

NUMERICAL SIMULATION OF TRANSONIC FLOWS BY A FLEXIBLE OPTIMISER EVOLUTION AGENT

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Abstract. *Our methodology is to use evolutionary algorithms (EAs) as a simulation and meshless method, because of the equivalence between verifying a numerical scheme for a nodal point and minimizing a corresponding objective function at this point. One advantage and powerful characteristic of our methodology is that mathematical knowledge and theories related to the problem can be efficiently and easily incorporated. The methodology proposed opens the possibility of solving the problem from a more local point of view as a domain decomposition problem. In its practical application, our aim is to obtain a good approximate solution of the exact solution, and from the good and useful information thereby made available, we can use a traditional simulation method to refine results, but starting the application of the method in a more appropriated way, for example, starting with an appropriate mesh. In this paper we continue our open research line. Here we get numerical results when a divergent nozzle has supersonic upstream flow at the entrance and subsonic flow at the exit. The steady state solution contains a transonic shock. The results agree with the exact position and conditions of the shock line.*

1 INTRODUCTION AND BACKGROUND

We highlighted in previous works^{1,2} the capability and applicability of the Evolutionary Computation in solving an EDP non-linear boundary problem, as was the case when obtaining a solution for the stationary full potential flow problem. This involved the calculation of the speeds for transonic flow regime in the compressible and isentropic flow within a Laval nozzle, with maximum Mach number inside the nozzle approaching Mach 1 (and thus without the presence of a shock line). The problem was solved in 2D, and because the large search space required to find the optimal solution that minimized the objective function, we proposed consideration of our efficient optimiser as being appropriate for this case of large search space: namely, an Evolutionary Intelligent Agent-based software named Flexible Evolution Agent (FEA)^{3,4,5}. A procedure associated with parallelization⁶ was made too. In all these previous works, the numerical scheme was a simple central difference approximation of first-

order derivatives of the following non-linear differential equation for the velocity components without dimensions $(u', v') = (u/c_0, v/c_0)$:

$$\left[\frac{\gamma+1}{2} u'^2 + \frac{\gamma-1}{2} v'^2 - 1 \right] \frac{\partial u'}{\partial x} + u'v' \frac{\partial u'}{\partial y} + u'v' \frac{\partial v'}{\partial x} + \left[\frac{\gamma+1}{2} v'^2 + \frac{\gamma-1}{2} u'^2 - 1 \right] \frac{\partial v'}{\partial y} = 0 \quad (1)$$

Our procedure was to generate candidate solutions for the velocity field and getting in many points to find the numerical values of the velocities that correspond to the global minimum of the fitness function (sum of all square numerical schemes associated with the points) for the whole domain.

2 FINDING THE POSITION OF THE SHOCK LINE AND THE VELOCITY AND DENSITY FIELDS

We point out in passing an important technical difference between the situation considered in this paper and the one in our previous work. Now we seek the capacity of our methodology to localize the position of the shock line – transonic shock and steady flow – when the nozzle has supersonic upstream flow at the entrance.

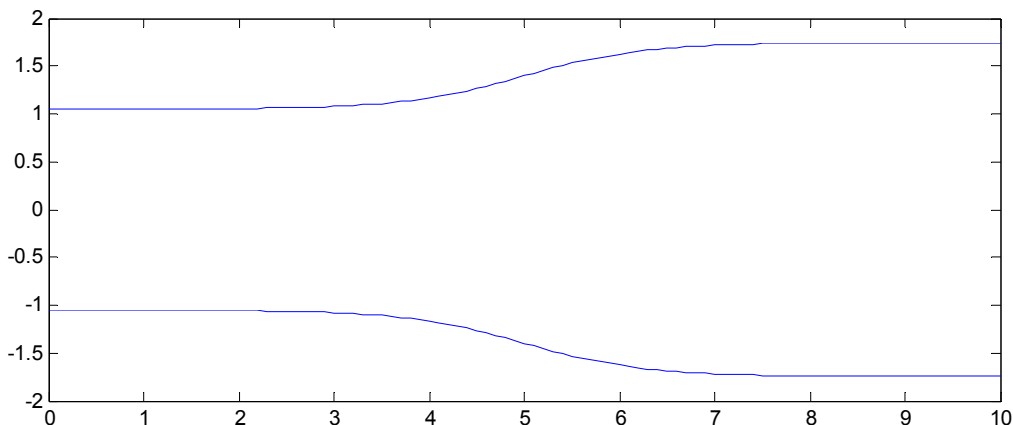


Figure 1: Geometry of the nozzle.

In the applications shown in this paper, the geometry of the nozzle is the same provided from H.M. Glaz and T.P. Liu⁷ (Fig. 1), which is given by:

$$A(x) = 1.398 + 0.347 \tanh(0.8x - 4), \quad 0 \leq x \leq 10 \quad (2)$$

We consider in the computational implementation two subdomains:

- The left subdomain (Ω^-) has supersonic flow at the entrance and $u \geq u_{entrance} > c_*$ on Ω^- , where c_* is the critical velocity.
- The right subdomain (Ω^+) has subsonic flow at the exit with $u_{exit} \leq u < c_*$ on Ω^+ , where c_* is the critical velocity.

The size of both subdomains was chosen so as to be large enough for both to include the shock zone. For each subdomain, we solve the transonic flow with consideration of isentropic flow and using Evolutionary Computation. The target is to obtain the velocity and density fields on the whole nozzle and to determinate the localization of the shock line.

On the position of the shock line, it is well known that if u has one discontinuity on the shock point (x_s) then:

$$u^-(x_s)u^+(x_s) = c_*^2 \quad (3)$$

$$\rho(u^-(x_s))u^-(x_s) = \rho(u^+(x_s))u^+(x_s) \quad (4)$$

Eq. (3) is the Prandtl-Meyer relation.

Physical entropy condition⁸: The density increases across the shock in the flow direction: $\rho(u^-(x_s)) < \rho(u^+(x_s))$.

The flow cases considered in this paper are close to being quasi-one-dimensional. Thus for the divergent nozzle with supersonic entrance considered, we can ensure that the mass flow: $\rho(u(x))u(x)A(x)$, is conserved at each point x .

3 NUMERICAL IMPLEMENTATION AND APPLICATIONS

First we consider the formulation given by the equations:

$$\text{div}(\rho u A) = 0 \quad (5)$$

$$\rho(u) = \rho_0 \left[1 - \frac{\gamma - 1}{2 c_0^2} u^2 \right]^{\frac{1}{\gamma - 1}} \quad (6)$$

where c_0 is the speed of sound in normal conditions and $\gamma = 1.4$ for the air.

3.1 Evolutionary computation and a first implementation

To solve this problem with evolutionary algorithms we have designed the following algorithm:

Step 1: Read the input data (boundary conditions and GA parameters).

Step 2: Set $cont = 0$;

Step 3: Set $cont = cont + 1$;

Step 4: Run the GA on Ω^-

Step 5: Run the GA on Ω^+

Step 6: Find the shock point $x_s \in \Omega^- \cap \Omega^+$ as the point where the following conditions are minima:

$$\left| u^-(x_s)u^+(x_s) - c_*^2 \right| \quad (7)$$

$$\left| \rho(u^-(x_s))u^-(x_s) - \rho(u^+(x_s))u^+(x_s) \right| \quad (8)$$

- Step 7:* Set $\Omega_s^- = [0, x_s) \subset \Omega^-$ and $\Omega_s^+ = (x_s, 10] \subset \Omega^+$.
- Step 8:* Evaluate the fitness function (ff) over all the points of the domain $\Omega_s = \Omega_s^- \cup \Omega_s^+$.
- Step 9:* If $cont < max_cont$ or $ff \geq error$ go to Step 3. Otherwise go to Step 10.
- Step 10:* Run the FEA on the whole domain $\Omega = \Omega_s \cup \{x_s\}$.
- Step 11:* Save results and stop.

The pseudocode of the GA is the following:

- Step 1_{GA}:* The limit values of the variables are established (they are different depending on whether we are at Step 4 or Step 5 of the preceding algorithm).
- Step 2_{GA}:* If $cont = 1$, the initial population is randomly generated. Otherwise, the initial population is generated from the best solution obtained at the corresponding Step 4 or Step 5 of the preceding algorithm when $cont = cont - 1$.
- Step 3_{GA}:* Set $cont_{GA} = 0$;
- Step 4_{GA}:* Set $cont_{GA} = cont_{GA} + 1$;
- Step 5_{GA}:* The individuals are repaired in such a way that they verify the hypotheses of the problem:
- If the algorithm is at Step 4: the velocities are ordered from lowest to highest and the densities from highest to lowest.
 - If the algorithm is at Step 5: the velocities are ordered from highest to lowest and the densities from lowest to highest.
- Step 6_{GA}:* Evaluate the fitness function.
- Step 7_{GA}:* Order the population from lowest to highest value of the fitness function.
- Step 8_{GA}:* Selection: Tournament selection operator (2:1).
- Step 9_{GA}:* Crossover: Antithetic crossover operator.
- Step 10_{GA}:* Mutation: Smooth mutation over one variable randomly chosen.
- Step 11_{GA}:* If $cont_{GA} < max_cont_{GA}$ go to Step 4_{GA}. Otherwise leave.

The chromosomes considered are:

- $(u_1, u_2, \dots, u_{nleft-1}, \rho_1, \rho_2, \dots, \rho_{nleft-1})$ when Ω^- is split in $nleft$ points x_i with $x_i = ih$, $0 \leq i < nleft$ and $h = |\Omega^-| / (nleft - 1)$. Here $u(x_0) = u_{entrance}$ and $\rho(u(x_0)) = \rho_{entrance}$.
- $(u_1, u_2, \dots, u_{nright-1}, \rho_1, \rho_2, \dots, \rho_{nright-1})$ when Ω^+ is split in $nright$ points x_i with $x_i = x^- + ih$, $0 < i \leq nright$, $x^- = 10 - |\Omega^+| - h$ and $h = |\Omega^+| / (nright - 1)$. Here $u(x_{nright}) = u_{exit}$ and $\rho(u(x_{nright})) = \rho_{exit}$.

The fitness function considered is the following one:

$$\sum_{i=1}^{n \text{ var}} f(x_i) \quad (9)$$

where $nvar = \begin{cases} nleft - 1 & \text{if we are running the GA on } \Omega^- \\ nright - 1 & \text{if we are running the GA on } \Omega^+ \end{cases}$, and

$$f(x_i) = w_1 \left\{ \rho(u(x_i)) - \rho_0 \left[1 - \frac{\gamma-1}{2c_o^2} u(x_i)^2 \right]^{\frac{1}{\gamma-1}} \right\}^2 + w_2 [\rho(u(x_i))u(x_i)A(x_i) - \rho(u(0))u(0)A(0)]^2 \quad (10)$$

When the FEA is run the shock point conditions (7) and (8) are added to the fitness function as follow:

$$\sum_{i=1}^{nvar} f(x_i) + 10^{-10} \left[(u^-(x_s)u^+(x_s) - c_*^2)^2 + (\rho(u^-(x_s))u^-(x_s) - \rho(u^+(x_s))u^+(x_s))^2 \right] \quad (11)$$

3.2 Results

In order to validate our methodology, we consider the same data at the entrance and the exit of the divergent nozzle given in one example from Keppens⁹. Thus we have:

Boundary conditions at the entrance: $M_{entrance} = 1.28$ (Mach number); $u_{entrance} = 2$ and $\rho_0 = 1.0156$.

Boundary conditions at the exit: $\rho_{exit} = 0.8$.

From these boundary conditions, c_0 , c_* and $\rho_{entrance}$ are determined, and also the flow conditions at the exit of the nozzle under isentropic consideration.

In this test case we defined: $\Omega^- = [0, 6]$ and $\Omega^+ = [4, 10]$ and considered only 101 equidistant points on the whole domain, $nleft = nright = 61$, $max_cont_{GA} = 100$, $max_cont = 100000$ and $error = 10^{-9}$. The fitness function was $7.874707 \cdot 10^{-10}$ at $cont = 24990$ and the shock point error was $1.438531 \cdot 10^{-1}$. Once obtained the shock point, FEA was run 100000 generations on the whole domain. The fitness function and the shock point error decreased to $5.531903 \cdot 10^{-10}$ and $1.321814 \cdot 10^{-5}$, respectively. Fig. 2 shows the solution obtained. The position of the shock point was well captured: $x_s = 5.1$, close to the exact position, and the results agree with the Prandtl-Meyer relation. Moreover, the range of values of the Mach number is compatible with the isentropic flow consideration. We have a shock in the numerical applications that is not strong.

3.3 A second implementation

Chen and Feldman^{10, 11} delighted that “*the uniform velocity state at the exit of the an infinite nozzle in the downstream direction is uniquely determined by the supersonic upstream flow at the entrance, which being sufficiently close to be an uniform flow*”. In this context but for a finite nozzle, now we consider the test case where the boundary conditions at the exit are: $\partial_x u = 0$ and $\partial_x \rho = 0$. This is consistent according to the geometry of the nozzle that has a constant section at the exit (see Fig. 1).

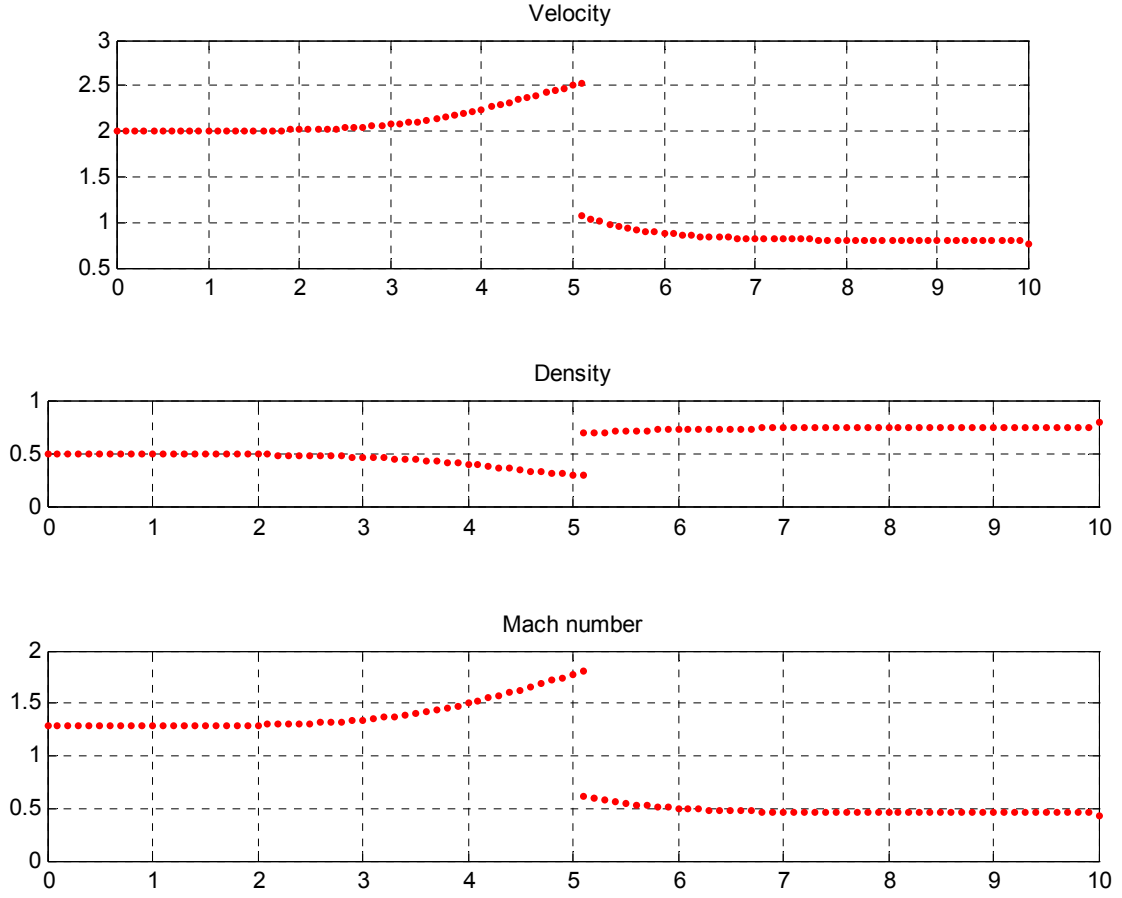


Figure 2: Velocity, density and Mach number in the nozzle.

Now the chromosomes considered are:

- $(u_1, u_2, \dots, u_{nleft-1}, \hat{\rho}_1, \hat{\rho}_2, \dots, \hat{\rho}_{nleft-1})$ when Ω^- is split in $nleft$ points x_i with $x_i = ih$, $0 \leq i < nleft$ and $h = |\Omega^-| / (nleft - 1)$. Here $u(x_0) = u_{entrance}$ and $\hat{\rho}(x_0) = \rho_{entrance} / \rho_0$.
- $(u_1, u_2, \dots, u_{nright-1}, \hat{\rho}_1, \hat{\rho}_2, \dots, \hat{\rho}_{nright-1})$ when Ω^+ is split in $nright$ points x_i with $x_i = x^- + ih$, $0 < i < nright$, $x^- = 10 - |\Omega^+| - h$ and $h = |\Omega^+| / (nright - 1)$. Here $[\partial_x u]_{exit} = 0$ and $[\partial_x \rho]_{exit} = 0$.

And the fitness function terms are:

$$f(x_i) = w_1 \left[\hat{\rho}(u(x_i)) u(x_i) A(x_i) - \frac{\rho(u(0))}{\rho_0} u(0) A(0) \right]^2 + w_2 [\partial_x (\hat{\rho}(u(x_i)) u(x_i) A(x_i))]^2 \quad (12)$$

where:

$$\hat{\rho}(u) = \left[1 - \frac{\gamma - 1}{2c_o^2} u^2 \right]^{\frac{1}{\gamma - 1}} \quad (13)$$

The first-order derivatives are approximated by:

- A backward difference approximation on Ω^- :

$$\partial_x g(x_i) = \frac{g(x_{i-2}) - 4g(x_{i-1}) + 3g(x_i)}{x_i - x_{i-2}} \quad (14)$$

- A forward difference approximation on Ω^+ :

$$\partial_x g(x_i) = \frac{-g(x_{i+2}) + 4g(x_{i+1}) - 3g(x_i)}{x_{i+2} - x_i} \quad (15)$$

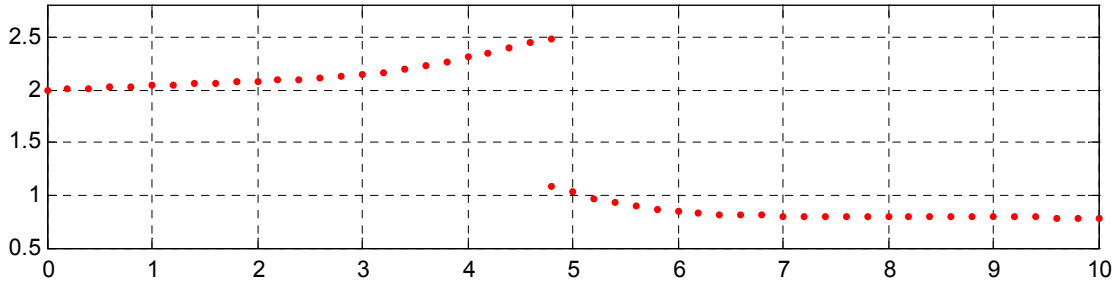


Figure 3: Velocity in the nozzle.

In this test case we defined: $\Omega^- = [0, 6]$ and $\Omega^+ = [4, 10]$ and considered only 51 equidistant points on the whole domain, $n_{left} = n_{right} = 31$, $max_cont_{GA} = 100$, $max_cont = 20000$. Fig. 3 shows the solution obtained. The position of the shock point was well captured: $x_s = 4.9$, very near to the exact position, and the results agree with the Prandtl-Meyer relation.

4 CONCLUSIONS

We proposed a simple and efficient method for calculating transonic flows that captures the steady state solution and the position of the line shock with a good accuracy. The results of the numerical examples shown in this paper provide strong evidence on the effectiveness of this new methodology as an alternative to use a traditional method. In this paper a simple genetic algorithm is capable to find remarkable solutions but, as we have delighted, the use of our FEA provides more refined solutions.

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