Avoiding obstacles in Underwater Glider Path Planning

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***Abstract*—Underwater gliders are strongly influenced by ocean currents due to their low surge speed. Gliders may drift significantly from their expected trajectories, making path planning a real concern for this type of vehicle, as it may reduce the time taken to reach a given waypoint or save power. In such a dynamic environment it is not easy to find an optimal solution or any such requires large computational resources. In this paper, we present a path planning scheme for this kind of underwater vehicle combining a technique inspired by a variant of the A* algorithm with an optimization process revealing the physical vehicle motion pattern. This method models a glider affected by the ocean current speed and direction, and generates a path according to predefined objectives. The combination of these two techniques allows static or dynamic obstacle avoidance, frequently demanded in coastal environments, with land areas, strong currents, shipping routes, etc. The method can easily be configured and adapted to various optimization problems, including missions in different operational scenarios. This planner shows promising results in realistic simulations, including ocean currents that vary considerably in time, and provides a superior performance over other approaches that are compared in this paper.**

***Index Terms*—Path planning, underwater gliders, avoiding obstacles.**

I. Introduction

Robotic Unmanned Underwater Vehicles (UUV) have demonstrated to be a valuable tool for a wide range of applications, including structure inspection, environmental monitoring and control or security. Since the possibilities of human intervention are quite limited during the robot mission, these vehicles can be conceived as physical agents that must perform their tasks with a high level of autonomy. In fact, they are commonly known as Autonomous Underwater Vehicles (AUV). However, it is hard to accomplish this goal as a consequence of the dynamism and uncertainty characteristic of the state of both the vehicle and its environment, estimated with a separate model each.

Automatic path planning constitutes a key capability because underwater robots are usually commanded in terms of goal navigation waypoints to be hit or target regions to be explored. Compared to ground mobile robotics, the underwater scenario is much more challenging, since operating conditions can vary notably even on reduced areas and over a relatively short period of time. In the particular case of ocean gliders, all the mentioned difficulties are magnified.

A glider (see Figure 1) is a type of UUV that operates by modifying its buoyancy in a cyclic pattern. This results in a vertical impulsion that is transformed into an effective but low surge speed by means of the combined effect of internal mass displacements and the vehicle wings and tail orientation, resulting in up/down slope or climb/dive transects. In terms of power consumption, the glider saw-tooth profile is very efficient, since the gravity is used as the power source for propulsion, that is the most critical task of UUVs autonomy. Besides processing and communication, the batteries are only used intensively during a small fraction of the cycle time to change the vehicle buoyancy, using an electric pump; and, much less demanding, to modify the vehicle attitude and bearing angle while submerged using low consumption actuators. Ocean gliders have been applied successfully in Maritime Research, and they are expected to become one of the reference technologies as observational tool in the coming years [16].

Their low surge speed make gliders far more influenced by ocean currents than other UUVs that can overcome them. Gliders may drift significantly from its intended trajectory, making path planning a crucial tool for this type of vehicles, as it might reduce the time spent to reach a given waypoint or save power. The coastal area is specially demanding as currents strength is normally higher and there is a potential risk of collision.

A. Related Works

Path planning for unmanned underwater vehicles has been a subject of interest for researchers since the introduction of these robotic platforms. Different approaches have been developed applying techniques that include searching algorithms based on artificial intelligence, potential field modeling, multi-

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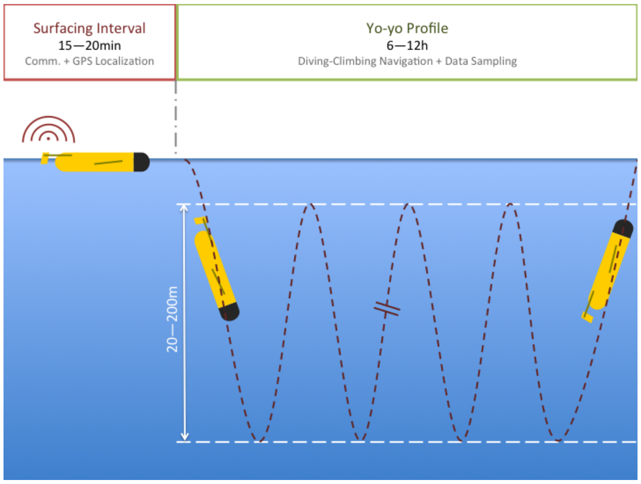

Fig. 1. Glider saw-tooth navigation pattern.

objective optimization, etc. Some of the most relevant, in our opinion, are summarized in the following.

An influential set of planners has evolved from the A* algorithm [6] as a basis, and they operate on graphs and grids. For example, Carroll et al. [3] apply this strategy on a quad-tree search space. More recently, Garau et al. [5] propose another alternative incorporating ocean currents on a uniform grid discretization.

The approaches based on minimization of energy functions are also worth commenting. As good examples, we can cite the work of Kruger et al. [9], that includes the time as an extra dimension in the search space, or Witt et al. [20], that incorporate modeling of time-varying obstacles using potential fields. The problem of local minima has been tackled by means of strategies based on particle swarms, simulated annealing, or genetic algorithms. In other proposals, the currents are modeled as continuous time functions, as is the case of the non-linear trajectory generation or NTG method [11] applied over B-Splines of Zhang et al. [22].

Other alternatives that also make use of continuous models are described on the works of Petres et al. [13], [14]. Later, this line has been extended to deal with the presence of strong currents [18].

Evolutionary computing has also been successfully applied to this type of problems. A significant example can be found in [1], where genetic algorithms are used for AUV trajectory planning in environments characterized by time-varying currents.

The high dimensionality of the search space has led to random exploration based approaches. The rapid random trees or RRT [10] [17] are a good example of this, and have been applied to the case of route planning for AUVs [19] and gliders [15]. The problem is that this method has application only on static maps.

Moqin et al. [12] propose an iterative optimization process for glider path planning. However, the focus of that work is centered on the waypoint precision enhancement, and not in optimal path planning. Furthermore, only static ocean currents are considered.

Finally, in the last years a line that has received a lot of attention from researchers is the use of multiple vehicles in a coordinated mission. Some relevant examples include [21] and [2], that face the problem of adaptive sampling of oceanic variables by means of gliders fleets.

B. Motivation

Our work has been organized around two main topics, being the analysis of path planning requirements and the study cases. Regarding the former, we have identified several factors that, in our humble opinion, should be assessed at the design phase of a path planner for ocean gliders. The path planning for underwater glider presents clear differences in relation to the classic planning problem:

- The currents field directly affects the movement of the vehicle so that the cost of displacement is variable and anisotropic at different points in space.
- The currents field is variable in time, so that it can be not be guarantee the optimality of the result without revisiting already visited nodes.
- The vehicle has one degree of freedom, the bearing, with the additional restriction that is only updated in discrete times.

For these reasons, most of classical approaches in the path planning are not directly applicable to this problem. Many path planners apply a certain form of discretization, either on the trajectory or command space, to reduce the computational cost. However, the downside of discretization lies in the presumably degradation of the quality of the results, that might lead to unrealistic trajectories.

The execution time is another factor which is often under-stated due to the typical long duration of glider missions and immersion periods. Although this is generally true, it is not the case when the path planner must respond within a reduced time interval to face an unforeseen situation.

In this paper we analyze the path planning problem in specially troublesome scenarios, mainly coastal, that include static and dynamic obstacles such as strong currents, land areas or heavy traffic shipping routes. There, the planner pursues the maximization of the distance traveled towards a distant way-point —or, in other words, the minimization of the remaining distance to reach it— over a short and known period of time. This corresponds to a leg/stage range planning with a maximum duration of three or four days and a typical trajectory length around 100 km. For this temporal horizon, ocean current forecasts of high temporal resolution are used. These forecasts can be obtained from some Regional Oceanic Models (ROM) with hourly outputs. The ROM are forecast systems of currents and other oceanographic variables that is based on numerical models. In such configuration, the path planning problem is clearly performed in a time-varying scenario.

In our previous works [7], [8] we presented a path planning technique for underwater gliders using optimization processes. This technique is based on the discretization of time instead of space. Taking into account that a glider can not change the bearing while is submerged, this method permits to reflect accurately the vehicle navigation pattern without lost of information because of discretization processes. It offers good results in offshore trajectories, however we have detected that in some cases the planner do not give good solutions in the presence of obstacles (land coast, strong currents against the vehicle steering, ...).

In this work, we have searched to include a previous phase to optimization process that allows to locate zones free of obstacles in the trajectory to the waypoint, with the necessity of low commotional cost. To solve this problem, here, we introduce a novel path planning technique for underwater gliders in troublesome coastal environments that combines a technique inspired in A* search with optimization. The method is quite flexible, as it can be applied to a number of other optimization problems with few adaptation or configuration. It shows promising results in realistic simulations, under highly time-varying ocean currents. The proposal gives a superior performance when compared with other approaches.

This paper is organized as follows: the next subsections include a revision of UUV path planning approaches and the motivation of this work. Then, in section II, the proposed method is described in detail. Section III presents the experiments carried out to validate our path planning algorithm. Finally, section IV contains the conclusions extracted from this work.

II. Path Planner

As commented in the introduction, gliders propel themselves by changing their buoyancy and transforming the resultant vertical motion, of continuous dives and climbs, into a surge movement by means of the combined action of the internal mass displacement and the external control planes. These cycles are repeated typically for 6-12 hours periods, called transects or stints. Once a stint is finished, the vehicle surfaces to communicate the status and data gathered to the control room and receive new orders, commonly the next waypoint or bearing. After 15-30 minutes at the surface, the next immersion period starts. An important fact is that gliders do not communicate while submersed, and the on-board navigation system simply tries to keep the last commanded heading or bearing during the whole stint.

Following the requirements analysis, we have developed an optimization process [7], [8] for the core of our planner. It operates accepting the commanded glider bearings as variables, which are then iteratively optimized to find the path of minimal cost. With this election, the benefit is twofold, avoiding discretization and allowing for a physically realistic simulation.

This approach produces acceptable results for static, moderate-strength ocean currents. However, as indicated previously, we are interested in short-term coastal navigation. There, and due to the complexity of the environment and the coupled nature of the process variables, the optimization can easily get trapped in local minimum or lead to wrong paths, including collisions (Figure 2). To overcome such limitations, our new path planner, that we call Optimization with Intelligent Initialization, integrates a bootstrap module based on A* search, that generates an appropriate initial set of values to start the optimization phase.

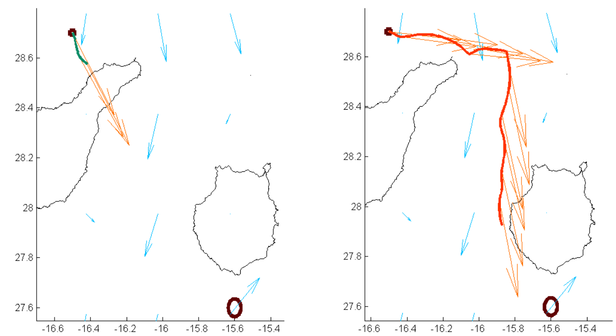

Fig. 2. Response in the presence of obstacles for the optimization method (left) and the optimization with intelligent initialization method (right). In both cases, the path ends after 4 days. Generated trajectories by glider bearings at each surfacing, with time-varying currents (ocean currents that exceed the glider speed $v_g = 0.4\mathrm{m/s}$ are highlighted) from the start point to the goal point . ends after 4 days period

The cost function of the optimization process is computed on the basis of a stint simulator that reproduces the glider trajectory combining the commanded bearing with the nominal glider speed and ocean currents. For this purpose, out simulator applies a simple glider kinematic model. The figure 3 illustrates the strong influence of ocean currents on the resultant glider trajectory, as a consequence of its relative low surge speed. Also, it is observed the high variability of currents orientation in only 3 days.

The number of variables to optimize is a function of the stint and the total path durations. As an example, a 4-day mission would require 12 variables for transect of 8 hours. In most cases, the final value returned by the objective function is computed as a distance metric.

A. The Algorithm

In our algorithm, the optimization takes a vector of bearings as variables. As previously commented, the glider trajectory is simulated on the basis of the real behavior, selecting the control bearings sequentially and keeping it constant for a whole stint. The path planner algorithm integrates two phases: initialization and optimization. The objective function minimized in these processes is the Euclidean distance between the position at the last surface and the target point.

1) Initialization: In our method, we define a search space generated from coarse-grain simulations using the CTS-A* method [4], a variant of A* planner that uses the constant bearing vehicle stint as operator for node transitions instead of a regular equispaced grid. Thus, the initialization considers a set of angularly equispaced radial vectors emanating from the starting point and simulates a constant bearing glider trajectory for each one, generating successive stints until the mission time expires (Figure 4). Then, each trajectory is divided into a set of turning points candidates at surfacing locations uniformly spaced out. Recursively, a new set of trajectories is generated for each one, simulating for the remaining mission time. As an heuristic, an optimistic estimation of the combined glider-current velocity is computed, allowing to prune non promising trajectories.

In practice, we have observed that it suffices to divide the trajectory in a single turning point. This is a direct consequence of the short path planned, since in a 4-day journey a glider might travel up to approximately $100\text{-}150\mathrm{km}$.

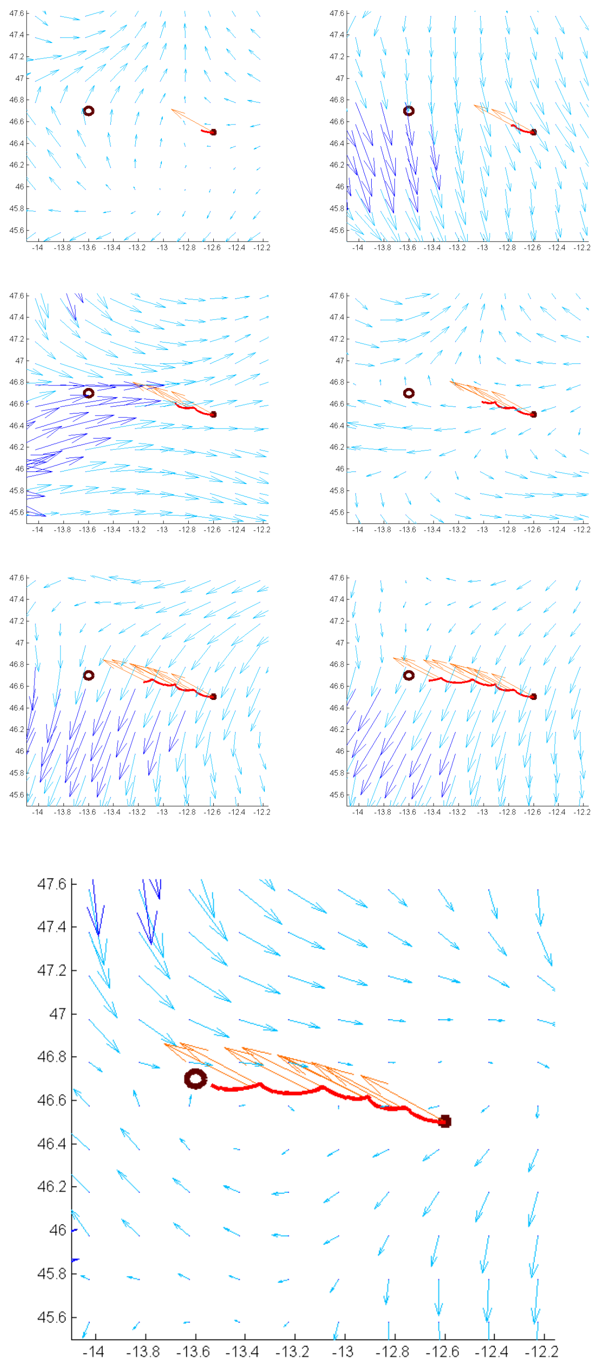

Fig. 3. Snapshots of the optimal trajectory and glider bearings at each surfacing, simulated for a 3-days period with time-varying currents (ocean currents that exceed the glider speed $v_g = 0.4\mathrm{m/s}$ are highlighted) from the start point to the goal point .

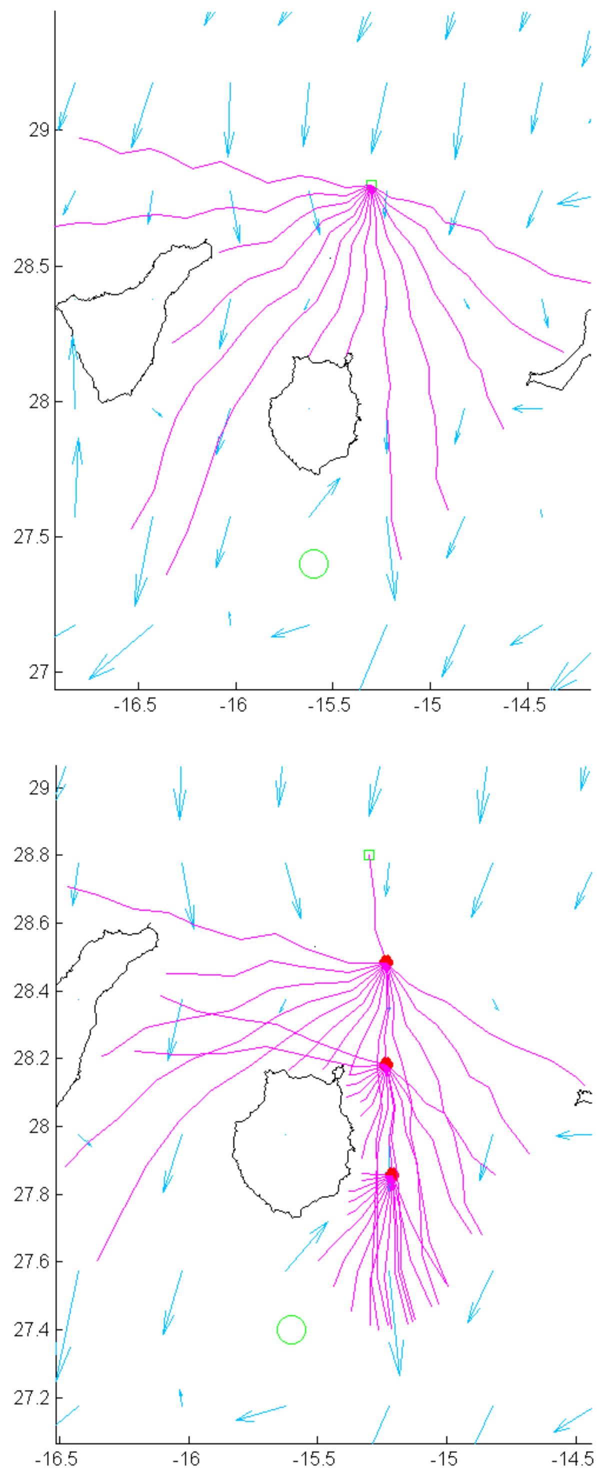

Fig. 4. First (top) and second (bottom) level of of the initialization process.

2) Optimization: In this phase, the algorithm takes the initial bearings and applies successive glider stints simulations trying to minimize the distance to target from the end of trajectory as cost function.

III. Experimental Results

We have carried out several simulations for the path planner presented in this paper using Matlab® to validate the proposal and test its performance. The results have been compared with the ones obtained with other methods.

We have simulated different missions in the Canary Islands coast, using ocean current maps from the ESSEO-CAN model. This is a ROM model that gives hourly outputs structured in four 24h sets. The simulations described in this paper were configured for a glider speed of 0.2-0.4 m/s and a stint of 8 hours.

The general objective of the simulations have been to obtain the trajectory that leaves the vehicle closer to a goal point navigating for 4 days. For the methods based in the bearing optimization, this requires a total of 12 variables. Figure 3 illustrates one example of the typical results obtained in these tests.

A. Alternative methods

The alternative methods selected for the comparison are briefly described below:

- **Direct to goal**: This is the trivial solution to the problem. At each surfacing the next bearing is computed as the direction of the goal point.
- **Standard A***: Adapted to operate over an uniform grid of ocean currents, using optimistic time estimations to reach the target waypoint as heuristic.
- **CTS-A***: This method [4] is a variant of classic A* where the times between two consecutive surfacings are kept constant. It is based on the discretization of the bearings that can be commanded at each surfacing, and is suitable for both static and dynamic ocean current maps.
- **Optimization**: Our previous method [7], [8] uses the direction to the goal point as initial guess for all bearings that are the variables of the optimization process.

B. Comparative tests

To compare the performance of each path planning method we have simulated the trajectories towards a distant waypoint using ROMs for a 4-day forecast horizon. A total of 45 cases have been generated and evaluated. We have divided the cases in two situations and analyzed them separately. The first set of cases correspond to coastal trajectories while the second one includes only trajectories in offshore scenarios.

Two measures are computed for the comparison of the methods: path quality and computational cost. We have established as a quality measure for the generated trajectories (the lower the better), the remaining distance from the final glider position to the target point.

We should comment here that the A* results require a special consideration, since the method used in the trajectory generation produces unrealistic non-constant surfacing times that are dependent on the grid size. That is to say, the surfacing points in A* do not correspond with the surfacing points of the glider.

TABLE I
Difference of the remaining distance to reach the goal with respect to the Init-Optim method. Mean and standard deviation within brackets, both in km. Simulations run for a glider speed $v_g = 0.4\text{m/s}$.

Scen	Optim	CTSA*	A*	Direct
Total	10.3 (21)	5.2 (6)	8.5 (18)	42.4 (46)
Coast	19.6 (26)	5.8 (7)	5.3 (7)	67.4 (39)
Ocean	0 (0)	6.5 (4)	9.1 (6)	13.6 (24)

The computational cost is also an important factor to be considered, as sometimes it is necessary to obtain a path in a few minutes. For example, when an unforeseen risky situation occurs while the glider is surfacing, a new bearing needs to be computed before the glider initiates a new transect.

Regarding the algorithms' parameters used in the comparison, we have selected the same equivalent discretization level for each method, when applicable. For example, the spatial grid for A* and CTS-A* is fixed to 1/20 degrees. The CTS-A* algorithm has been configured to use a division of $20°$ in the bearings rose. For our new approach we have used a division of $5°$ for the initialization phase, inserting a candidate turning point each 3 surfacings, the equivalent to one day of navigation.

Table I shows the mean and standard deviation of the difference of the remaining distance to the goal between each method and our new approach, labeled as Init-Optim in the tables. The global result for all cases and the mean in each environment (near the coast, offshore) is shown separately. The average distance traveled by the glider at 0.4 m/s has been 120 km. Table II shows the computing time for each method, measured on a Intel(R) Core(TM) 2 Quad processor computer running at 2.5 Ghz.

Compared with the previous optimization method (Optim) we can observe that the new approach gets approximately the same results when no obstacles are present, while it shows an important improvement when there is a need to avoid obstacles. Regarding A* and CTS-A* methods we have observed that, in general, we can obtain better quality in the path with less computational cost. On the other hand, we have verified that the computational cost of the new method when the route is free of obstacles is approximately half the value when the obstacles are present.

Figure 5 shows two of the cases include in the previous analysis. The distance required to reach the waypoint after 4 days is presented. It must be noted that the currents vary on time and only the last snapshot of them is shown in the figure.

Table II shows the computing time for each method. Our new proposal consumes twice as long as the version with simple initialization. Still, the computational cost is low.

To test the performance of the algorithms on adverse conditions, the simulations were repeated using a glider at

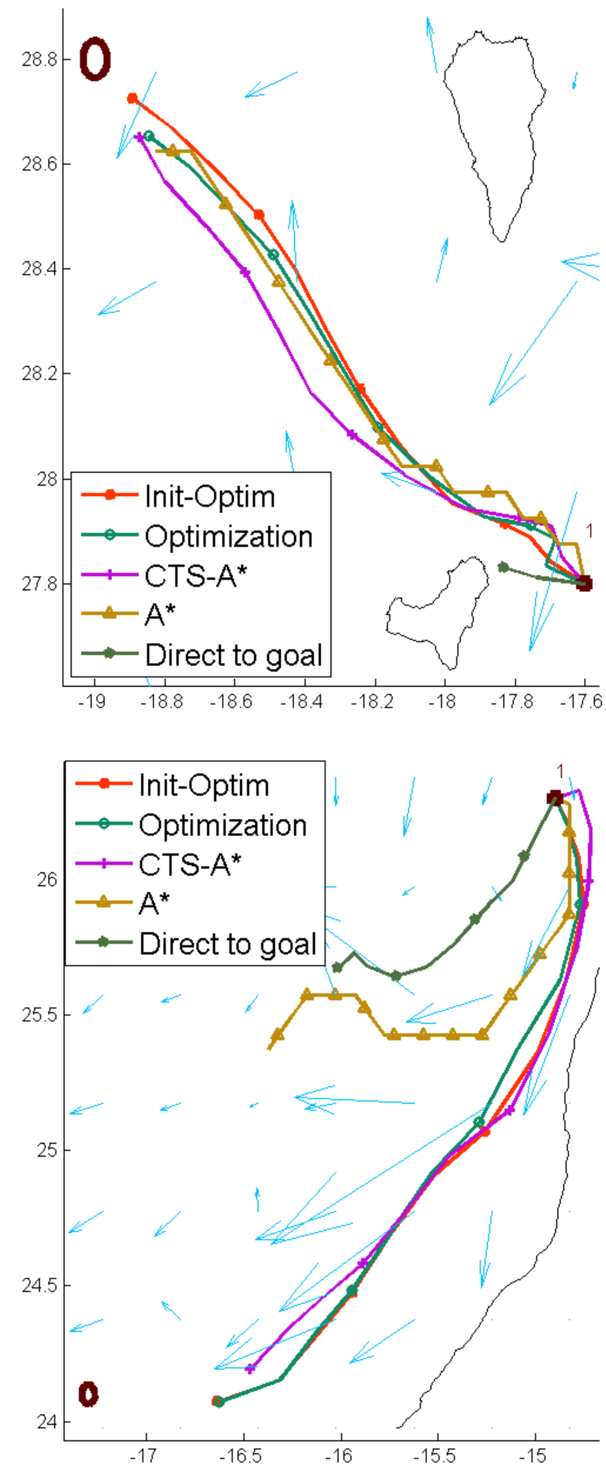

Fig. 5. Two comparatives of trajectories simulated for a 4-days period with time-varying currents (ocean currents that exceed the glider speed $v_g = 0.4$m/s are highlighted) from the start point ● to the goal point **O**. Top graphic: Total Distance = 176.5 km. Distance remaining to reach the goal point: Init-Optim: 13.3 km; Optimization: 22.1 km; CTS-A*: 20.6 km; A*: 25.9 km; Direct to goal: 157.1 km. (stop in land). Bottom graphic: Total Distance = 343.4 km. Distance remaining to reach the goal point: Init-Optim: 67.4 km; Optimization: 68.8 km; CTS-A*: 85.1 km; A*: 169.4 km; Direct to goal: 217.6 km.

TABLE II

COMPUTATIONAL TIME. MEAN AND STANDARD DEVIATION WITHIN BRACKETS, BOTH IN SECONDS. GLIDER VELOCITY AT 0.4 M/S AND 0.2 M/S.

Methods	0.4 m/s	0.2 m/s
Init-Optim	26 (10)	24 (12)
Optim	15 (11)	12.5 (10)
CTS-A*	477 (93)	105 (28)
A*	55 (11)	12 (4)
Direct to goal	<1 (0)	<1 (0)

TABLE III

DIFFERENCE OF THE REMAINING DISTANCE TO REACH THE GOAL WITH RESPECT TO THE INIT-OPTIM METHOD. MEAN AND STANDARD DEVIATION WITHIN BRACKETS, BOTH IN km. SIMULATIONS RUN FOR A GLIDER SPEED $v_g = 0.2$m/s.

Scen	Optim	CTSA*	A*	Direct
Total	6.2 (13)	9.9 (10)	13.5 (49)	18.0 (29)
Coast	16.3 (17)	7.0 (7)	10.5 (13)	30.4 (29)
Ocean	0.2 (1)	11.5 (10)	15.3 (19)	10.5 (27)

0.2 m/s (Table III). The basic version of the optimization method reduces the difference due to the fact that the obstacles are in the same point and the Optimization with Intelligent Initialization method covers less distance. A* and CTS-A* obtain worse results due to the use of discretization in their implementations and in some cases they are not able to avoid obstacles, so it has a high standard deviation. On the other hand, while the two versions of optimization keep their times, A* and its variant reduce notably their cost. In the first group, the process is the same, as they need to optimize the same number of variables, while in the second one the search area has less extension and the number of nodes visited is reduced.

Figure 6 shows an example of one these cases with the glider at 0.2 m/s. This example shows how to obtain some more kilometers can be significant to find the right path to reach the target.

Finally, the influence of some algorithm parameters has been analyzed. If we reduce the division of the bearing rose from 20° to 5° in Init-Optim, the results are improved in a 4% at a cost of duplicating the computational cost. Similarly, if we use a search grid of double resolution in A*, the results are improved in a 2%, but the computational cost is 5 times higher.

IV. CONCLUSIONS

We have described a novel path planning algorithm for gliders based on combination of A* search and optimization process that offers promising results in realistic simulations. The pattern of displacement of the gliders (the bearing can not be changed while submerged) allows to easily adapt our method to problems where there is a temporal discretization, in which the size of each time window coincides with the duration of the stints. In addition, our method uses a continuous representation of the bearing space using an optimization method and eliminating the problems discussed. Furthermore, it incorporates an initialization phase that allows for obstacle avoidance, at a negligible computational time penalty. This

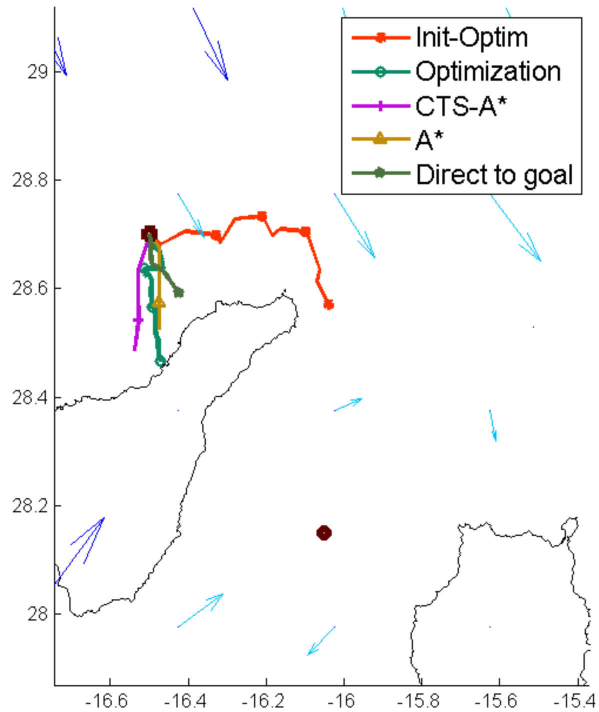

Fig. 6. Two comparatives of trajectories simulated for a 4-days period with time-varying currents (ocean currents that exceed the glider speed $v_g = 0.2\mathrm{m/s}$ are highlighted) from the start point to the goal point. Top graphic: Total Distance = 75.2 km. Distance remaining to reach the goal point: Init-Optim: 46.7 km; Optimization: 54.2 km (stop in land); CTS-A*: 60.7 km (stop in land); A*: 58.9 km (stop in land); Direct to goal: 61.3 km (stop in land).

heuristic-guided process generates a coarse initial solution that is then refined using optimization. In sum, our method is suitable for dynamic scenarios with obstacles or forbidden areas, making it a practical tool for coastal environments.

The method shows a superior performance when compared with other alternative approaches. In general, classical A* or variants, like the CTS-A* algorithm analyzed here, do not find a path better than iterative optimization methods. Notice that an improvement of $5\text{-}10\mathrm{km}$ in trajectories of three days can have a huge impact in many glider missions, e.g. it might reduce the economical cost of data collection after the mission of several weeks. Anyhow, it is in the computational cost where the latter clearly outperforms the former.

Finally, the solution presented in this paper is valid for this particular problem, but would not have the same benefits if it is applied in the path planning of other kind of vehicle because there is not a temporal discretization in their pattern of displacement.

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