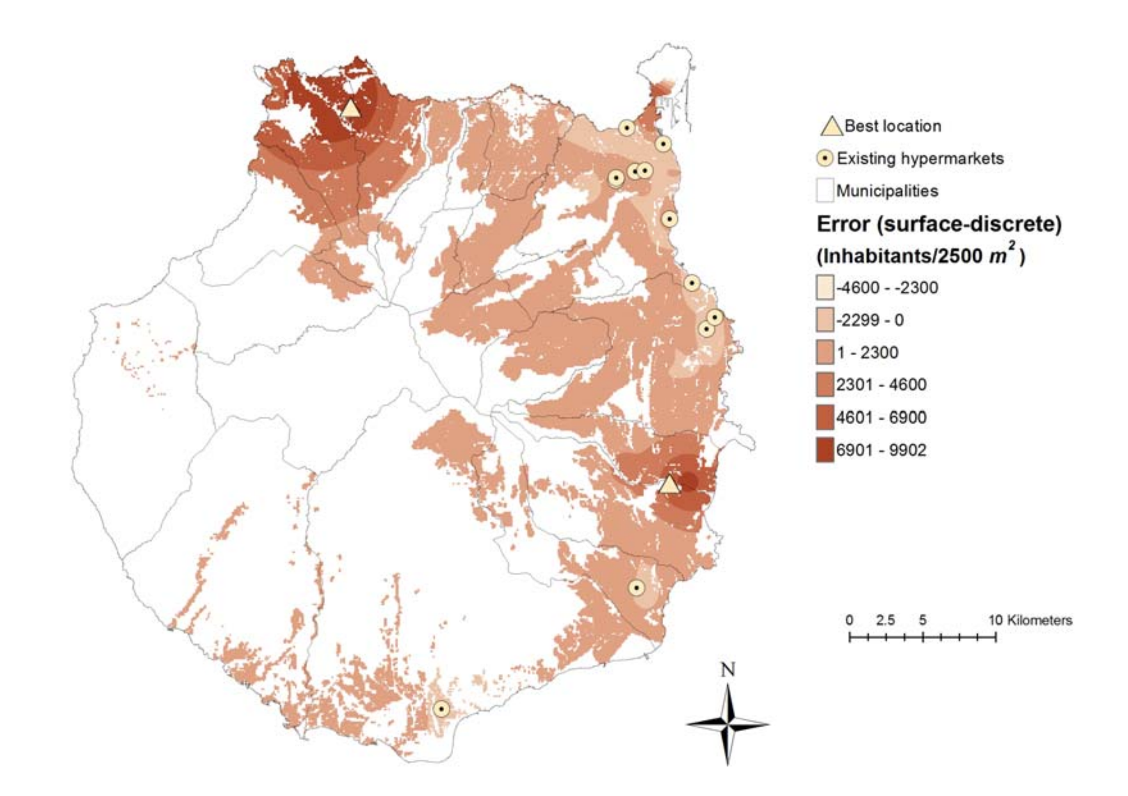

Figure 4. Error map for a new 2500 m^2 hypermarket and $\lambda = 2$ for the discrete model

Figure 5 shows the best locations for the new store for the different scenarios. In fact the number of location is greater but for clarity these have been aggregated. The maximum distance between a best location and a point marked on the map is 180 meters. Table 2 presents the correspondence between the location for the scenarios and the points in the map (columns three, four and six). The new store is always located in a zone where no hypermarkets already exist, 25 out of 27 are in the north-western part of the island around an imaginary line of about 4800 *m*., and two are located in the southeast. The distance (in meters) between the solution using the population surface model and the uniform and discrete population distributions are presented in columns five and seven, respectively. Note that in the surface model, the best locations for $\lambda = 0.5$ and $\lambda = 1$ always coincide and only the location for $\lambda = 2$ varies. When $\lambda = 0.5$ the solutions for the uniform and discrete model coincide, but for the other values of λ the locations appear interchanged (except for 2500 m^2 that always coincide). The distance between the uniform and discrete best locations to the best location for the surface model when $\lambda = 0.5$ and $\lambda = 1$ varies between 1367 and 1680 meters. The most significant differences occur when $\lambda = 2$, especially, when the size of the new hypermarket is 2500 m^2. In this case, the difference between the best locations is around 33 *kms*.

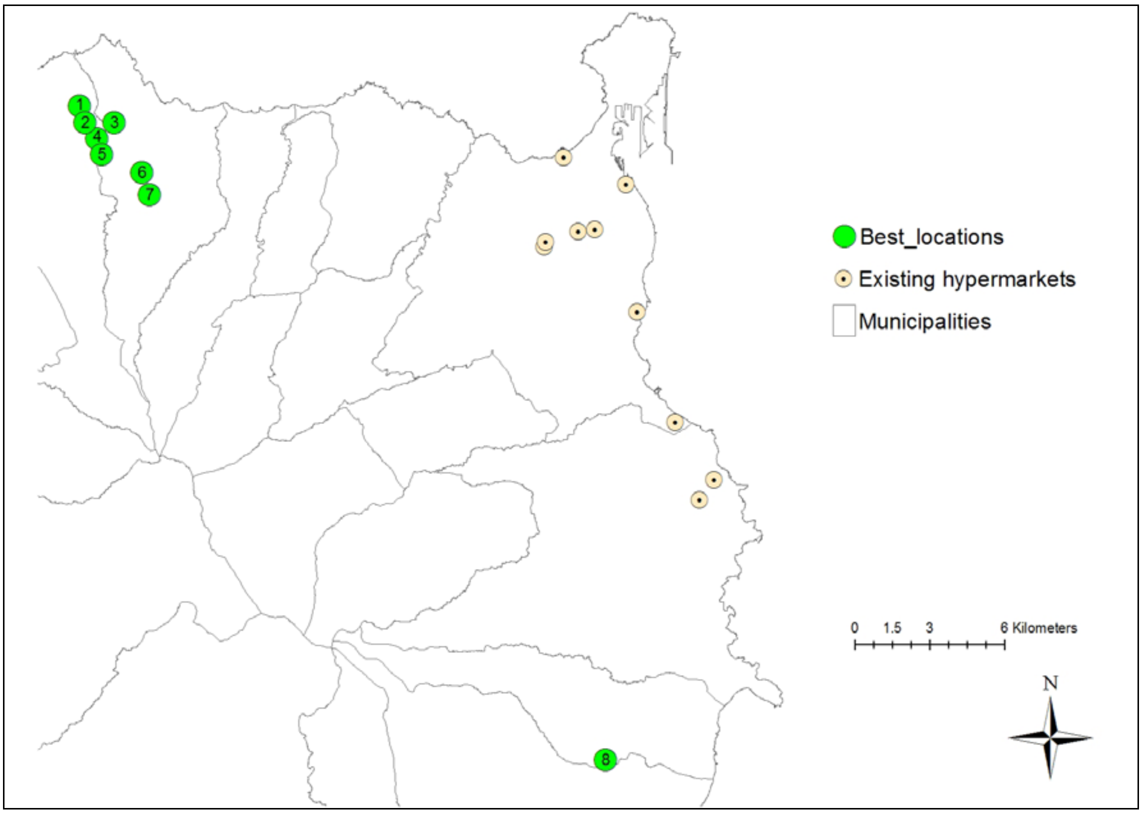

Figure 5. Best locations for the new hypermarket for the different scenarios

Size (m^2)	λ	Population surface	Uniform distribution	Distance PS-U (m)	Discrete distribution	Distance PS-D (m)
2500	0.5	1	3	1652	3	1672
	1	1	3	1493	3	1557
	2	2	8	32984	8	32930
5000	0.5	1	3	1655	3	1680
	1	1	6	1367	3	1434
	2	4	3	2367	6	2151
7500	0.5	1	3	1588	3	1641
	1	1	7	1475	3	1445
	2	5	3	2560	7	2440

Table 2. Best locations for the different scenarios

Table 1 shows the difference among the different models in relation with the optimal values obtained. But, what happen when the firm locates the new store following a model that is not the best? If the new hypermarket is located using the discrete demand model, the firm has an expected capture of C_d, but in fact the best approximation to the capture at this point is that obtained using the surface population model, C_{sd}. The percentage error experimented by the new firm with respect its expected capture is given by $e_d = 100\frac{|C_{sd} - C_d|}{C_d}$. Of course, if the demand distribution chosen is the uniform model, the corresponding error is $e_u = 100\frac{|C_{su} - C_u|}{C_u}$. Table 3 shows the percentage error that occurs when the new firm chooses either the uniform or the discrete demand model instead the surface population model.

Note that, the average errors for the problems solved in this paper are 4.33 % and 3.59 % for the uniform and discrete demand models, respectively. The maximum errors obtained are 10.67 % (for the uniform case) and 9.5 % (for the discrete case). Using both, the uniform and the discrete demand models, the capture is always underestimated with respect to the capture obtained with the surface model.

Size (m^2)	λ	Pixel size	C_u	C_{su}	e_u	C_d	C_{sd}	e_d
2500	0.5	100	26264.3	26512.2	0.944	26431.6	26509.4	0.294
		50	26217.6	27298.2	4.122	26431.6	27295.0	3.266
	1	100	30769.1	30996.3	0.738	31035.6	30992.0	0.141
		50	30714.2	33325.4	8.502	31035.6	33275.7	7.218
	2	100	45452.1	49720.8	9.392	46300.5	49575.8	7.074
		50	45385.2	49780.1	9.683	46300.5	49587.0	7.098
5000	0.5	100	50664.6	51148.2	0.955	50849.3	51134.6	0.561
		50	50563.7	52605.8	4.039	50849.3	52596.6	3.436
	1	100	57597.9	58036.1	0.761	57845.3	58020.7	0.303
		50	57491.1	61825.0	7.538	57845.3	61735.6	6.725
	2	100	75117.1	75473.1	0.474	75002.3	75467.7	0.621
		50	74002.0	81894.4	10.665	75002.3	82122.2	9.493
7500	0.5	100	73385.7	74111.6	0.989	73714.6	74102.1	0.526
		50	73311.7	76173.7	3.904	73714.6	76154.9	3.310
	1	100	81789.2	82461.7	0.822	82058.0	82445.5	0.472
		50	81585.8	87140.2	6.808	82058.0	87157.9	6.215
	2	100	101047.7	101550.0	0.497	100847.2	101544.0	0.691
		50	100795.7	107970.8	7.118	100847.2	108112.1	7.204

Table 3. Percentage of errors with respect to the population surface models

In this case, the precision of the population map (pixel size) is very important, the average errors when the pixel size is 50 are 6.93 % and 5.00 % (for uniform and discrete models, respectively) while when the pixel size is 100 these errors are 1.73 % and 1.19 %, respectively. Finally, the table shows that the error increases when the value of parameter λ increases, that is, when customers increase their resistance to travel for shopping.

4. CONCLUSIONS

In this paper, we have used a GIS based model to solve the planar Huff problem. The use of these tools allows us to improve the traditional model into two ways. First the inclusion of forbidden regions in the feasible region is quite easy and it does not imply an increase of the complexity of the problem. Second, different demand distributions are considered in addition to the aggregated model: the population surface model and the uniform demand distribution model. In these two new models the population is represented as a raster map where each pixel has associated a value corresponding to the population living in the area that it covers. In the uniform model, the demand is distributed uniformly along the administrative units. In the population surface model, the population is distributed along the inhabited areas of each administrative unit taking into account the different land uses.

A real example is presented where a new hypermarket is located on the island of Gran Canaria (Spain). Different decay distance functions have been considered in order to take into account different customers perceptions of the distance. The study was also made considering three possible sizes for the new store and two precision levels for the population maps.

For the studied example the maximum capture is higher the greater customers' resistance to travel for shopping, and of course, the greater the size of the new store. The results obtained show that an increase of the precision in the population map implies an important improvement in the solution for the surface population model. The change of the pixel size from 100 *m.* to 50 *m.* implies that the error produced both in the discrete and the uniform models are doubled. There is not a significant difference in the error achieved by the uniform and discrete model (an average of 2.47 % and 2.40 %, respectively) when a pixel size of 50 *m.* is used.

We suppose that the best approximation for the capture of the new store is obtained using the population surface model and we analyze what happens when the firm locates the new store following a model that is not the best. Then, we show how both the uniform and the discrete demand models always underestimate the capture, achieving a maximum error at the best location for these models of 10.67 % and 9.49 %, respectively.

In most of the scenarios solved, the best location for the new store is sited around an imaginary line of about 4800 *m.* in the north-western area of the island, far from the existing facility. Only when a new store of 2500 m^2 is located and customers present a low predisposition to travel to shopping, the location for the discrete and uniform models varies around 33 *kms.* from the population surface solution.

Using these capture maps, not only the best locations can be pinpointed but we can also compare the suitability of the different zones and calculate error maps to compare the results obtained using different models. These maps can be used within a multi-criteria decision environment in combination with other criteria (minimization of the distance to the distribution centres or to the main roads, etc.) as Suárez-Vega et al. (2011) have proposed.

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