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NUMERICAL MODELLING OF BEACH SAND TRANSPORT PHENOMENA BY WIND FLOW.

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ABSTRACT.

This work shows a way for simulating the natural process of moving sand of beach due the action of the wind. Two aspects must be considered about this natural fluid-particle system: the wind field adjustment and the transport and the deposition of the sand which will be represented in terms of particles concentration.

firstly, we estimate a three dimensional wind field from experimental data adjusting with finite element technique, a simple interpolated one such that final field obtained is incompressible, imposing a boundary condition of impenetrability over the ground.

On the other hand, the distribution of sand concentration is approached solving a classical convection-diffusion equation. In this case, the terminal velocity of the particles due to gravity must be introduced into the model.

INTRODUCTION.

Nowadays, the existence of some beaches is seriously menaced by the irrational allocation of industries which may have important ecological effects. The preoccupation of the governments is increasing day by day, and, at last, it seems like they are taking into account the information provided by numerical simulation for their decisions.

This is the case of the zone of Fuerteventura island that we try to study. The products of the sea erosion arrive to a 'source' shore due to the sea currents and the wind, and they form a sandy ground. Here we suppose that the periodical contribution of sand is known. There are many statistical and geological works about this question, and some of them about several beaches in Canary Island (see for example [4] and [5]). We have call source shore to that one because some of that sand is transported by the wind across several places of the island and finally it is accumulated on the opposite shore. This fact makes possible the continuity of the beaches allocated there, which have a negative increase of sand accumulation by their own shores.

Our question is if it is possible to allocate a factory of sand exploitation near the source shore, without meaning the end of the existence of the receiving beaches. In any case, the limits of this exploitation must be fixed.

MODEL DEVELOPMENT OF WIND FIELD .

Here, we shall consider a Lagrangian three dimensional model for local scale. For a given domain $\Omega \subset R^3$, with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, we are looking for a velocity field $\mathbf{u}$ that adjusts the wind field $\mathbf{u}_0$ obtained interpolating experimental data provided by the measurement stations. For the horizontal components of the velocity we have:

$$u_{0x} = \frac{\sum_{i=1}^{n} u_{ix} \frac{1}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}} \qquad u_{0y} = \frac{\sum_{i=1}^{n} u_{iy} \frac{1}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}} \tag{1}$$

n being the number of measurement stations, u_{ix} and u_{iy} the horizontal velocities measured at station i, respectively, and d_i the distance from the considered point to the i-th station.

The vertical component u_{0z} is obtained from an exponential function of the horizontal ones:

$$u_{oz} = u_{op}\, e^{-\delta z} \tag{2}$$

being $u_{0\rho} = \left(u_{ox}^2 + u_{oy}^2\right)^{1/2}$, z the height from the ground, and δ a parameter depending on meteorological conditions.

The expected field must verify a null divergence condition in the whole domain, and a impenetrability condition through the ground (Γ_1), (Γ_2 represents a free boundary and the condition on it will be imposed later):

$$\begin{aligned} div\,\mathbf{u} &= 0 \qquad in\ \Omega \\ \mathbf{u}\cdot\mathbf{n} &= 0 \qquad on\ \Gamma_1 \end{aligned} \tag{3}$$

A least square functional has been formulated in order to find $\mathbf{u}$ for a known $\mathbf{u}_0$:

$$J(\mathbf{u}) = \frac{1}{2}\int_\Omega (\mathbf{u}-\mathbf{u}_0)^t\ \mathbf{P}\ (\mathbf{u}-\mathbf{u}_0)\, d\Omega \tag{4}$$

P being a diagonal matrix in this case.

Then, the problem may be formulated as:

'Find $\mathbf{u} \in \mathrm{K}$ such that:

$$J(\mathbf{u}) = \min_{\mathbf{v}\in \mathrm{K}} J(\mathbf{v})$$

$$\mathrm{K} = \left\{ \mathbf{v}; div\, \mathbf{v} = 0, \mathbf{v}\cdot\mathbf{n}|_{\Gamma_1} = 0 \right\} \tag{5}$$

This problem is equivalent to find the saddle-point for the Lagrangian:

$$L(\mathbf{v},q) = J(\mathbf{v}) + \int_\Omega q\; div\, \mathbf{v}\; d\Omega \tag{6}$$

Applying Lagrange multiplier (λ) technique and introducing the conditions over **u**,

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{P}^{-1}\vec{\nabla}\lambda \qquad in\ \Omega \tag{7}$$

$$-div(\mathbf{P}^{-1}\vec{\nabla}\lambda) = div\,\mathbf{u}_0 \qquad in\ \Omega \tag{8}$$

$$-\mathbf{P}^{-1}\frac{\partial\lambda}{\partial\mathbf{n}} = \mathbf{n}\cdot\mathbf{u}_0 \qquad on\ \Gamma_1 \tag{9}$$

$$\lambda = 0 \qquad on\ \Gamma_2 \tag{10}$$

The finite element method applied to this problem give us the values of λ, and from equation (7), **u** can be obtained.

NUMERICAL MODELLING OF SAND TRANSPORT.

Into the equation of transport of particles in a fluid, we must introduce the effect of the terminal velocity of the particles due to the gravity. It can be showed from an equilibrium of forces (see [10]) that the terminal velocity of a particle can be formulated as:

$$u_t = -\left[\frac{2m_p g(\rho_p - \rho_g)}{AC_D\rho_p\rho_g}\right]^{1/2} \tag{11}$$

where m_p is the mass of the particle, ρ_p the density of the particle, ρ_g the density of the atmospherical air, A the transversal frontal area, g the local acceleration of gravity, and C_D the deposition coefficient.

It is well known in fluids mechanic studies, that C_D is a characteristical function of the form of the particle. The particles of sand may have a lot of forms and sizes, not only for its original formation but also for the union of some of them. To simplify the model, it is suitable to suppose spherical particles. Then, the terminal velocity is:

$$u_t = -\left[\frac{4d_p g(\rho_p - \rho_g)}{3C_D\rho_g}\right]^{1/2} \tag{12}$$

d_p being the diameter of the particle (for sand we will consider from 100 to 1000 μm).

For laminar regime and smaller particles, equation (11) can be simplified again. In our case, and for Reynolds numbers bigger than 0.5, it is necessary to use the experimental data of C_D versus Re to approach u_t for a given d_p. Anyway, there are some experimental graphics to obtain directly u_t (see for example [10]).

The velocities field for the particles is then:

$$\hat{u}_x = u_x \qquad \hat{u}_y = u_y \qquad \hat{u}_z = u_z + u_t \tag{13}$$

From the velocities field $\hat{\mathbf{u}}$ (calculated or interpolated for each step of time), the convection-diffusion equation can be formulated for the transport of sand in the atmosphere:

$$\frac{\partial c}{\partial t} + \hat{\mathbf{u}} \cdot \vec{\nabla} c - div(\mathbf{K}\vec{\nabla} c) = f \qquad in\ \Omega \tag{14}$$

c being the concentration of particles of sand, $\mathbf{K}$ the diffusivity coefficients matrix, and f the sources of sand (positive or negative).

The boundary conditions are fixed from the experimental observations. We can assume a given value for the concentration in that zones with invariable behavior (Γ^1), and free condition for the rest of the boundary (Γ^2):

$$c = c_0(x,y,z) \qquad on\ \Gamma^1 \tag{15}$$

$$\frac{\partial c}{\partial \mathbf{n}} = 0 \qquad on\ \Gamma^2 \tag{16}$$

A classical technique for solving this problem is the finite element method, using a time discretization based on a characteristic method, and prismatic elements of triangular base with six nodes for the spatial discretization.

THREE DIMENSIONAL MESH GENERATION.

The phenomena of transport and deposition of sand takes place near the ground, so the three dimensional grid need no high order of discretization in z direction. This may suppose a problem to obtain meshes of good quality. In any case, the problem is complicated and it is a discussion object.

Here, we propose the construction of a mesh with prismatic elements of triangular bases, obtained from a two dimensional mesh of the topography.

Once we have the discrete data of the topography, a projection of them over the xy plane can be easily done. This two dimensional domain is up to mesh using front advance technique. If we project this mesh over the topography and the pseudo-topographies obtained by dividing each vertical distance from each node on the ground to the corresponding at the top of the domain (we assume that the upper boundary of the three dimensional domain is a plane) by a fixed number which will give us the quantity of layers of mesh. Connecting each triangle with the corresponding upstairs, a three dimensional mesh as we expected is obtained. This mesh should be enough for solving our problem taking into account the condition of quality of the mesh. On the other hand, if a tetrahedric mesh is required, each element of the mesh before may be divided into three tetrahedric elements, meaning a more complicate structure of data due the mesh connectivities.

From the initial mesh, an adapted one can be achieved from an error indicator ε_i corresponding to each i-th element, moving the nodes in the grid such that the zones with high values for ε_i will have high density of nodes. ε_i is given by:

$$\varepsilon_i = \left[\frac{h_i^2}{24K_{\min}} \int_{\Omega_i} r^2 d\Omega_i + \frac{h_i}{24K_{\min}} \int_{\Gamma_i} \left\langle \mathbf{K} \frac{\partial c_h}{\partial \mathbf{n}} \right\rangle^2 d\Gamma_i \right]^{1/2} \tag{17}$$

where h_i is the diameter of the Ω_i element with boundary Γ_i, $K_{\min}$ is the minimal eigenvalue of $\mathbf{K} = \mathbf{K}(x, y, z)$, $\left\langle \mathbf{K} \frac{\partial c_h}{\partial \mathbf{n}} \right\rangle$ is the jump of flux for the numerical solution $c_h = c_h(x, y, z)$ on the elemental boundary Γ_i, and $r = r(x, y, z)$ is the residual of the steady equation,

$$r(x, y, z) = -div(\mathbf{K}\vec{\nabla} c_h) + \mathbf{u}\vec{\nabla} c_h - f \tag{18}$$

Obviously, the density function may be changed in order to perform the mesh depending on the considered case.

COMPUTATIONAL CONSIDERATIONS.

There are some computational considerations about this model. First of all, the results are depending on the accuracy of the meteorological observations. Only if the experimental data are reliable, the model can approach the real phenomena.

The time requirement for a numerical simulation is important to be minimized, much more if we talk about a non steady problem. Thus, the question of which iteration method to apply has a singular interest, due to that each step of time means to solve a linear system of equations, which is really the more expensive computational operation.

The linear system arising from the finite element discretization of the wind field adjustment problem is symmetric. Conjugate gradient-like methods seem to be the more efficient ones for sparse matrix. Anyway, they must be provided of a preconditioner to accelerate the convergence and sometimes to assure it (for ill conditioned systems).

When we try the non steady transport equation with finite element technique, the considering are similar, being usual high condition numbers.

For the steady transport equation, using finite element too, it is usually applied an artificial diffusion to obtain the stability of the numerical scheme. The system arising are nonsymmetric now, and GMRES method generally outperforms the other iterations (see [3]).

Another consideration is the vectorization/parallelization of the algorithms, which allows to provide a fast solver of these equations (see [3] and [9]).

CONCLUSIONS.

The numerical modelling of the transport phenomena of the beach sand is a complicated problem to solve. Some simplifications must be done to make possible a first model.

To verify the results and fix the error margins of the model, it is necessary to compare them with sand concentration realistic values at the studied domain.

This simplified model can be improve by a thermodynamic model that adjusts the real characteristic of the transport and deposition of solids particles, wet deposition removal processes in the equation of the conservation of the particles mass, the inclusion of the specific humidity, a specific study of the form and the size of the particles in the domain, and its consequences about the terminal velocity.

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