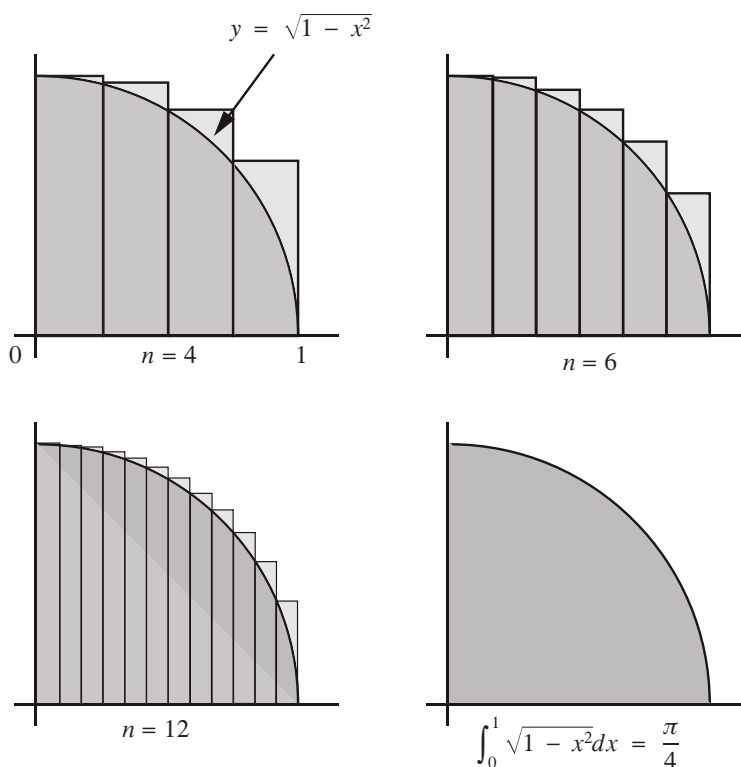


**106.24 Proof without words: a Riemann sum**

Claim:  $\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \frac{\sqrt{n^2 - k^2}}{n^2} = \frac{\pi}{4}$ .

*Proof:* The limit may be seen as a Riemann sum with a geometric meaning.



$$\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \frac{\sqrt{n^2 - k^2}}{n^2} = \lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \frac{\sqrt{1 - (\frac{k}{n})^2}}{n} = \int_0^1 \sqrt{1 - x^2} dx = \frac{\pi}{4}$$

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