Matlab based application for GPS observations

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Abstract

This application is a collection of programs used for the analysis of GPS (Global Positioning System) data. It uses the GPS broadcast carrier phase and pseudo-range observable to estimated three-dimensional relative positions of ground stations and satellite orbits. The software is designed as a toolbox for MATLAB.

It consider the precise positioning using the not difference model and the simultaneous use of phase and pseudo-range data observable.

All GPS phase and pseudo-range data input is via the RINEX format, and the orbital data are in SP3 format.

The software is composed of distinct modules, which perform the functions of preparing the data for processing, generating reference orbits for the satellites, computing the matrix of distinct models and performing a least squares analysis.

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Introduction

This work is a software package for the estimation of relative position of ground stations and satellite orbits, using GPS observations. The software is designed to run under MATLAB [1].

At the start of processing has available the broadcast ephemeris for the satellites observed; phase and pseudo-range observations. This information should be organized, one o more stations observing simultaneously the phases and pseudo-ranges of two or more satellites. The data and other information are prepared in the proper formats [2],[3].

The analysis software is composed of distinct modules, which perform the functions of preparing the data for processing, generating reference orbits for the satellites; detecting graphicaly breacks in the data, and performing a least squares analysis.

The directory structure in the analysis data is summarized below.

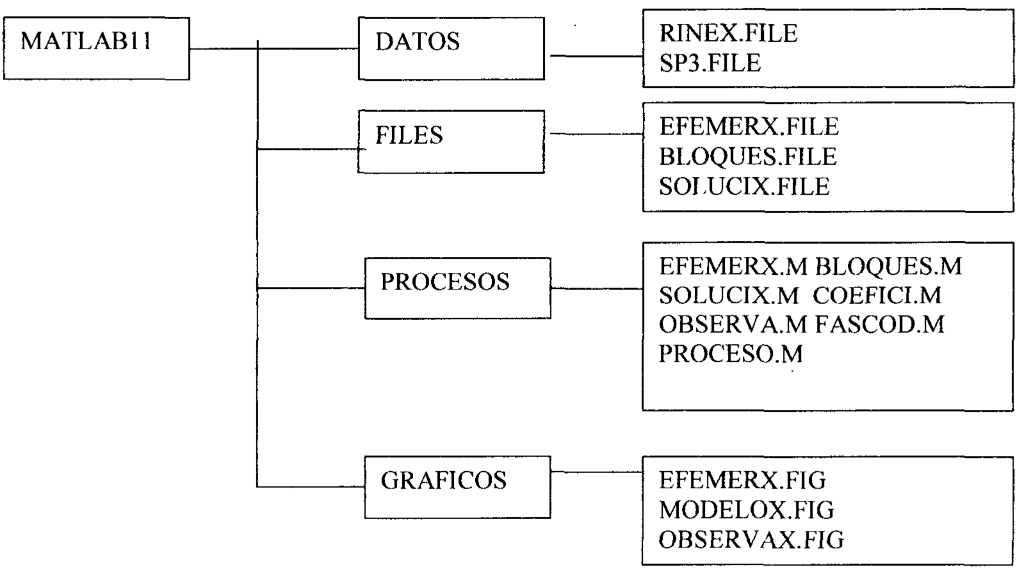

Fig. 1. Directory structure

DATOS: contains input data files.
FILES: contains output data files.
PROCESOS: contains source code.
GRAFICOS: contains output graphic

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Introduction to GPS measurements

High precision geodetic measurements with GPS are performed using that the carrier phase can be measured with sufficient precision that the instrumental resolution is a millimeter or less. For the highest relative positioning accuracies, carrier phase observation must be obtained simultaneously at each epoch from several stations, for several satellites and two frequencies L1(1575.42 Mhz), L2(1227.6 MHz).

A second type of GPS measurement is the pseudo-range using the code transmitted by the satellites.

For each station, an for each satellite we can obtain one equation of observation [4], this measurement is called NOT DIFFERENCE MODEL or Single Point Positioning.[5],[6], For a single satellite, differencing the signal received simultaneously at each two ground stations eliminates one unknown of the equation. This measurement is called SINGLE DIFFERENCE MODEL. Rather, we form a DOUBLE DIFFERENCE MODEL by differencing the between station differences also between satellites in this case we can to eliminate the receiver and satellite clock errors.

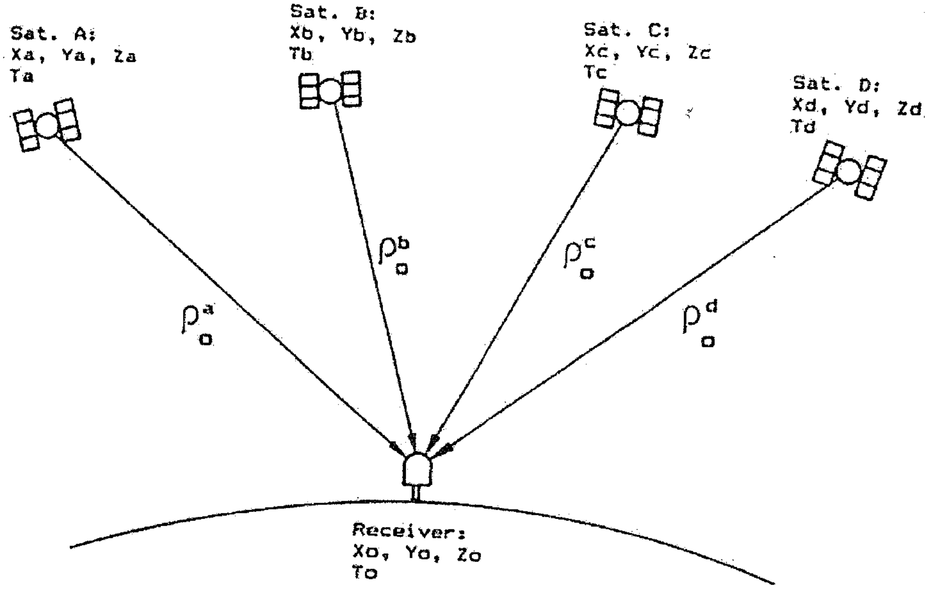

Fig. 2. Principle of single point positioning

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Modeling the satellite motion

A first requirement of any GPS geodetic experiment is an accurate model of the satellites motion. The tree dimensional accuracy of the estimated data, as a fraction of its length, is roughly equal to the fractional accuracy of the orbital ephemeris used in the analysis. The accuracy on the broadcast ephemeris computed regularly by the Department of Defense using pseudo-range measurements from 5 stations is typically 5-10 part in 10^7 (10-20 m.), by using phase measurements from a global network of over 50 stations, however, the International GPS Service For Geodynamics (IGS), is able to determine the satellites motion with an accuracy of 5-10 parts in 10^9 (10-20 cm.).

The motion of a satellite can be described, in general, by a set of six initial conditions; 3 of position and 3 of velocity or osculating Keplerian elements [7]; and a model for the forces acting on the satellite over the span of its trajectory. To model accurately the motion, we require knowledge of the acceleration induced by gravitational attraction of the sun, moon and higher order terms in the Earth's gravity field, and some means to account for the action of non-gravitational forces, due to solar radiation pressure and gas emission by the spacecraft batteries. For GPS satellites non-gravitational forces are the most difficult to model and have been the source of considerable research over the past 10 years.

In principle, a trajectory can be generated either by analytical expressions or by numerical integration of the equations of motion; numerical integration is almost always used, for both accuracy and convenience. The position of the satellite as a function of time is then read from a table of ephemeris generated by EFEMERX.M program.

Mathematical model of observations

The functional model relating the observations and parameter is non-linear. In the single point positioning we need at least four satellites simultaneously, the equations of observation for *r* receiver *s* satellites and *t* time for two type of GPS measurements are given by:

$$\phi_r^s(t) = \rho_r^s(t) + cdT_r(t) + \lambda N_r^s + cdt^s(t) + \delta_r^s(t) \qquad [1]$$

$$P_r^s(t) = \rho_r^s(t) + cdT_r(t) + cdt^s(t) + \delta_r^s(t) \qquad [2]$$

$\phi_r^s(t)$ phase observation

$P_r^s(t)$ *pseudo-range observation*

$dT_r(t)$ unknown, clock offset from GPS system time for receiver *r*

$\delta_r^s(t)$ unknown, propagation delay and other errors

$dt^s(t)$ unknown, clock offset from GPS system time for satellite *s*

N_r^s Ambiguity, there is an unknown integer number of cycles in the observed carrier phase at the first epoch

λ,c constants.

$\rho_r^s(t)$ distance is a function of $(x,y,z)_s$ coordinates of GPS satellite *s* and $(x,y,z)_r$ coordinates of receiver *r* is given by:

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$$\rho_r^s = \sqrt{(x_s - x_r) + (y_s - y_r) + (z_s - z_r)} \qquad [3]$$

$$\rho_r^s = \rho_0^s + \left(\frac{\partial \rho_r^s}{\partial x_r}\right)_0 dx_r + \left(\frac{\partial \rho_r^s}{\partial y_r}\right)_0 dy_r + \left(\frac{\partial \rho_r^s}{\partial z_r}\right)_0 dz_r \qquad [4]$$

this equation is linearized:

$$dx_r = x_r - x_0 \qquad \left(\frac{\partial \rho_r^s}{\partial x_r}\right)_0 = \frac{x_0 - x_s}{\rho_0^s}$$

$$dy_r = y_r - y_0 \qquad \left(\frac{\partial \rho_r^s}{\partial y_r}\right)_0 = \frac{y_0 - y_s}{\rho_0^s} \qquad [5]$$

$$dz_r = z_r - z_0 \qquad \left(\frac{\partial \rho_r^s}{\partial z_r}\right)_0 = \frac{z_0 - z_s}{\rho_0^s}$$

Where $(x,y,z)_0$ are the a priori coordinates of receiver.

Replacing in [1] or [2] if a user observes various satellites simultaneously, the equations are represented in matrix form by:

$$A\vec{x} = \vec{b} \qquad [6]$$

Generally, the number of observation equations is not equal to the number of unknowns, and we should make the matrix N of the normal equations, which has a certain structure which means that its elements in the matrix observe a certain mode of arrangement. This is specially true for the matrices where only a small number of elements and submatrices are non-zero. The structure of matrix N plays a decisive role in the storage and resolution of the normal equations.[8],[9],[10].

The methods for solving the normal equations fall into two basically different kinds, namely the direct method, and the iteration method.

The efficiency of a given matrix algorithm depends on many things. Rounding errors are in part what make the matrix computation area so nontrivial. When calculations are performed on a computer, each arithmetic operation is generally affected by roundoff error. This error arises because the machine hardware can only represent a subset of the real numbers. Another important aspect of finite precision arithmetic is the phenomenon of cancellation, this term refers to the extreme loss of significant digits when small numbers are additively computed from large numbers.

We conclude two important facts:

- Different methods for computing the same quantity can produce substantially different results.
- Whether or not an algorithm produce satisfactory results depends upon the type of problem solved and the goals of the user.

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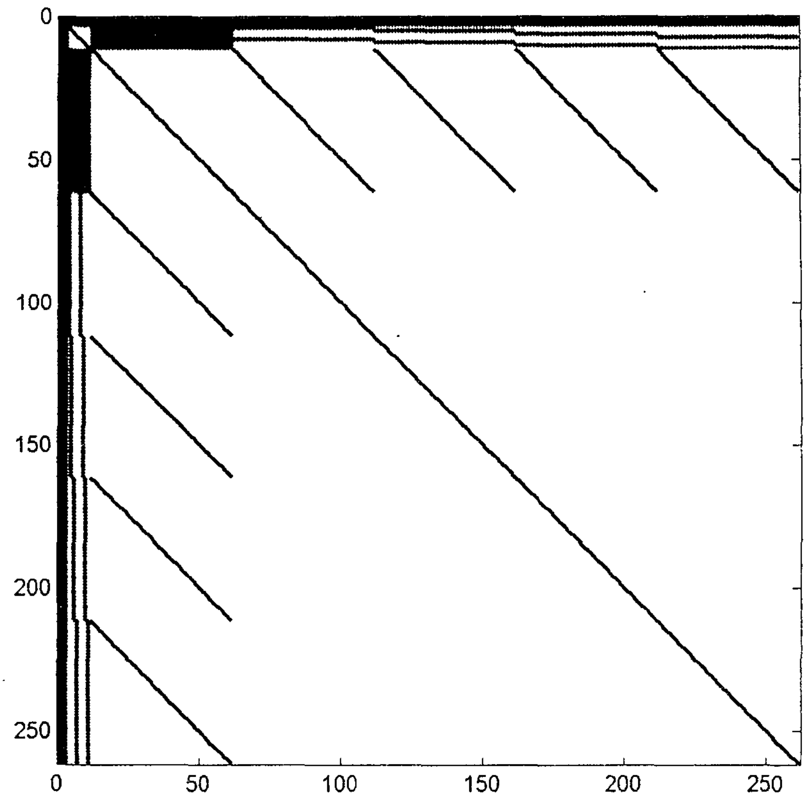

Fig. 3.. Normal equations structure

We used the powerful MATLAB software to aid in the development and study of that equations systems produced for GPS observation [11].

Module description

Listed below are explanations for each module in the application:

EFEMER1 Generate a file with coordinates and orbital radio for each satellite using as input file the navigation message. The number of observations epochs will be done like input data.

EFEMER2 Generate a file with coordinates and orbital radio for each satellite using as input file the SP3 format. The number of observations epochs will be done like input data.

COEFICI Generate a file with the distances at each observation epoch for each satellite, using the file above like input data.

OBSERVA Reordering data and generate a file with de carrier phase measurement and another with the pseudo-range.

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BLOQUES	Generate the blocks for each satellite which one include; BLOQUEA1 the position known, BLOQUEA2 ambiguity known, BLOQUEA3 and BLOQUEA4 clock offset known, BLOQUEA5 propagation delay known.
FASCOD	Make, using the blocks above, different model. When the matrix is formed obtain a graphical representation of her sparse
PROCESO	Perform a least squares estimate of station coordinates, ambiguities and clocks offset using different methods. Estimate the condition number and range of A.

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