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Applications of 3-D Automatic Triangulations for Wind Field Simulation

R. Montenegro, G. Montero, J.M. Escobar, E. Rodríguez and J.M. González-Yuste*

University Institute for Intelligent Systems and Numerical Applications in Engineering
University of Las Palmas de Gran Canaria, Spain

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http://www.dca.iusiani.ulpgc.es/proyecto0507/html

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- **Mesh optimisation: Untangling and smoothing**
- **Local refinement of tetrahedral meshes**
- **Wind field modelling and genetic algorithms**
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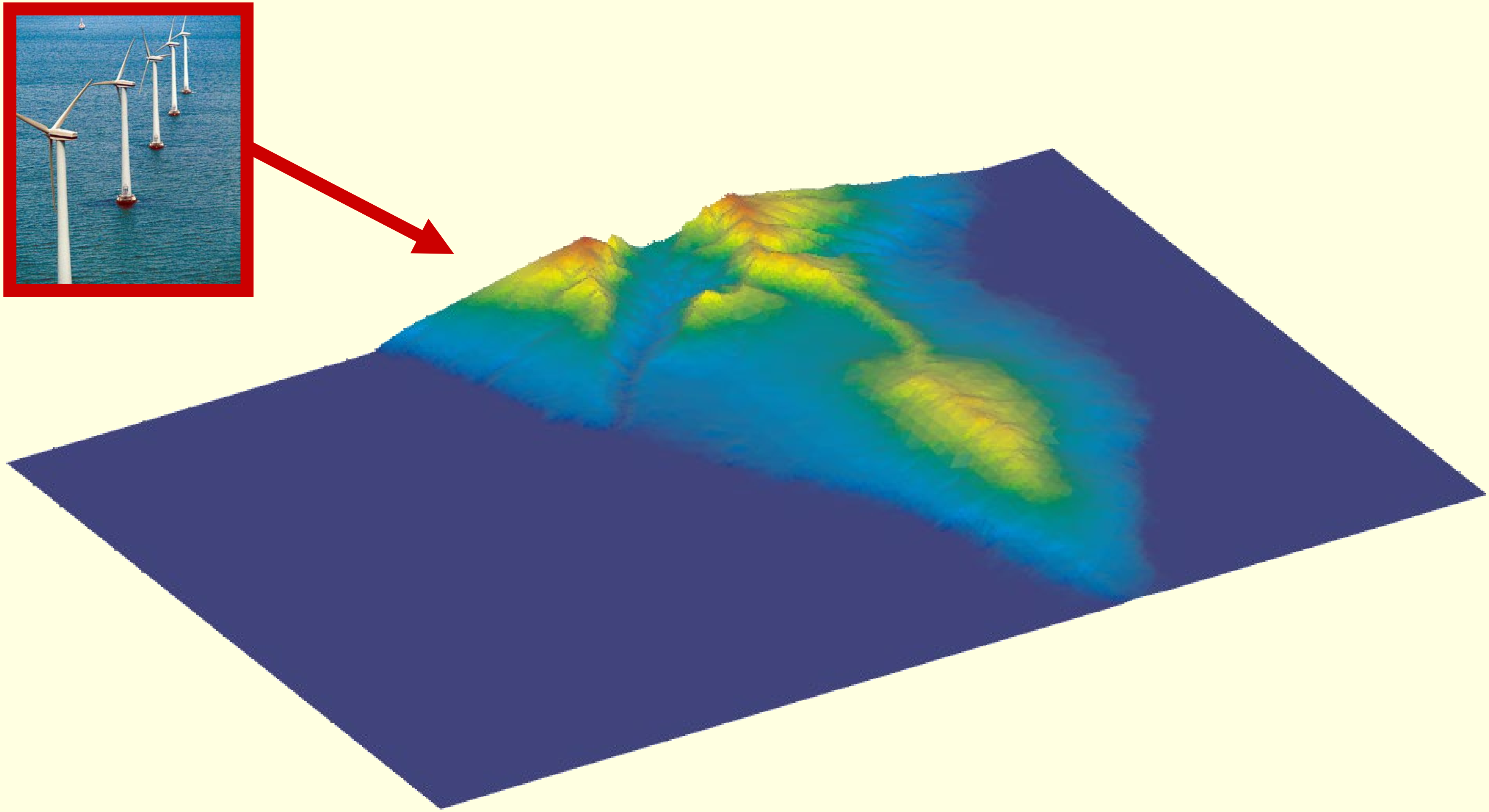
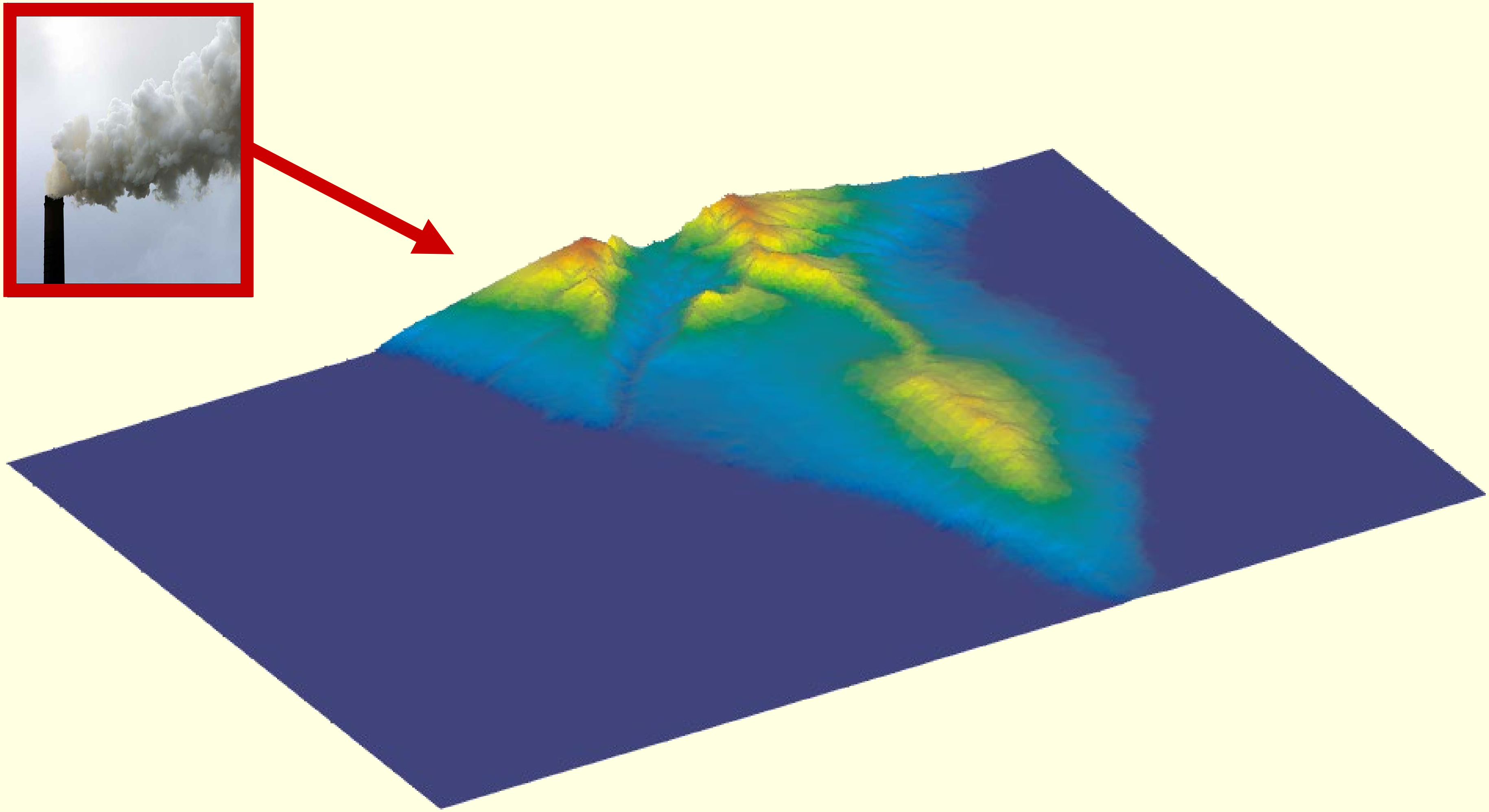
- **Conclusions and future research**

Motivation: Simulation of Wind Field



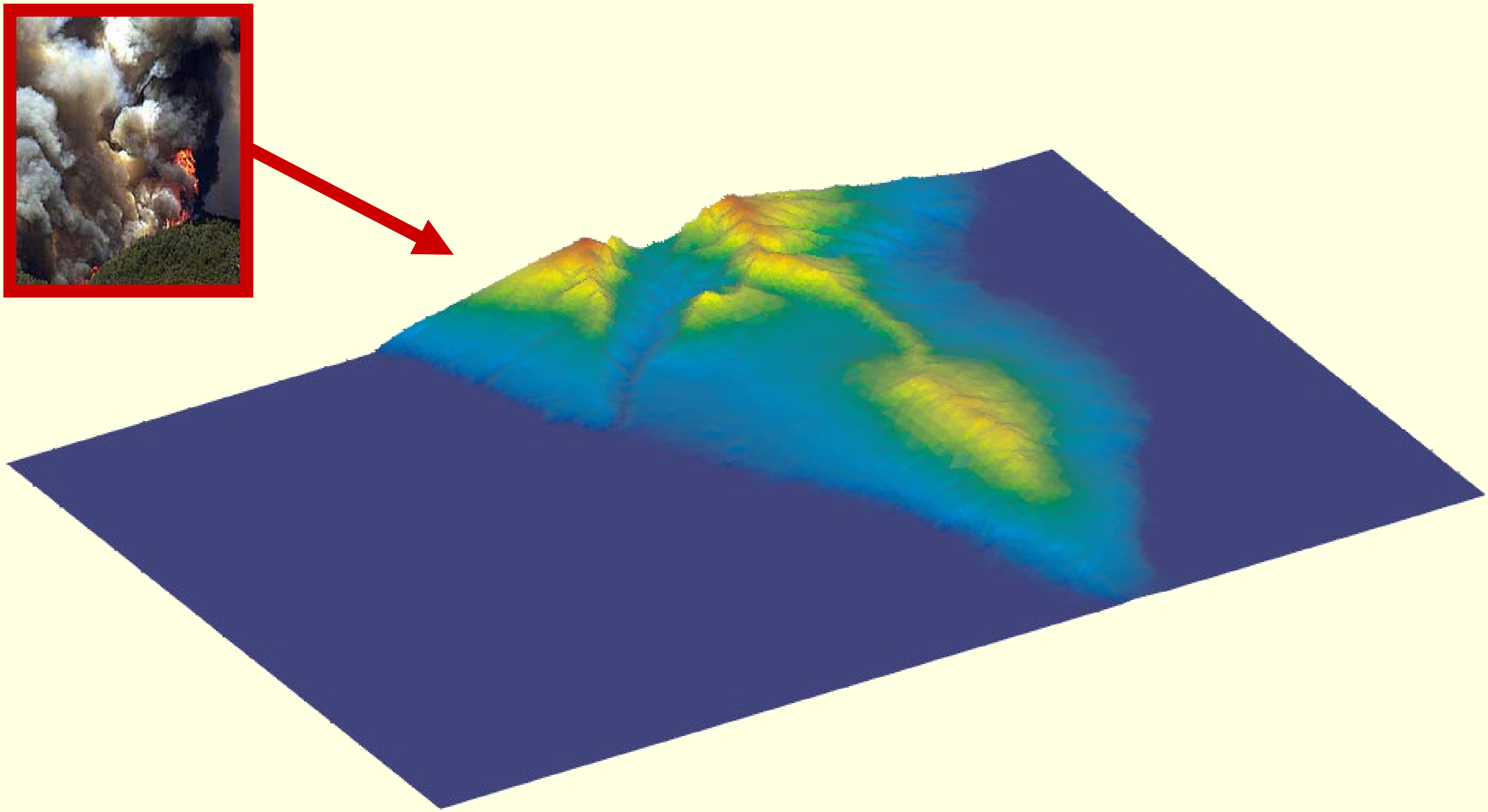
***South of Isla de La Palma (Canary Island):* Terrain surface obtained by digitalisation**

Motivation: Simulation of Atmospheric Pollution



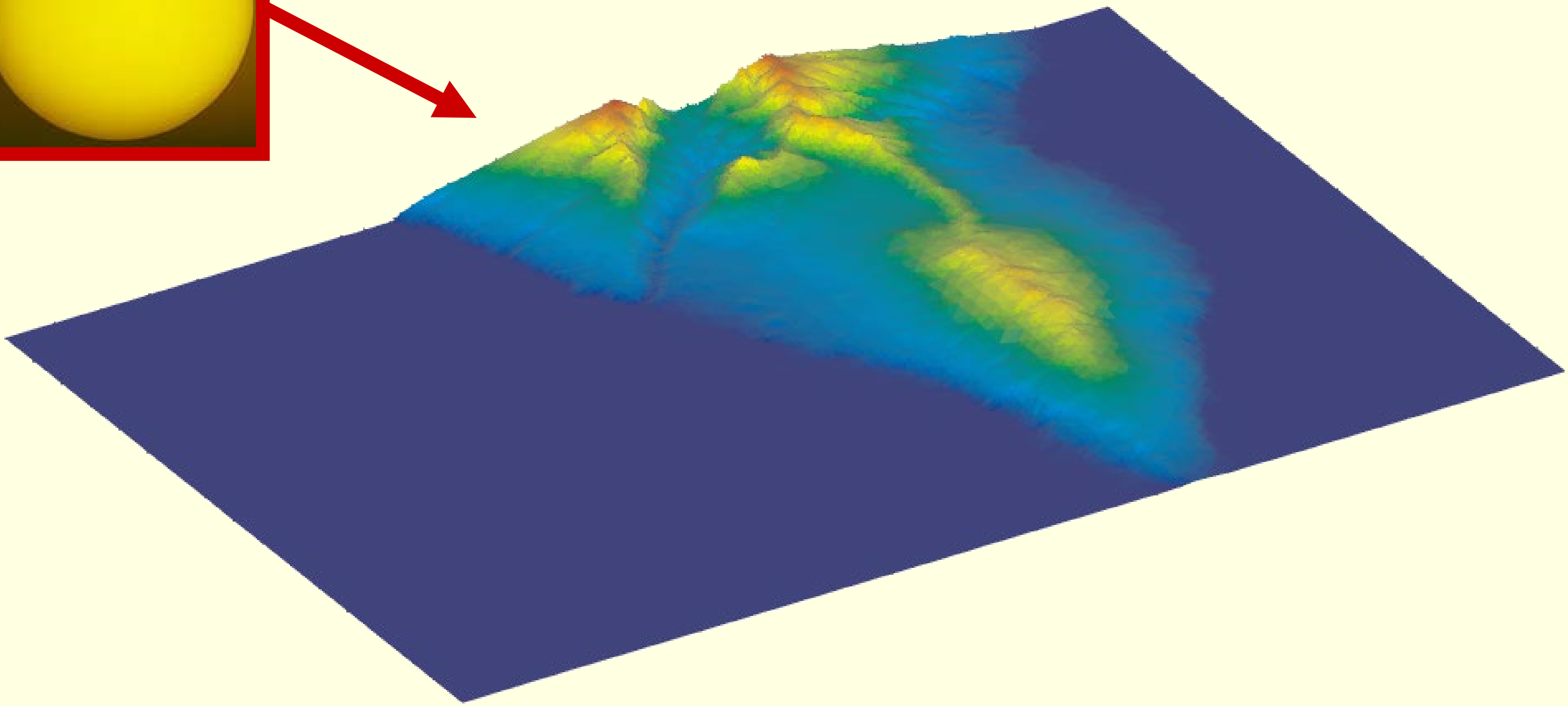
South of Isla de La Palma (Canary Island): ***Terrain surface obtained by digitalisation***

Motivation: Simulation of Fire Forest



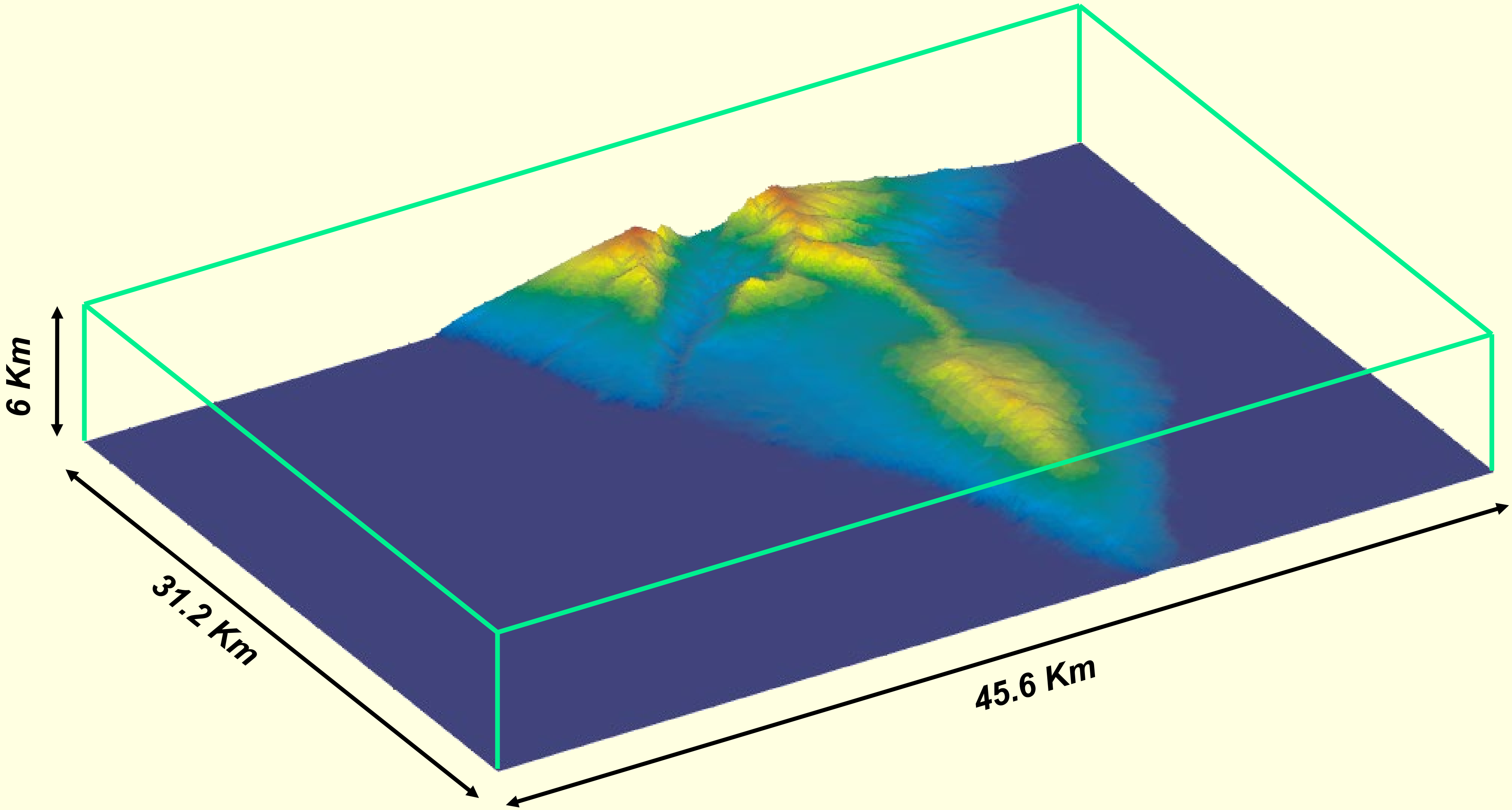
South of Isla de La Palma (Canary Island): ***Terrain surface obtained by digitalisation***

Motivation: Sunshine



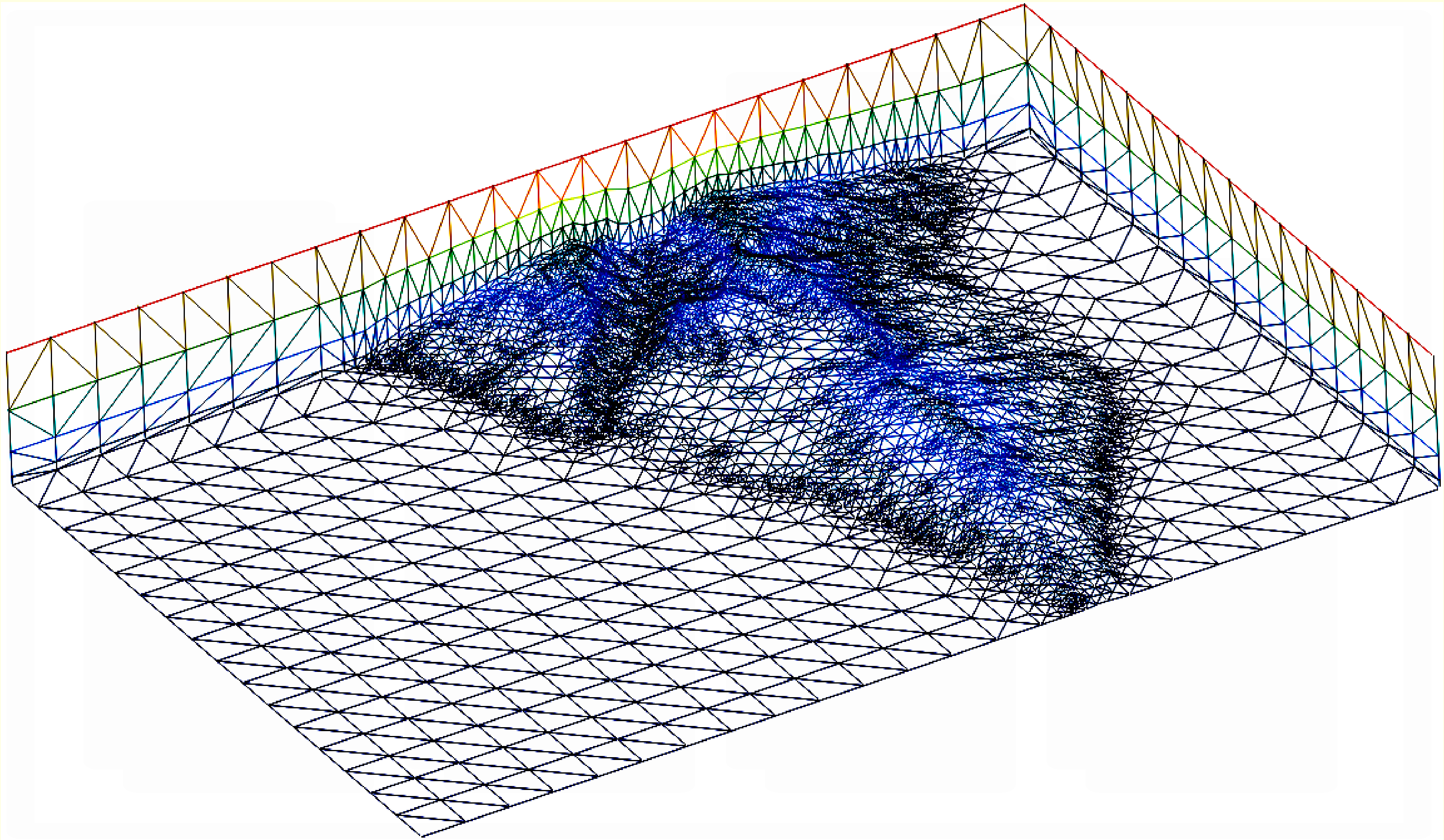
South of Isla de La Palma (Canary Island): Terrain surface obtained by digitalisation

Motivation: 3-D Domains over Complex Terrains

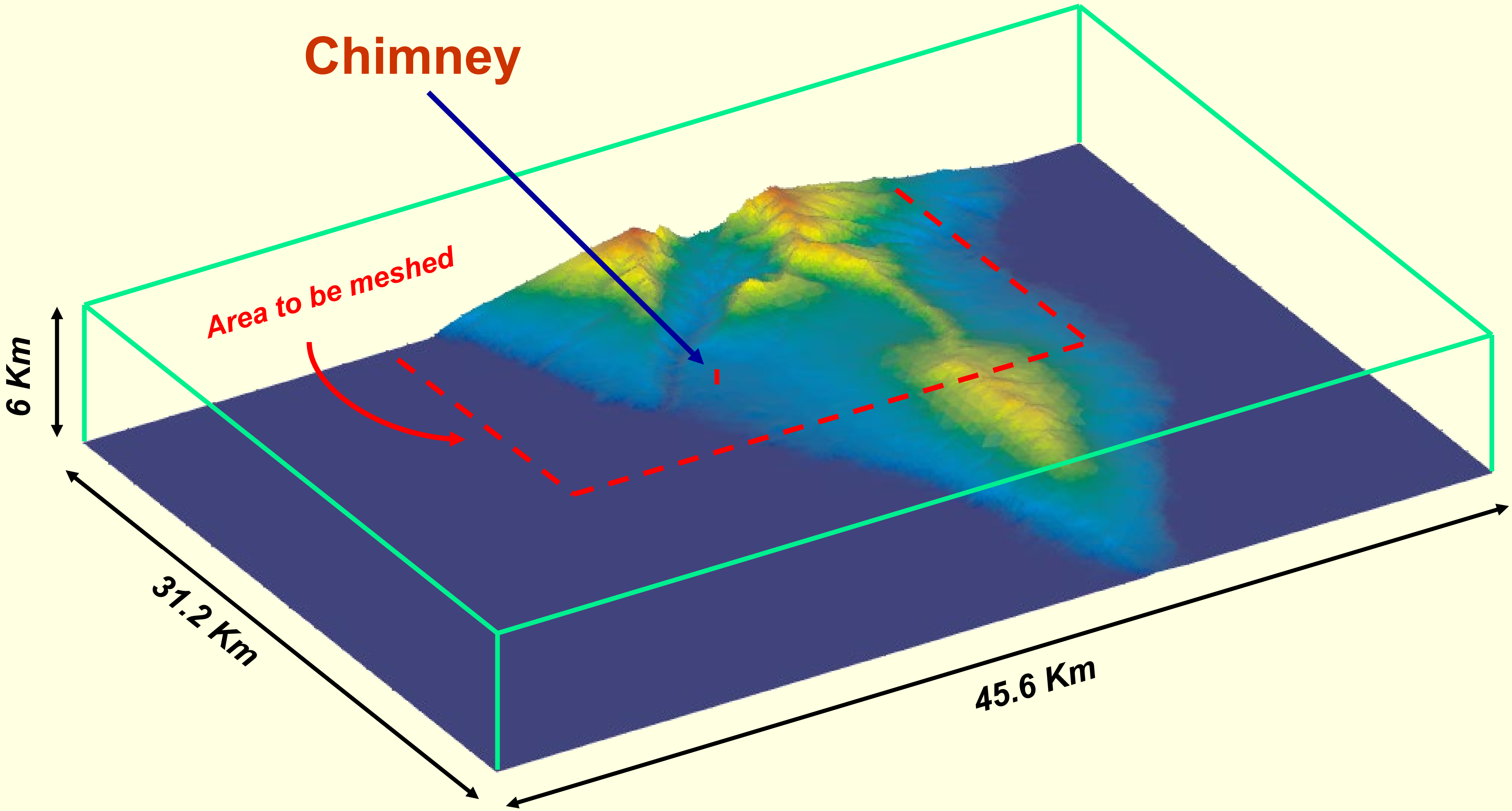


South of Isla de La Palma (Canary Island): ***Terrain surface obtained by digitalisation***

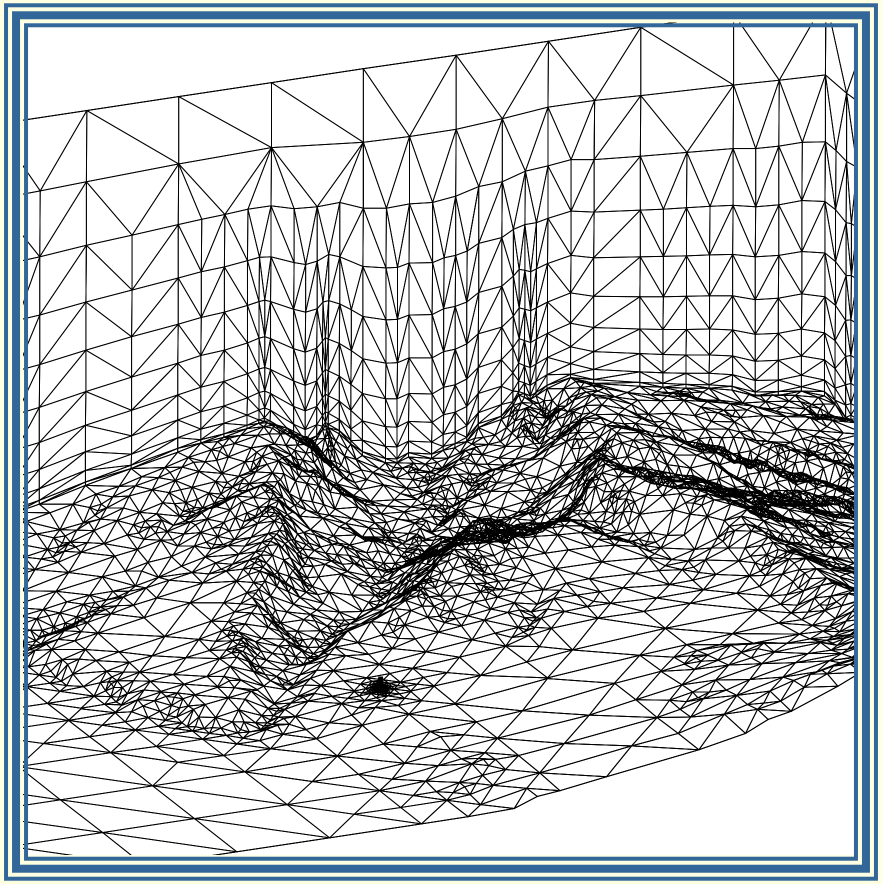
Motivation: Tetrahedral Mesh Generation

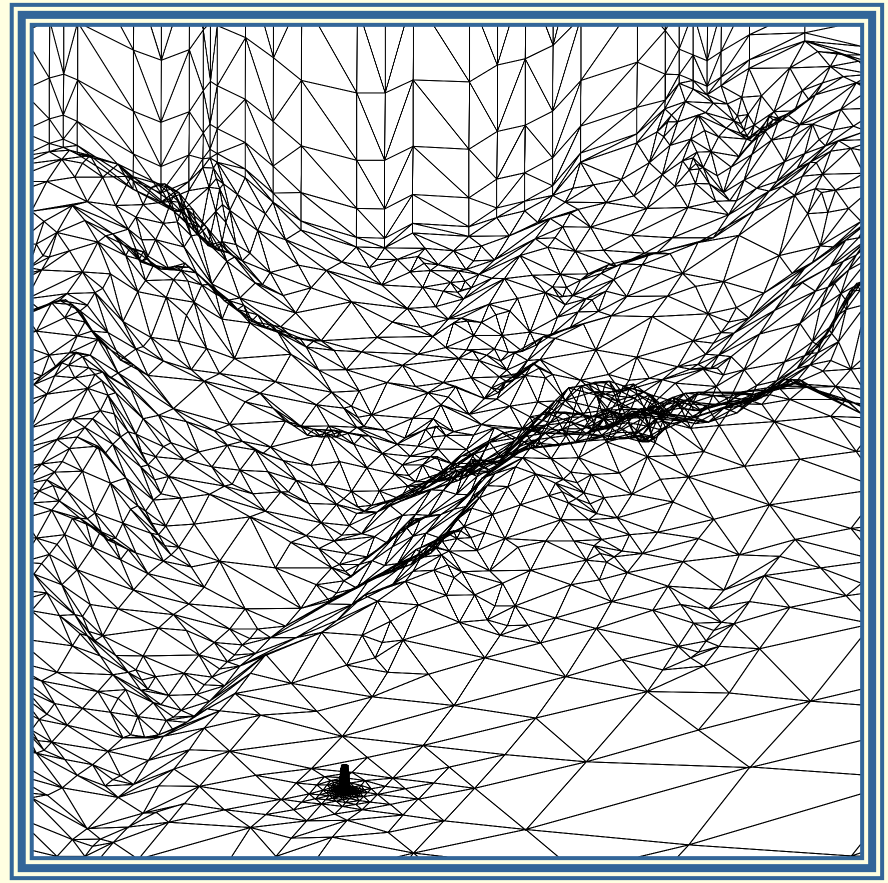


Motivation: Problem Increases with Domain Complexity



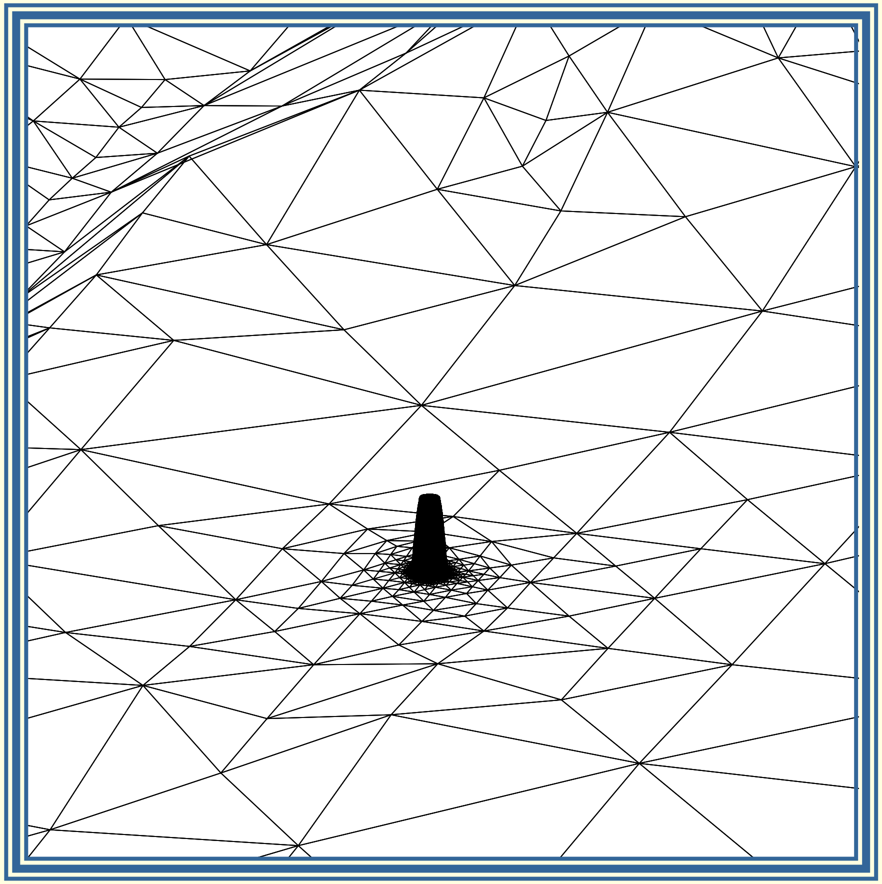
Motivation: Problem Increases with Domain Complexity

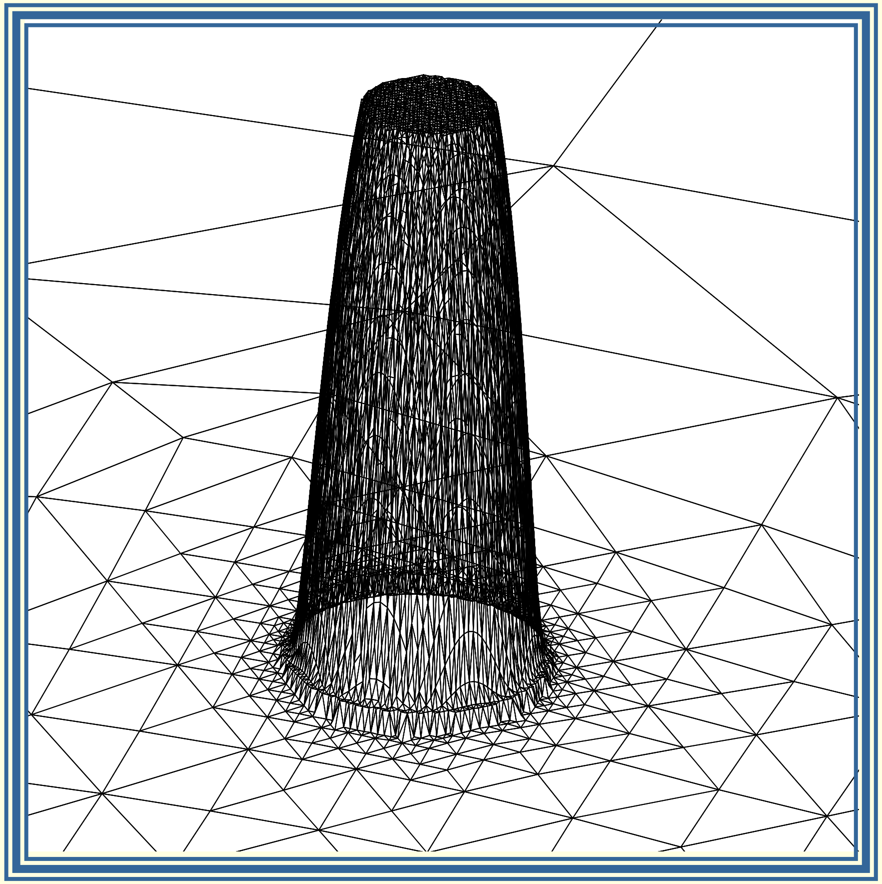




- ☐ ***Maximum size of mesh edges: 2 Km***
- ☐ ***Minimum size of mesh edges: 2 m***

Motivation: Problem Increases with Domain Complexity





- ☐ ***Maximum size of mesh edges: 2 Km***
- ☐ ***Minimum size of mesh edges: 2 m***

Main Stages of the Mesh Generation

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Adaptive discretization of the terrain surface

(2-D refinement/derefinement algorithm)

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Determination of the set of points
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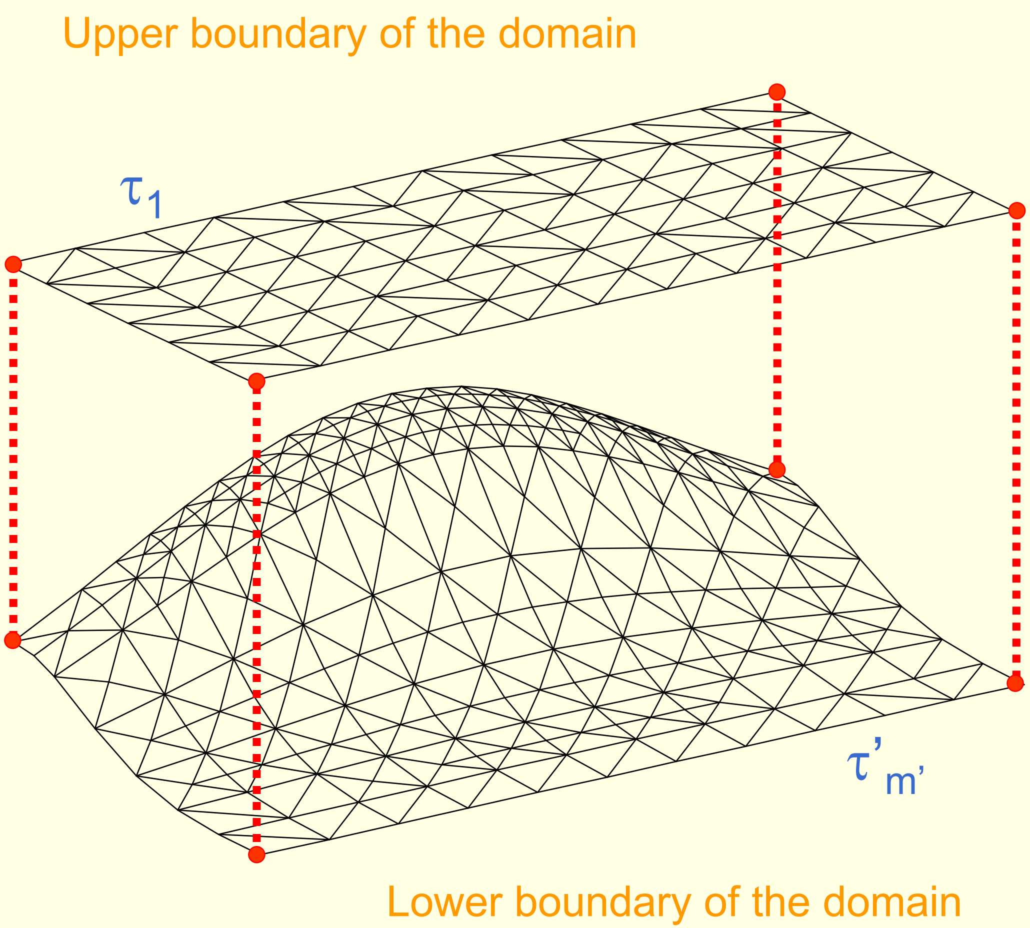
Mesh optimisation
(Simultaneous Untangling and smoothing)

Determination of the Set of Points



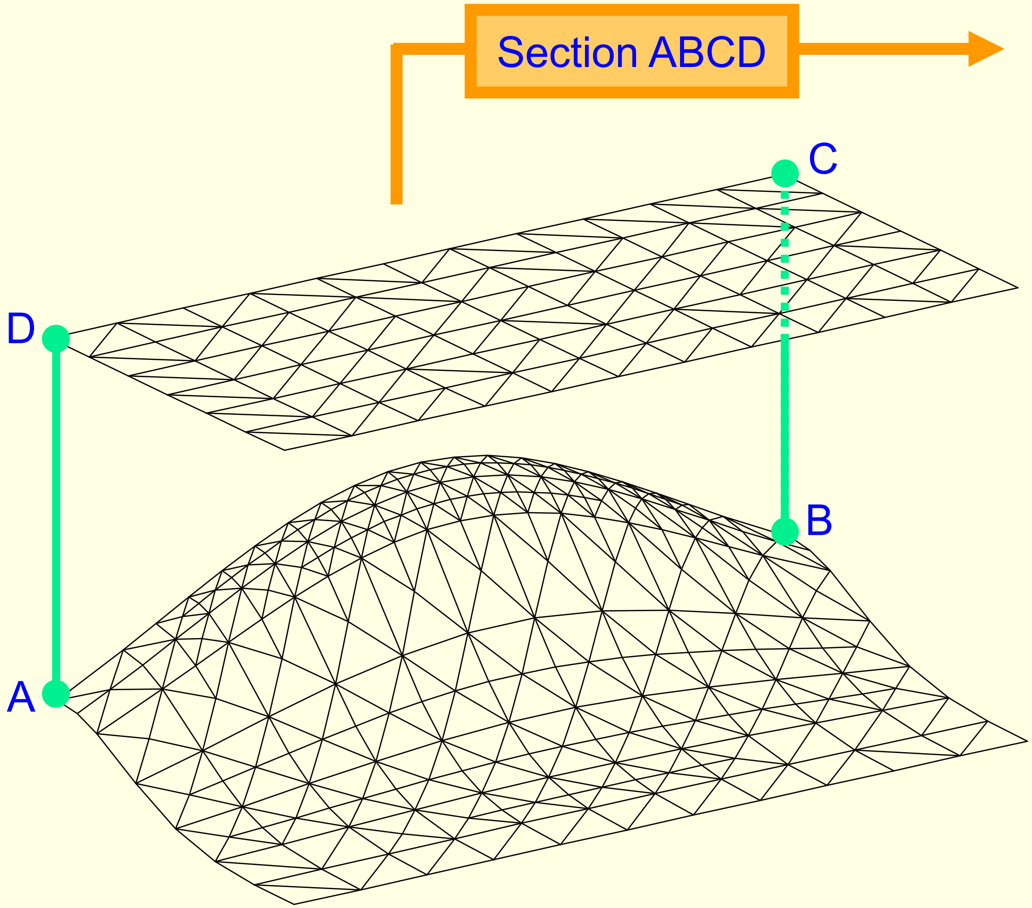
Three sub-sets:

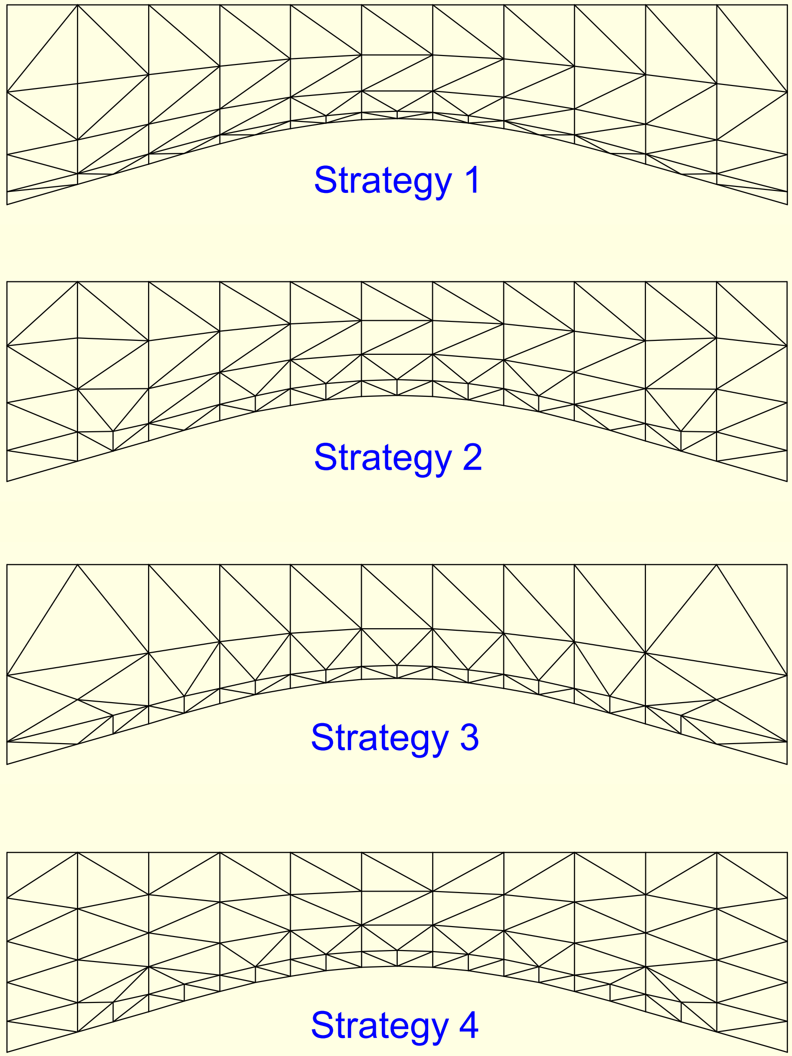
1) Uniform distribution of τ_1 for the upper boundary

2) Adaptive distribution of $\tau'_{m'}$ for the lower boundary

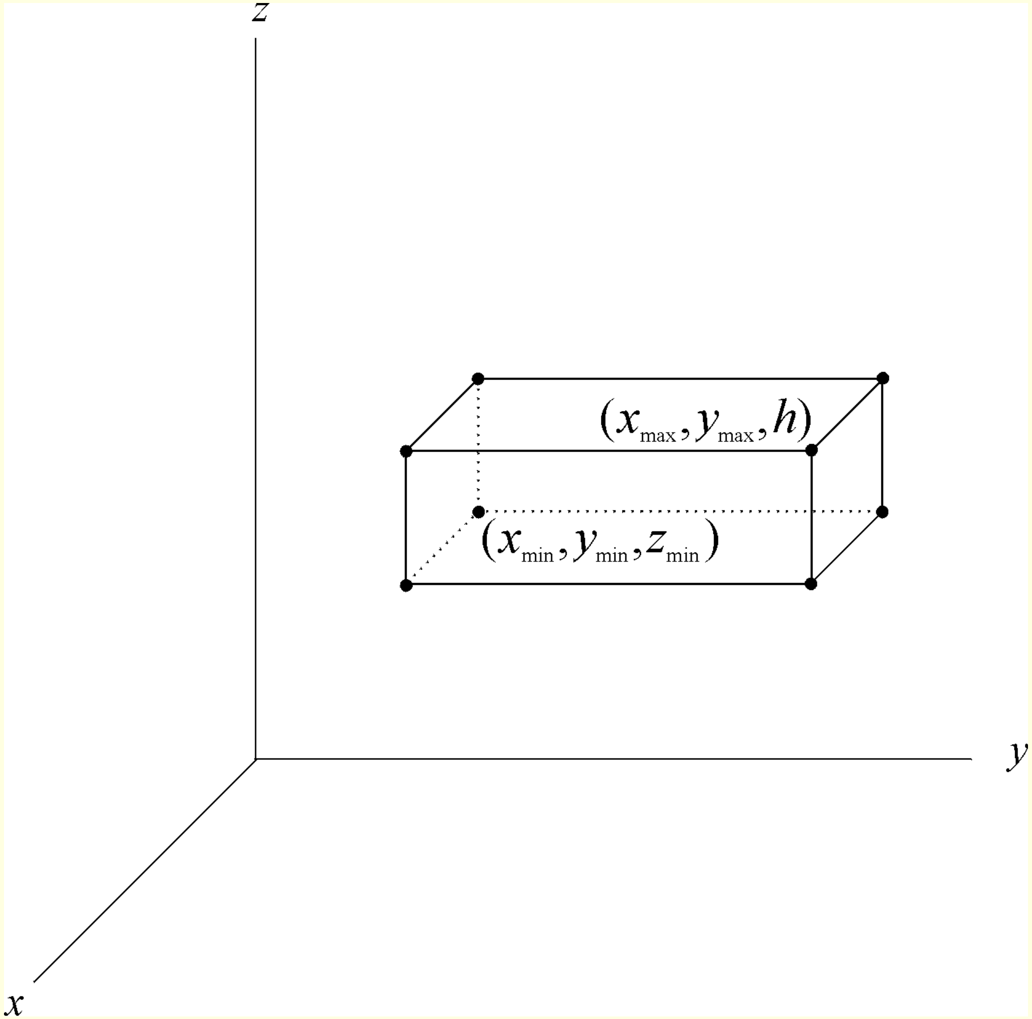
3) Distribution between both layers attending to the vertical spacing function

Determination of the Set of Points: Strategies





Generation of the 3-D Mesh



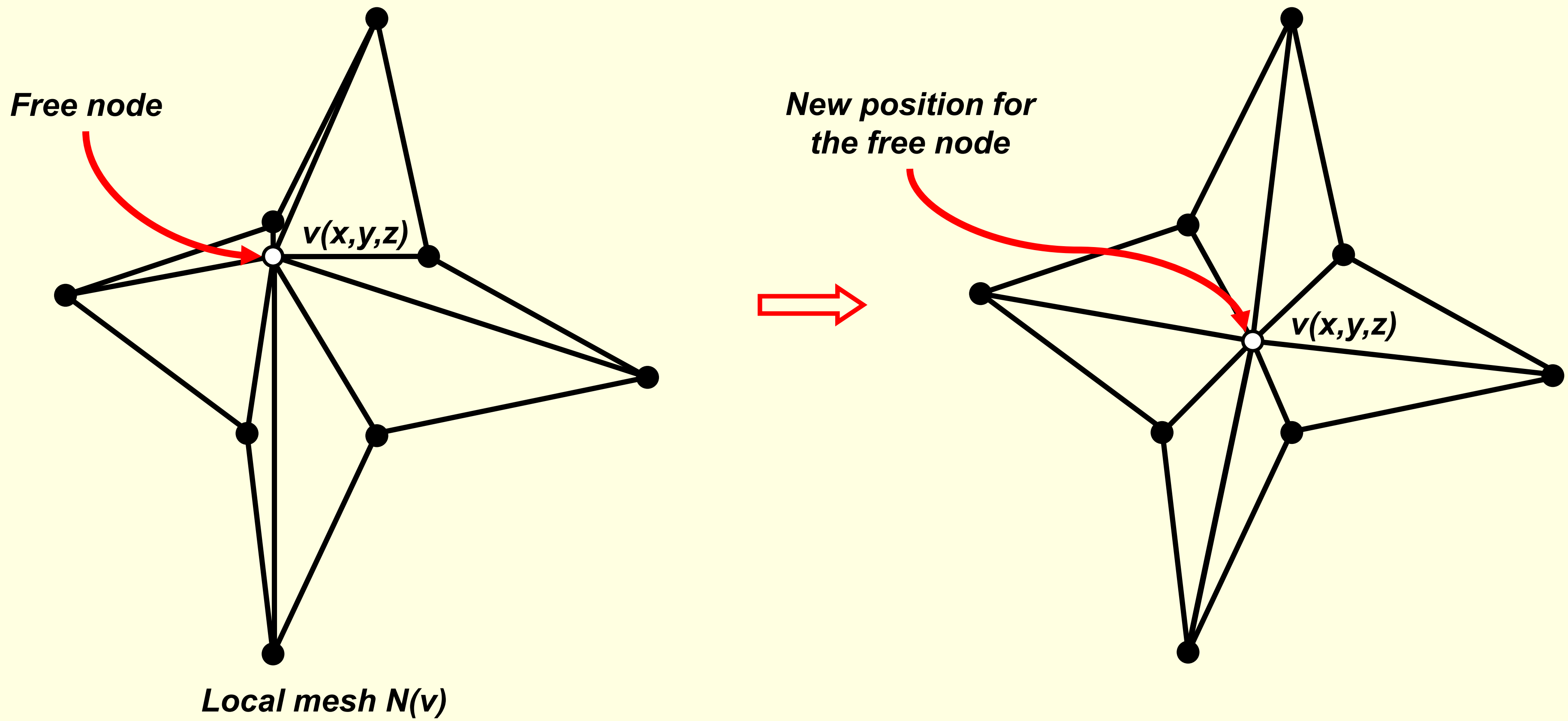
- Transformation of real domain to an auxiliary parallelepiped
- Delaunay triangulation in the parallelepiped
- Inverse transformation to the real domain (compression of the mesh)

- Advantage: Mesh conforming with the terrain surface
- Disadvantage: Possibility of mesh untangling

Simultaneous Untangling and Smoothing

Local optimisation

***Objective:* Improve the quality of the local mesh N(v) by minimising an objective function**



Iterative procedure over all the nodes of the mesh

Simultaneous Untangling and Smoothing

Usual Objective Function for the Local Mesh *N(v)*

$$\left|K_\eta\right|_p(\mathbf{x}) = \left[\sum_{m=1}^{M} \eta_m^p(\mathbf{x})\right]^{\frac{1}{p}}$$

$\mathbf{x}$ **is the position vector of the free node *v***

M **is the number of tetrahedra of *N(v)*** p **is 1 or 2, generally**

$\eta_m = \frac{1}{q_m}$ **is the objective function for the *m*-th tetrahedron of *N(v)***

$0 \le q_m \le 1$ **is an algebraic quality measure of the *m*-th tetrahedron of *N(v)***

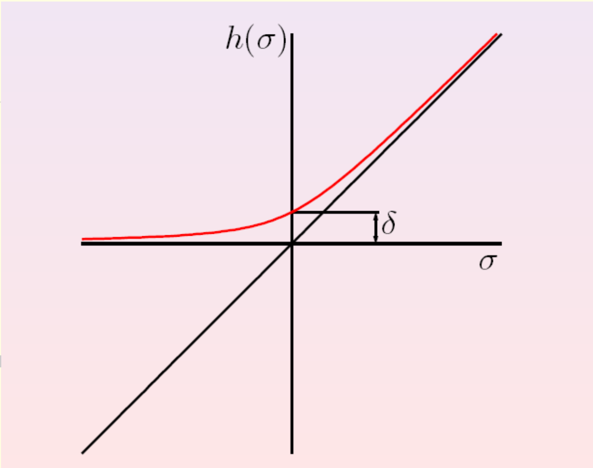
Simultaneous Untangling and Smoothing

Usual Objective Function for the Local Mesh *N(v)*

$$\left|K_\eta\right|_p(\mathbf{x}) = \left[\sum_{m=1}^{M} \eta_m^p(\mathbf{x})\right]^{\frac{1}{p}}$$

Original function:

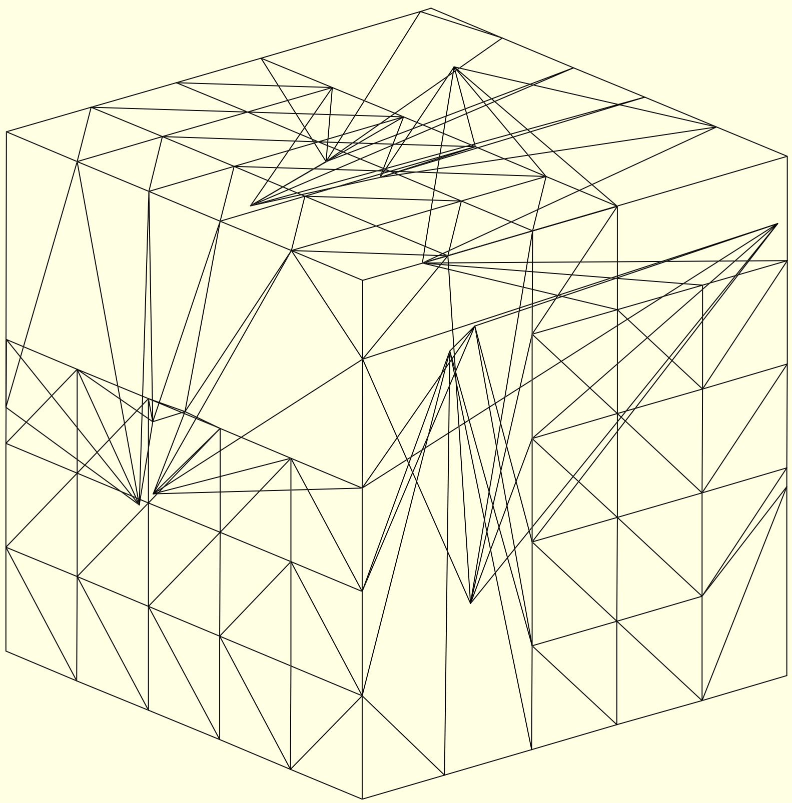
$$\eta_m = \frac{|S_m|^2}{3\sigma_m^{\frac{2}{3}}}$$



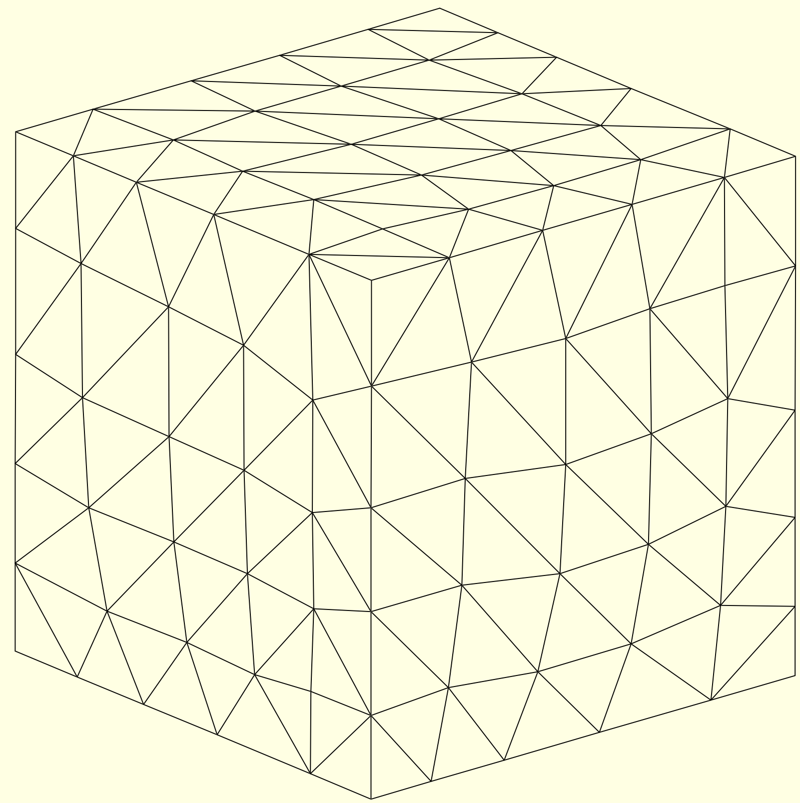
Modified function:

$$\eta_m = \frac{|S_m|^2}{3[h(\sigma_m)]^{\frac{2}{3}}}$$

Mesh Optimisation: Example 1

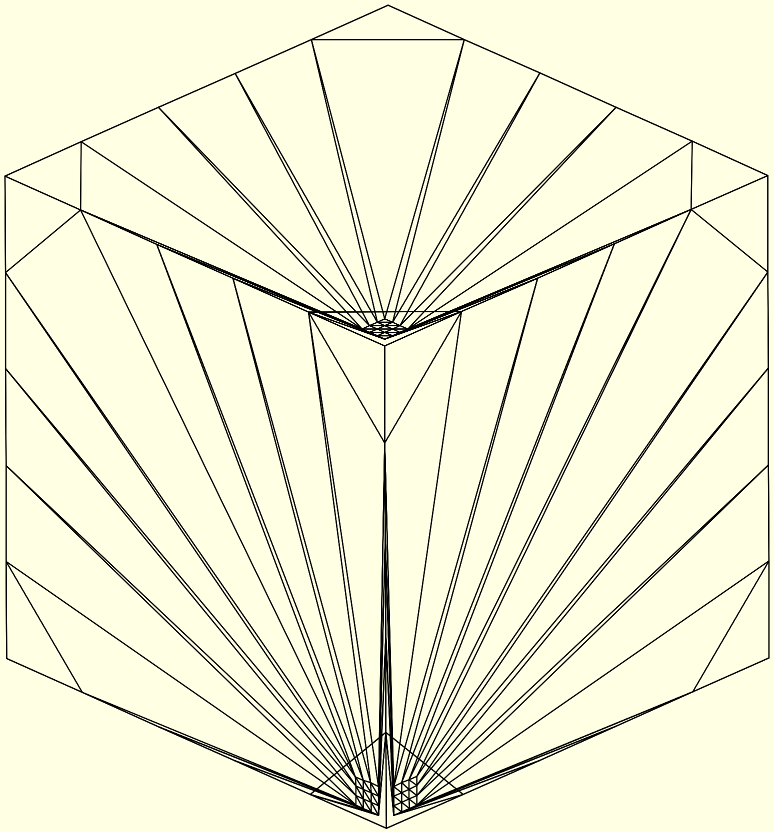


Before

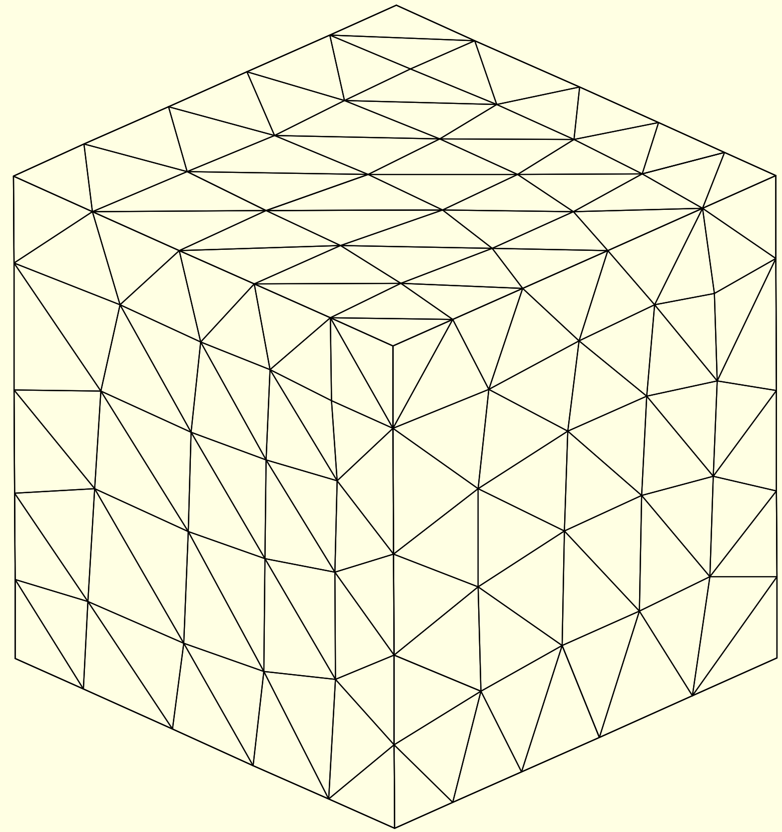


After

Mesh Optimisation: Example 2

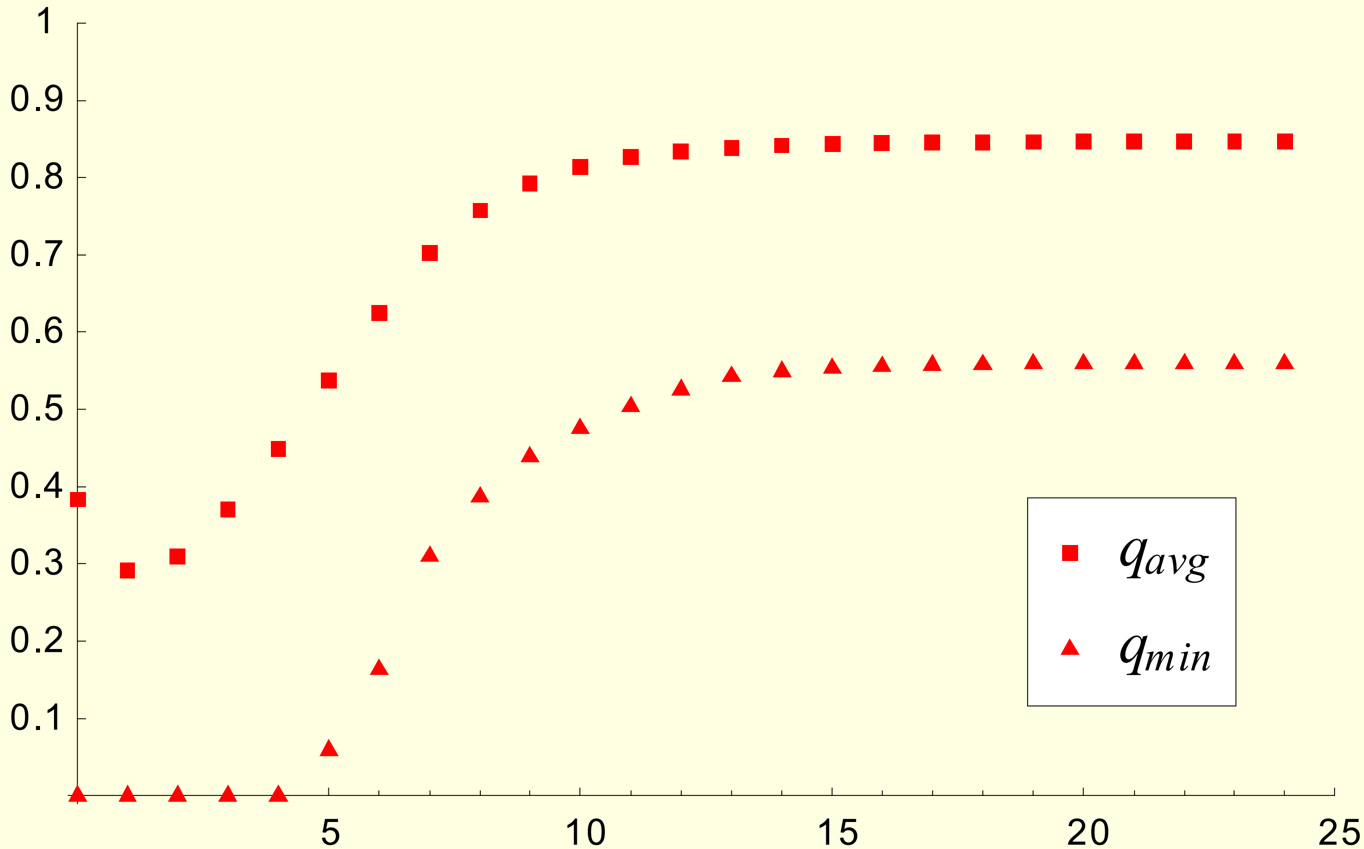


Before



After

Mesh Optimisation: Example 2

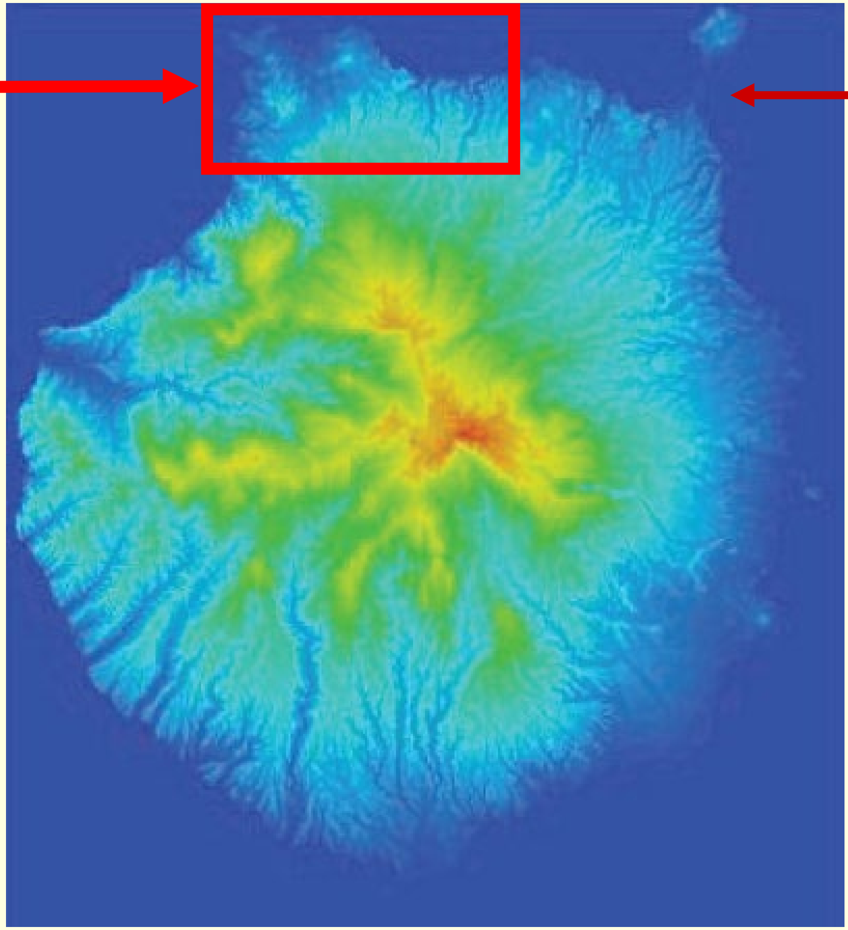


Evolution of the average quality q_{avg} and minimal quality q_{min}

in terms of the number of iterations of the optimisation process

Mesh Optimisation: Example 3

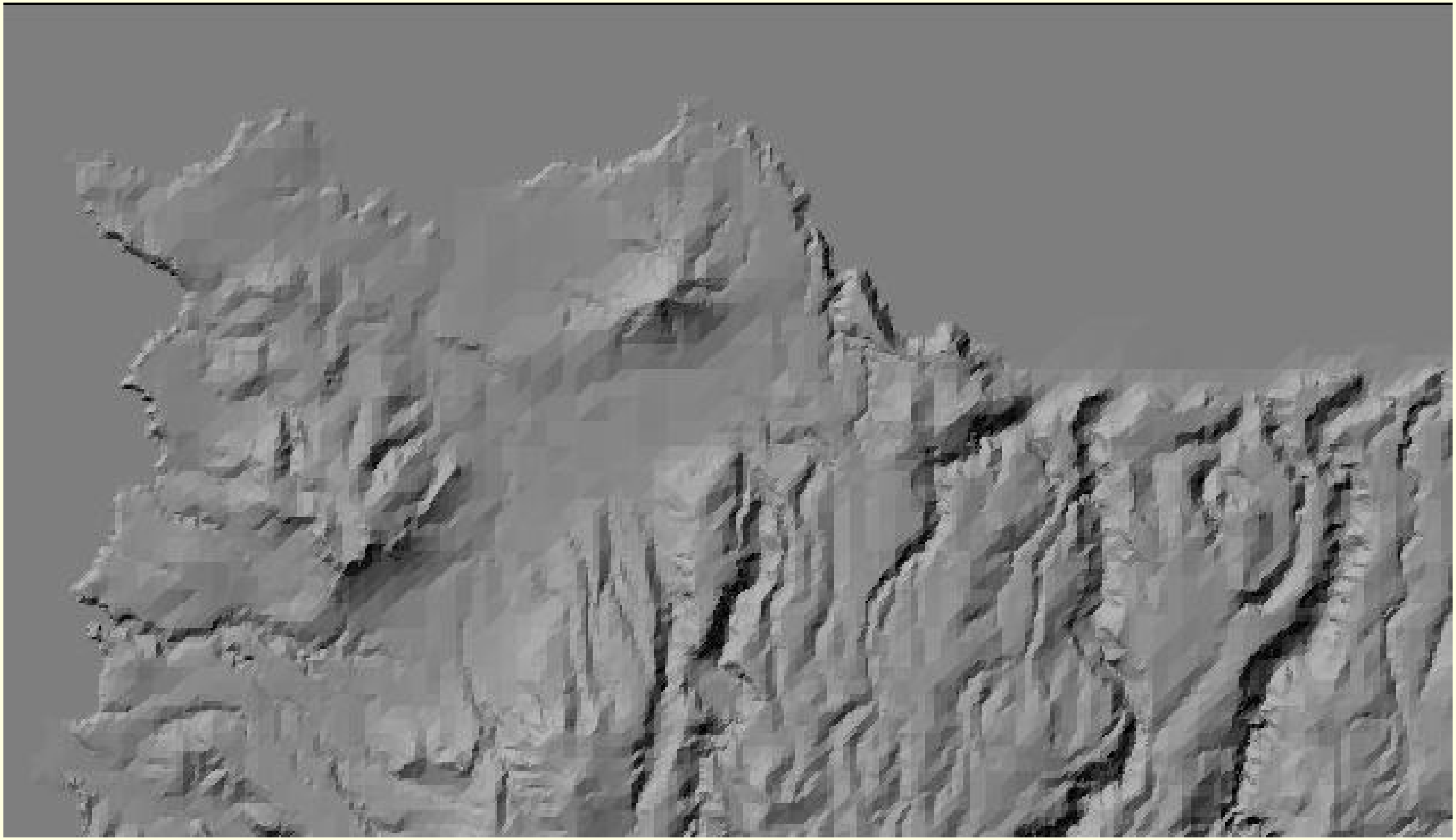
Area to be meshed



Las Palmas de Gran Canaria

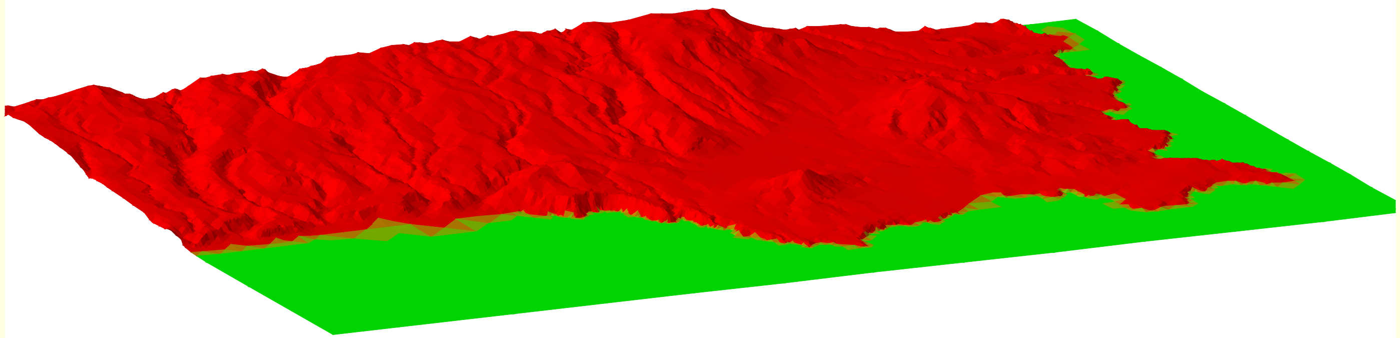
Isla de Gran Canaria

Mesh Optimisation: Example 3



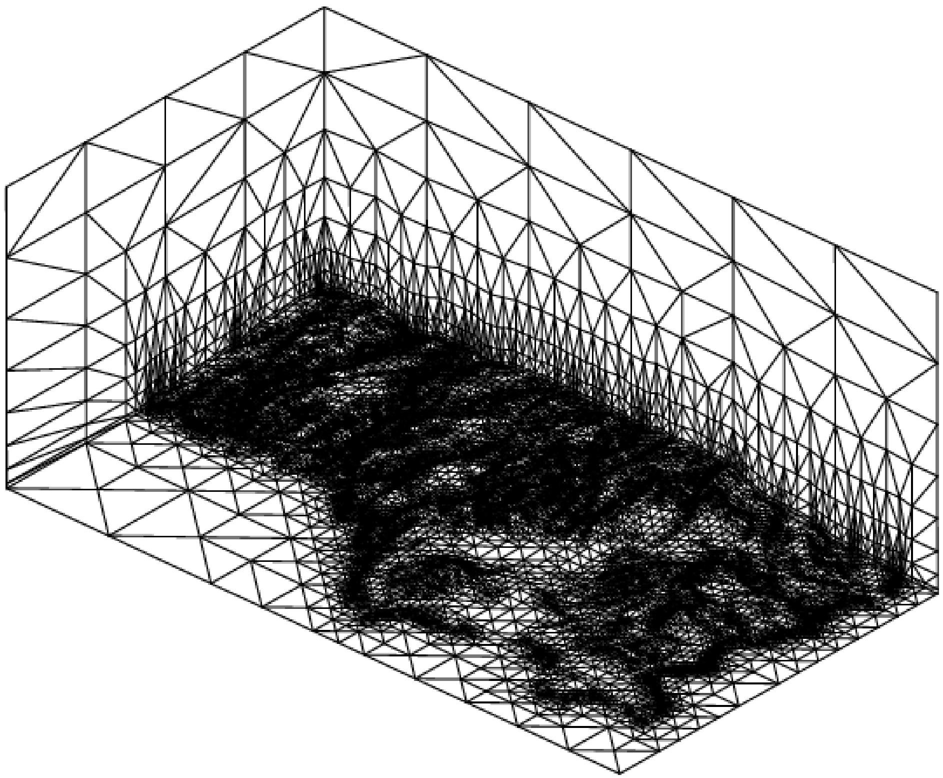
NW of *Isla de Gran Canaria*

Mesh Optimisation: Example 3

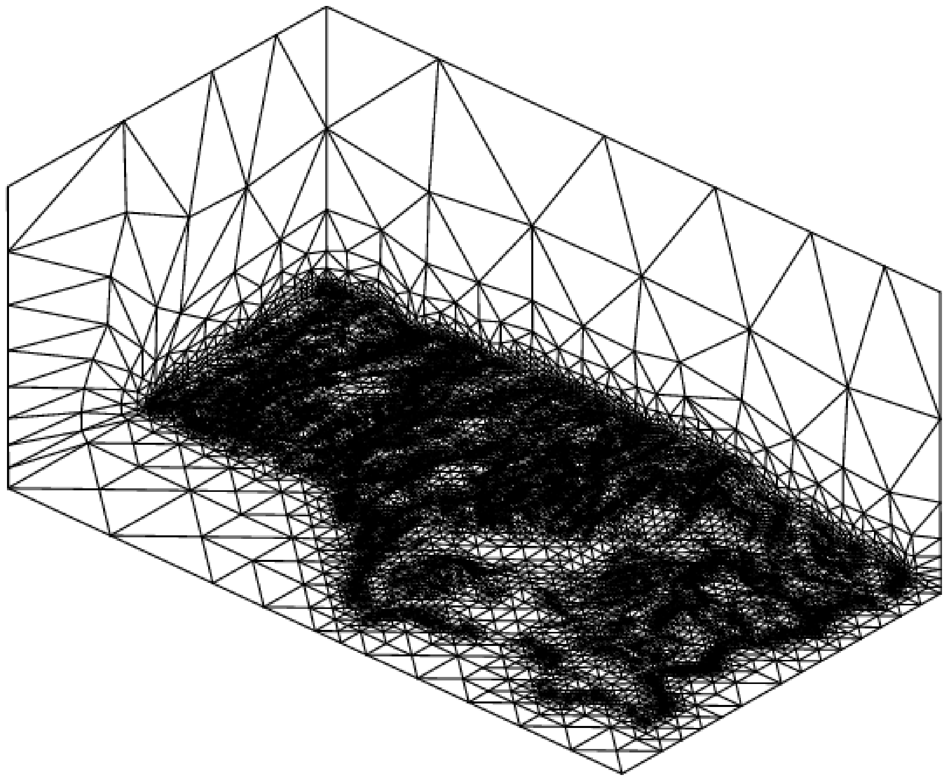


NW of Isla de Gran Canaria

Mesh Optimisation: Example 3



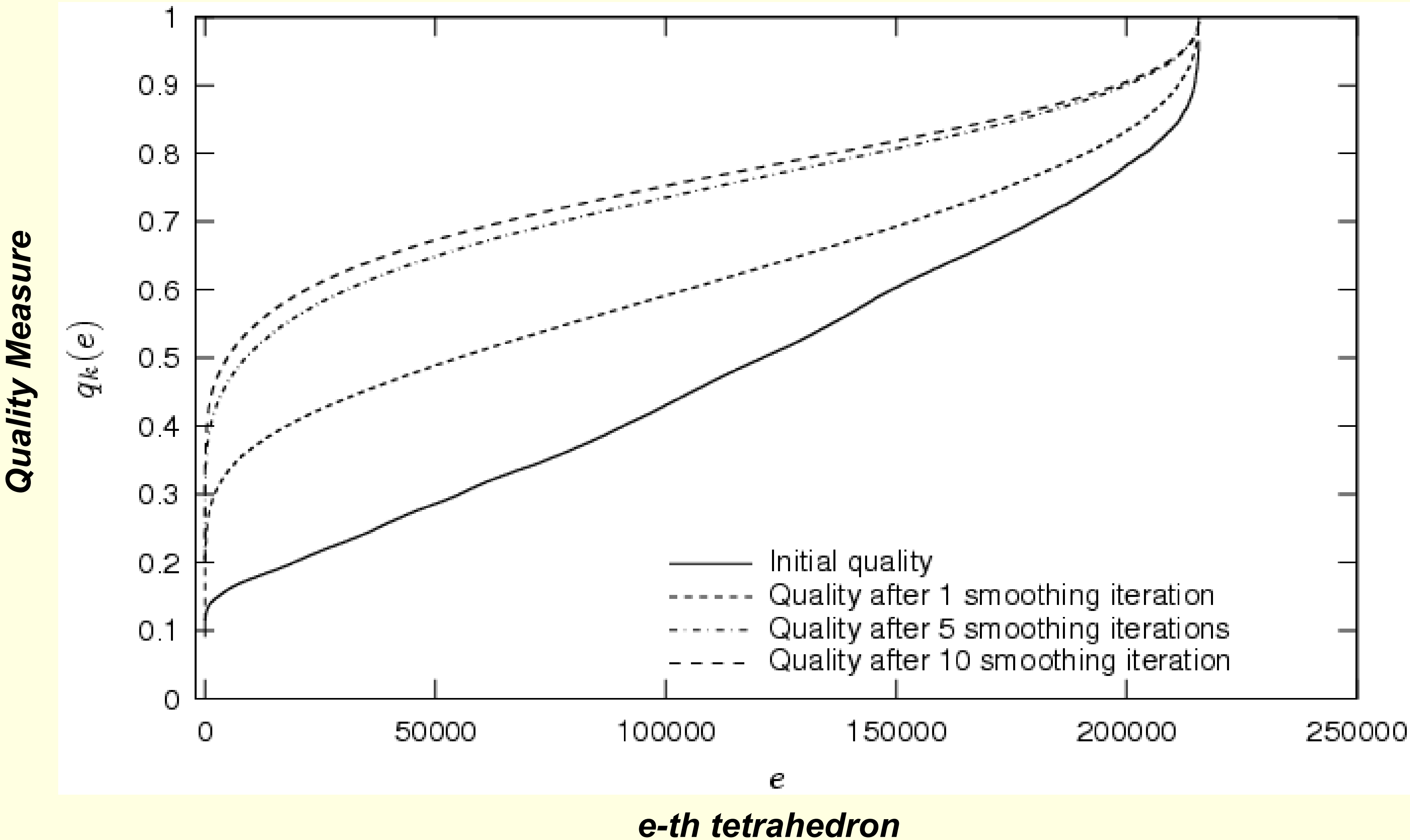
Before



After

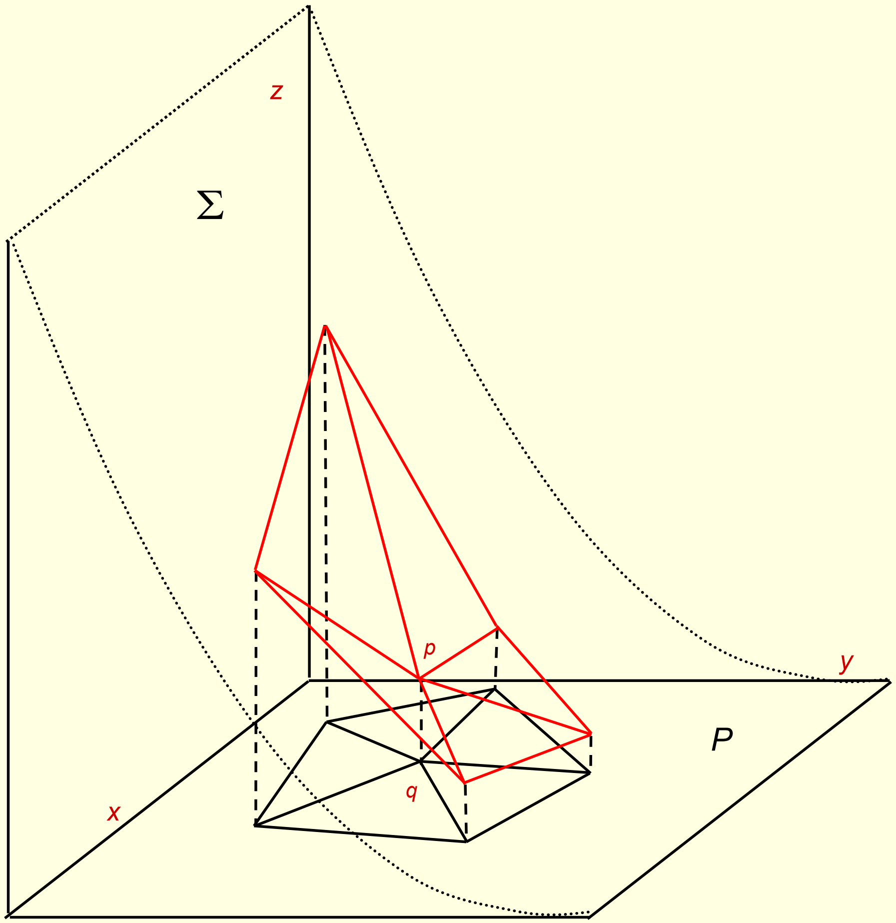
NW of *Isla de Gran Canaria*

Mesh Optimisation: Example 3

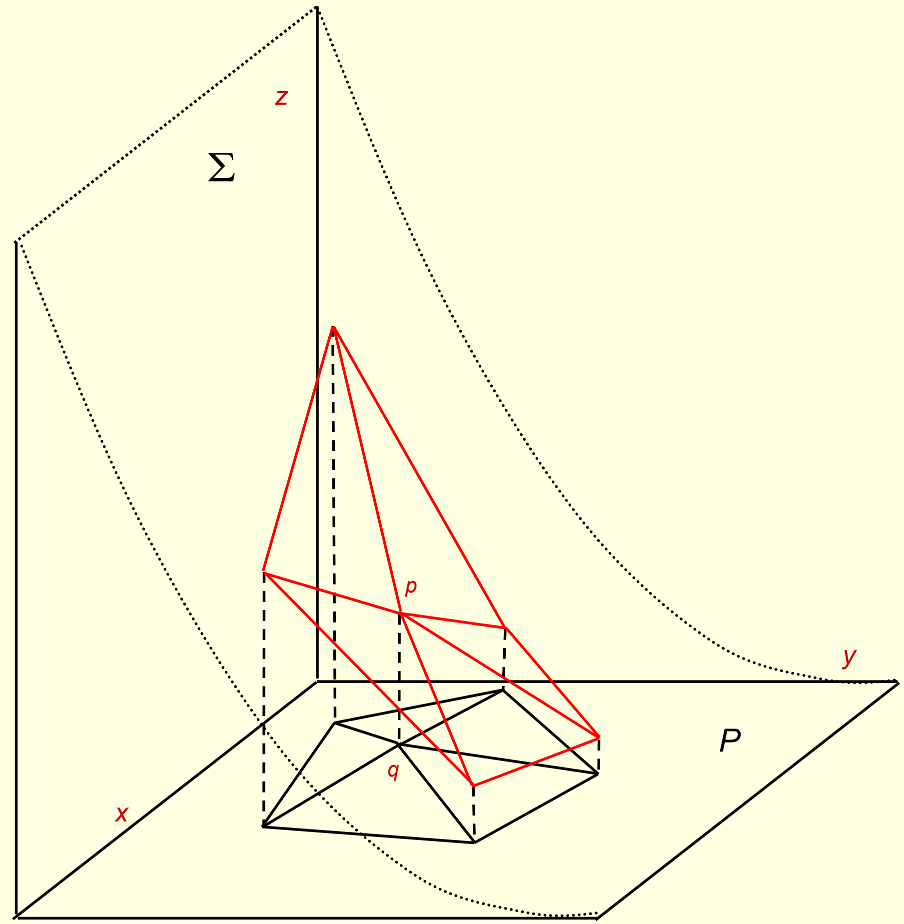


Quality curves for the rectangular area of the NW of *Isla de Gran Canaria*

Smoothing Surface Triangulations on a Plane

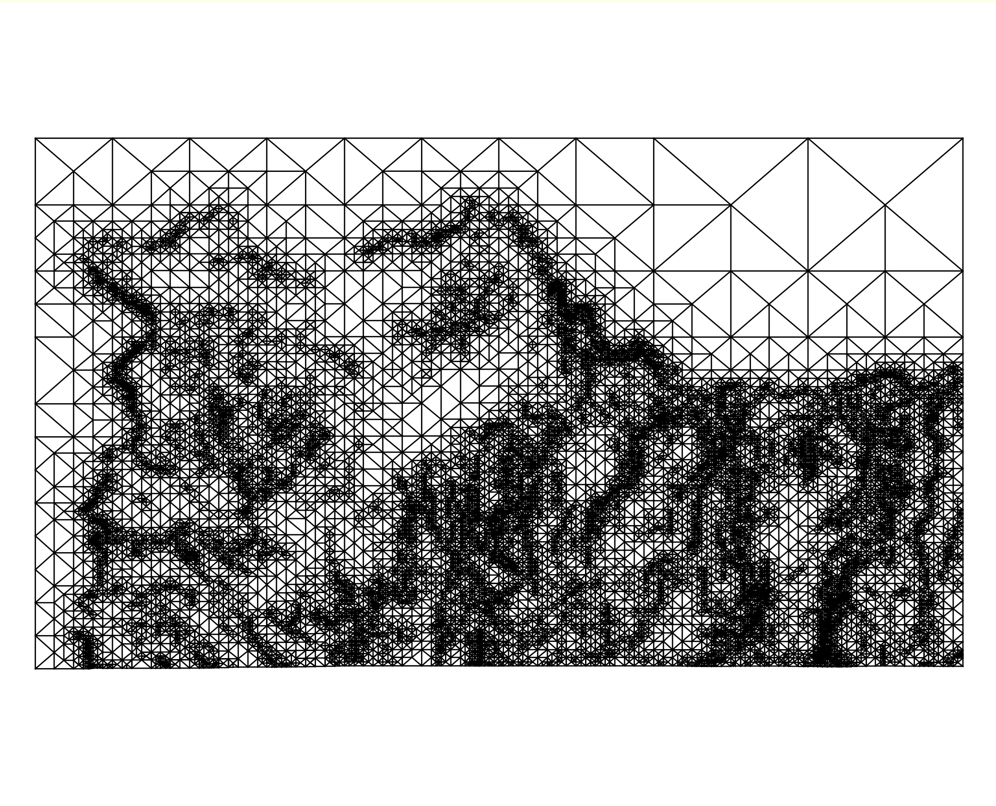


Initial mesh

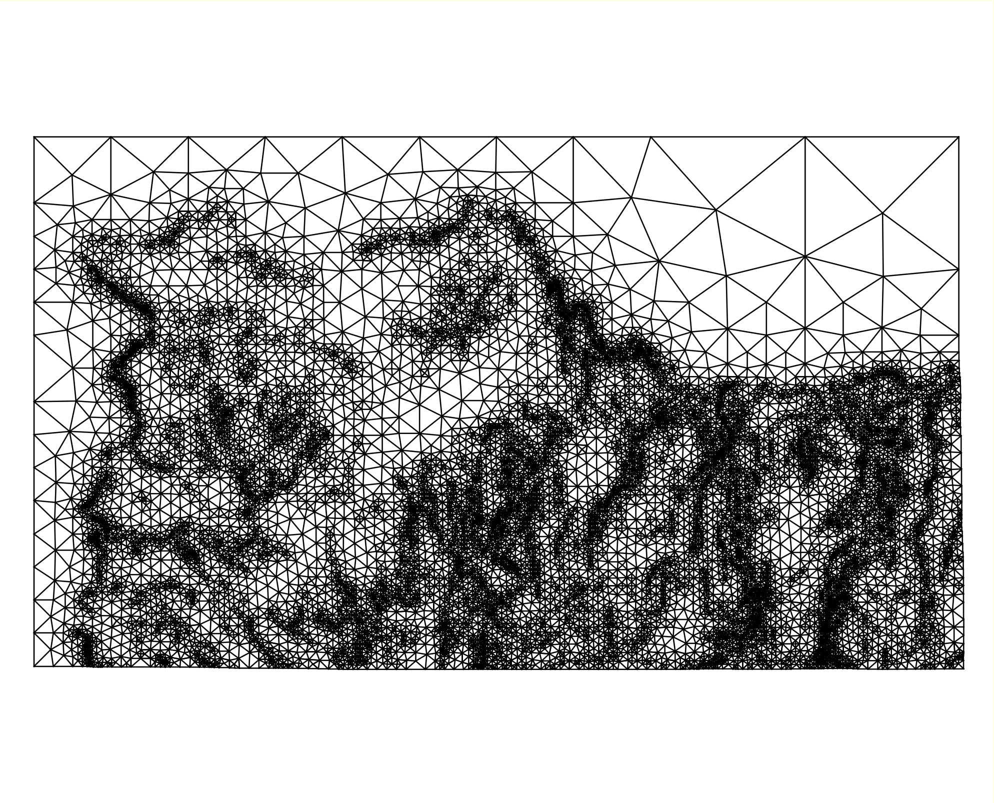


Optimized mesh

Applications of Smoothing Surface Triangulations

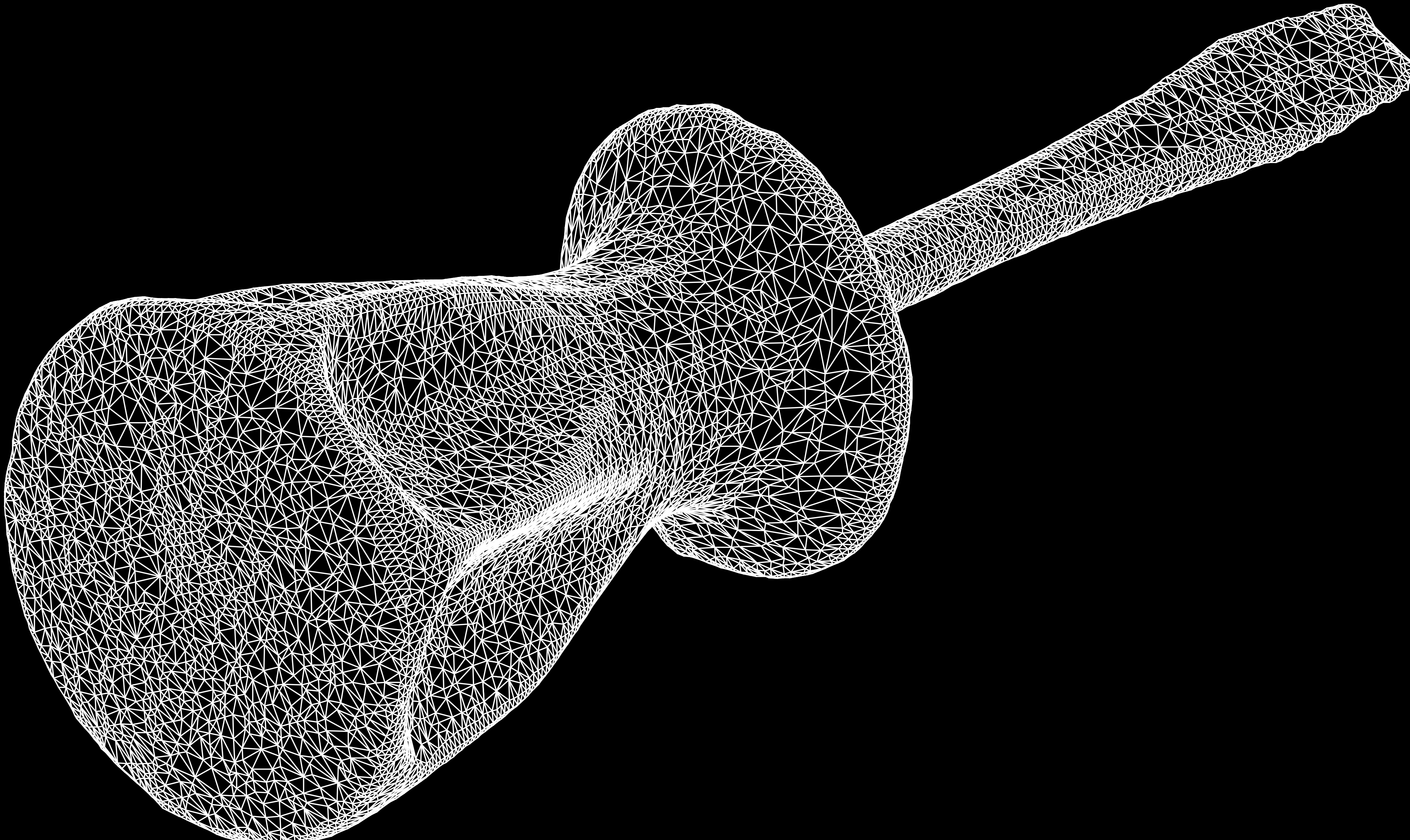


Before

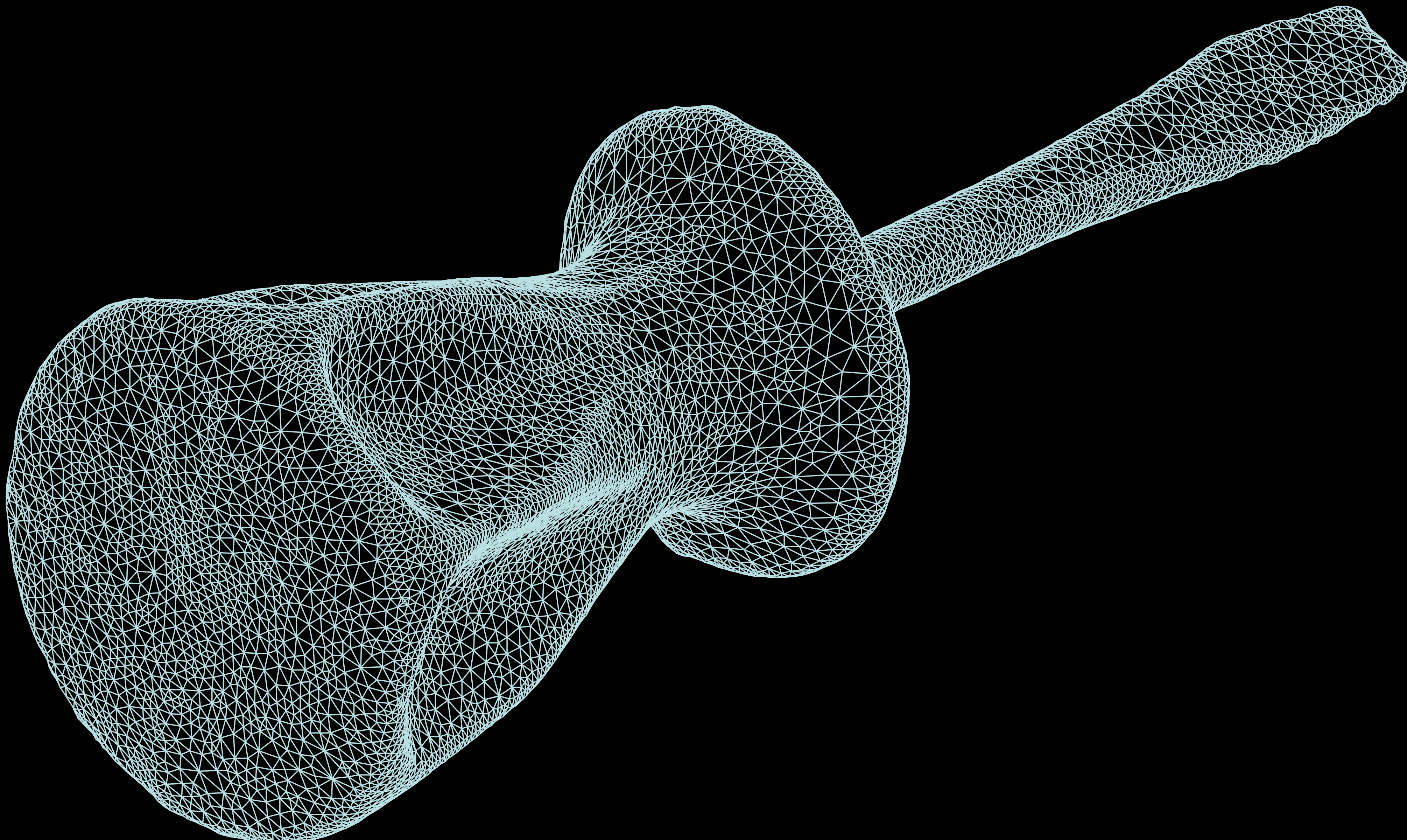


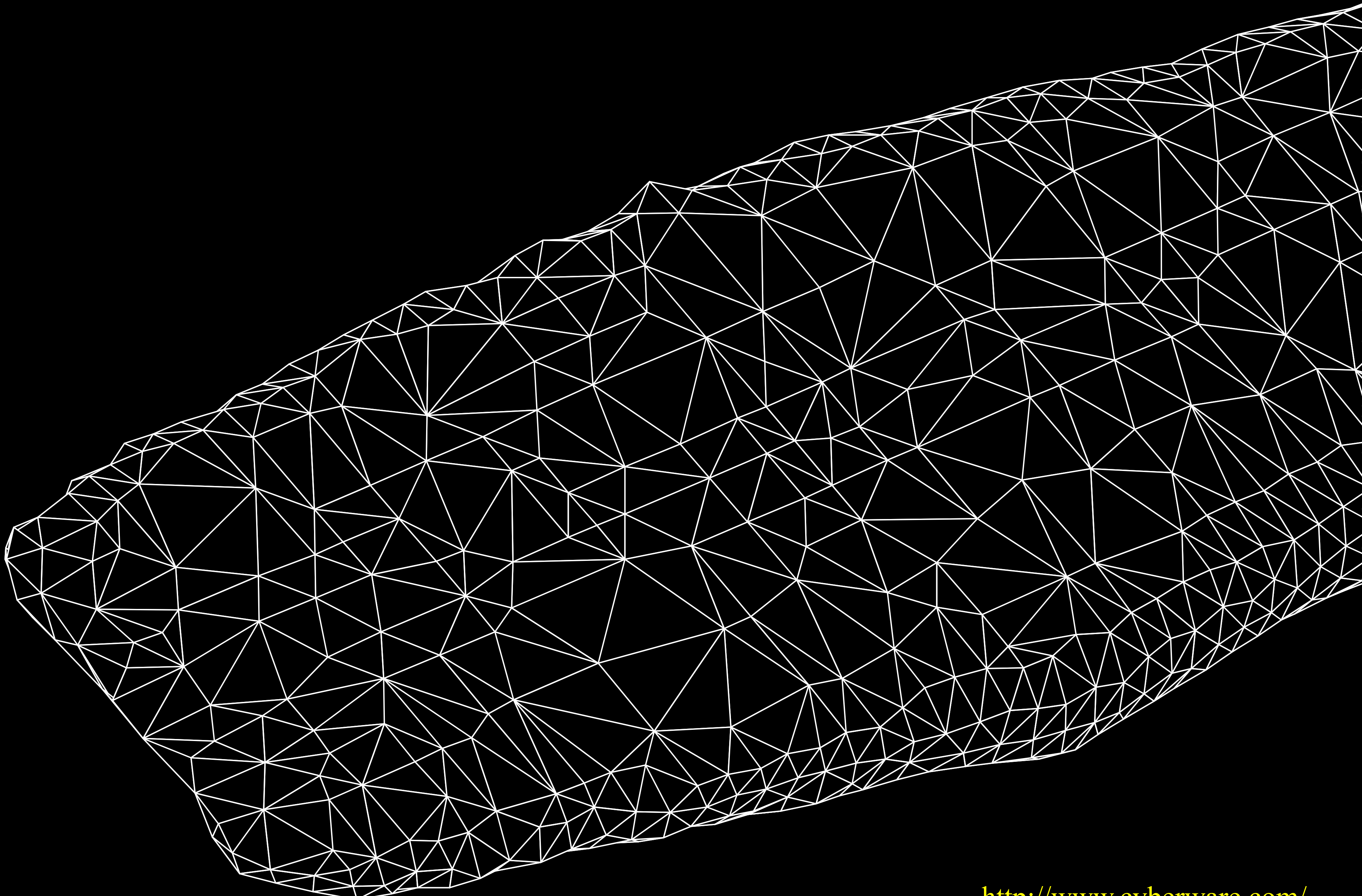
After

NW of *Isla de Gran Canaria*



http://www.cyberware.com/





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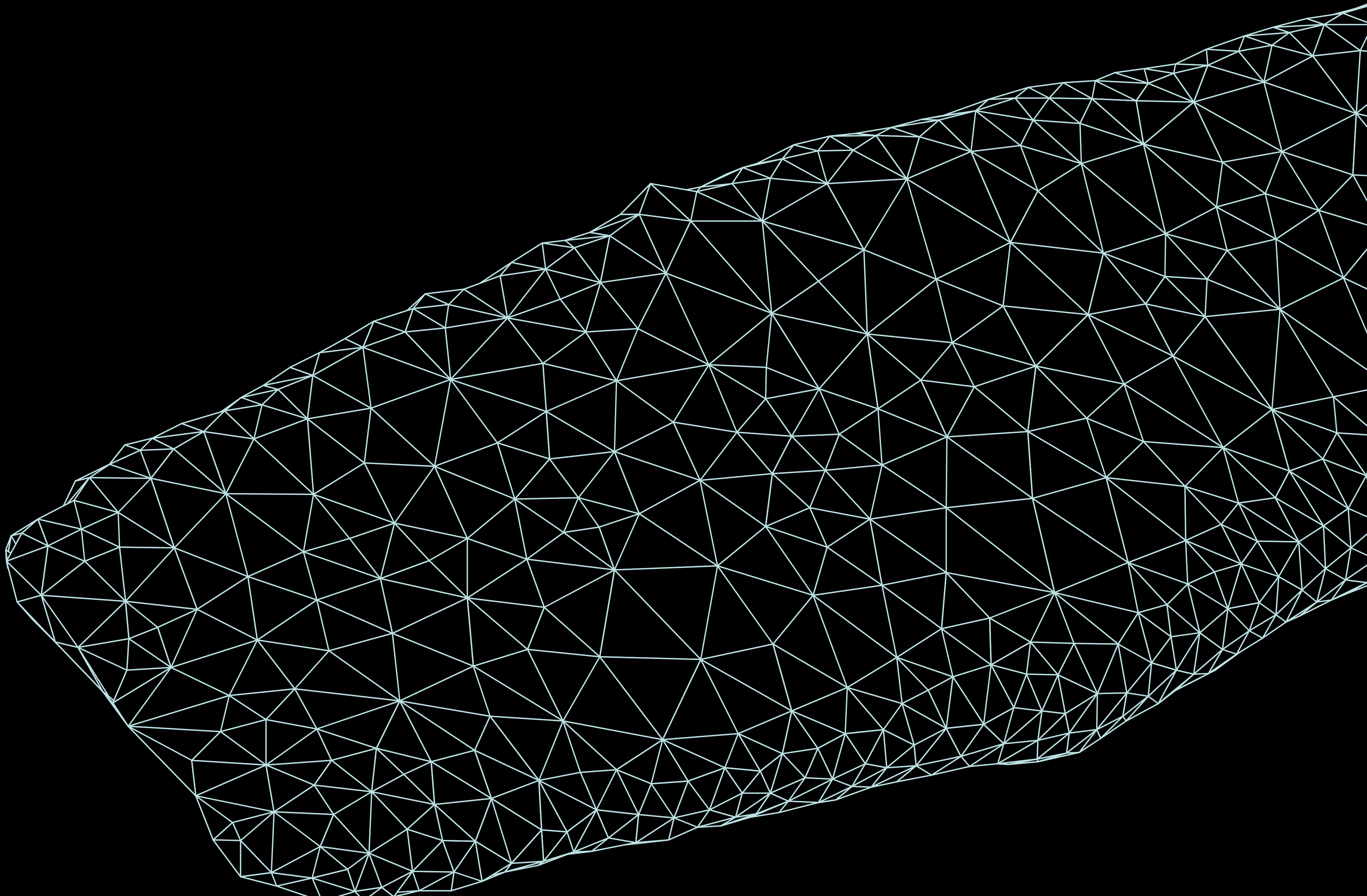
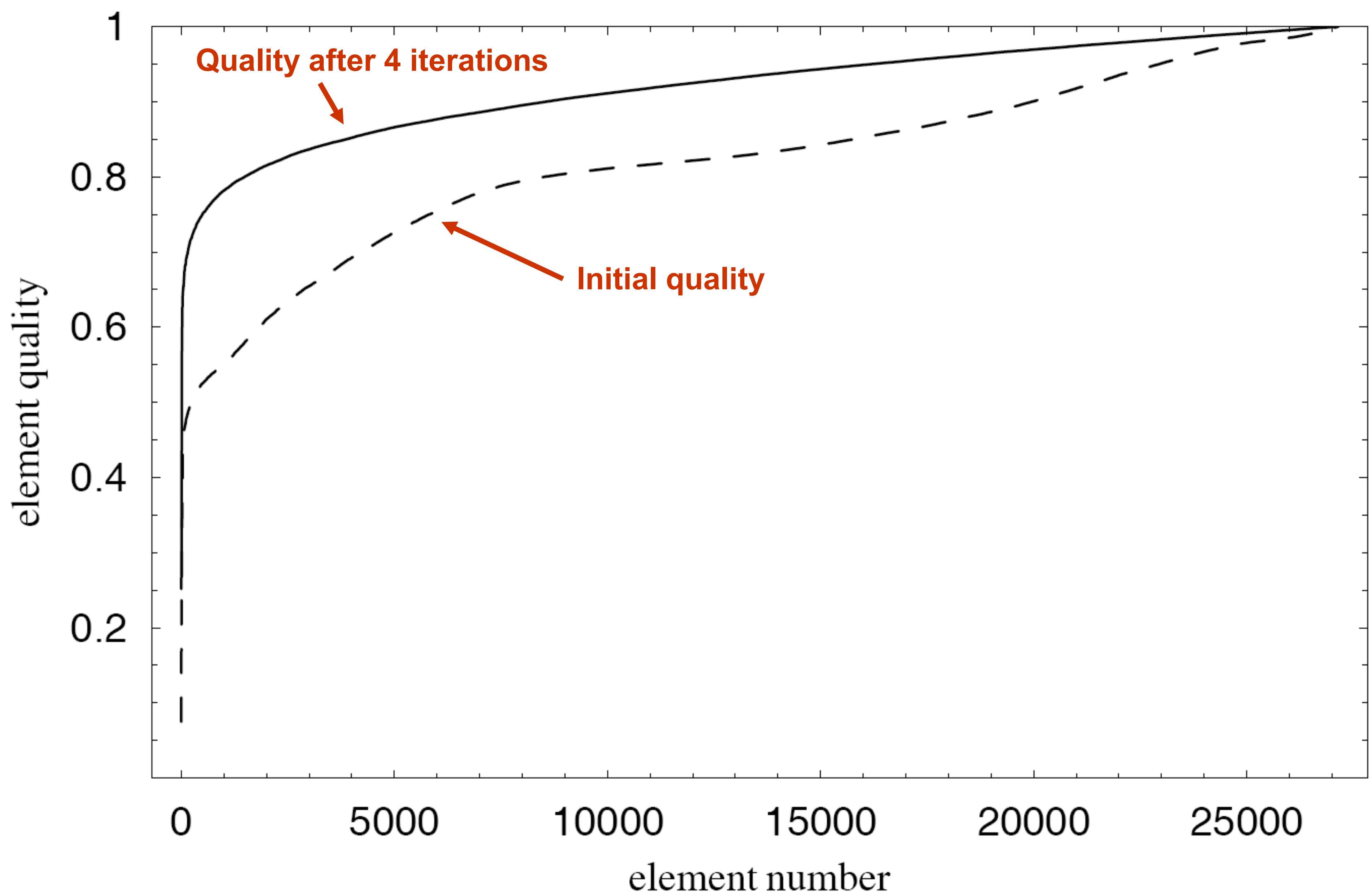
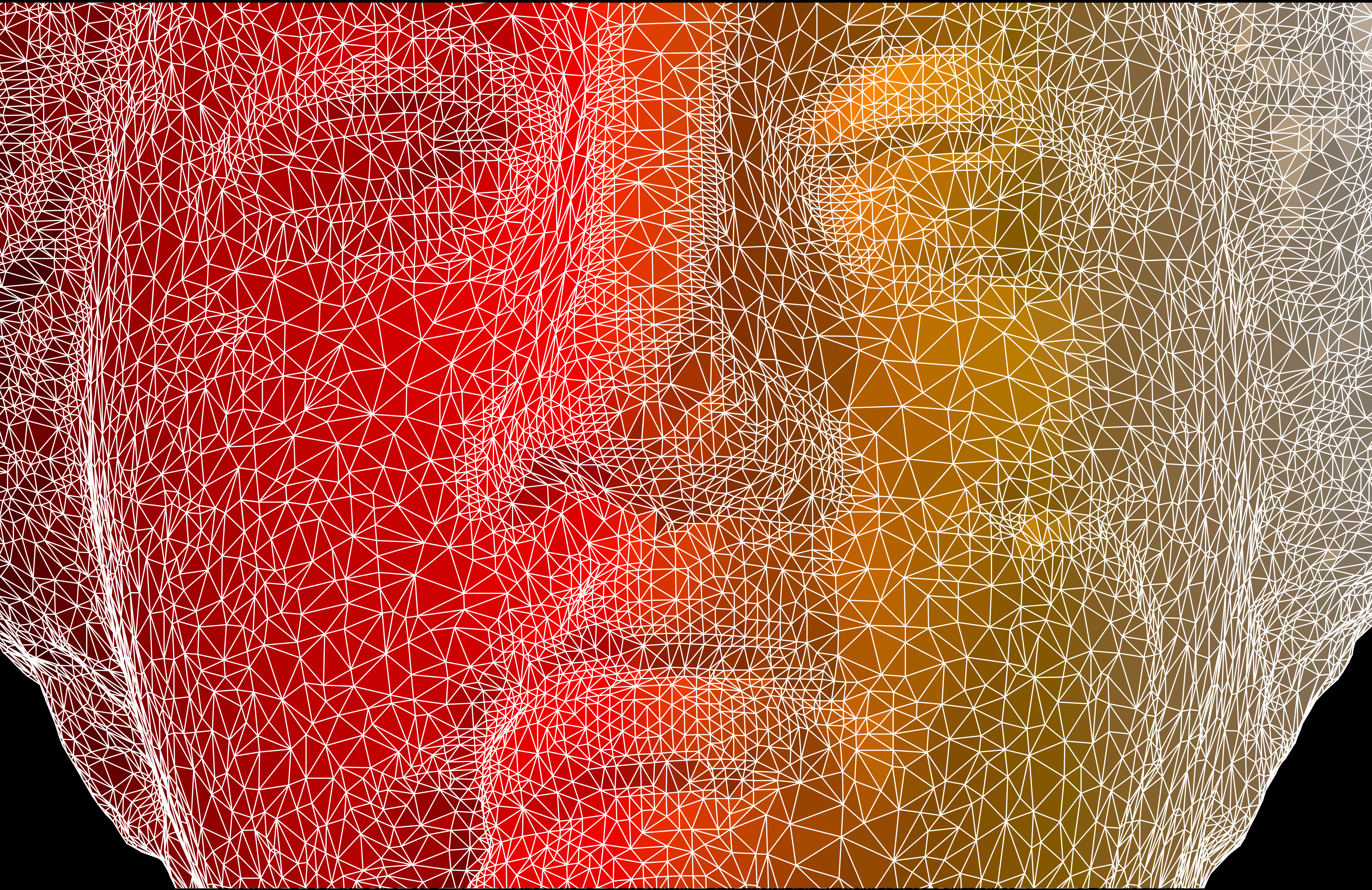
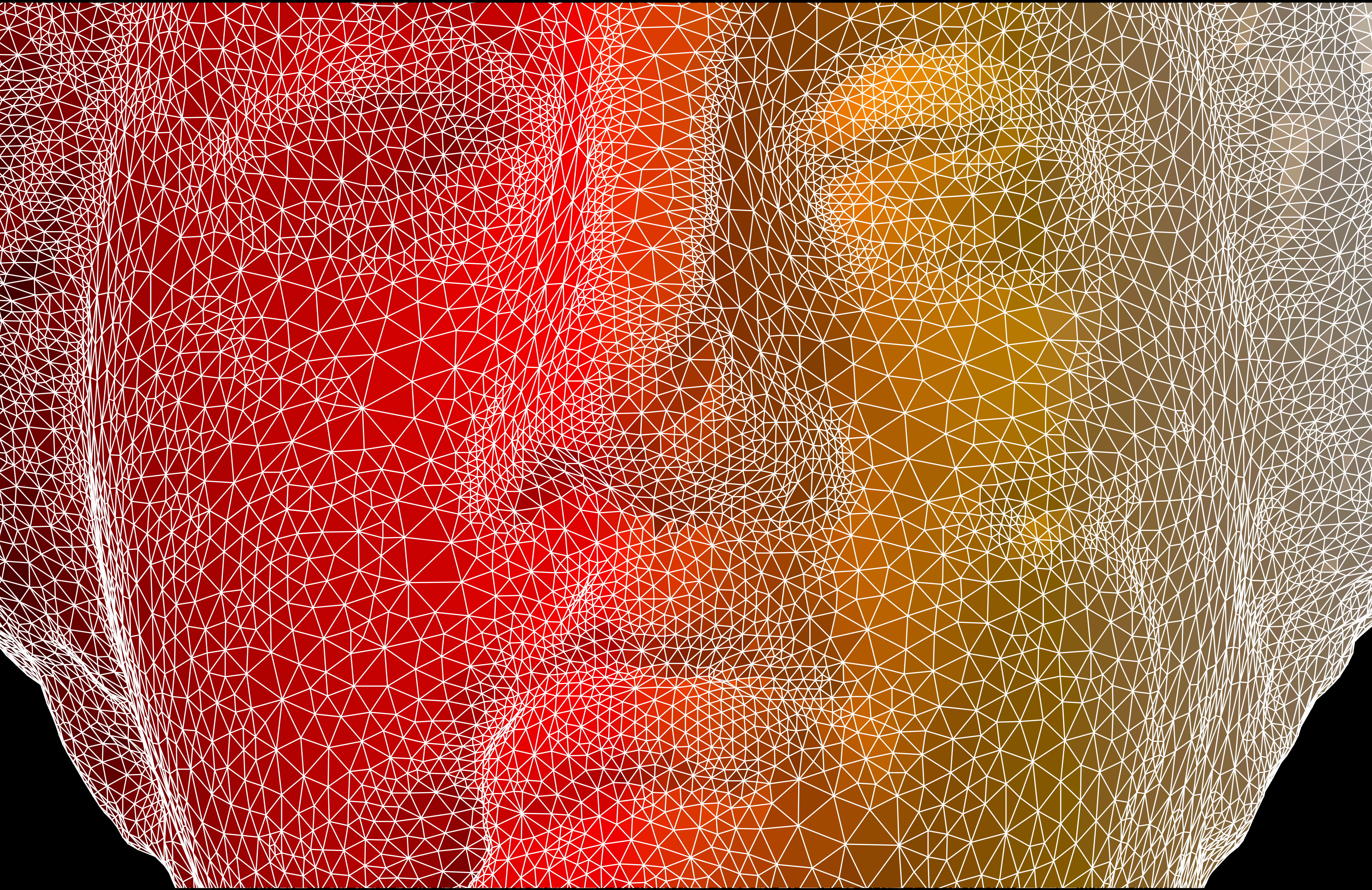


Applications

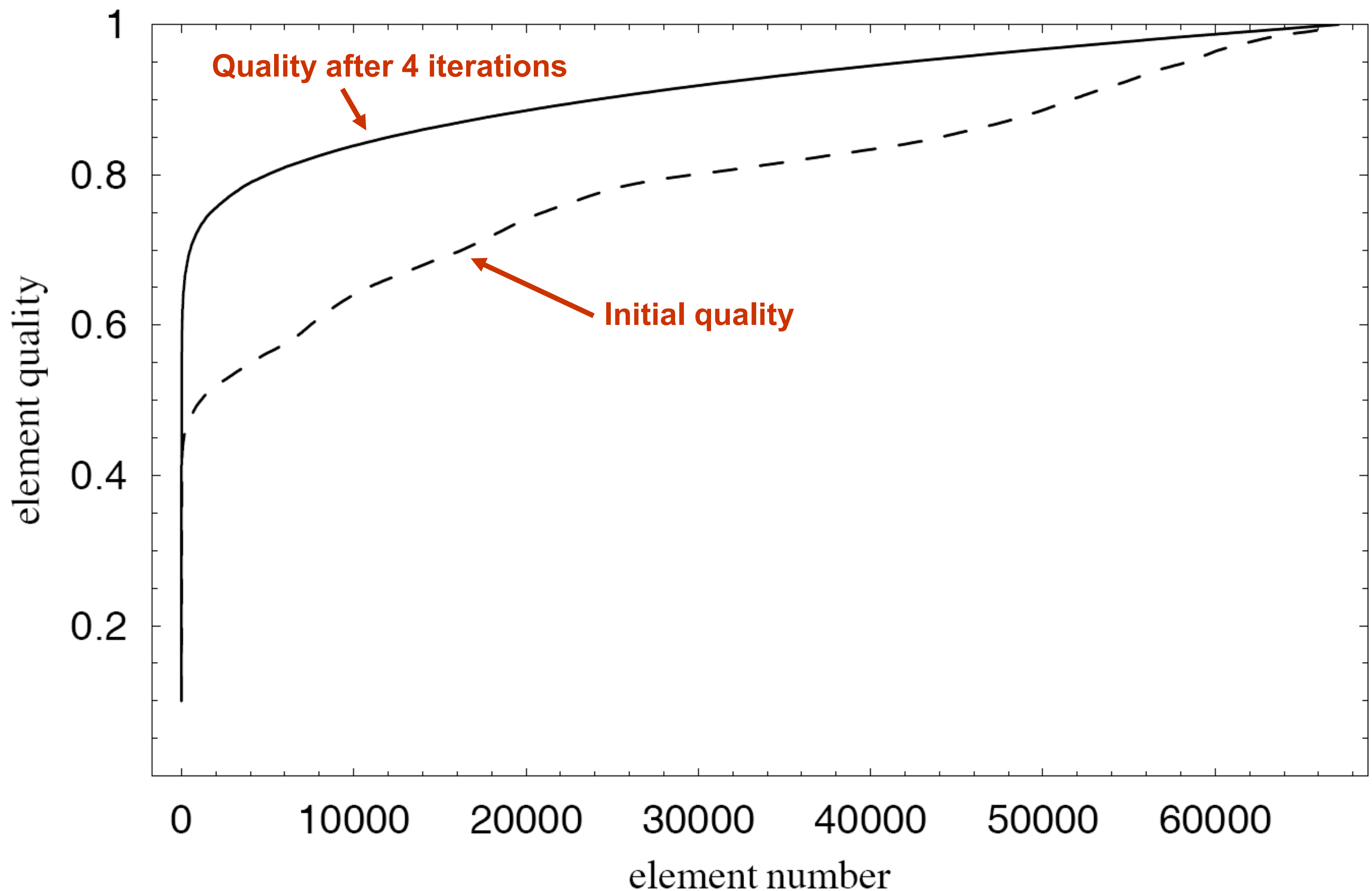


Quality curves for surface triangulations of the screwdriver





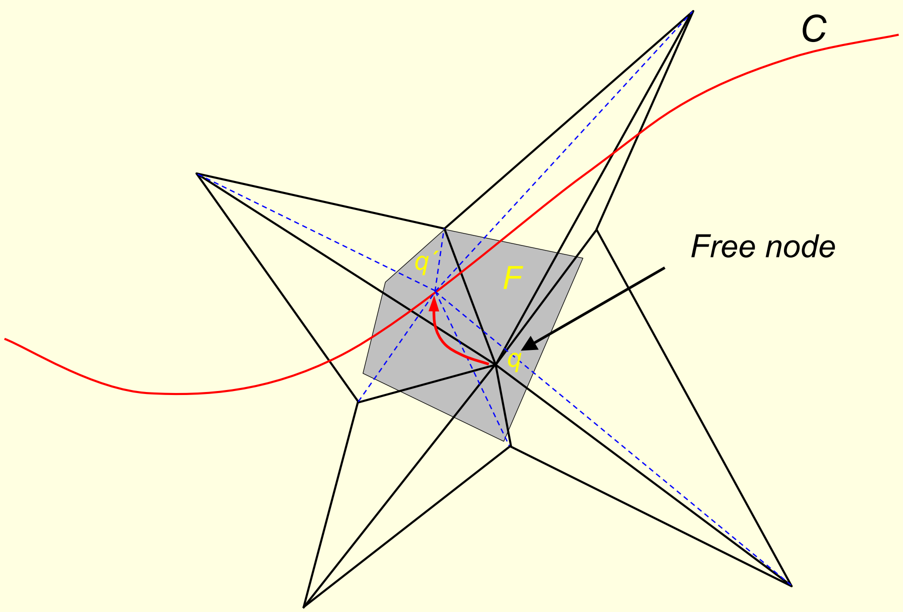
Applications



Quality curves for surface triangulations of Igea Artifact

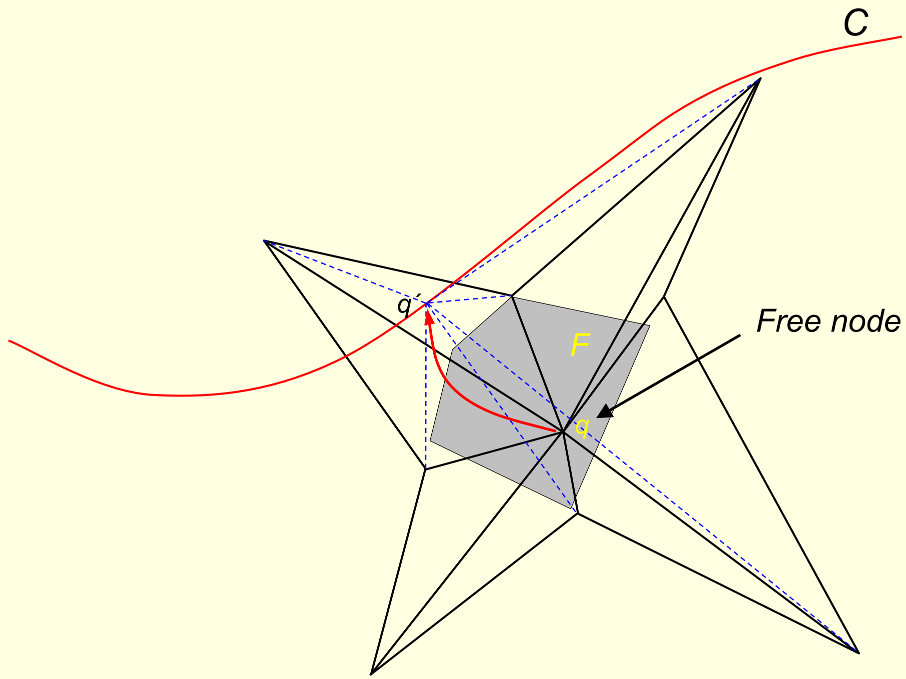
Mesh Smoothing to Match Curves

Curve C crossing the feasible region F



Free node q projectable on C

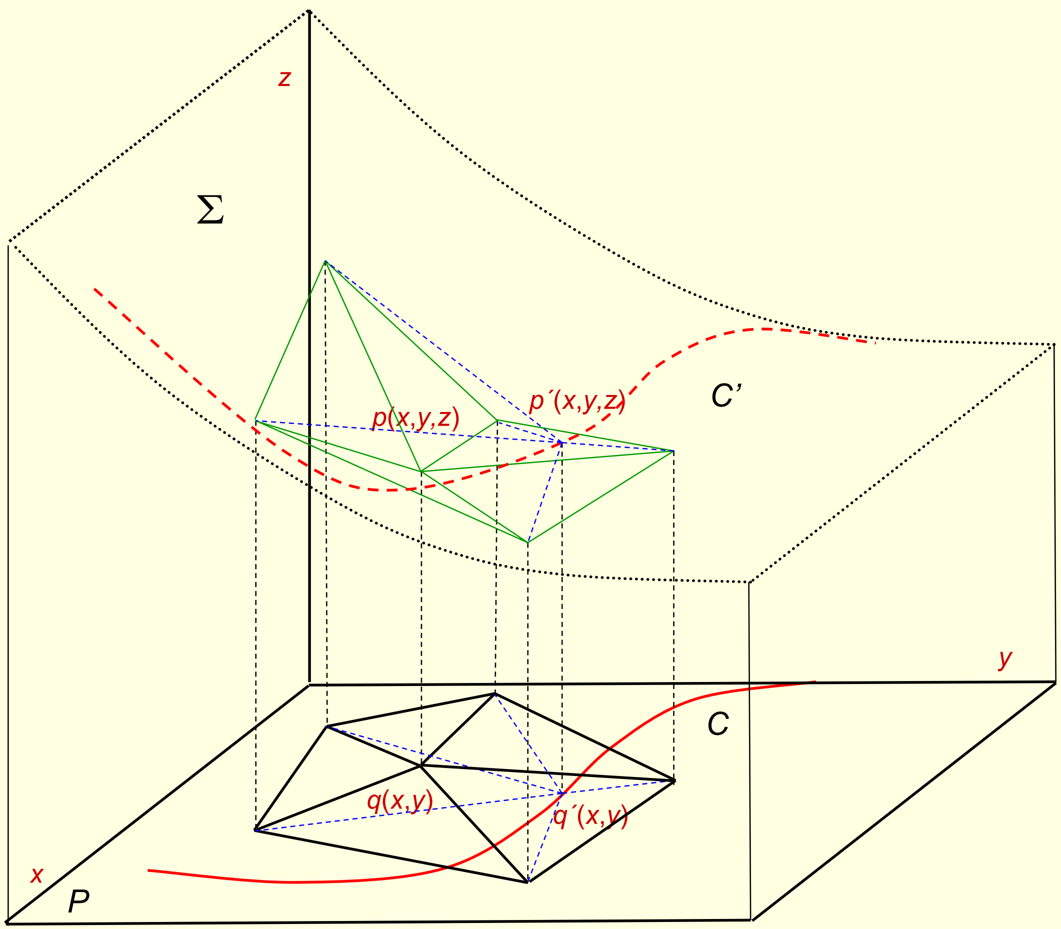
Curve C out of the feasible region F



Free node q not projectable on C

Searching of the optimal position of the node on the curve

Searching the Optimal Position of q on C

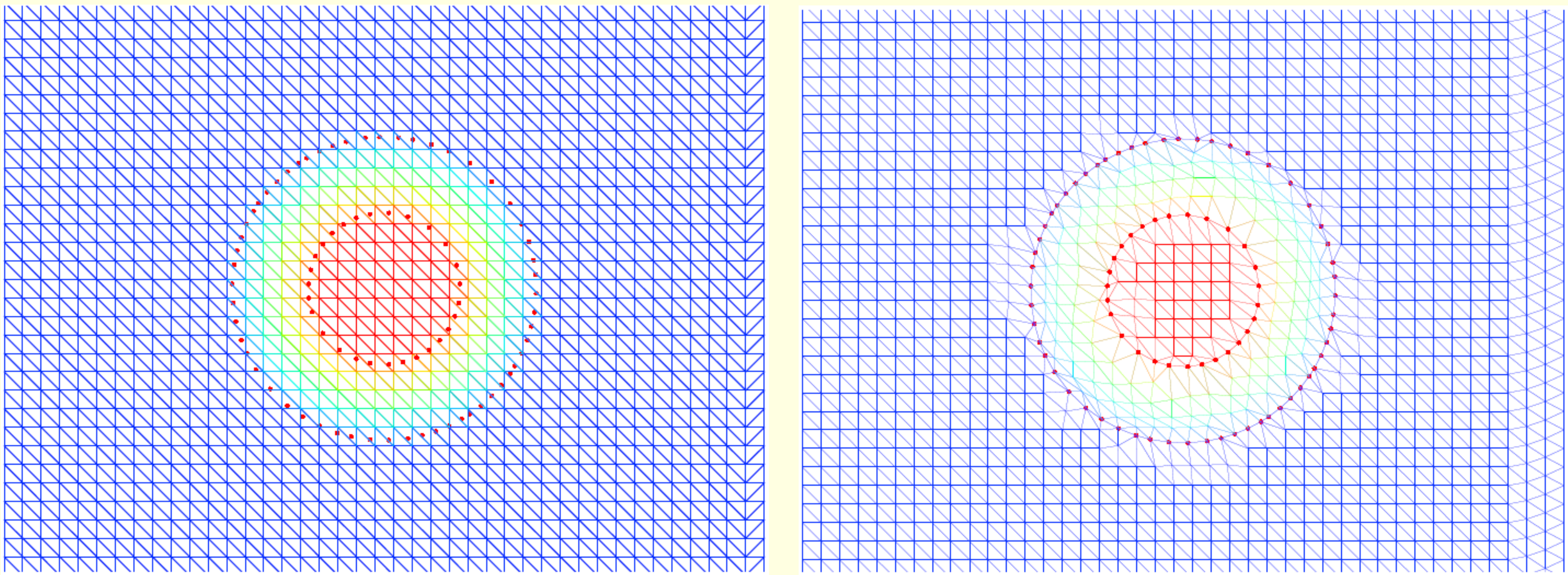


$N(q')$ not tangled

⇓

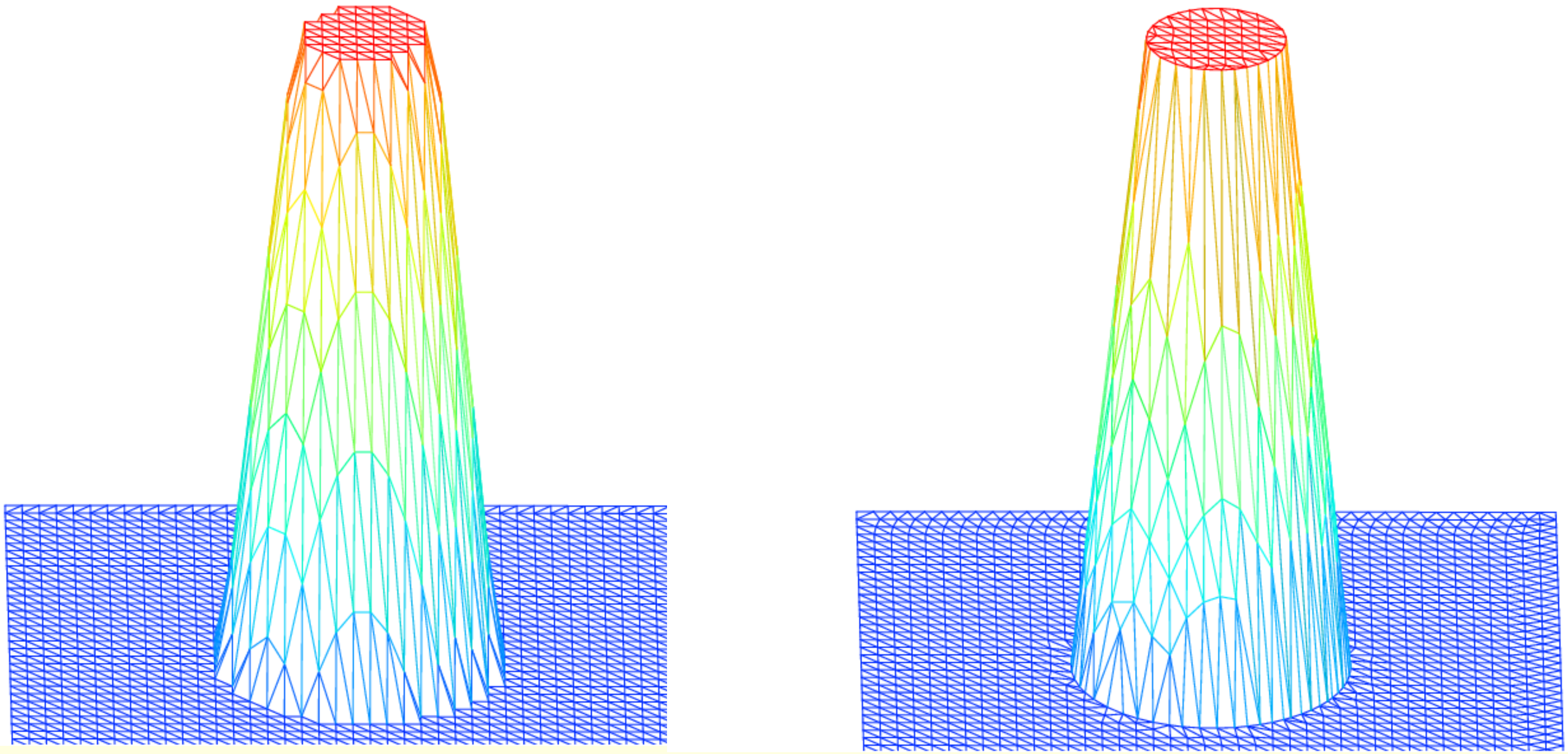
p projectable on C'

Applications for Matching Curves



Insertion of a chimney in a regular mesh. Vertical view

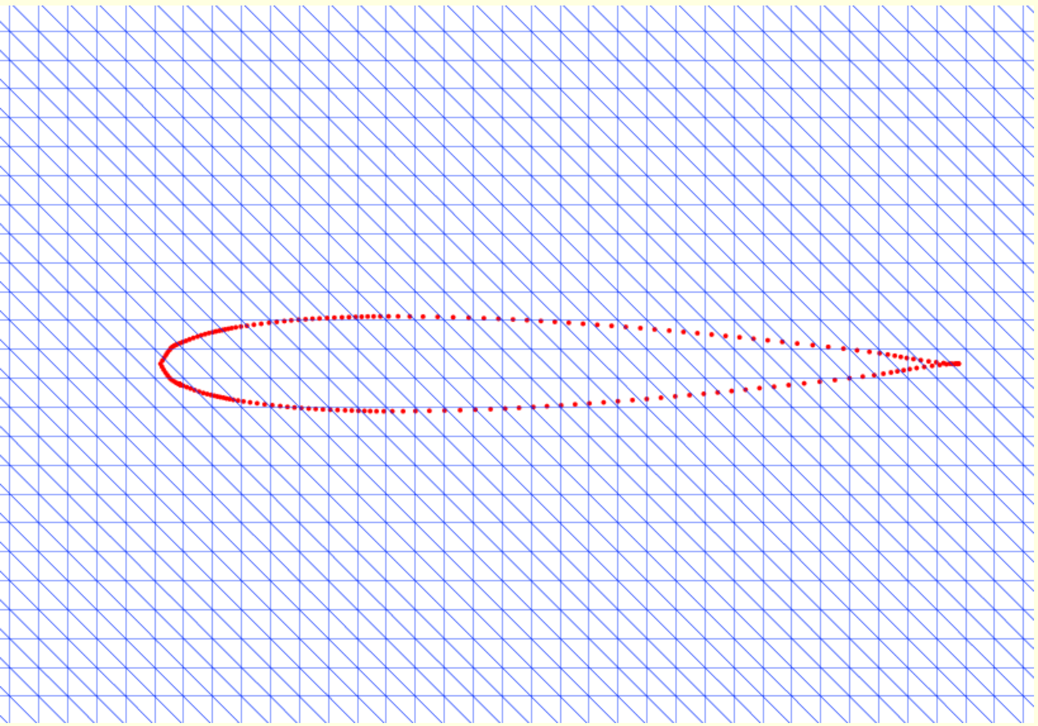
Applications for Matching Curves



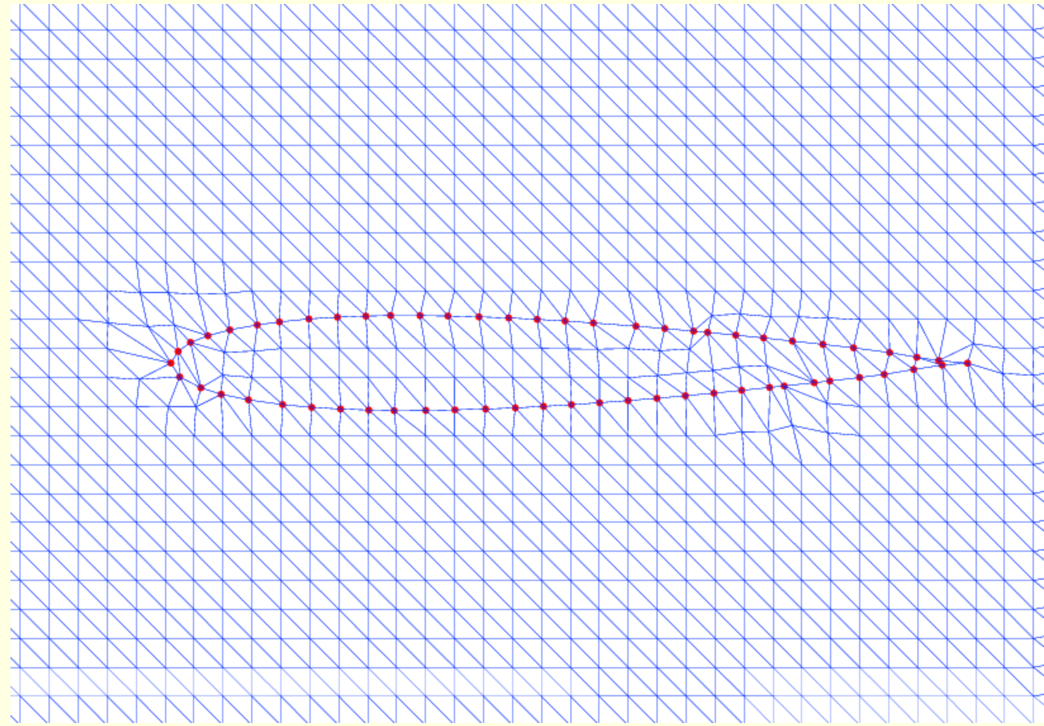
Insertion of a chimney in a regular mesh. Frontal view

Applications for Matching Curves

Insertion of the profile NACA012 in a regular mesh

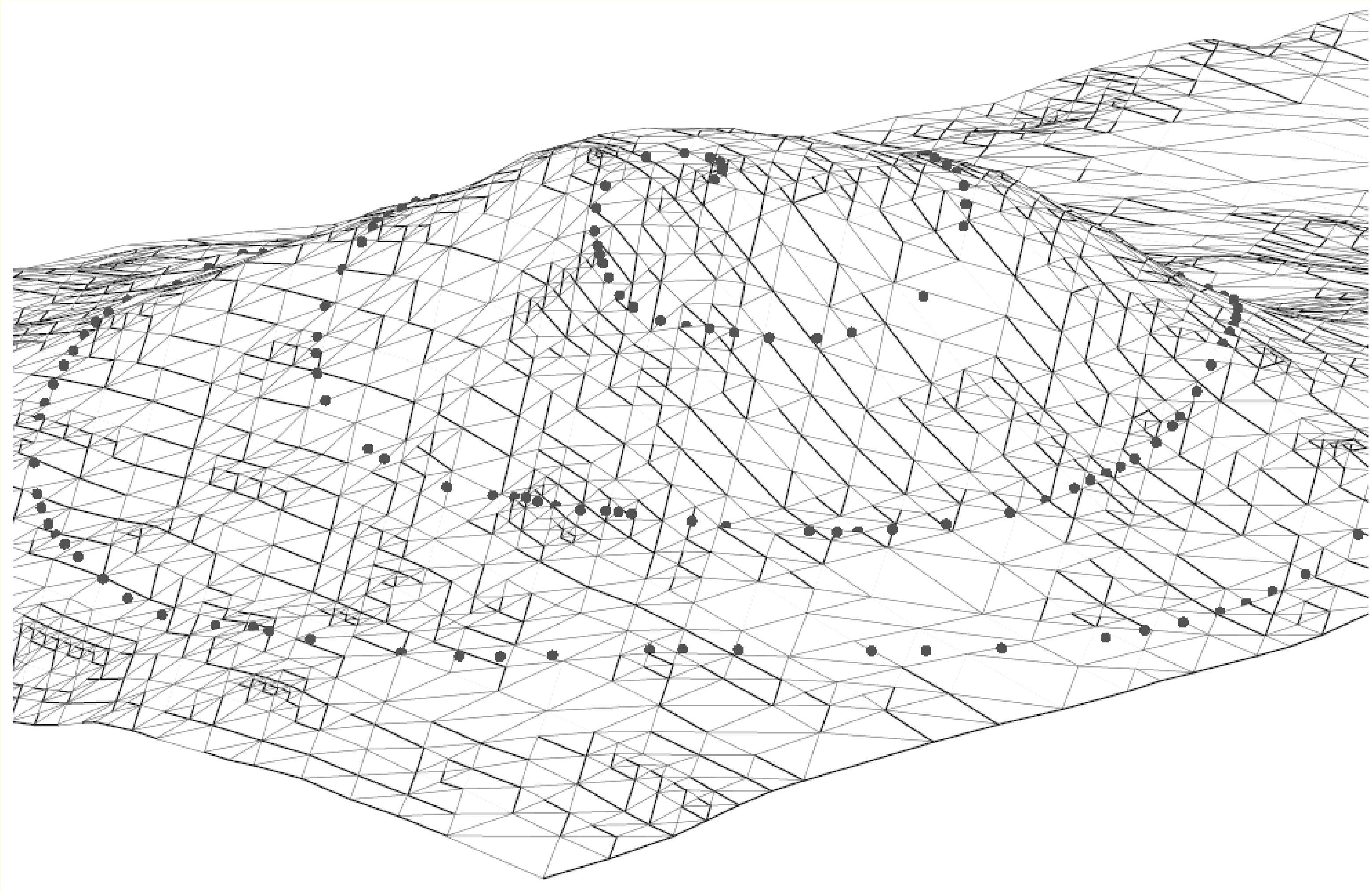


Set of points used to define the profile



Adapted mesh

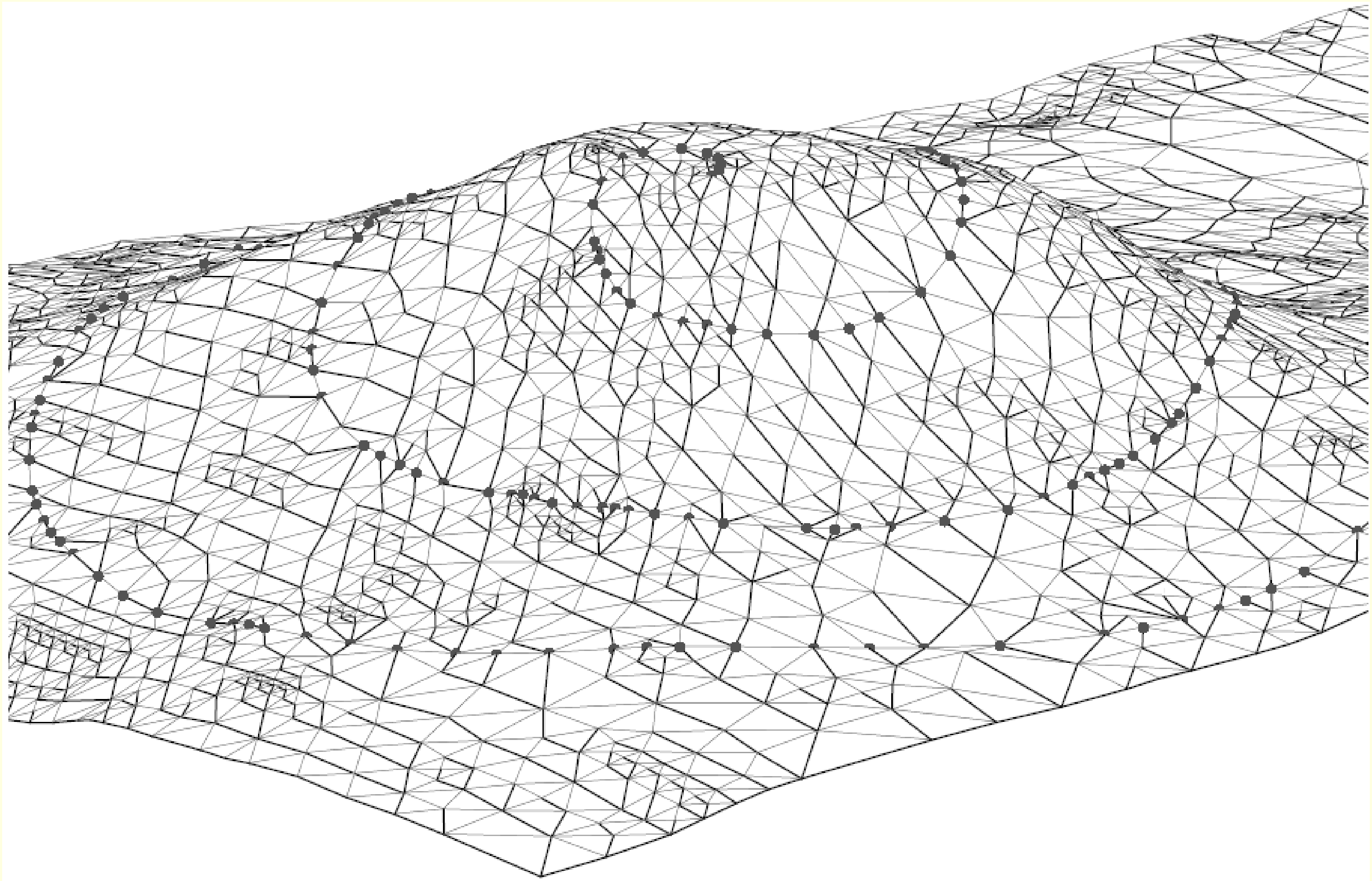
Applications for Matching Curves



Insertion of a spiral road (doted line) in a nested mesh of the Galdar Mountain.

Initial mesh

Applications for Matching Curves

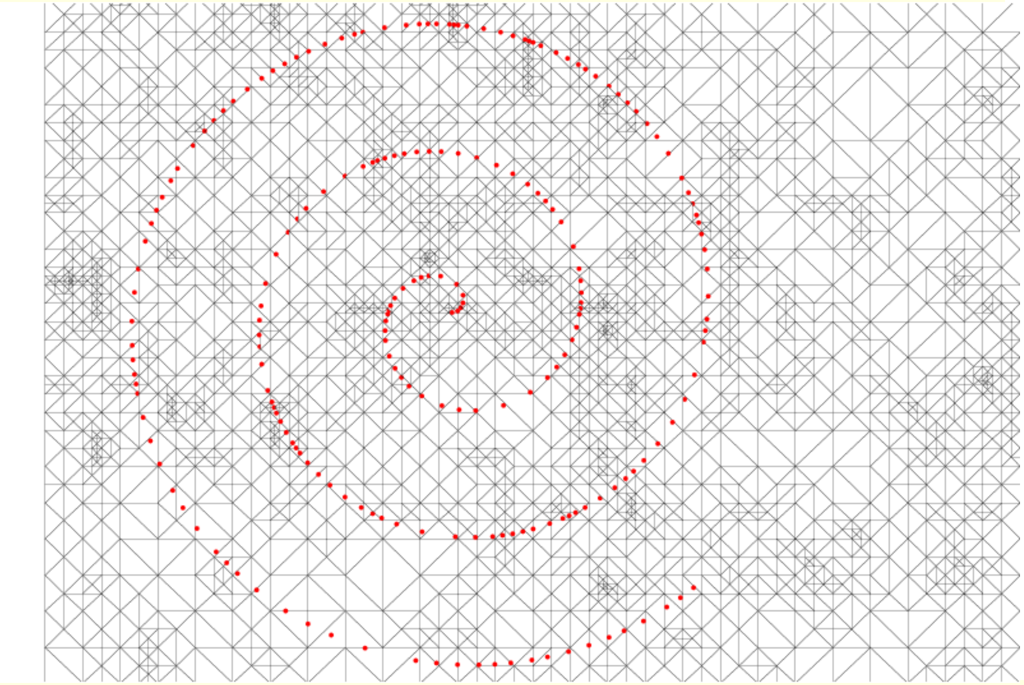


Insertion of a spiral road (doted line) in a nested mesh of the Galdar Mountain.

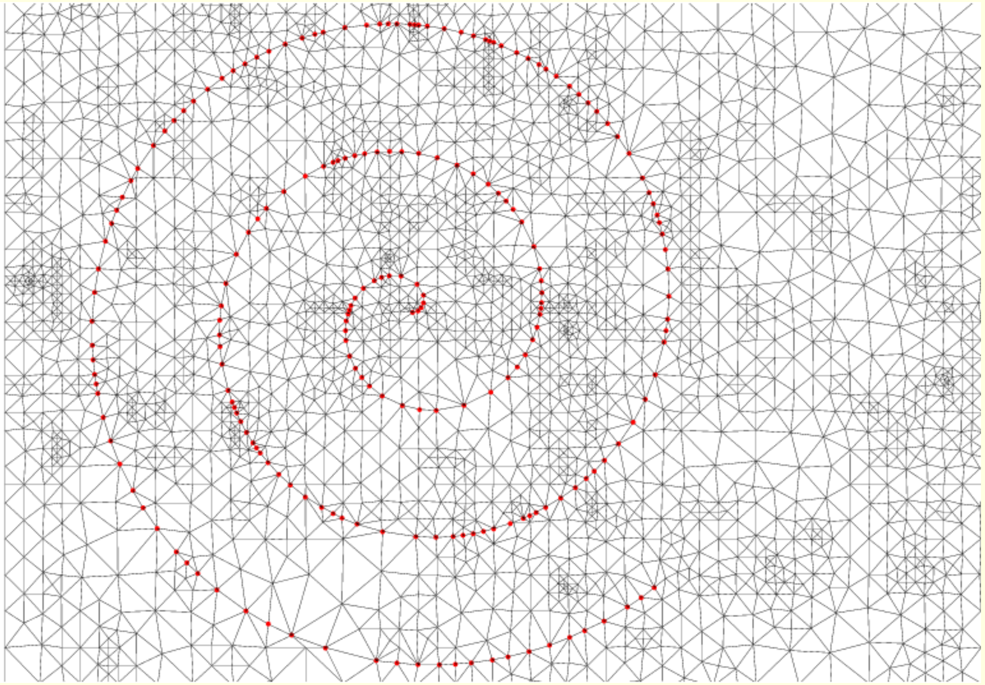
Adapted mesh.

Applications for Matching Curves

Spiral road in Galdar Mountain (a bird's-eye view).

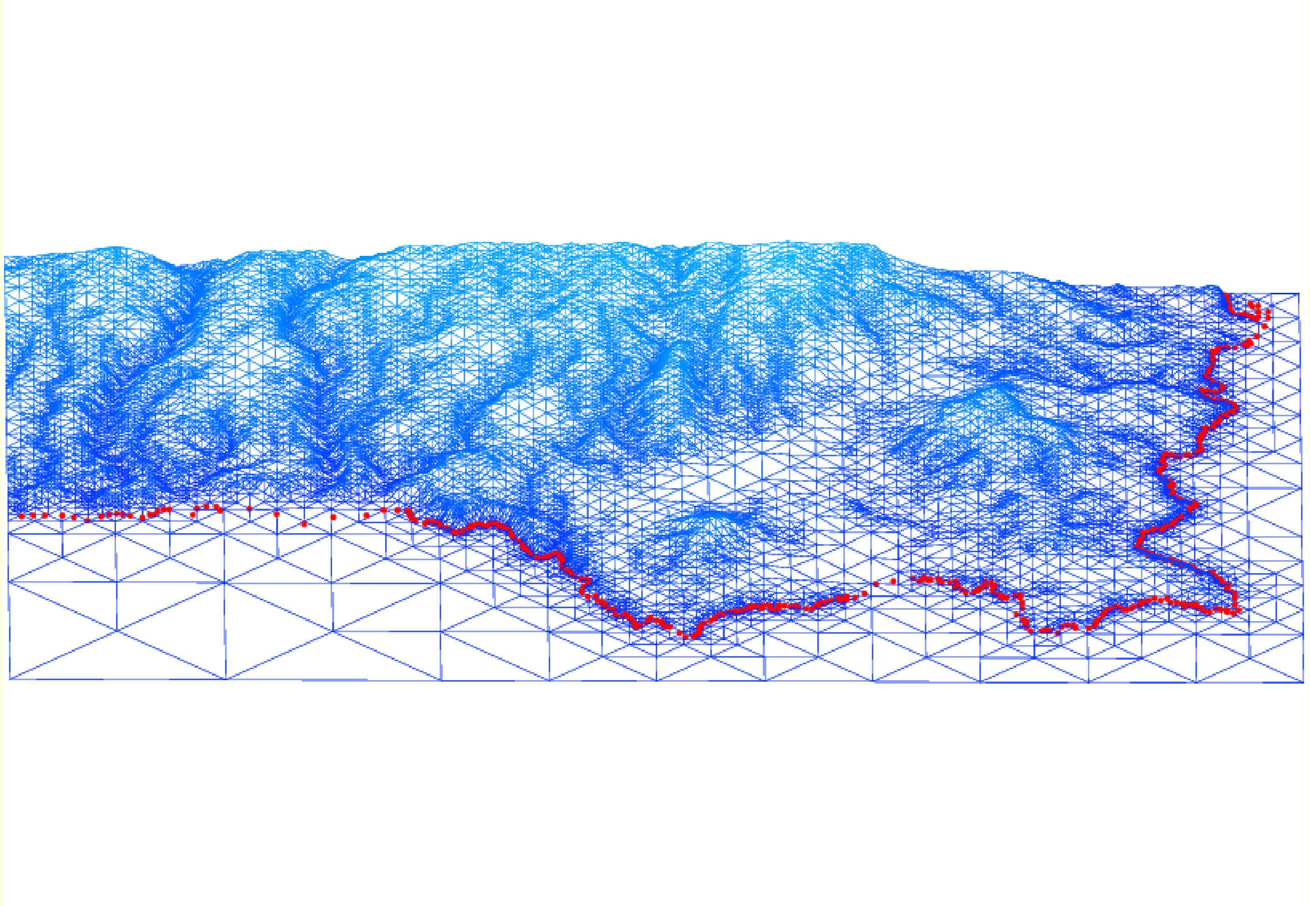


Initial nested mesh



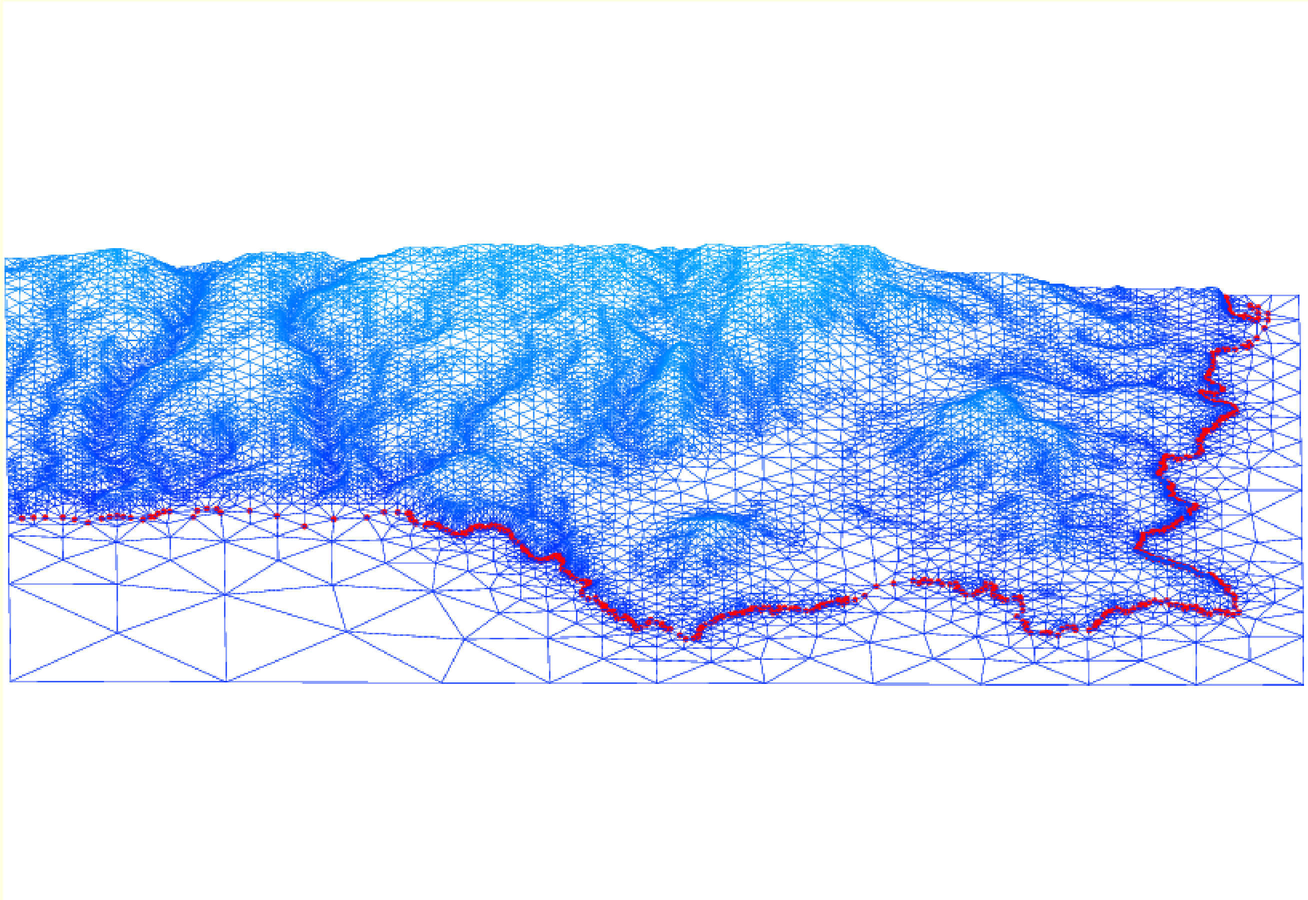
Final smoothed and adapted mesh

Applications for Matching Curves



Initial nested mesh of NO of Gran Canaria with points delimiting the coastal shore

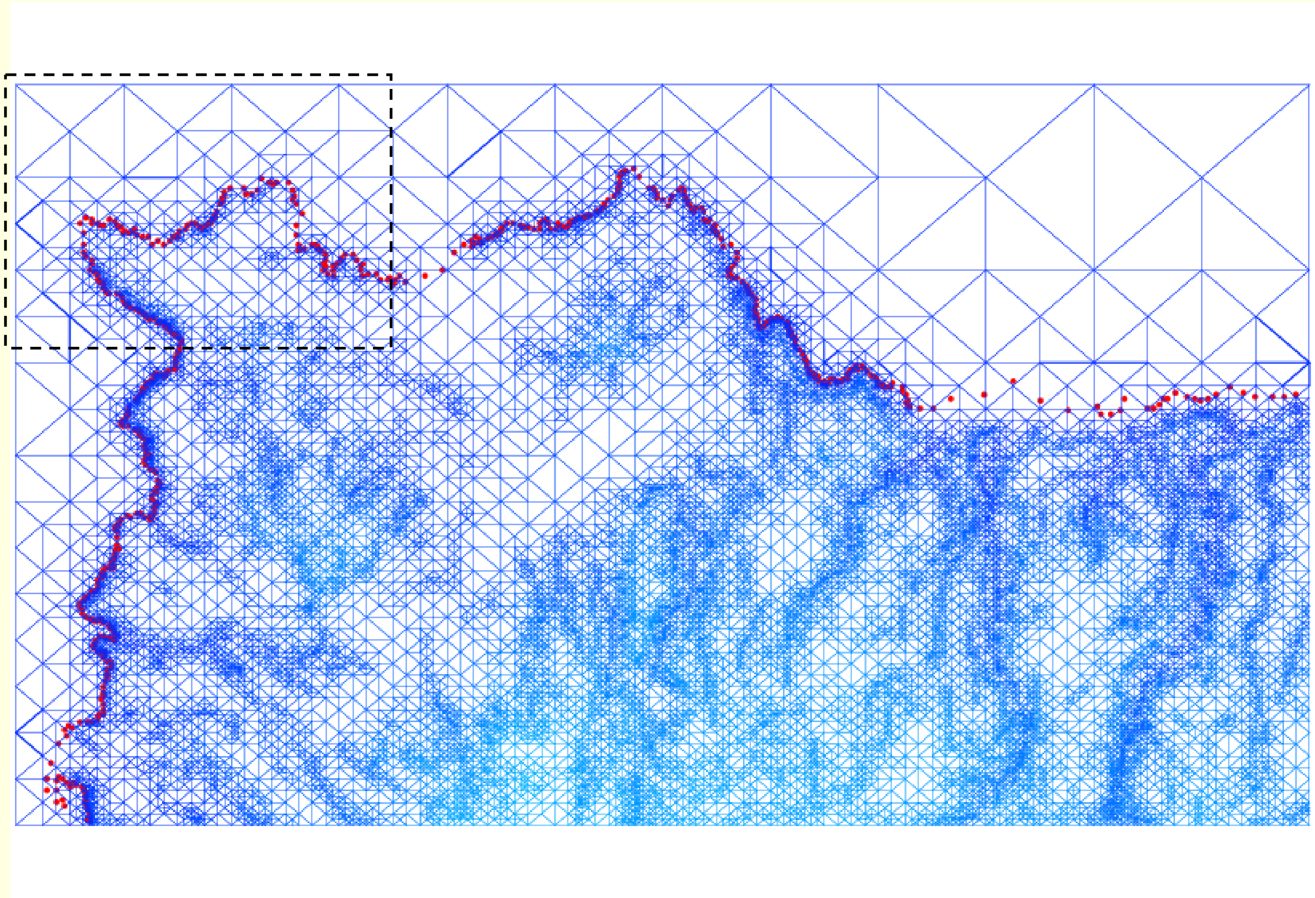
Applications for Matching Curves



Smoothed and adapted mesh of NO of Gran Canaria

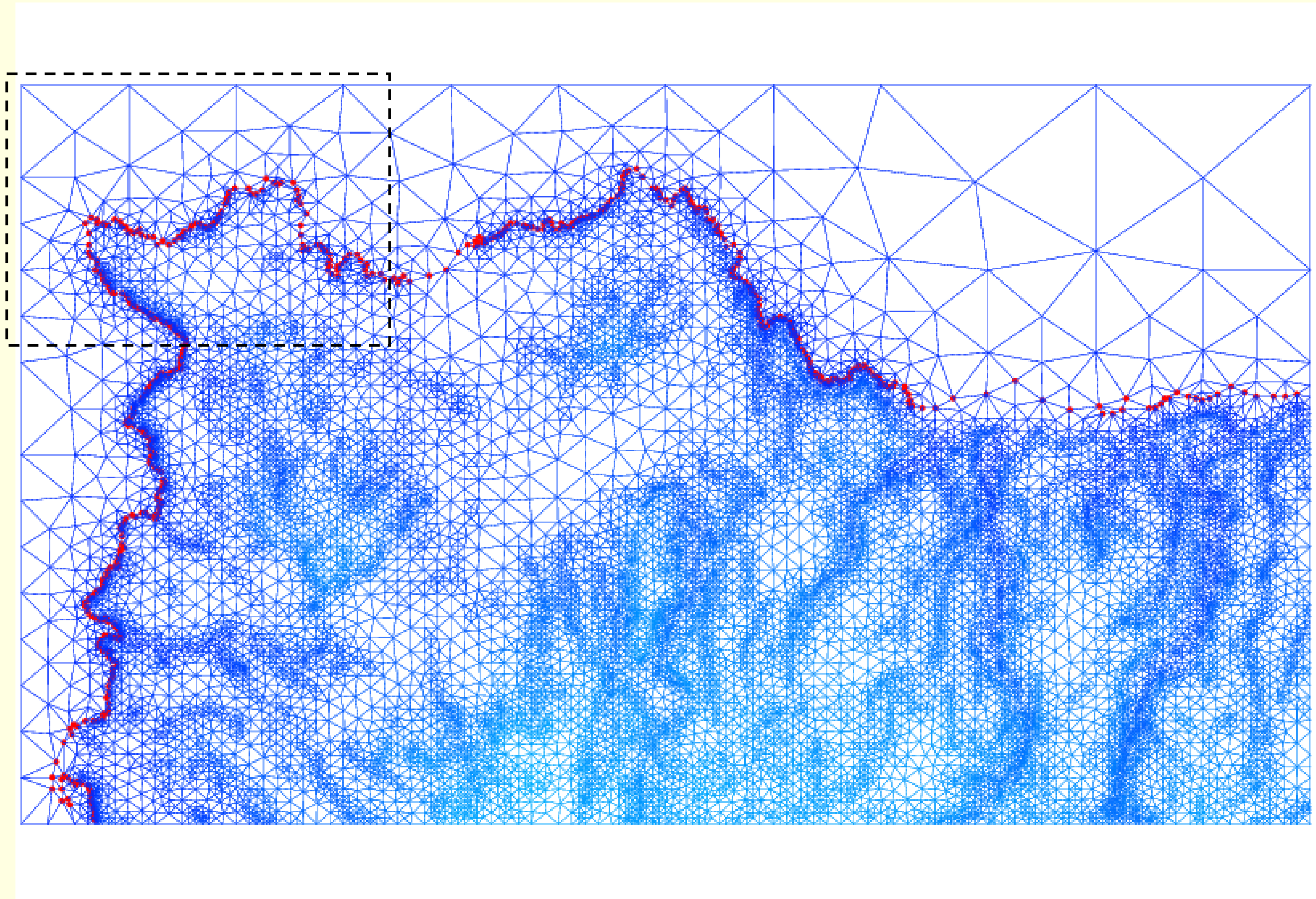
Applications for Matching Curves

A bird's-eye view of the NO of Gran Canaria. Initial mesh

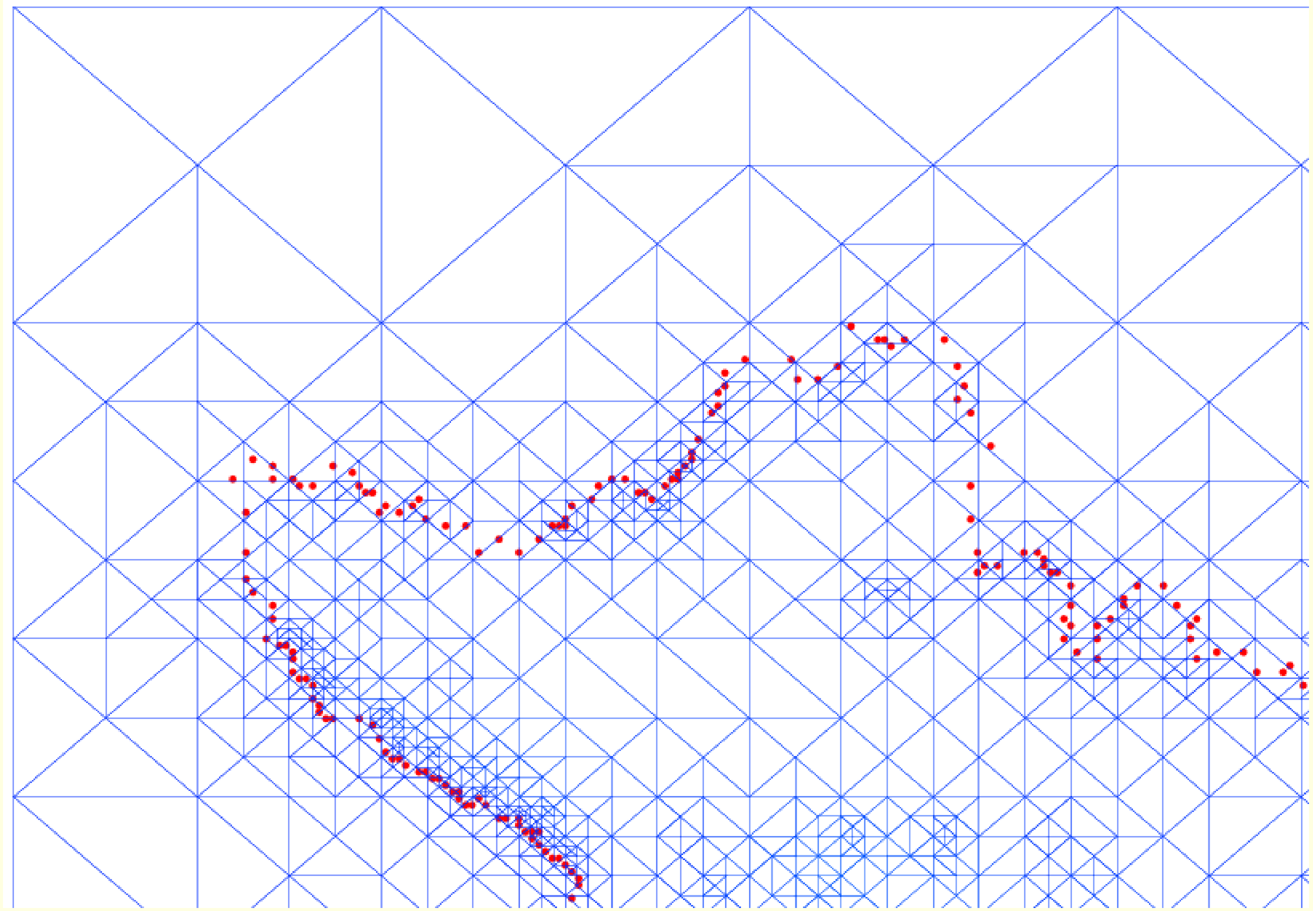


Applications for Matching Curves

A bird's-eye view of the NO of Gran Canaria. Final mesh

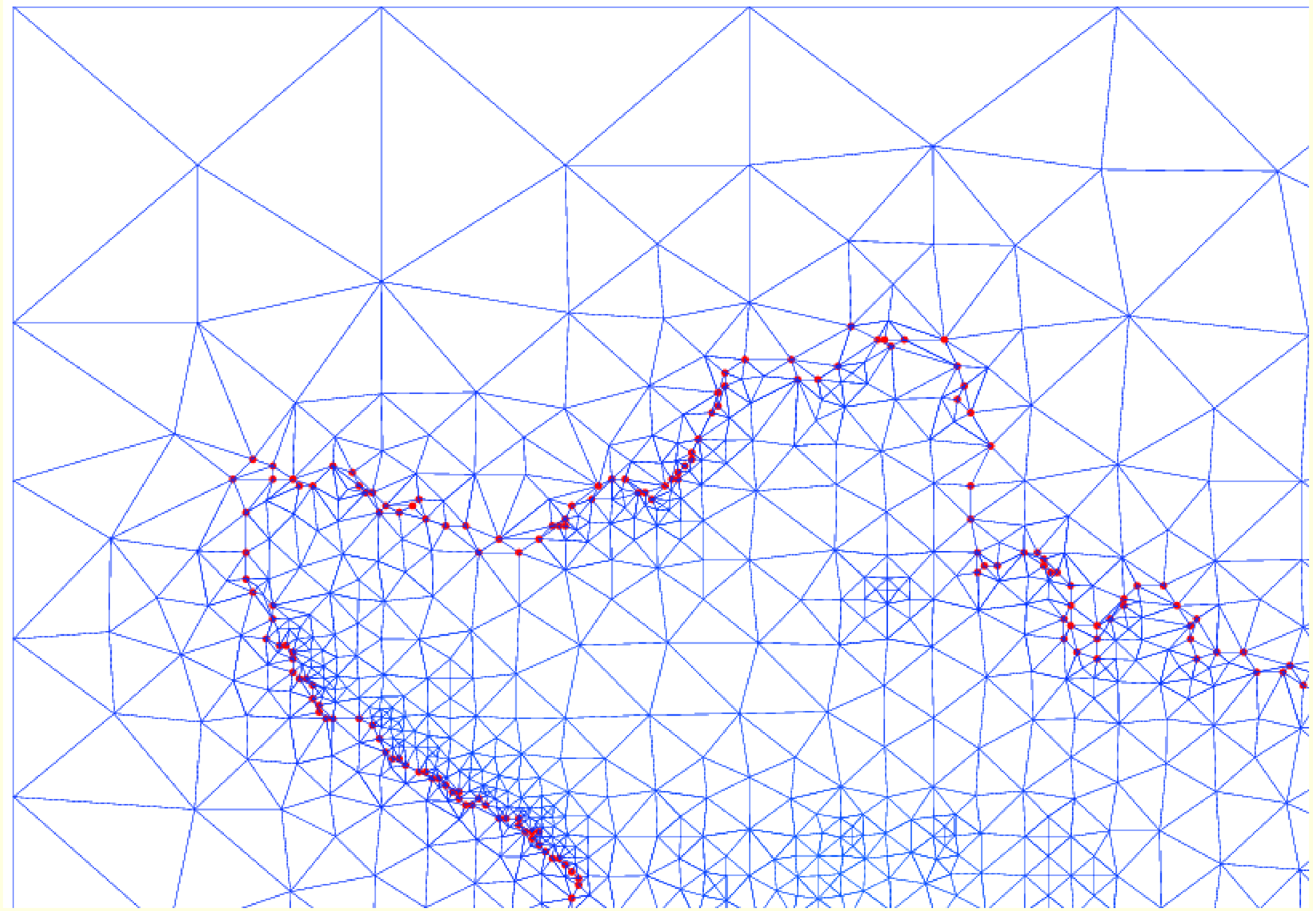


Applications for Matching Curves



A detail of the boxed region. Initial mesh

Applications for Matching Curves



A detail of the boxed region. Final mesh

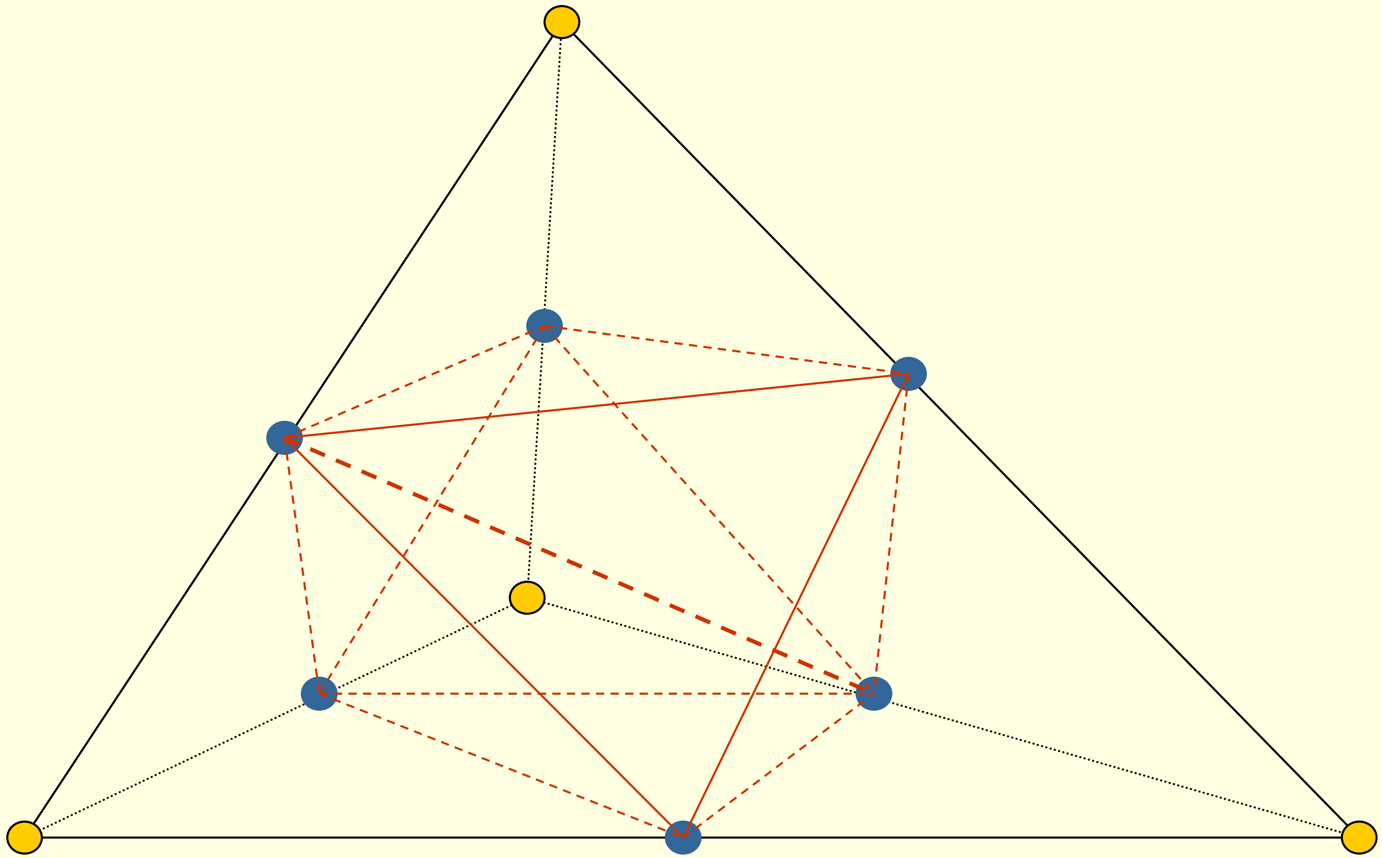
Local Refinement of Tetrahedral Meshes

- When we solve a problem using adaptive finite element method,
  - A local refinement algorithm is necessary for improving the numerical solution

Local Refinement of Tetrahedral Meshes

Tetrahedra with 6 new nodes

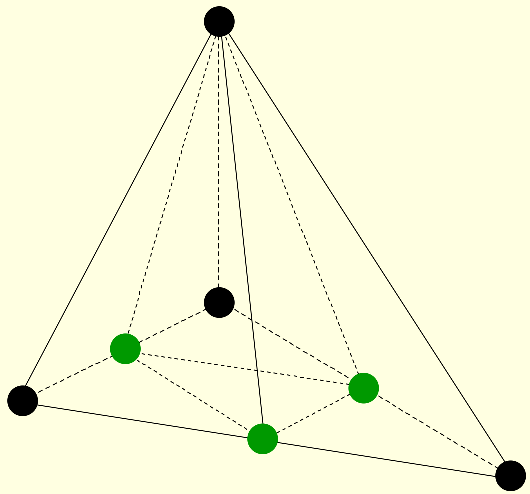
□ Subdivision in 8-subtetrahedra



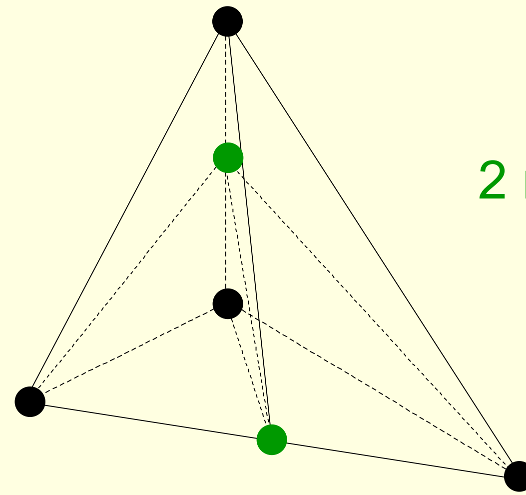
Local Refinement of Tetrahedral Meshes

Subdivisions to Assure Mesh Conformity

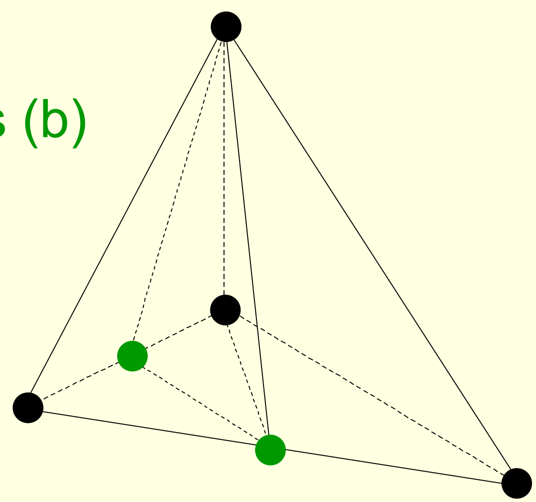
3 new nodes



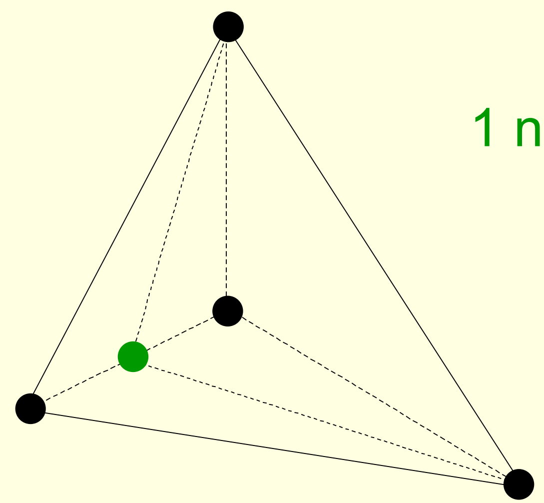
2 new nodes (a)



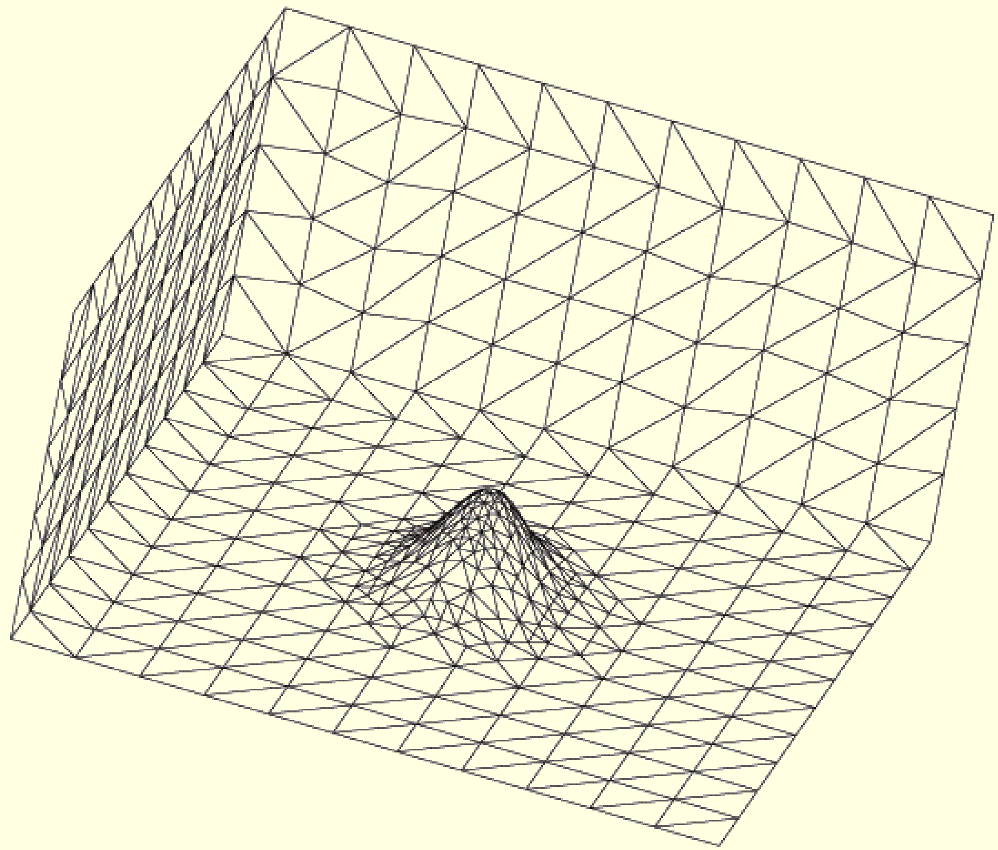
2 new nodes (b)



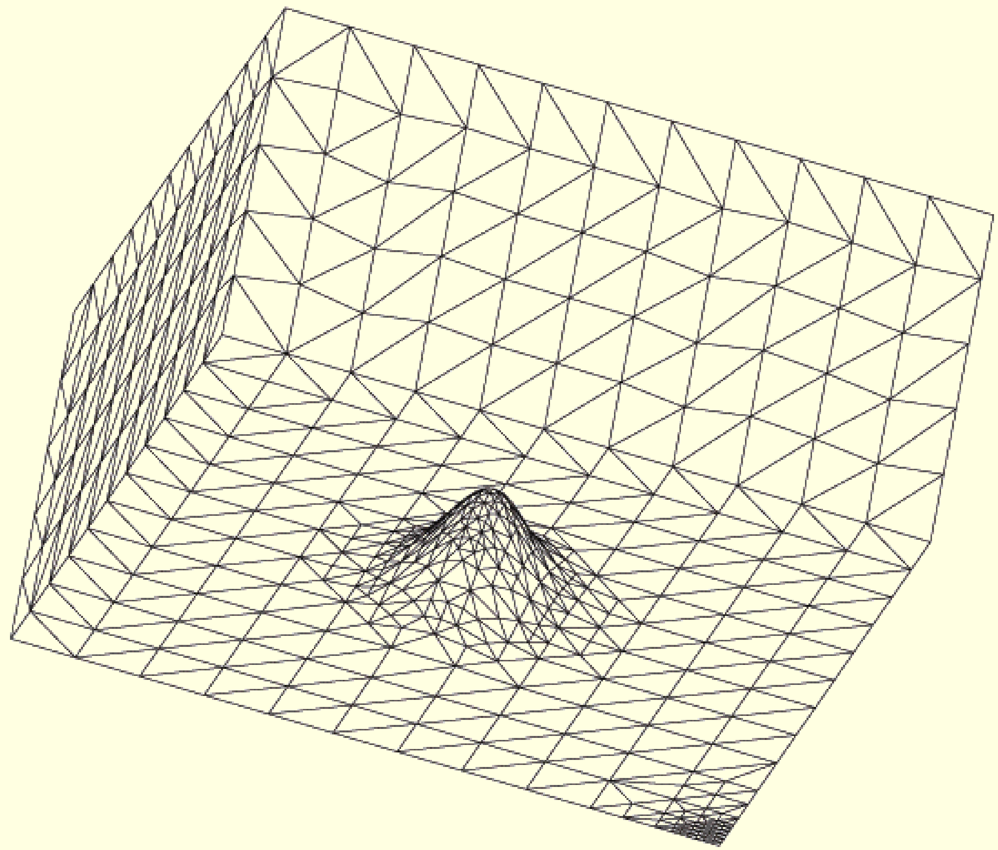
1 new node



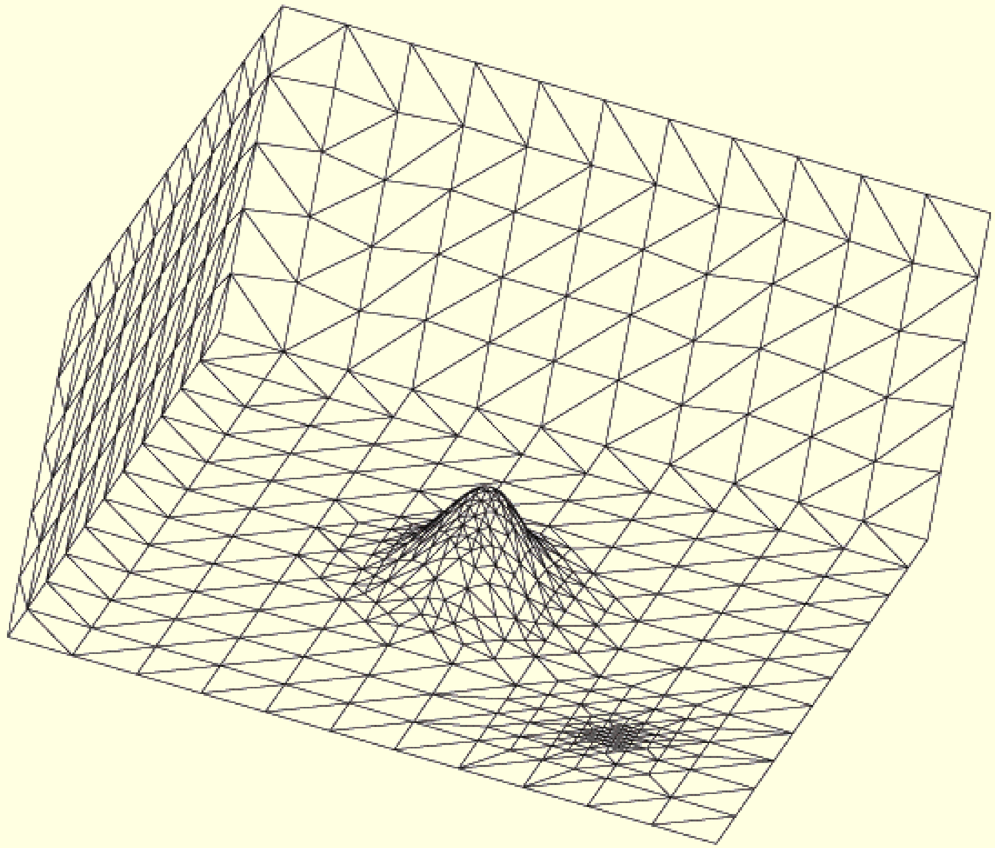
Application of Refinement/Derefinement Algorithm



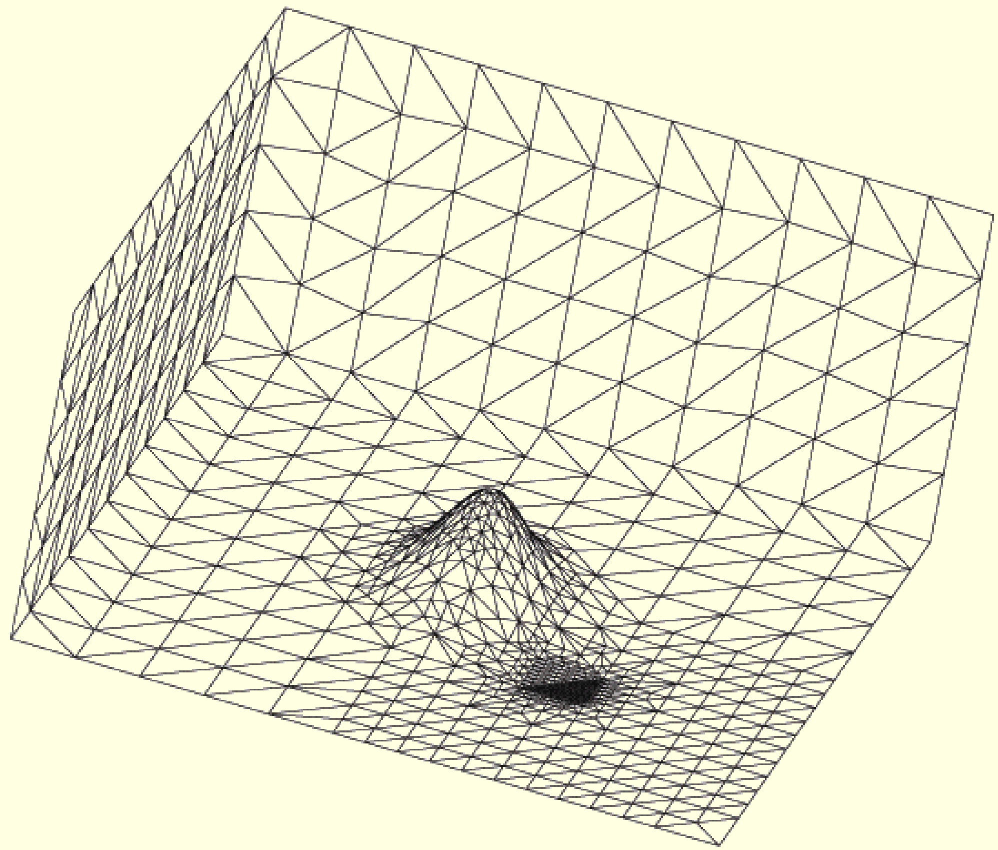
Application of Refinement/Derefinement Algorithm



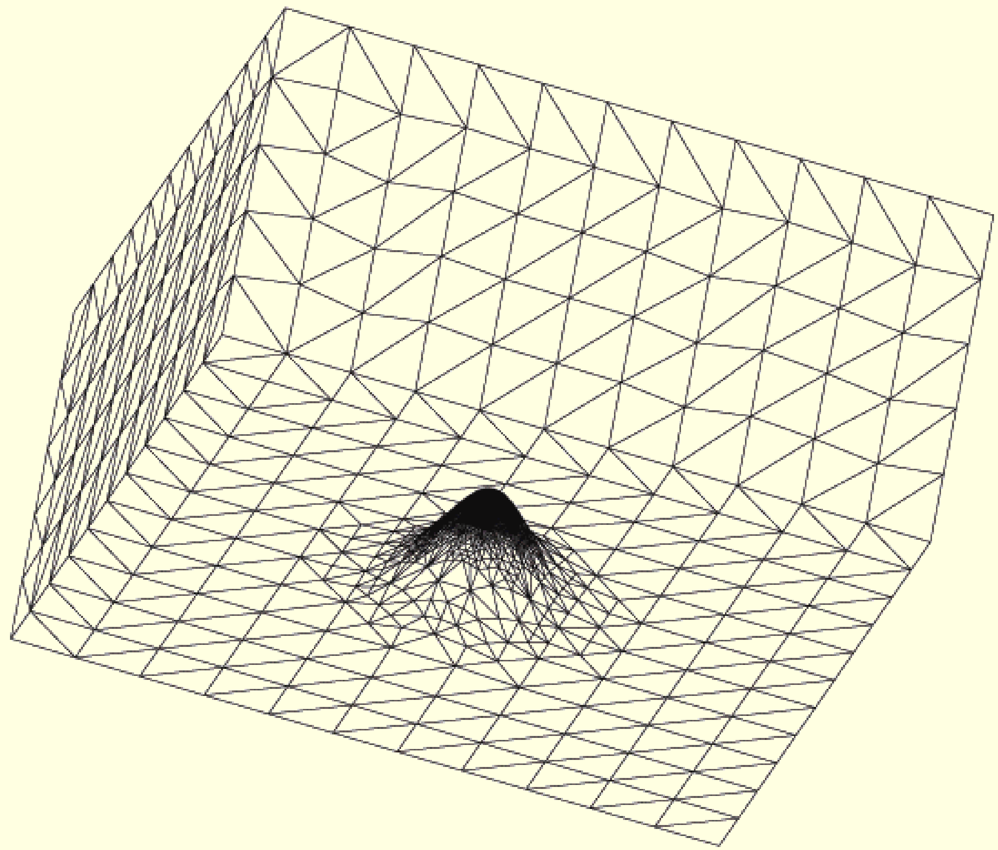
Application of Refinement/Derefinement Algorithm



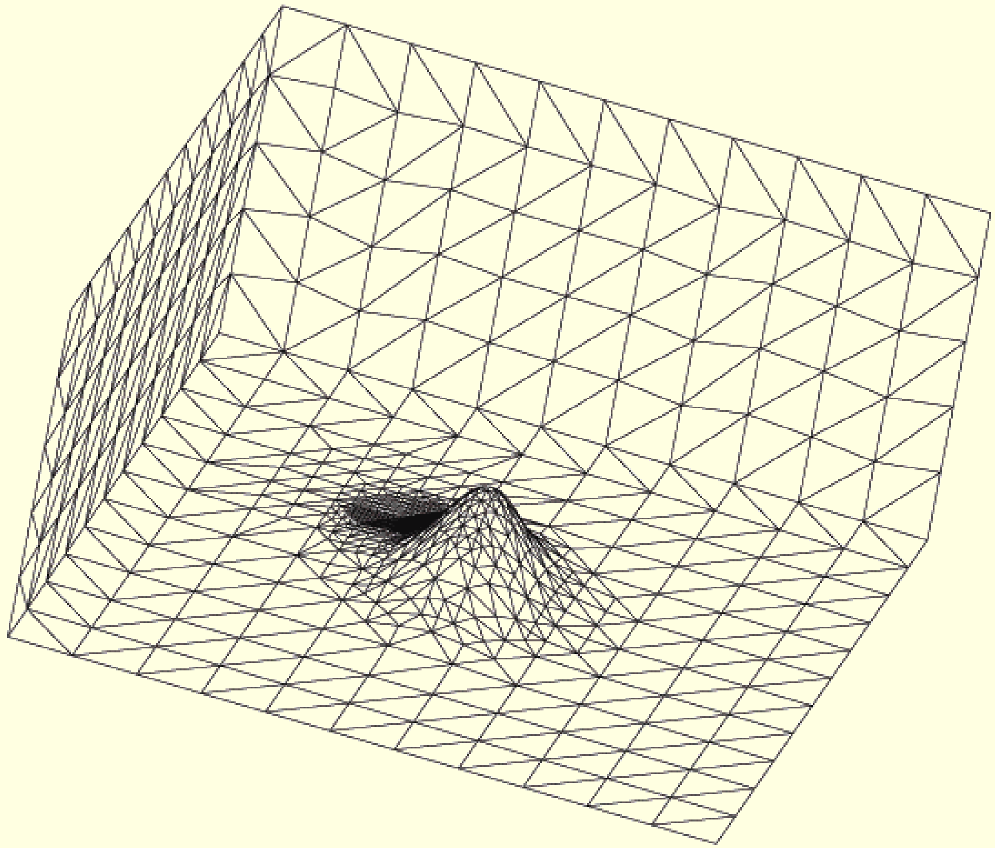
Application of Refinement/Derefinement Algorithm



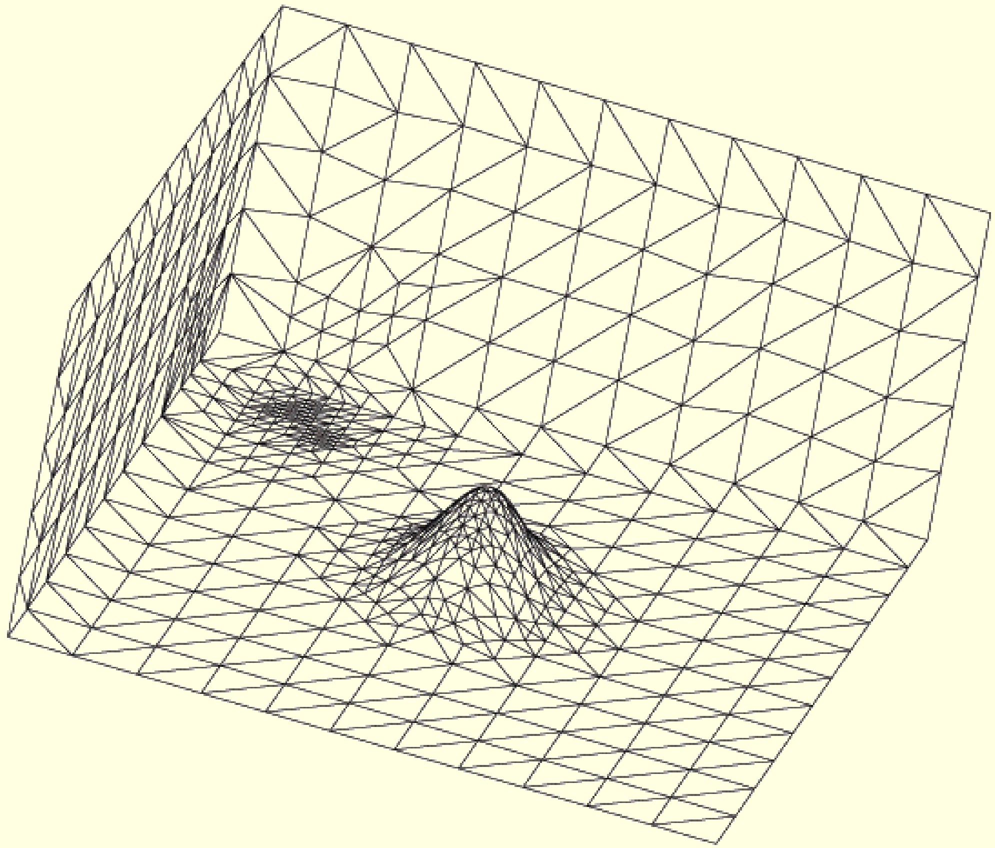
Application of Refinement/Derefinement Algorithm



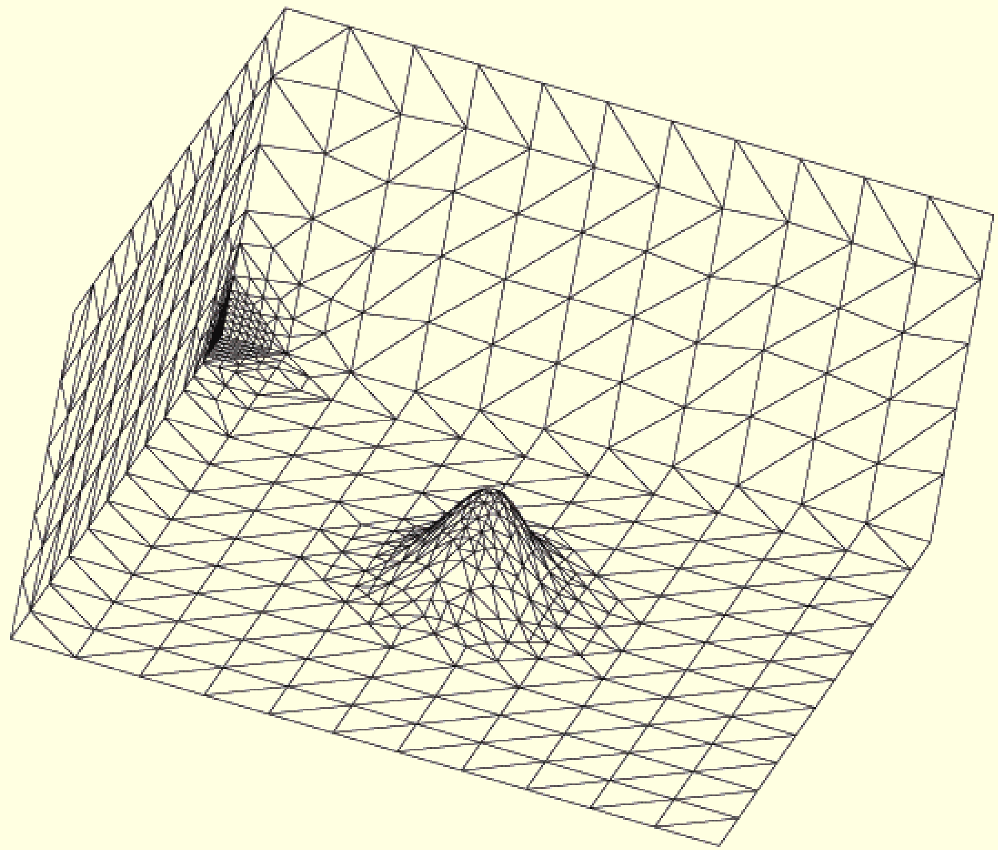
Application of Refinement/Derefinement Algorithm



Application of Refinement/Derefinement Algorithm



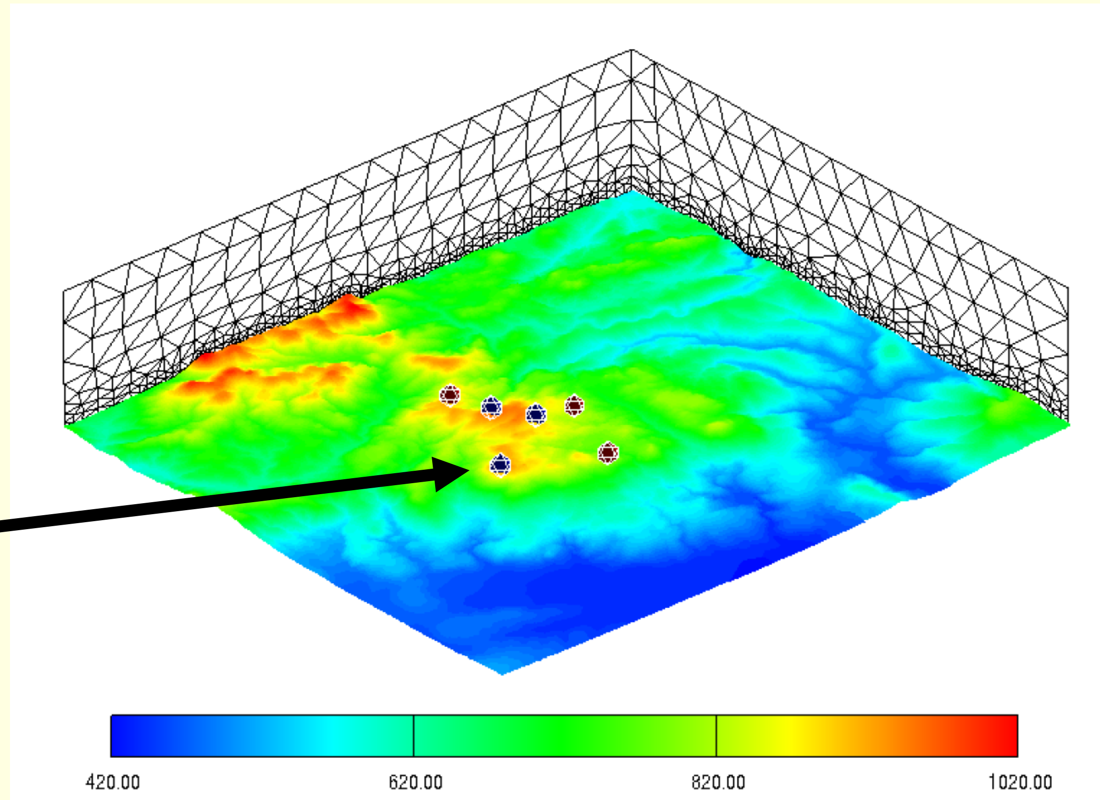
Application of Refinement/Derefinement Algorithm



Mass Consistent Model for Wind Field Adjustment

Let $\Omega \subset R^3$ be a domain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$

$\vec{u}_0$: observed wind, which is obtained with horizontal interpolation and vertical extrapolation of experimental measurements.



Mass Consistent Model for Wind Field Adjustment

1) Horizontal Interpolation

$$\vec{v}_0(z_m) = \varepsilon \frac{\sum_{n=1}^{N} \frac{\vec{v}_n}{d_n^2}}{\sum_{n=1}^{N} \frac{1}{d_n^2}} + (1-\varepsilon) \frac{\sum_{n=1}^{N} \frac{\vec{v}_n}{|\Delta h_n|}}{\sum_{n=1}^{N} \frac{1}{|\Delta h_n|}}$$

$$0 \le \varepsilon \le 1$$

Mass Consistent Model for Wind Field Adjustment

2) Vertical Extrapolation

Friction velocity: $\vec{v}^* = \frac{k \; \vec{v}_0(z_m)}{\log \frac{z_m}{z_0} - \Phi_m}$

Height of the planetary boundary layer: $z_{pbl} = \frac{\gamma \; |\vec{v}^*|}{f}$

$f = 2\omega \; \sin\varphi$ is the Coriolis parameter

ω is the Earth rotation and φ the latitude

γ is a parameter depending on the atmospheric stability

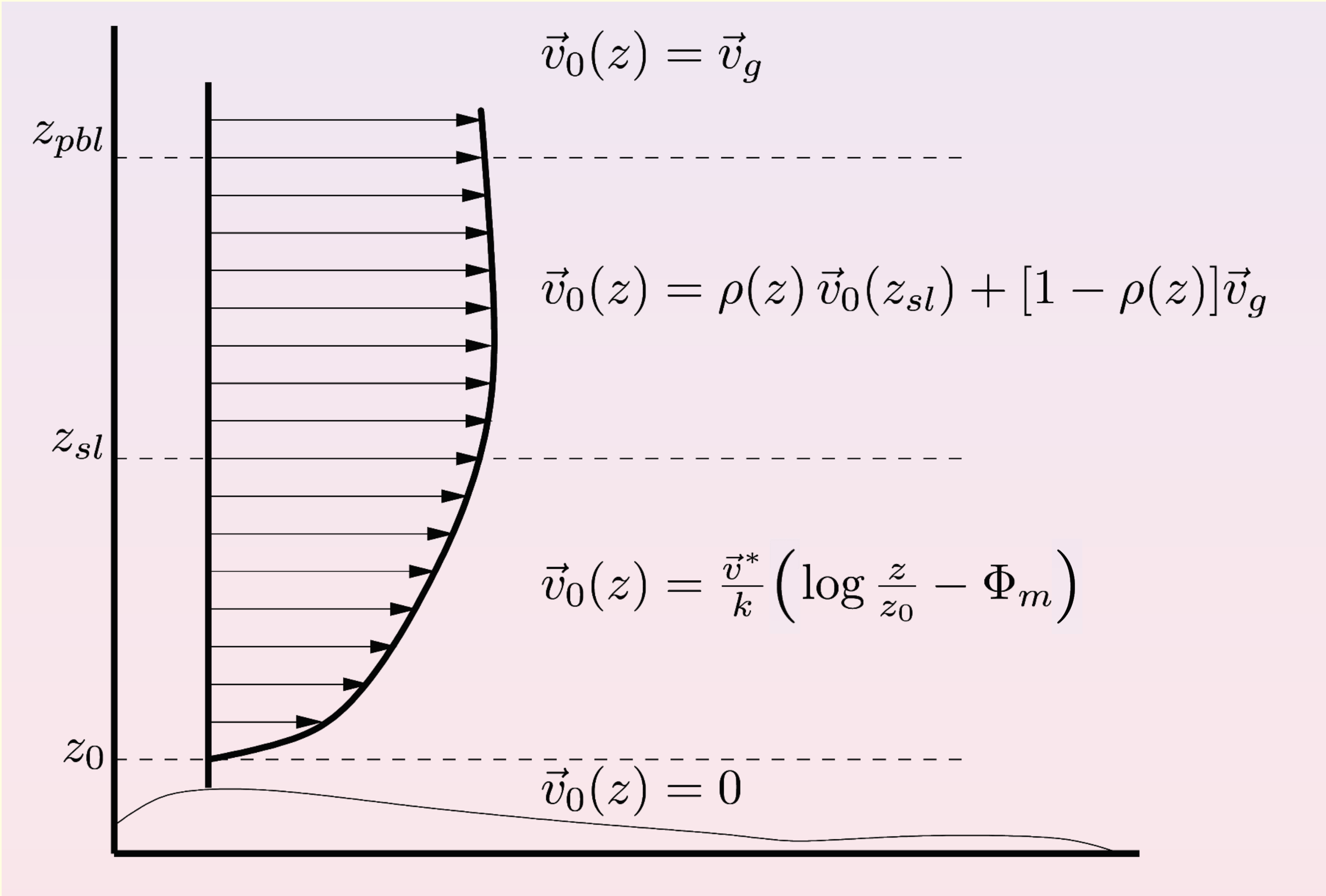
Mixing height:

- $h = z_{pbl}$ in neutral and unstable conditions
- $h = \gamma' \sqrt{\frac{|\vec{v}^*| \, L}{f}}$ in stable conditions

γ': constant of proportionality.

Height of surface layer: $z_{sl} = \frac{h}{10}$

Mass Consistent Model for Wind Field Adjustment



Mass Consistent Model for Wind Field Adjustment

Let $\Omega \subset R^3$ be a domain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$

$\vec{u}_0$: observed wind, which is obtained with horizontal interpolation and vertical extrapolation of experimental measurements.

Objective: find the velocity field $\vec{u}$ that it adjusts to $\vec{u}_0$ verifying

- Incompressibility condition in the domain: $\operatorname{div} \vec{u} = 0$ in Ω
- Impermeability condition on the terrain: $\vec{u} \cdot \vec{n} = 0$ on Γ_1

☐ Then, $\vec{u}$ is the solution of the least-square problem: Find $\vec{u} \in K$ verifying

$$\begin{cases} J(\vec{u}) = \min\limits_{\vec{v} \in K} J(\vec{v}) \\ K = \left\{\vec{v};\ \operatorname{div} \vec{v} = 0,\ \vec{v} \cdot \vec{n}\big|_{\Gamma_1} = 0\right\} \end{cases}$$

where $J(\vec{v}) = \frac{1}{2} \int_\Omega (\vec{v} - \vec{u}_0)^t P (\vec{v} - \vec{u}_0)$

$\alpha = \frac{\alpha_1}{\alpha_2}$

Mass Consistent Model for Wind Field Adjustment

- Lagrange multiplier technique is used to solve this problem. So, if we introduce

$$L(\vec{v},q) = J(\vec{v}) + \int_\Omega q \ \mathrm{div}\ \vec{v}$$

its saddle point $(\vec{u},\lambda)$ verifies the Euler-Lagrange equations:

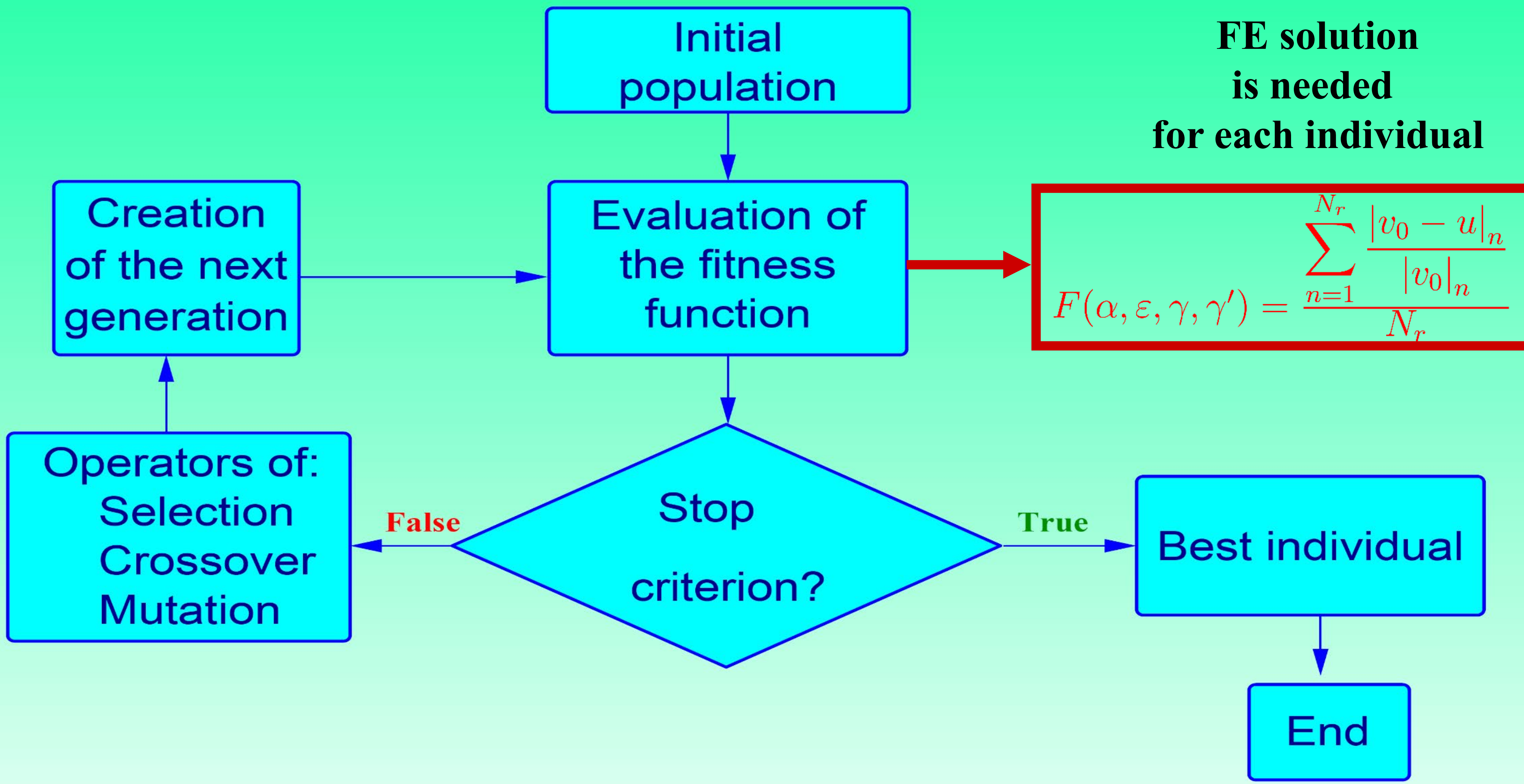
$$-\vec{\nabla}\cdot(P^{-1}\vec{\nabla}\lambda) = \vec{\nabla}\cdot\vec{u}_0 \qquad \text{in } \Omega$$

$$-P^{-1}\frac{\partial\lambda}{\partial\vec{n}} = \vec{n}\cdot\vec{u}_0 \qquad \text{on } \Gamma_1$$

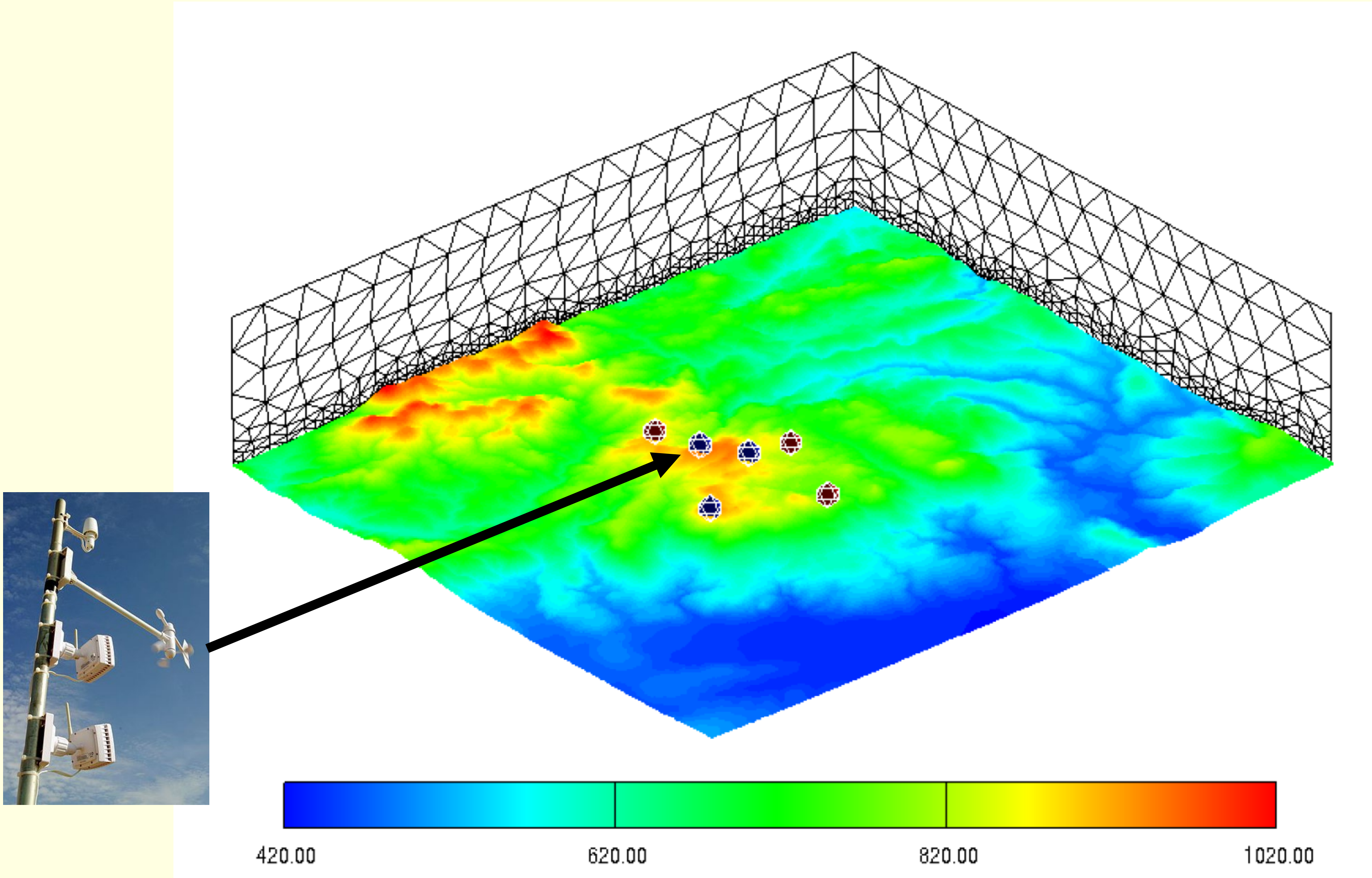
$$\lambda = 0 \qquad \text{on } \Gamma_2$$

and, finally, the adjusted velocity field is obtained by: $\vec{u} = \vec{u}_0 + P^{-1}\ \vec{\nabla}\lambda \quad \text{in } \Omega$

Parameter Estimation with Parallel Genetic Algorithm



Application in a Wind Farm

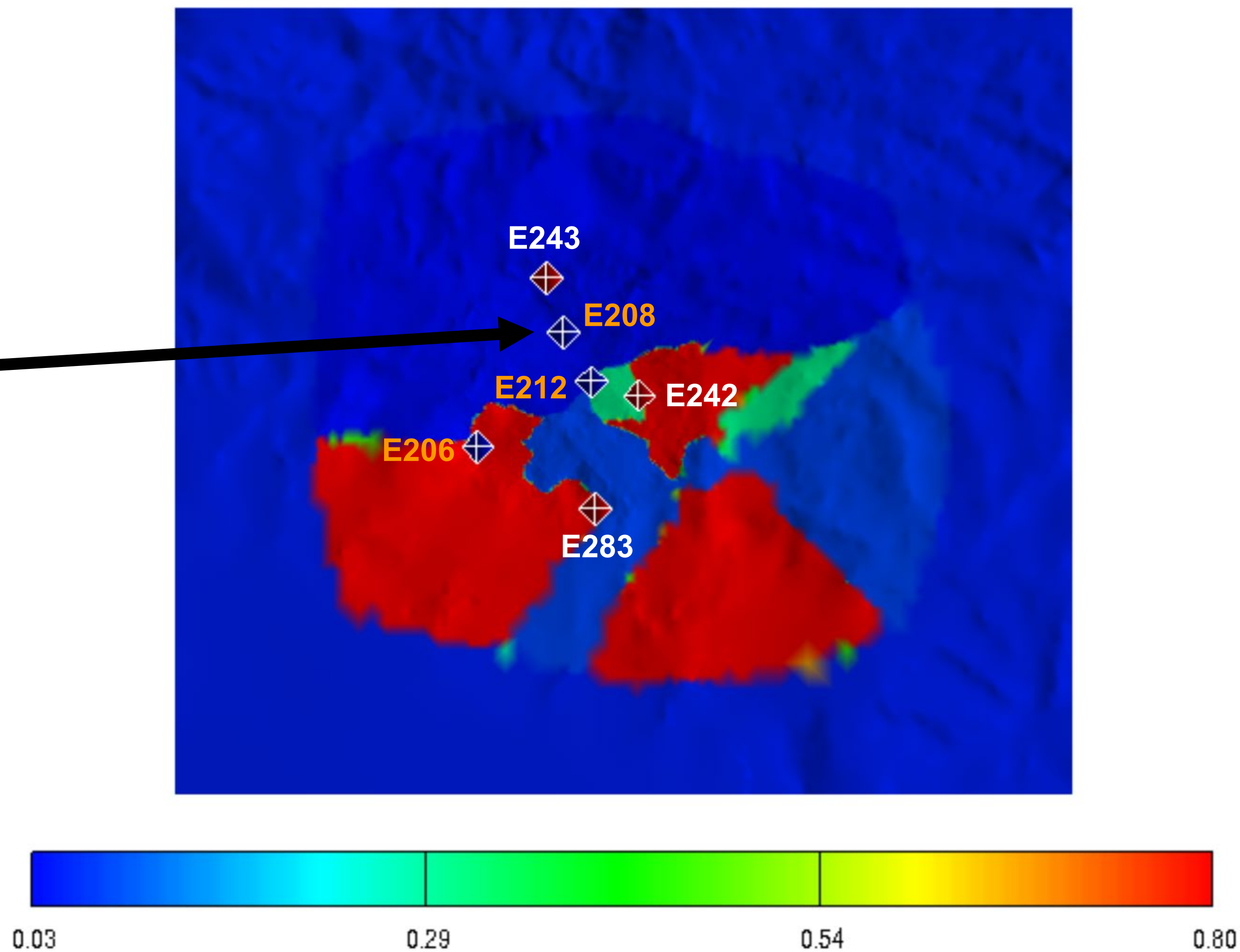


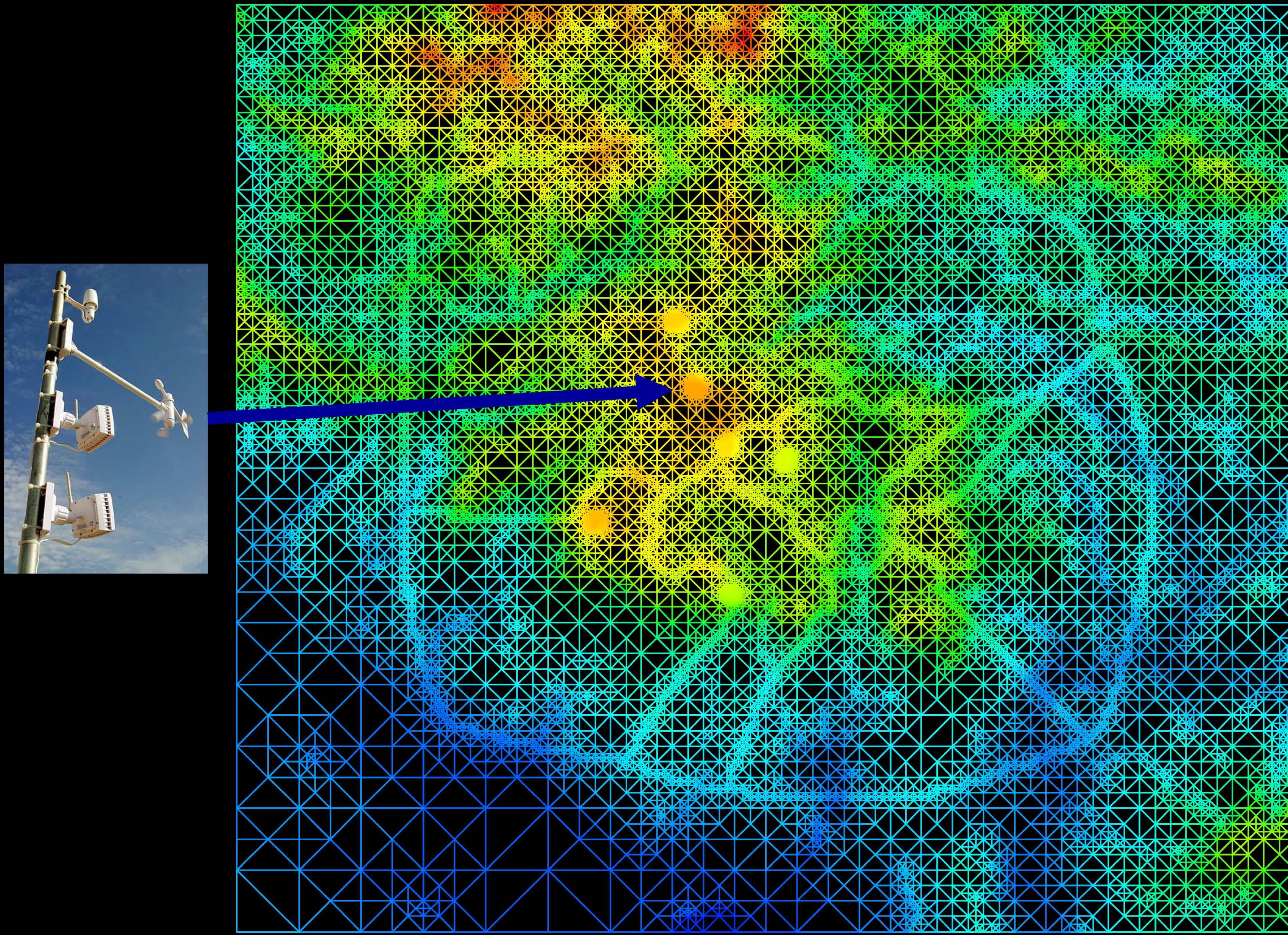
Application in a Wind Farm

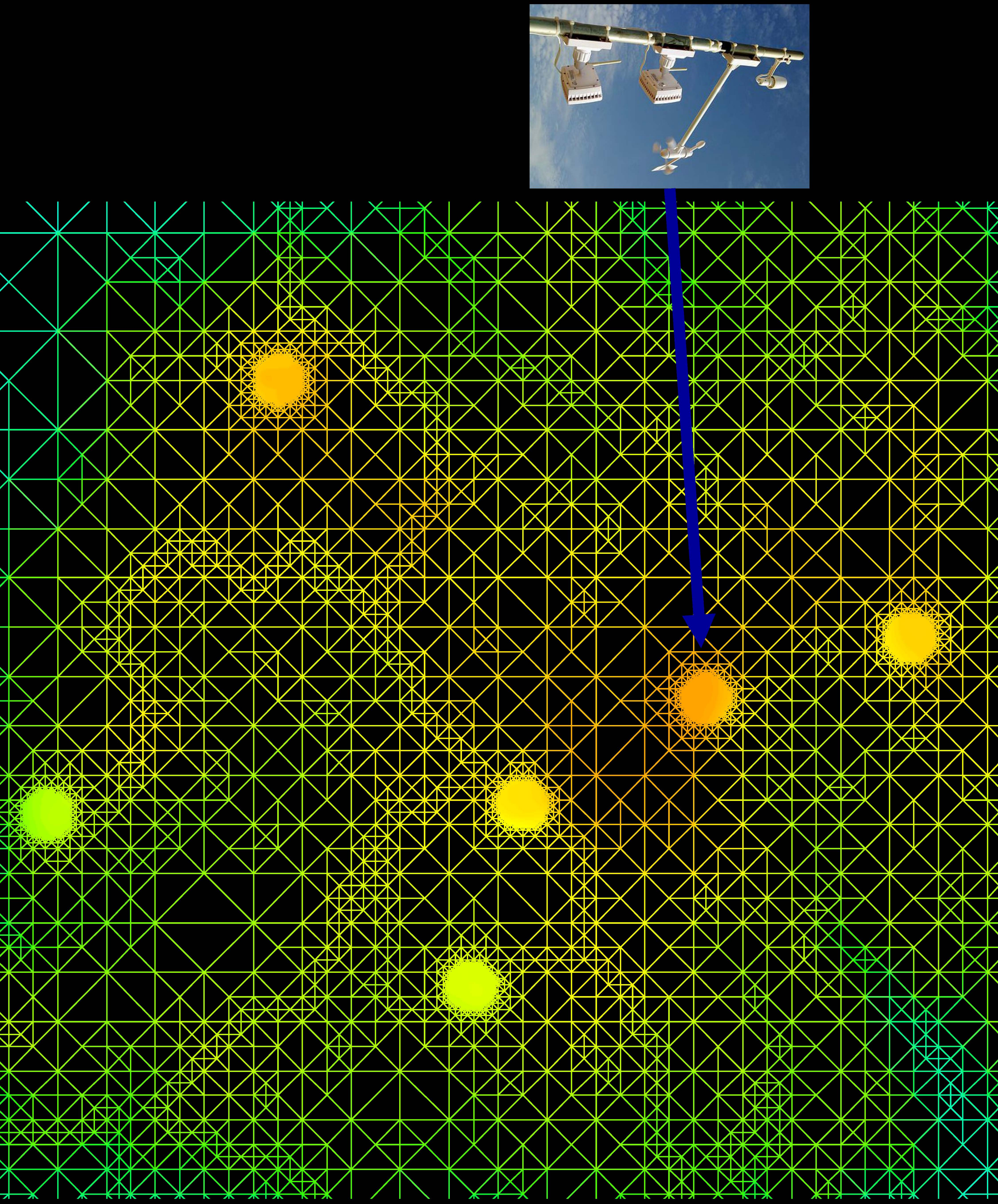
Province of Lugo (Spain)

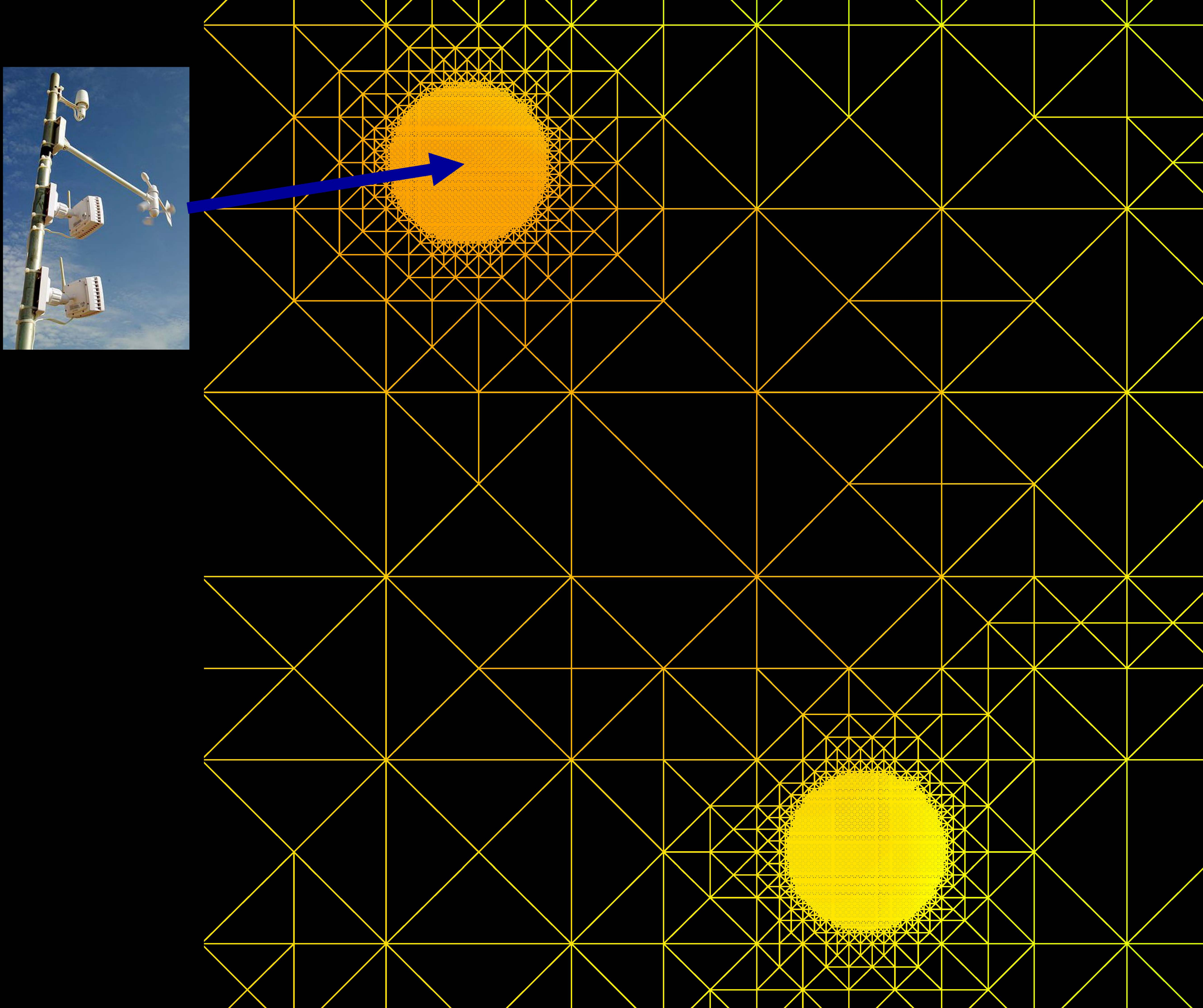
Application in a Wind Farm

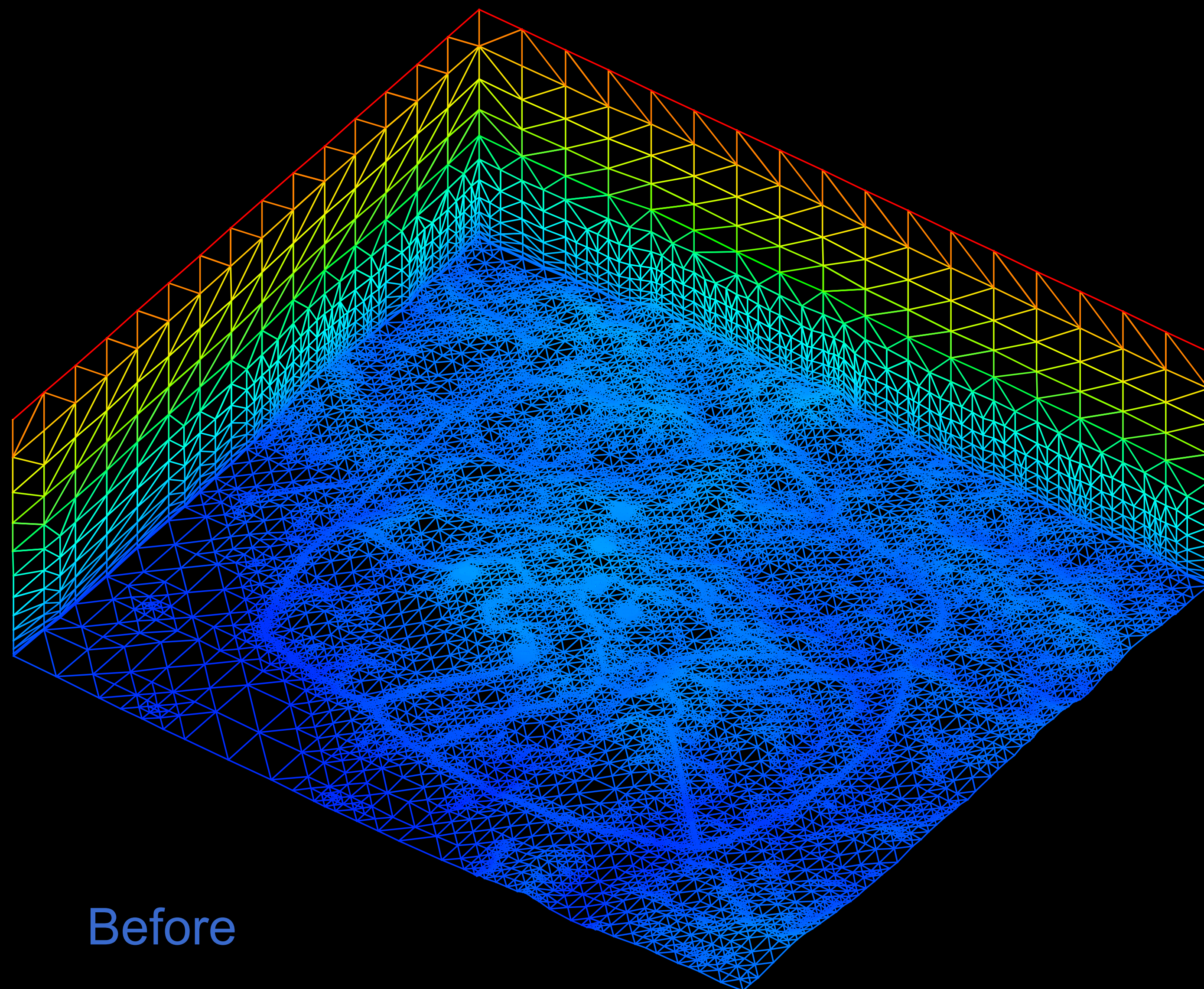
z_0: Roughness length (m)

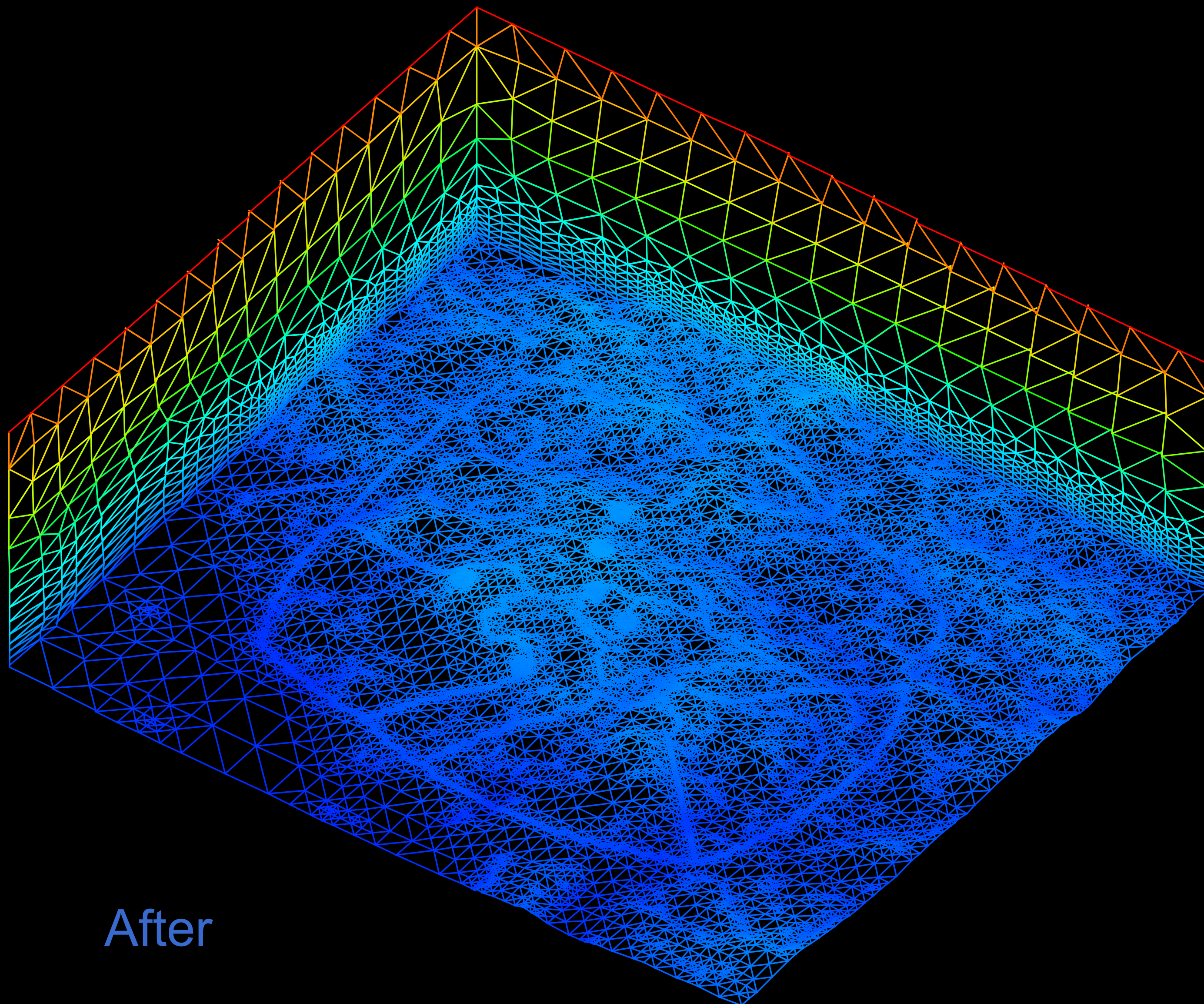








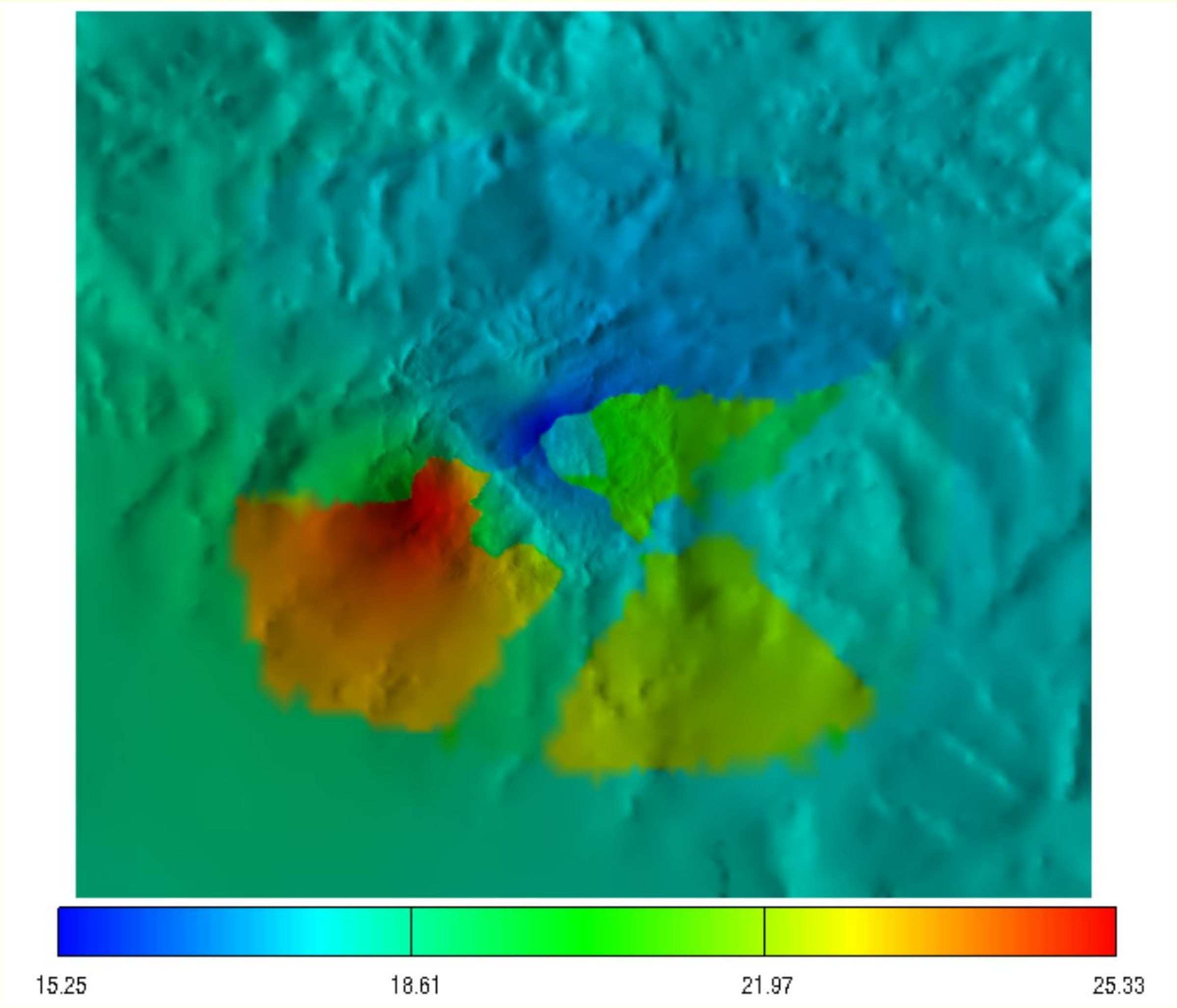

Before



After

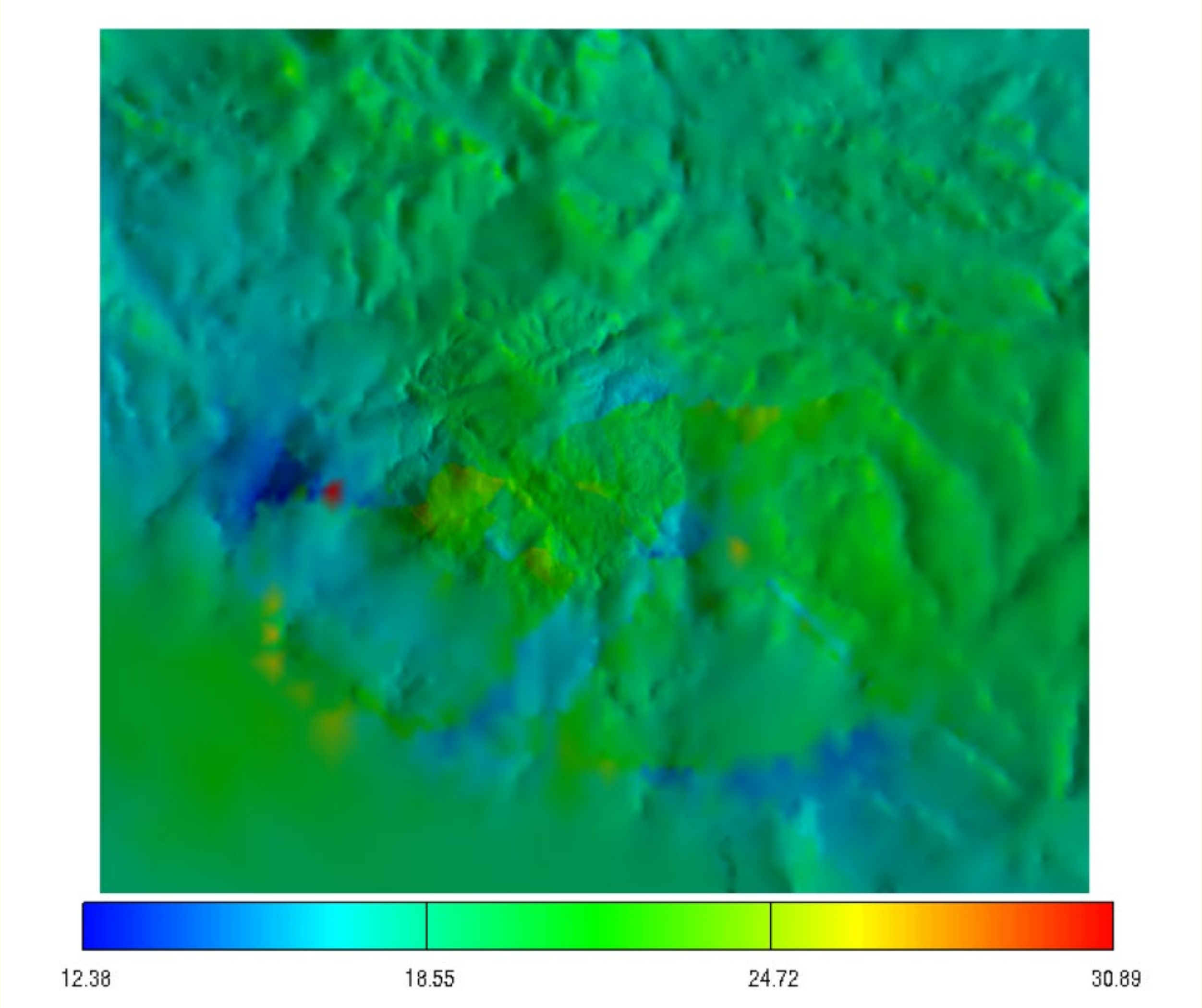
Application in a Wind Farm

Interpolated Velocity Field (m/s)

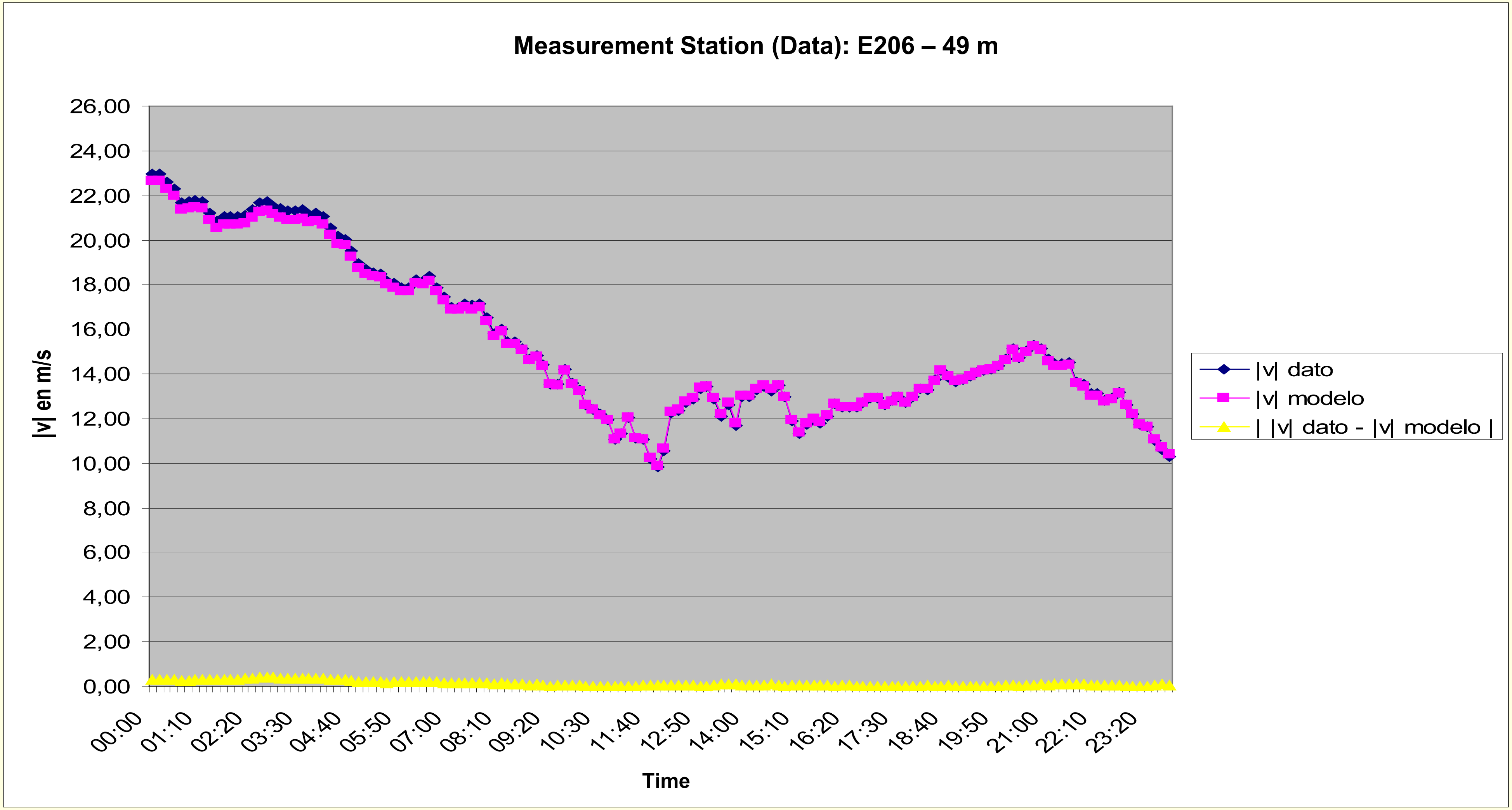


Application in a Wind Farm

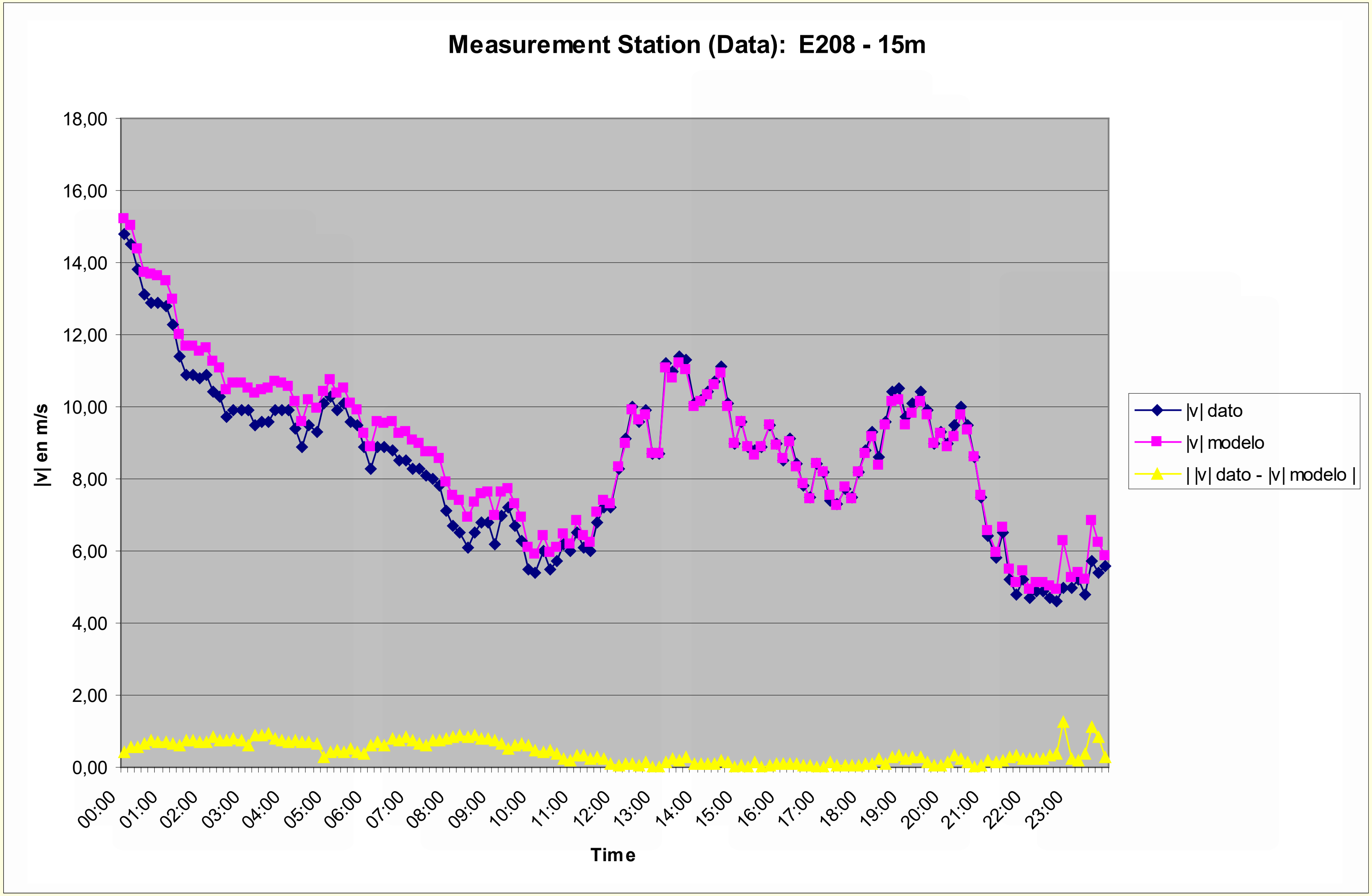
Adjusted Velocity Field (m/s)



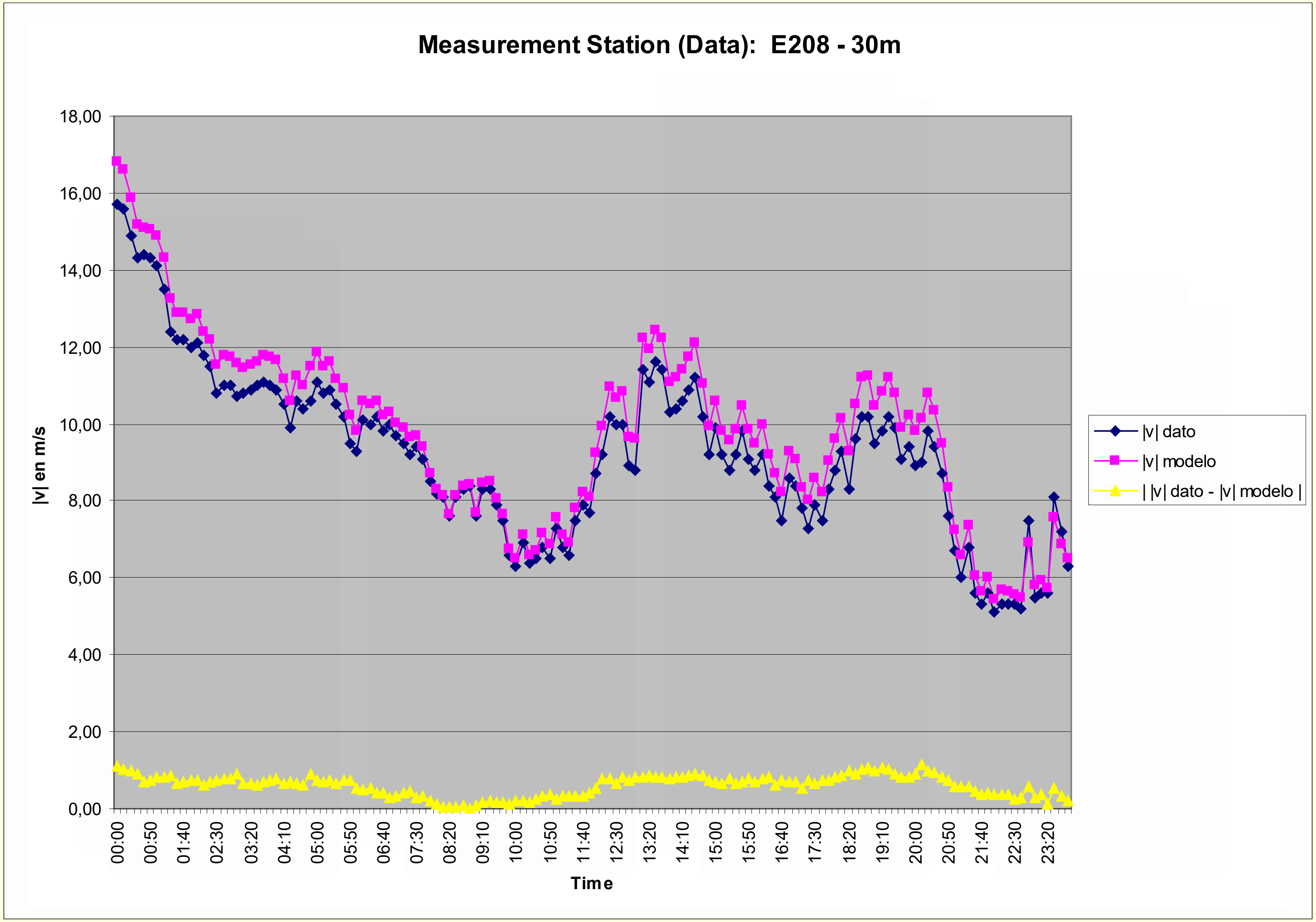
Application in a Wind Farm



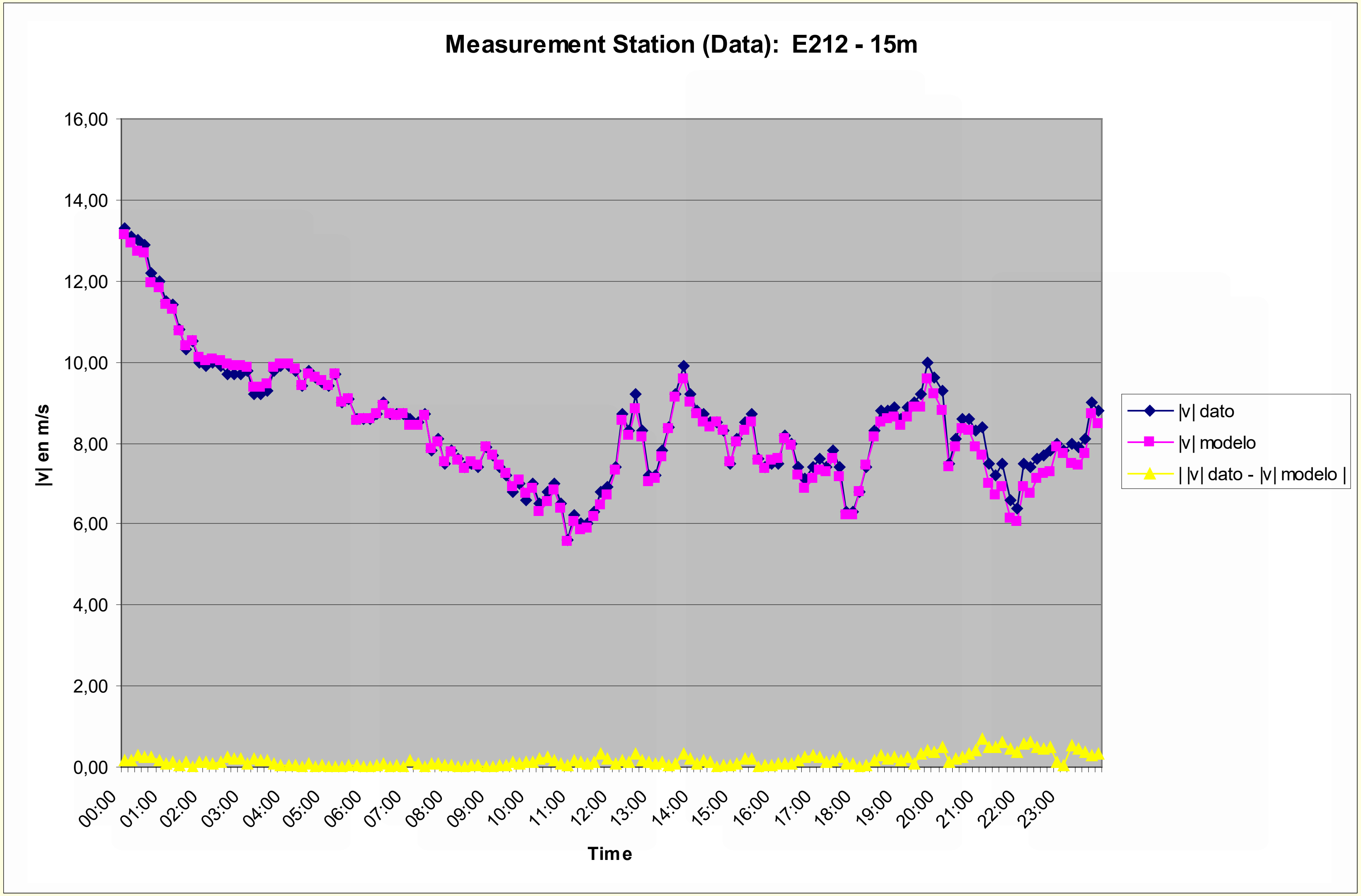
Application in a Wind Farm



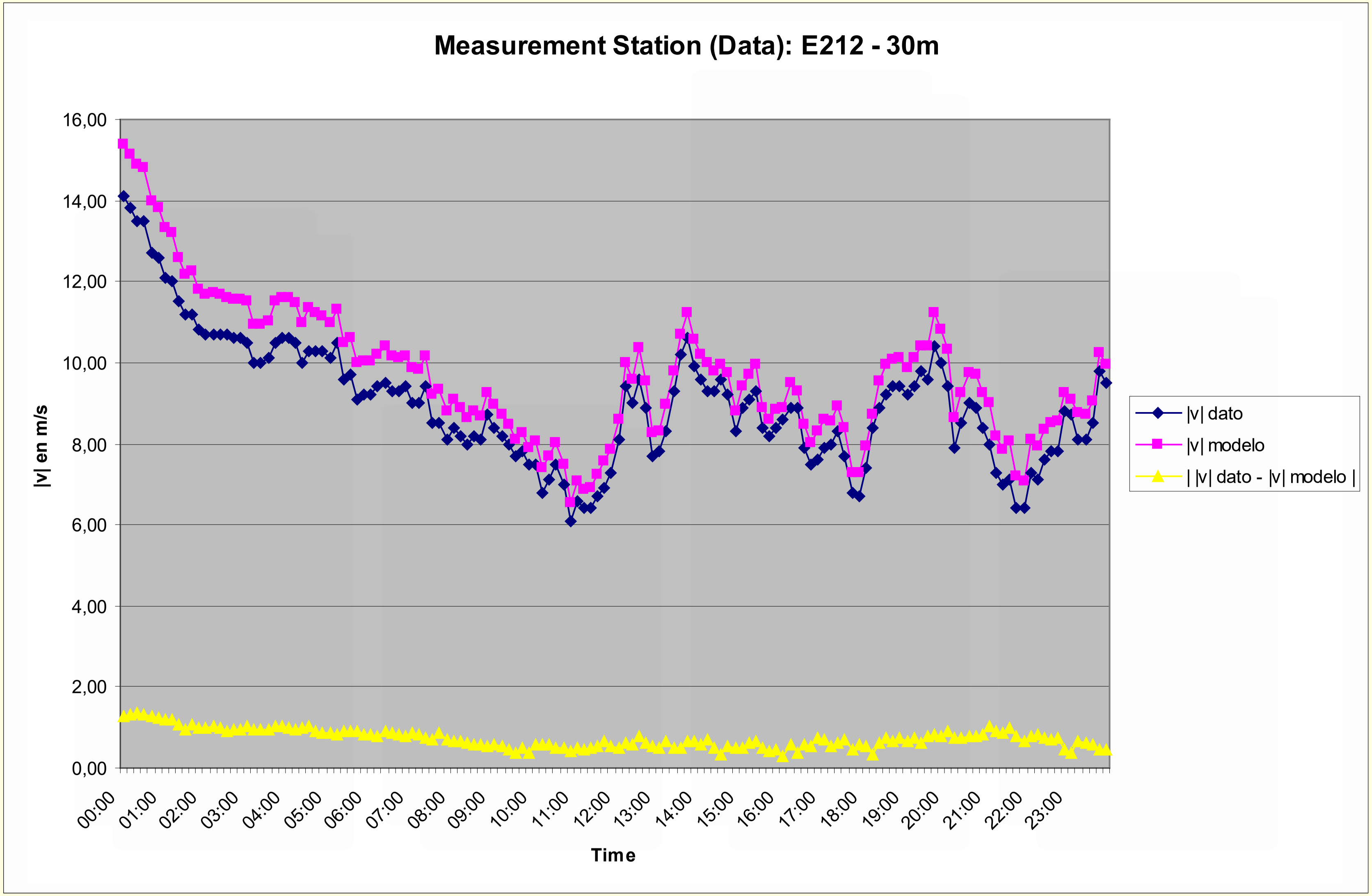
Application in a Wind Farm



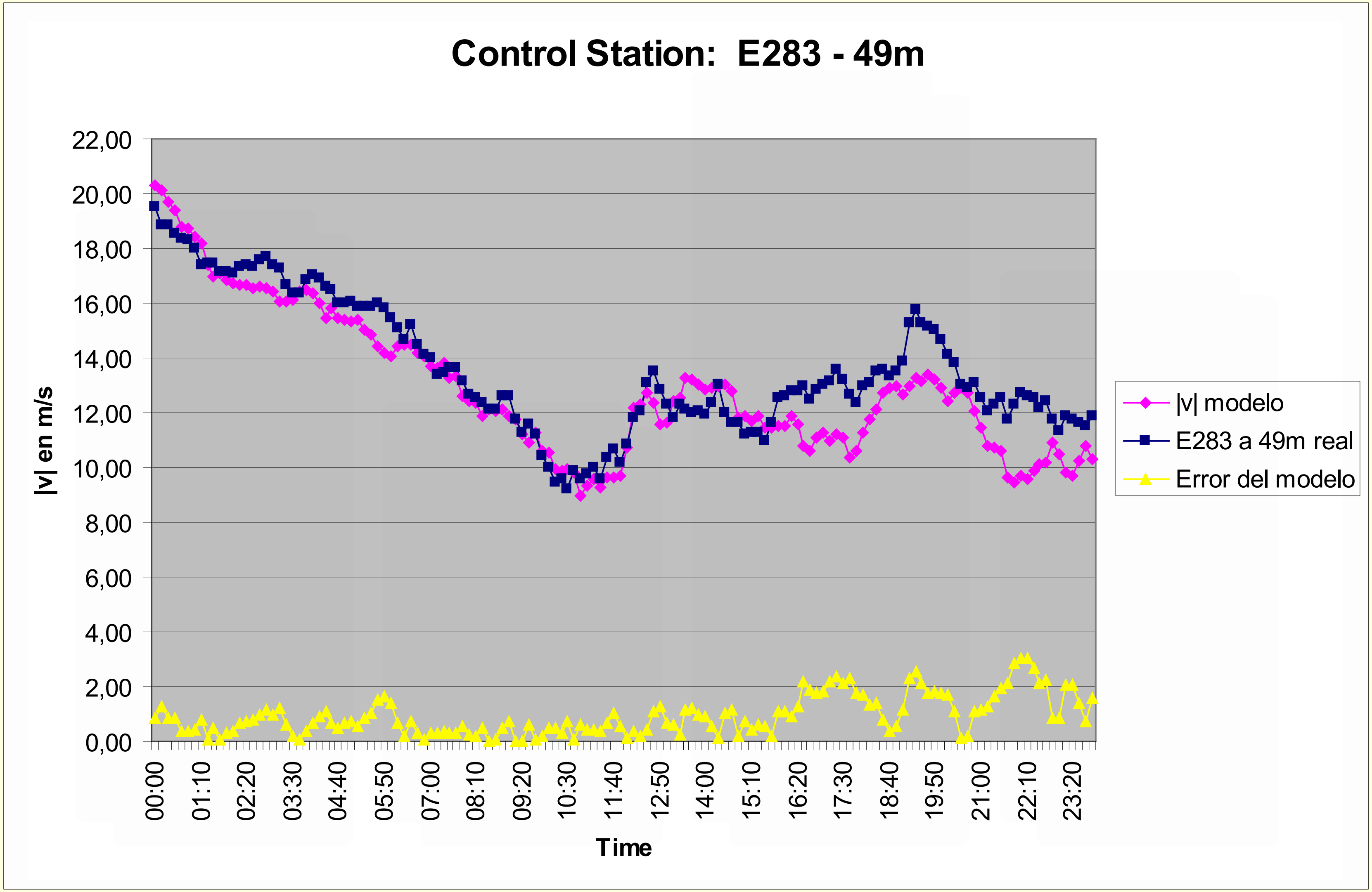
Application in a Wind Farm



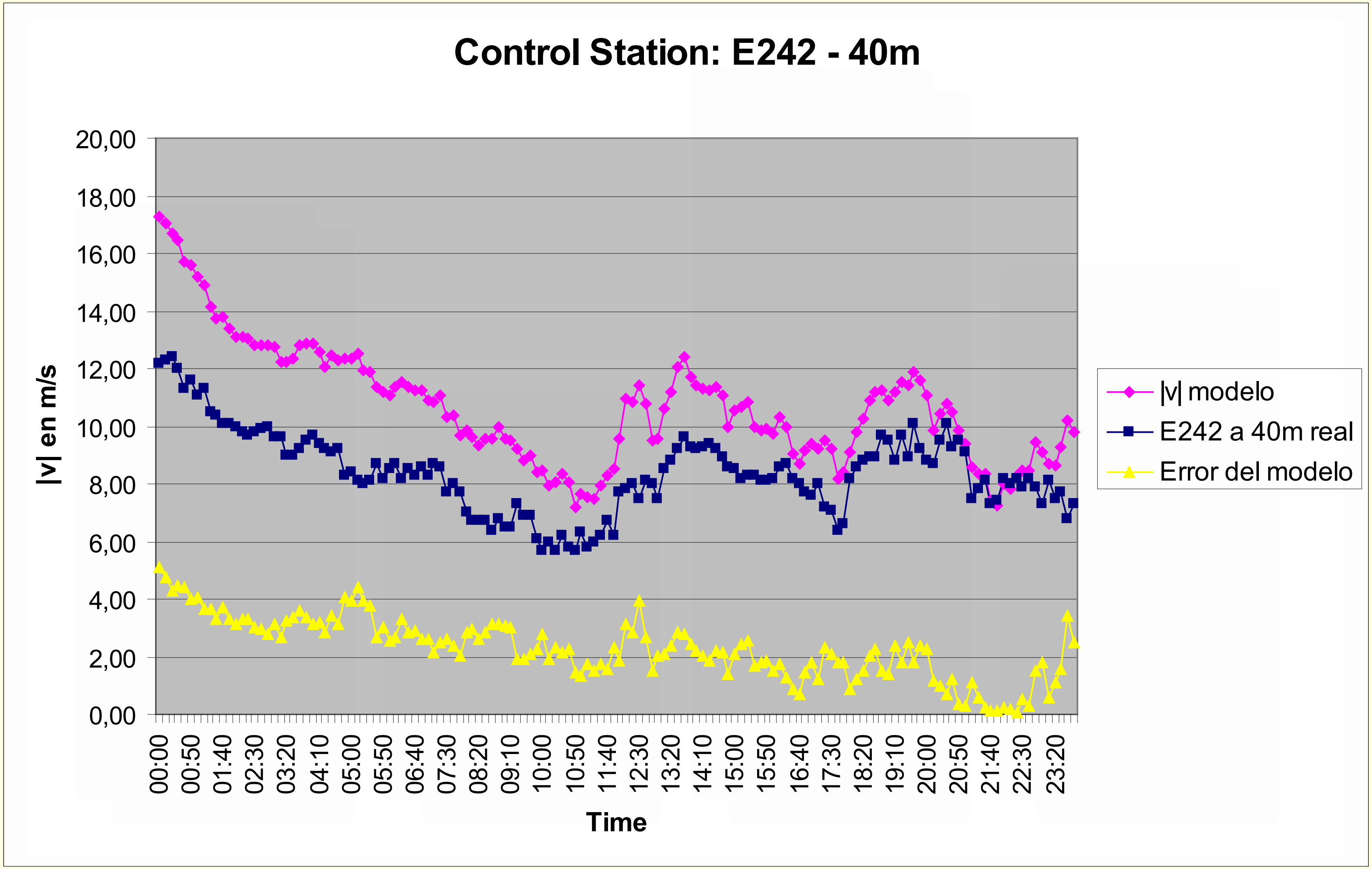
Application in a Wind Farm



Application in a Wind Farm



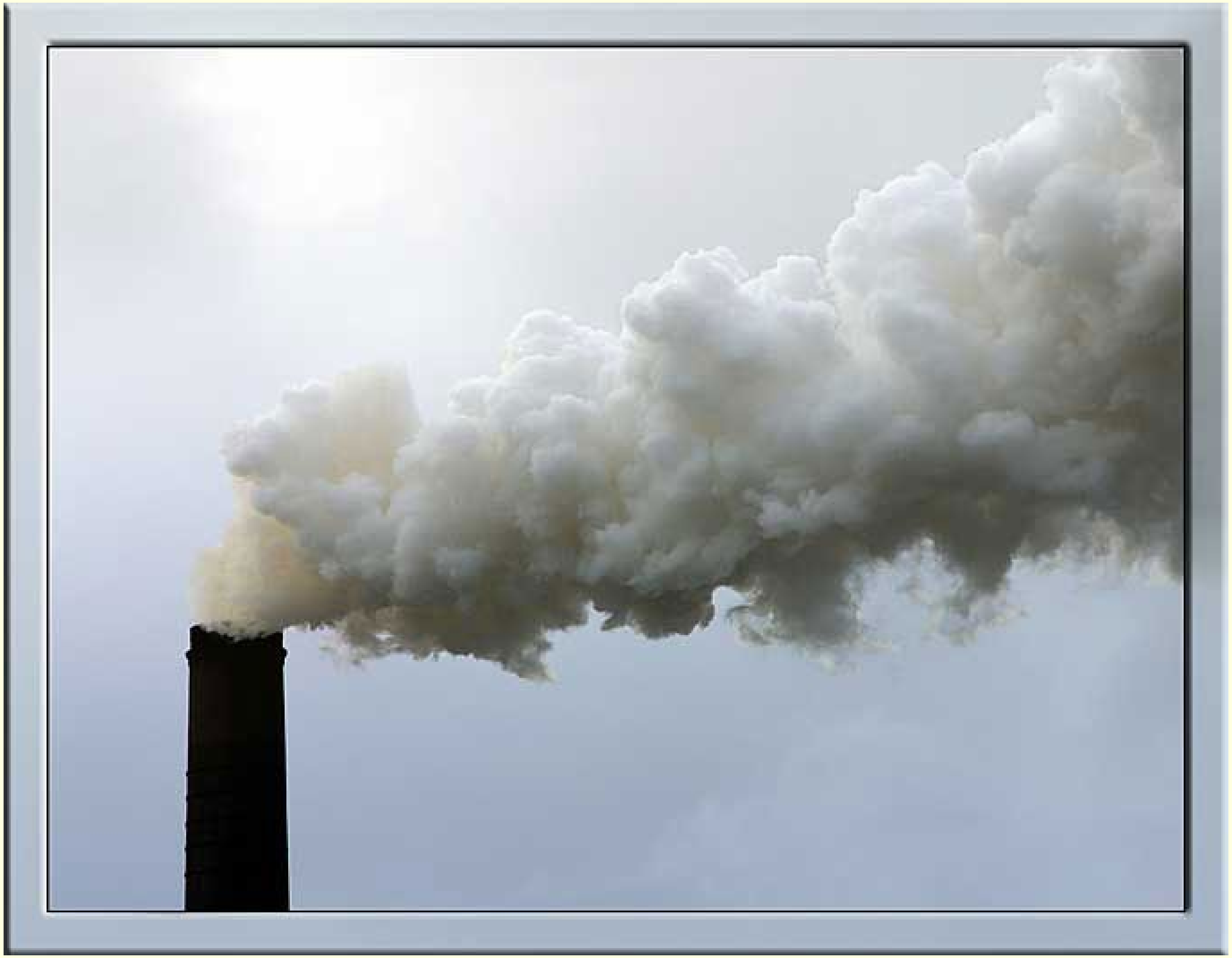
Application in a Wind Farm



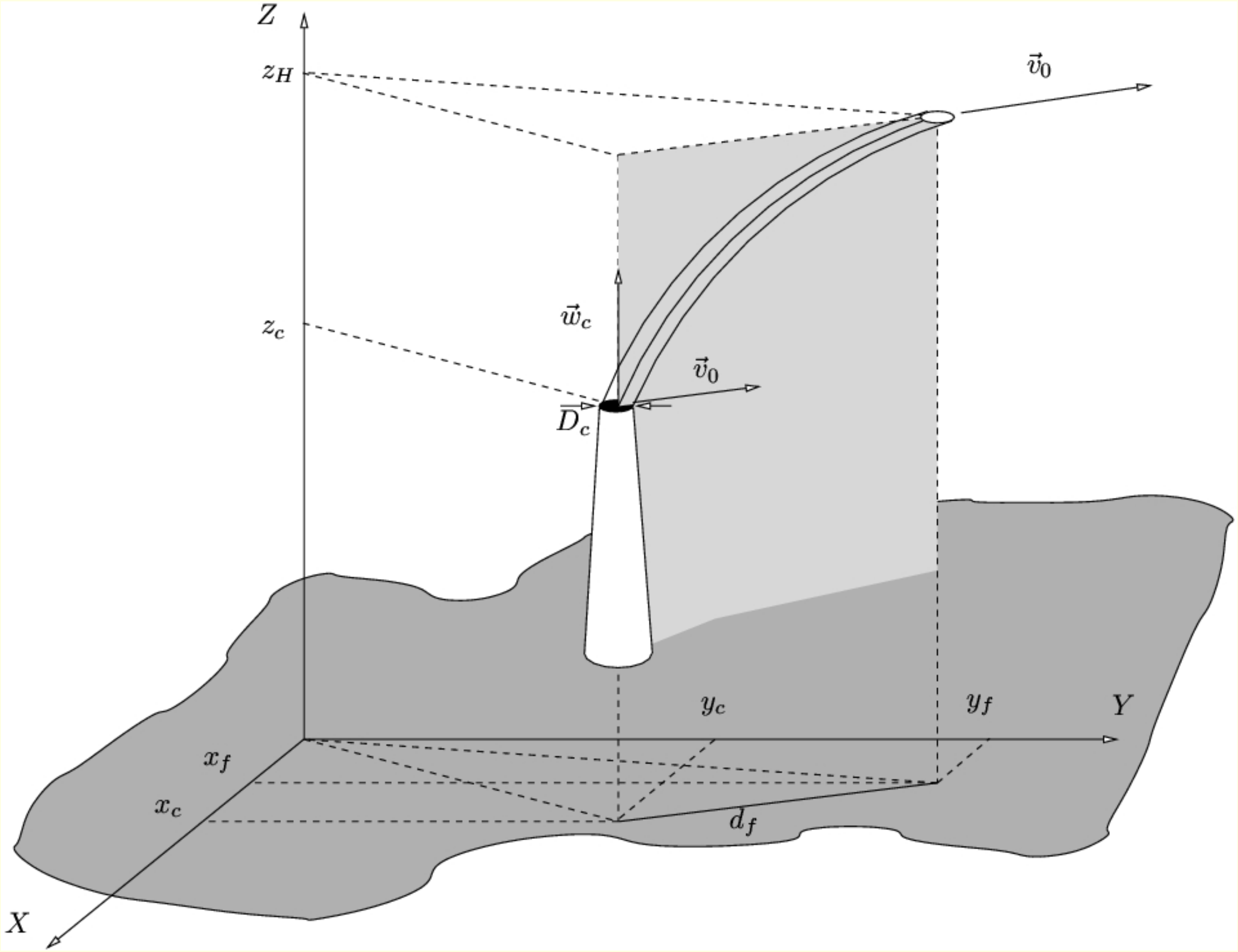
Application in a Wind Farm

Diferences |v_model - v_data| (m/s) - 14 parameter evaluations per day (21th March 2003)							
	|v| E206 a 49m	|v| E208 a 15m	|v| E208 a 30m	|v| E212 a 15m	|v| E212 a 30m	|v| E242 a 40m	|v| E283 a 49m
Mean Error	0,12	0,40	0,60	0,16	0,73	2,31	0,94
Max Error	0,41	1,26	1,14	0,69	1,38	5,09	3,04
Min Error	0,00	0,00	0,01	0,00	0,29	0,09	0,00
Variance	0,01	0,09	0,07	0,02	0,05	1,14	0,51

Gaussian Plume Model: Vertical Velocity Correction



Gaussian Plume Model: Predominant Buoyancy Rise

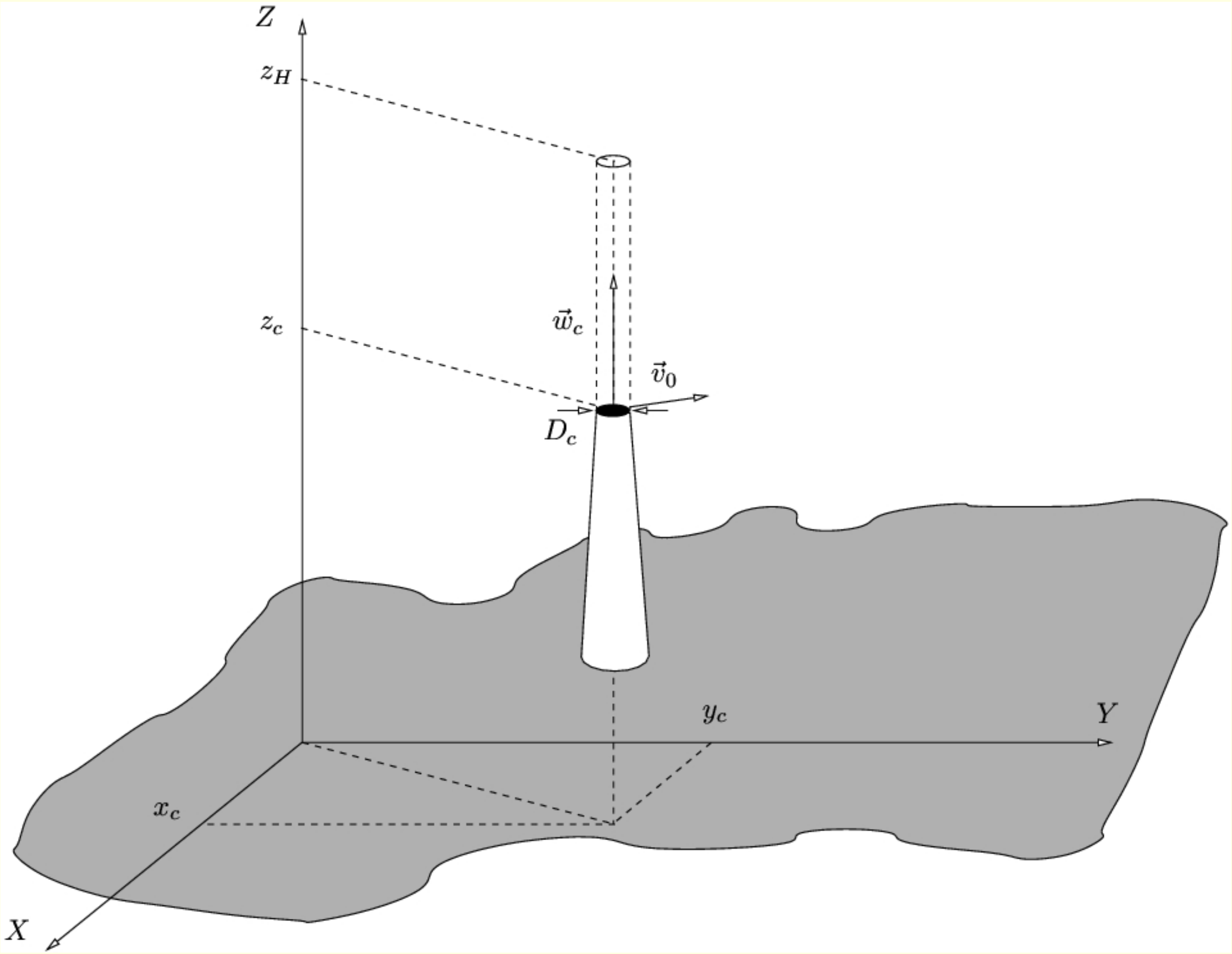


$$\frac{|\vec{w}_c|}{|\vec{v}_0(x_c, y_c, z_c)|} \leq 4$$

Vertical velocity correction

along the plume trajectory

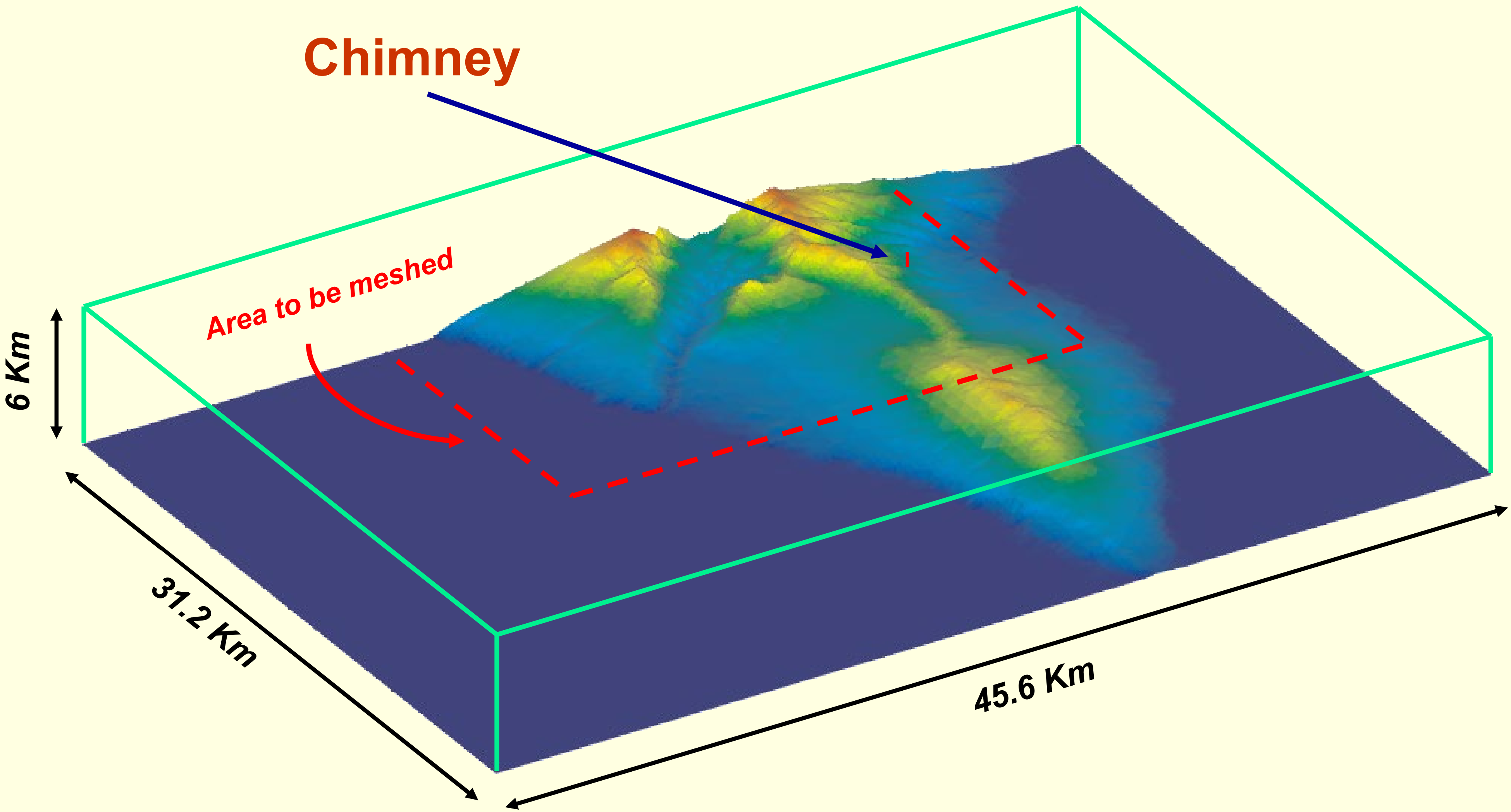
Gaussian Plume Model: Predominant Momentum Rise



$$\frac{|\vec{w}_c|}{|\vec{v}_0(x_c, y_c, z_c)|} > 4$$

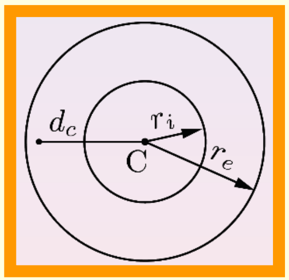
Vertical velocity correction along the plume trajectory

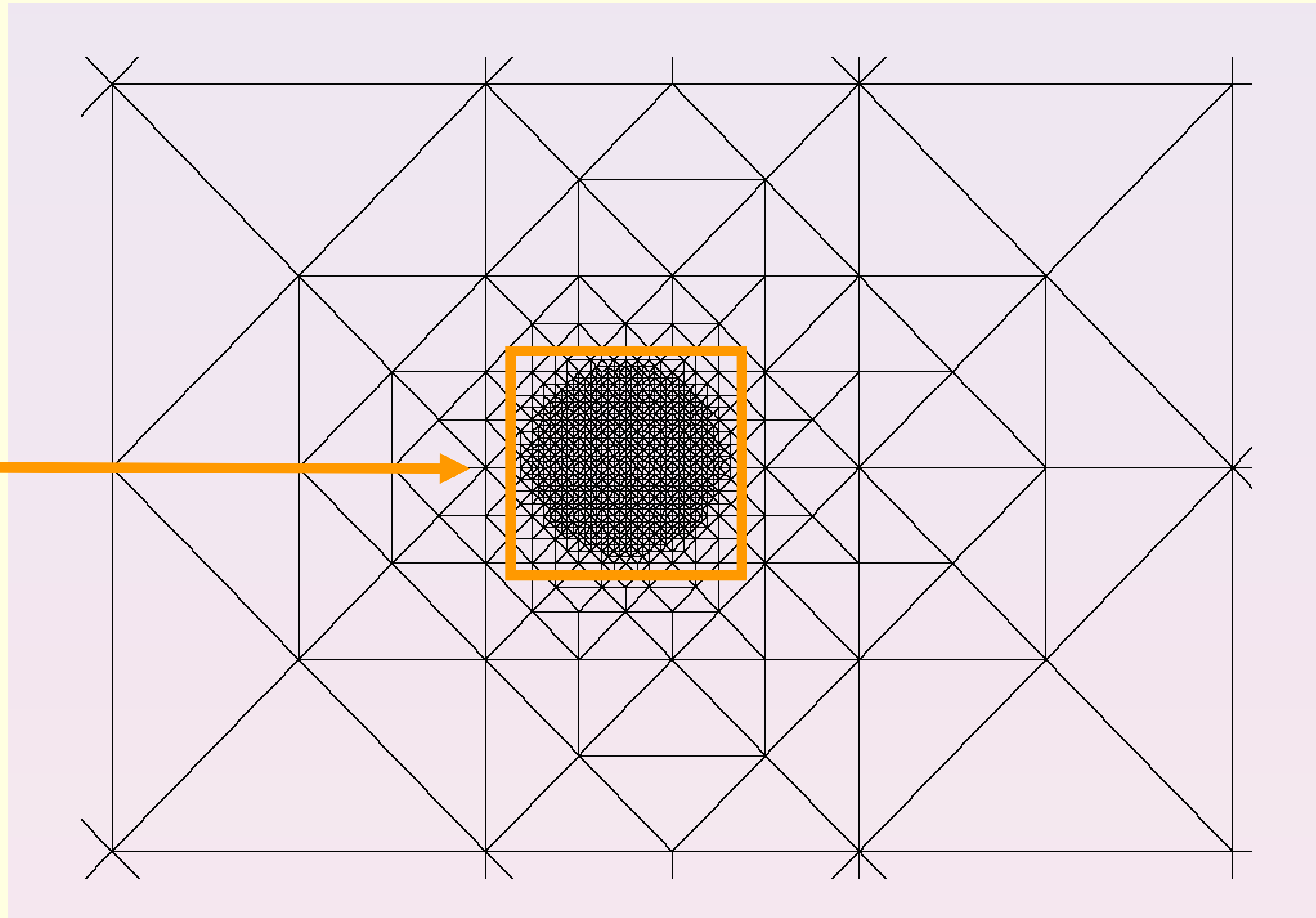
Wind Field Simulation Including Chimney Emissions



Wind Field Simulation Including Chimney Emissions

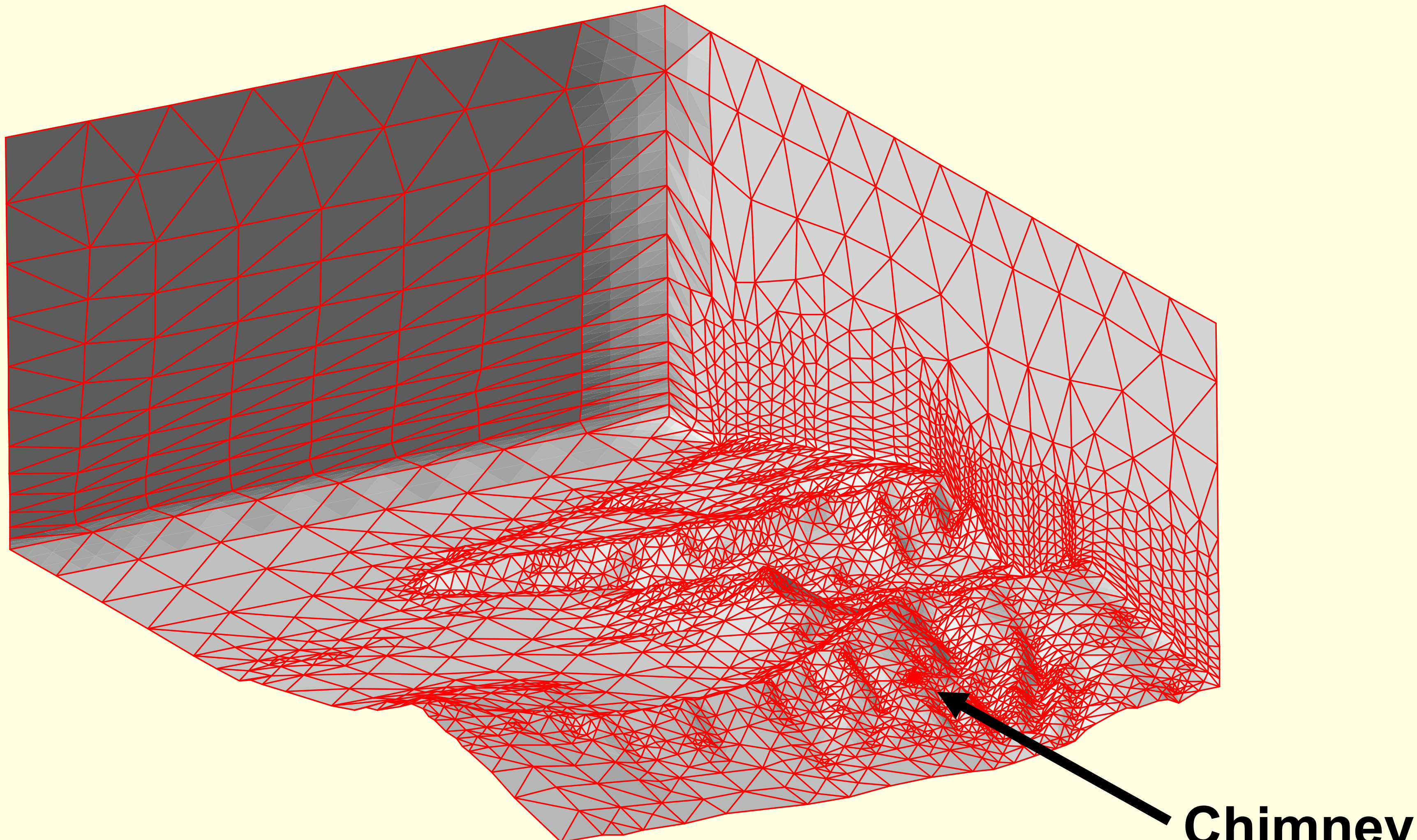
Chimney





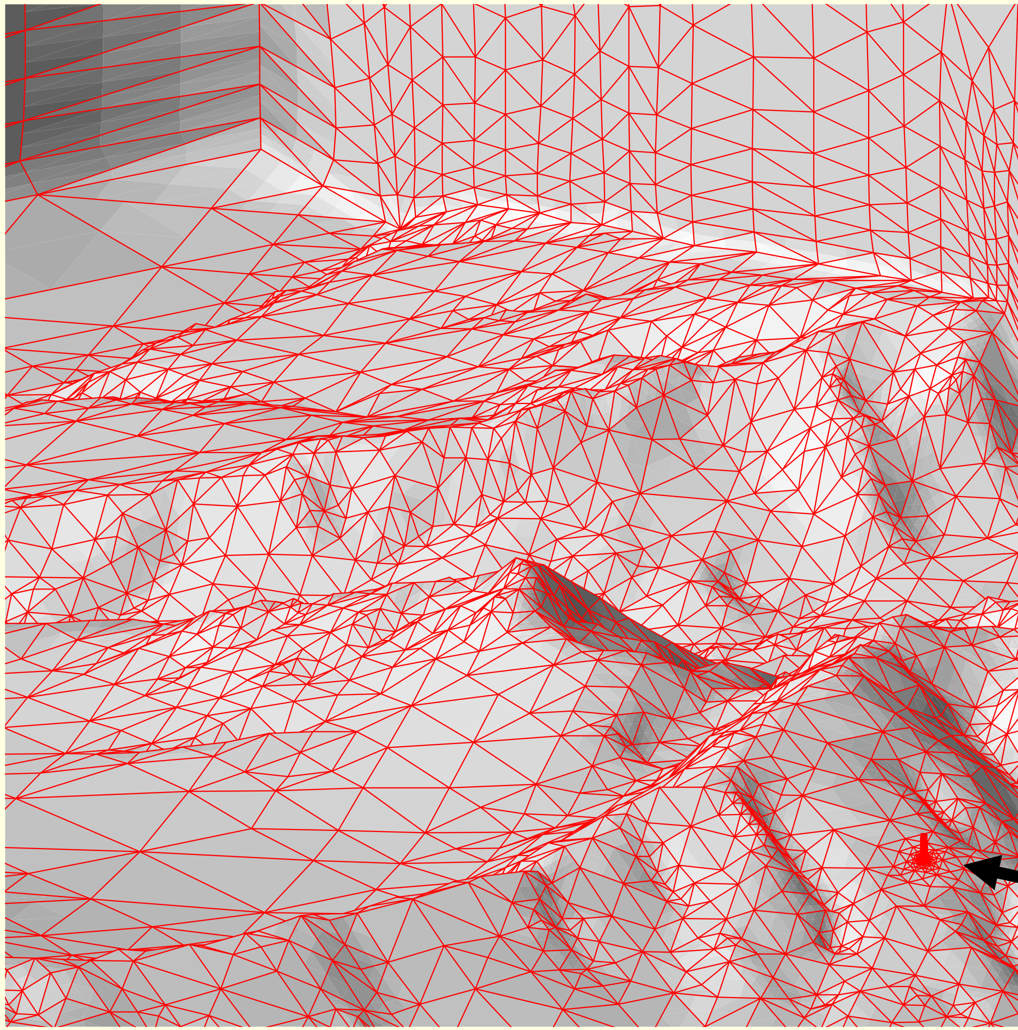
Automatic discretization in the surroundings of the chimney by using a 2-D refinement/derefinement algorithm

Wind Field Simulation Including Chimney Emissions



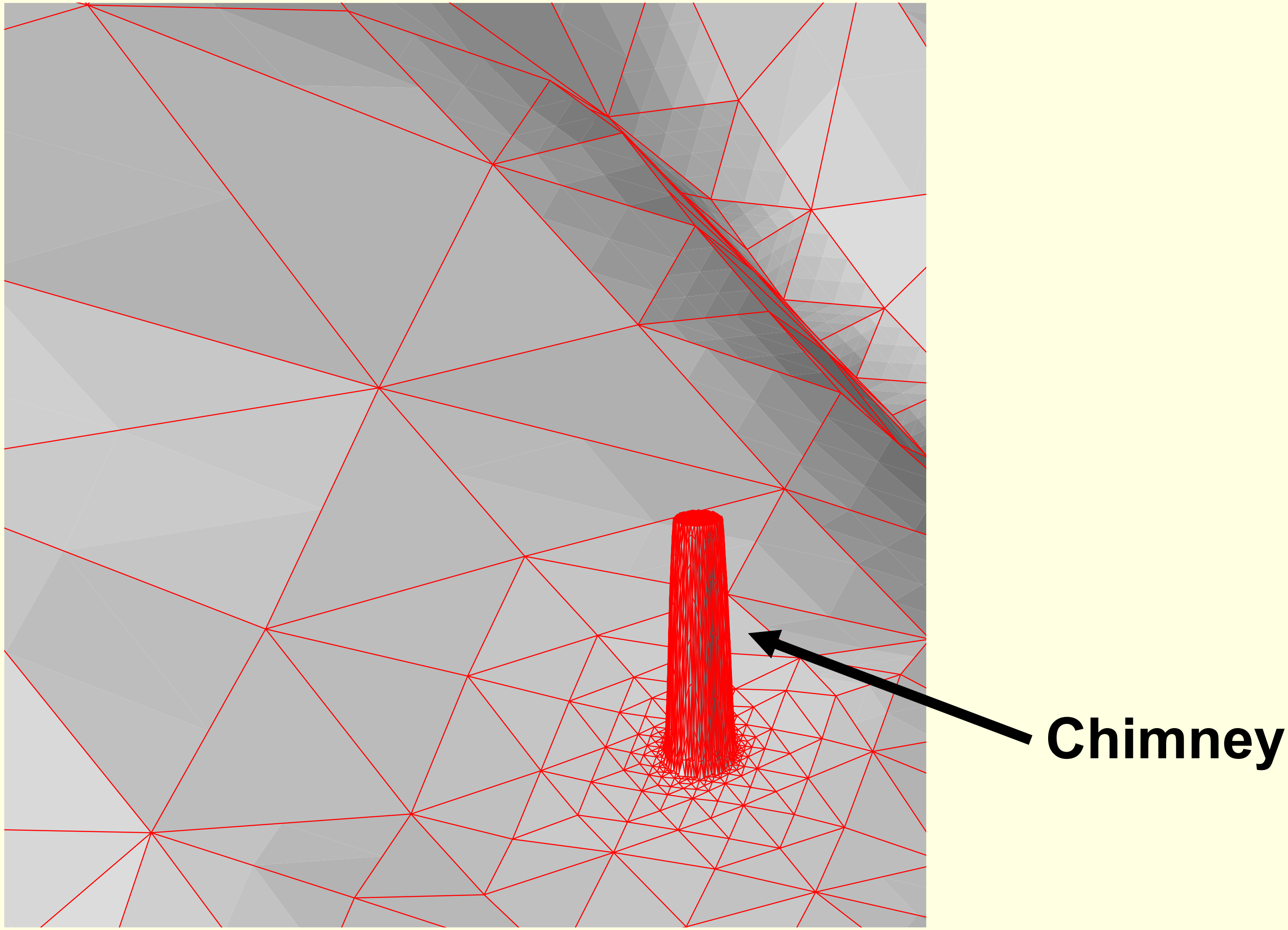
- ***Maximum size of mesh edges: 2 Km***
- ***Minimum size of mesh edges: 2 m***

Wind Field Simulation Including Chimney Emissions



- ***Maximum size of mesh edges: 2 Km***
- ***Minimum size of mesh edges: 2 m***

Wind Field Simulation Including Chimney Emissions



- ***Maximum size of mesh edges: 2 Km***
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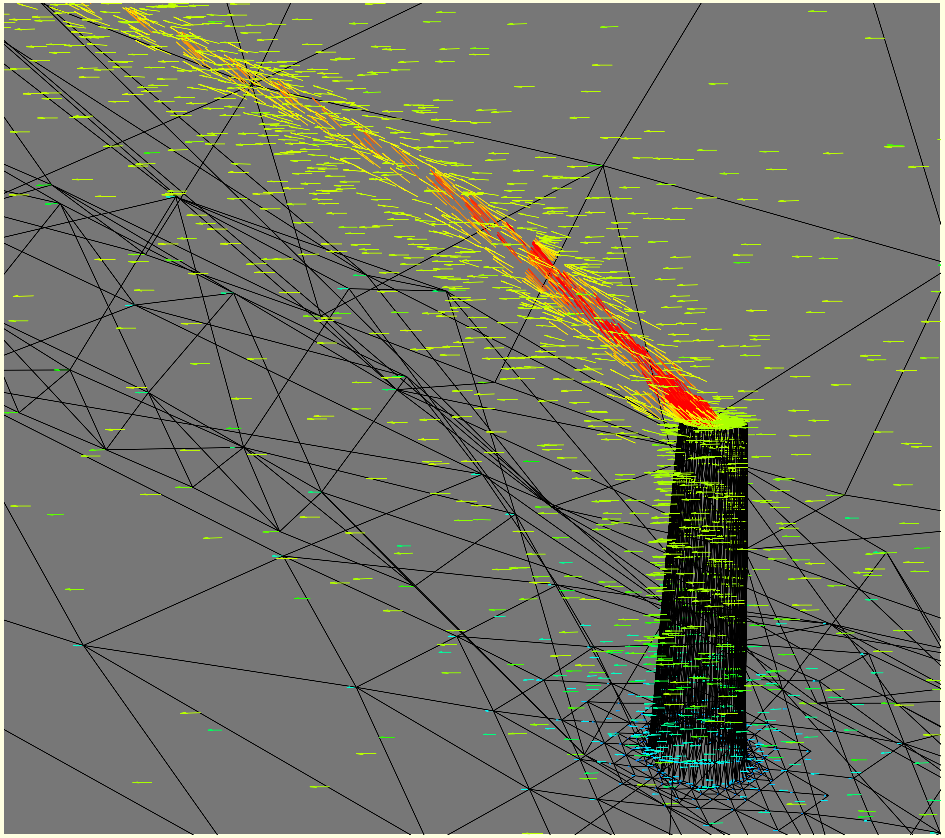
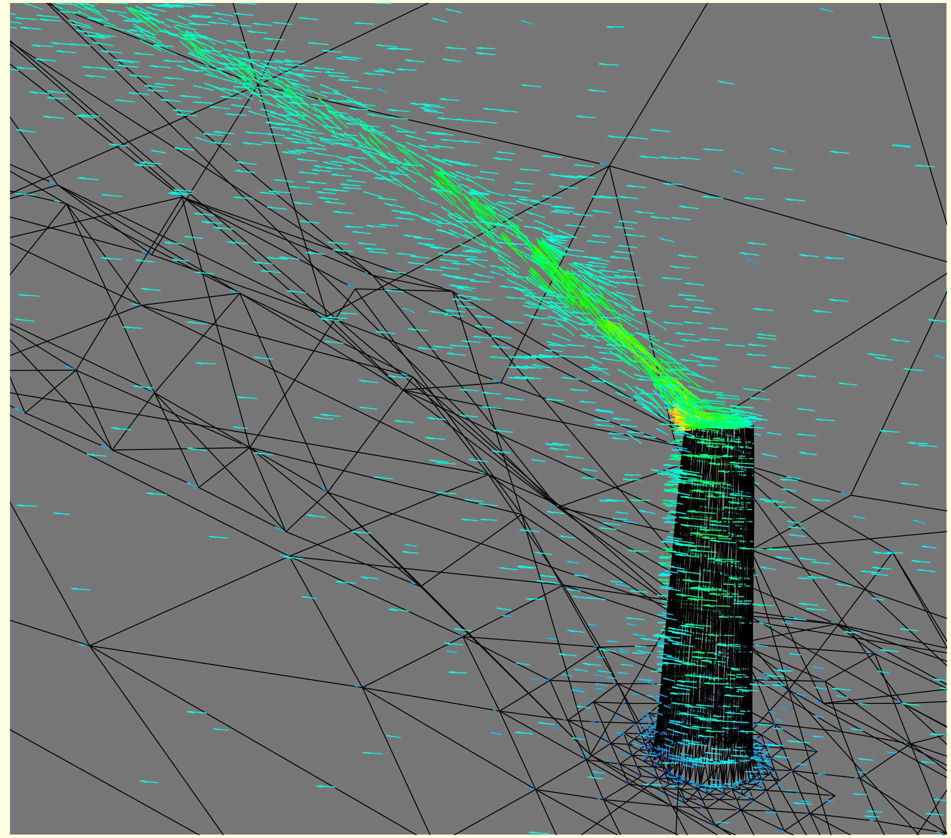
Wind Field Simulation Including Chimney Emissions

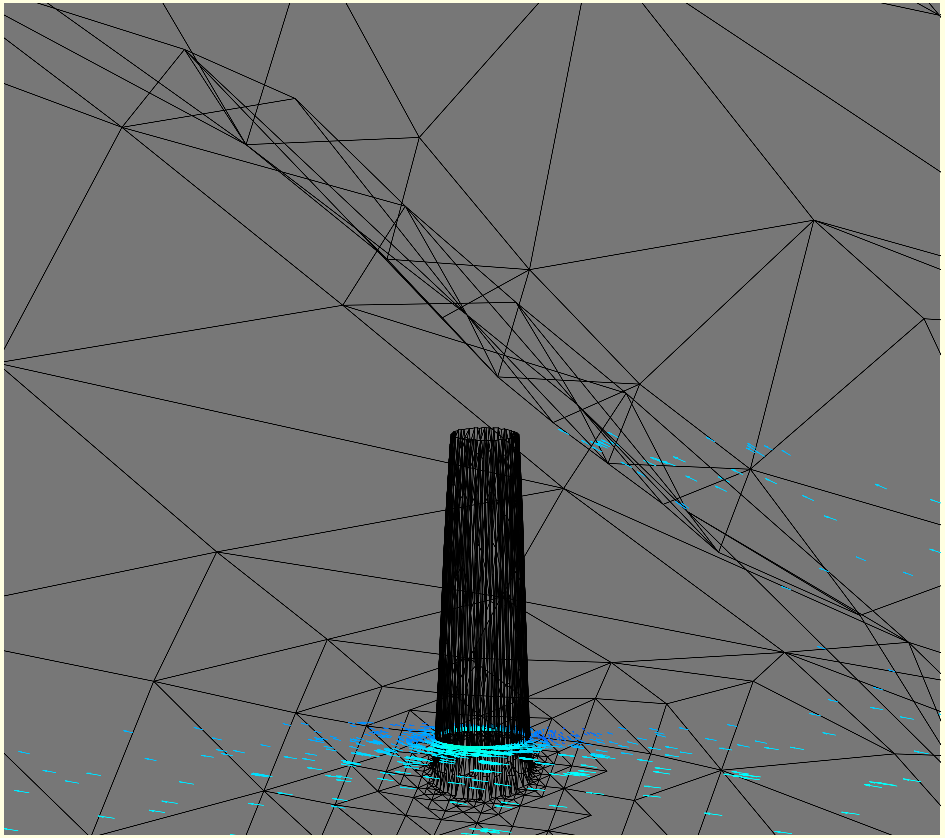


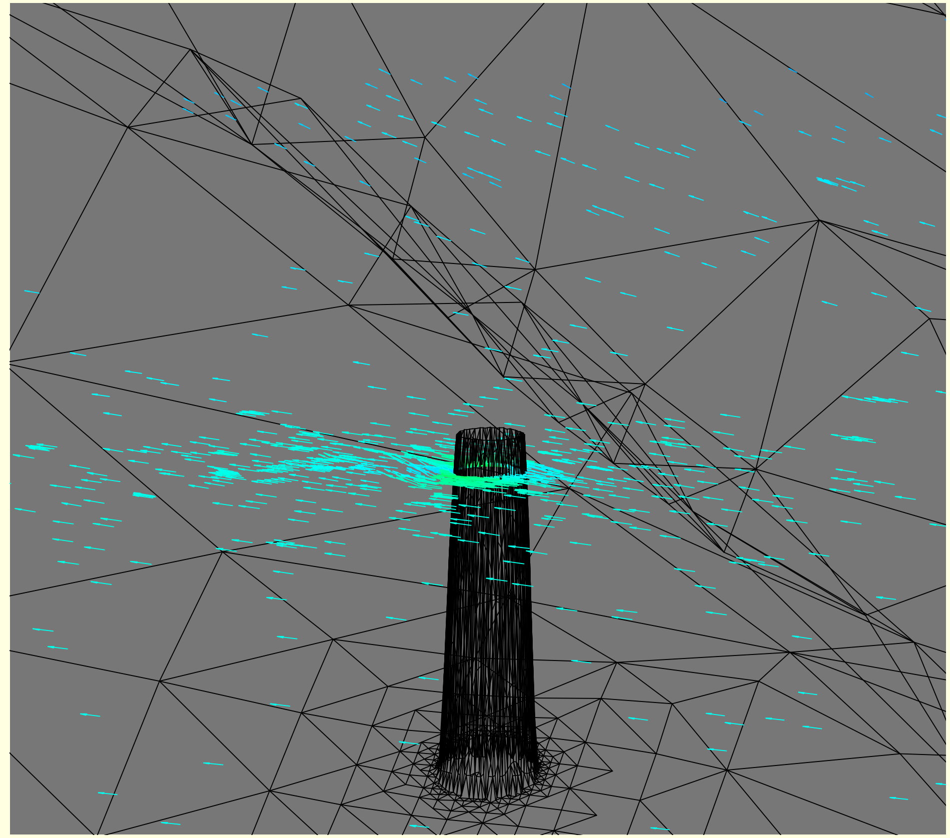
Initial velocity field



Adjusted velocity field

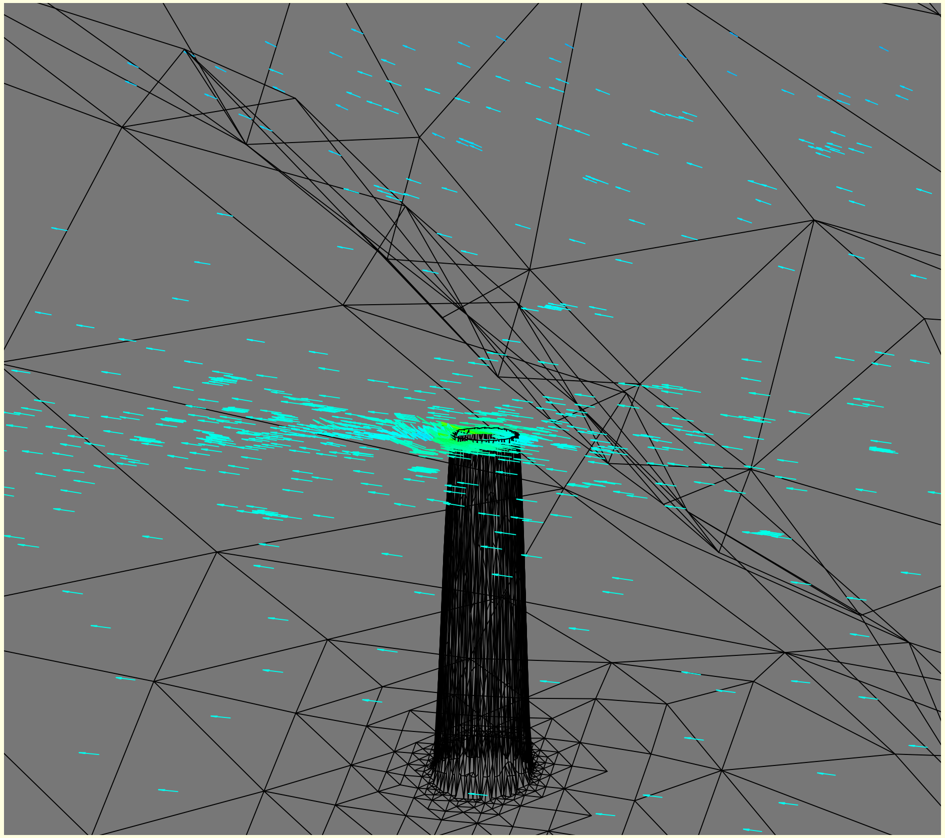
Wind Field Simulation Including Chimney Emissions

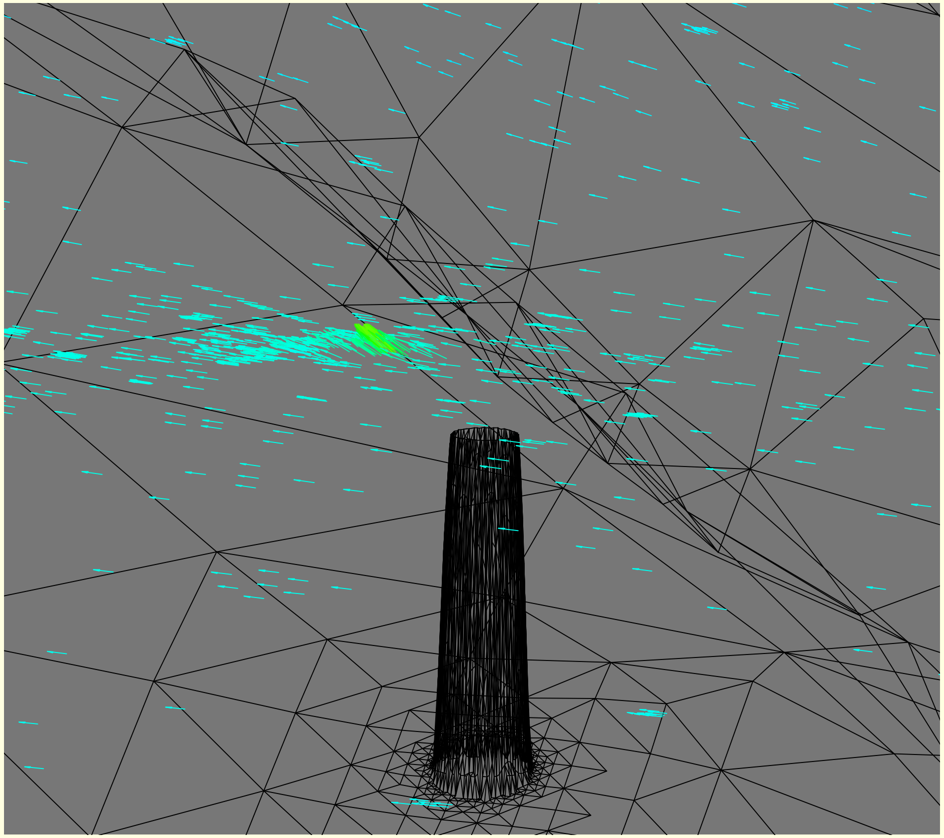




Adjusted velocity field over horizontal planes

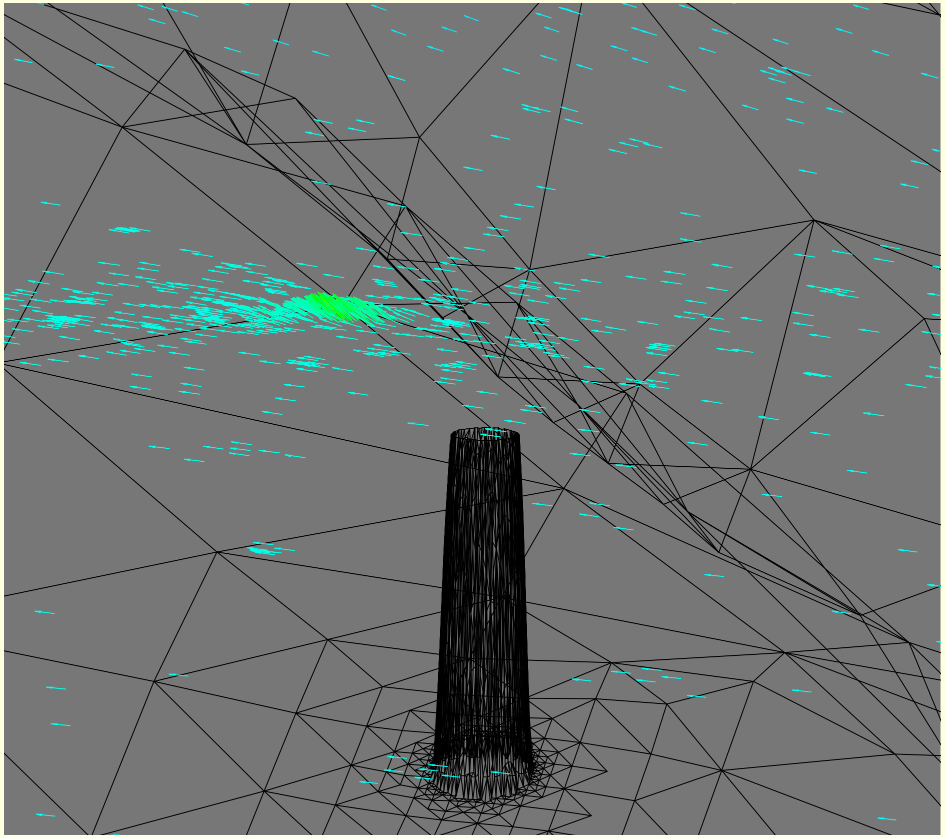
Wind Field Simulation Including Chimney Emissions

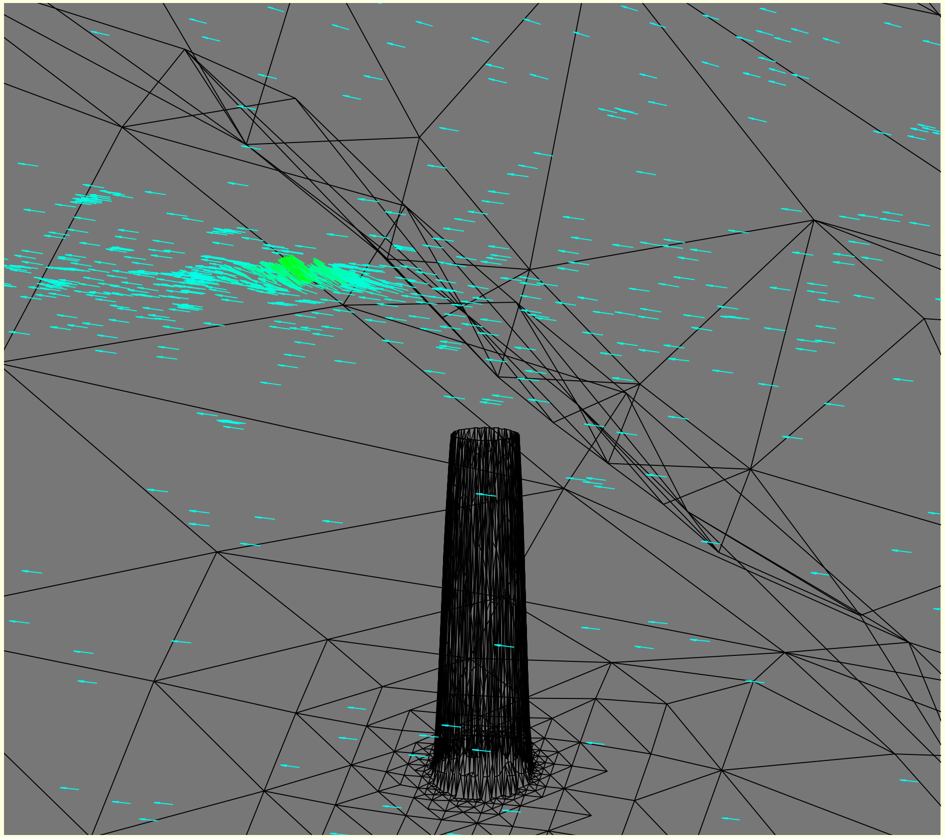




Adjusted velocity field over horizontal planes

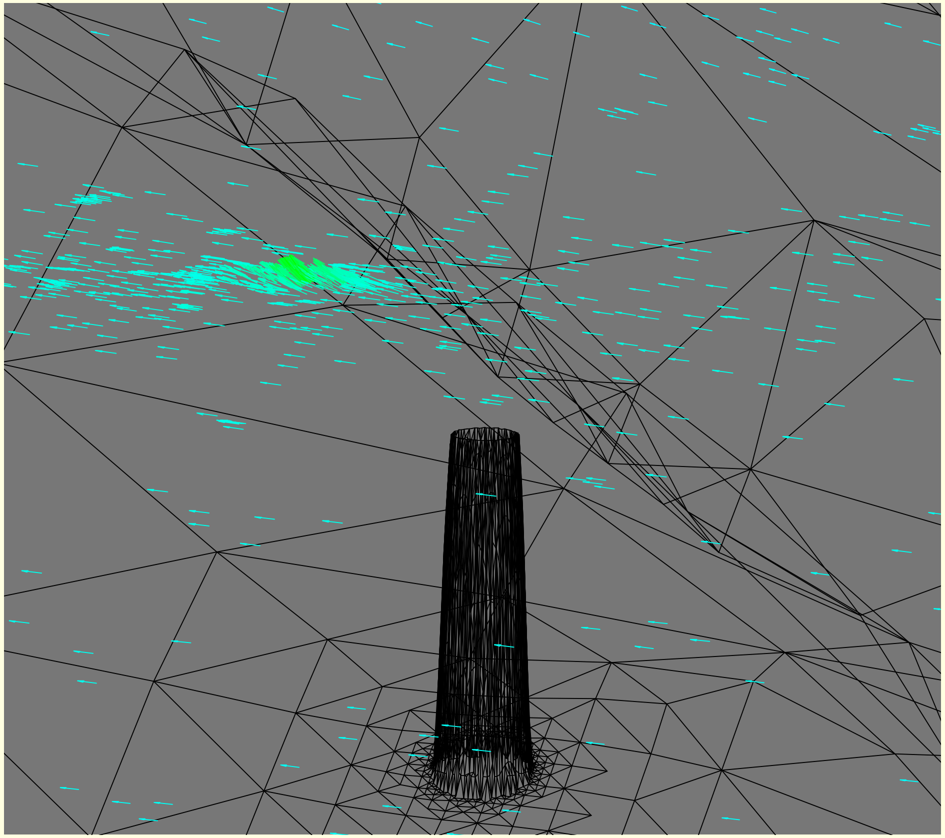
Wind Field Simulation Including Chimney Emissions

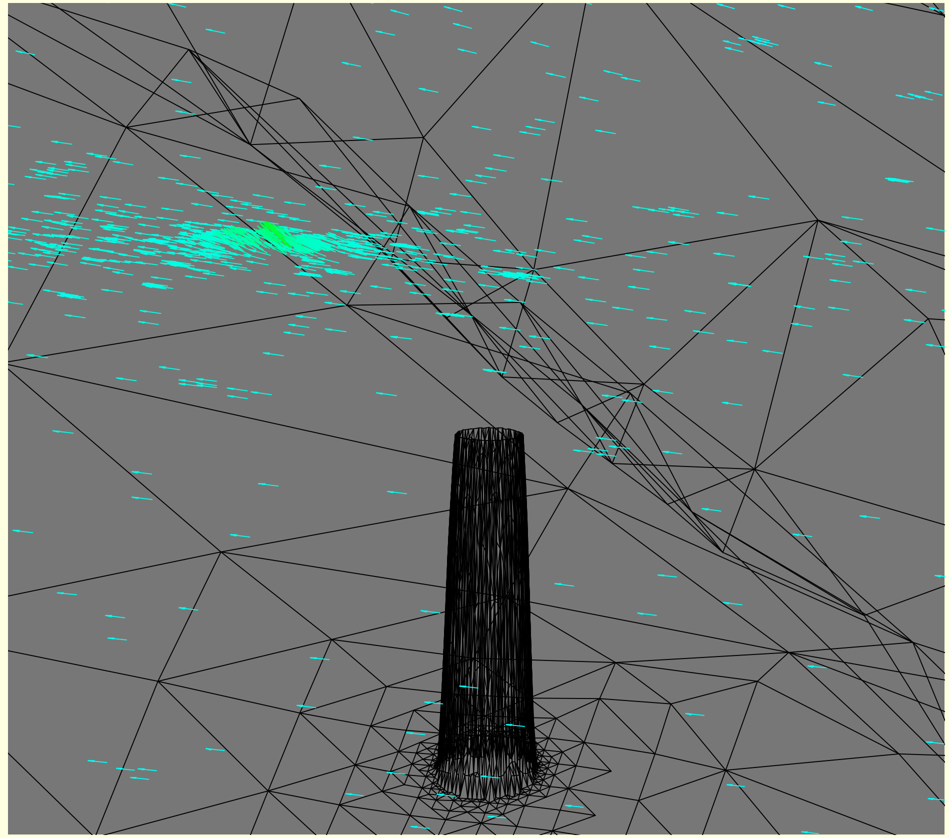




Adjusted velocity field over horizontal planes

Wind Field Simulation Including Chimney Emissions

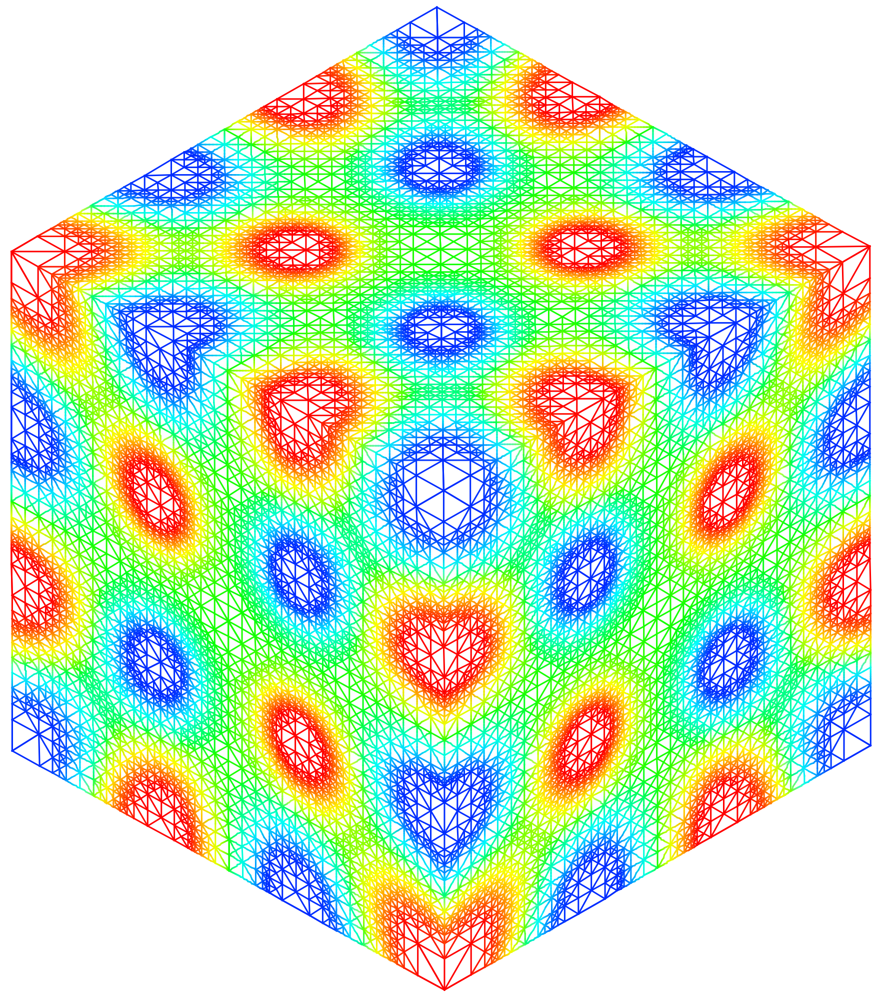


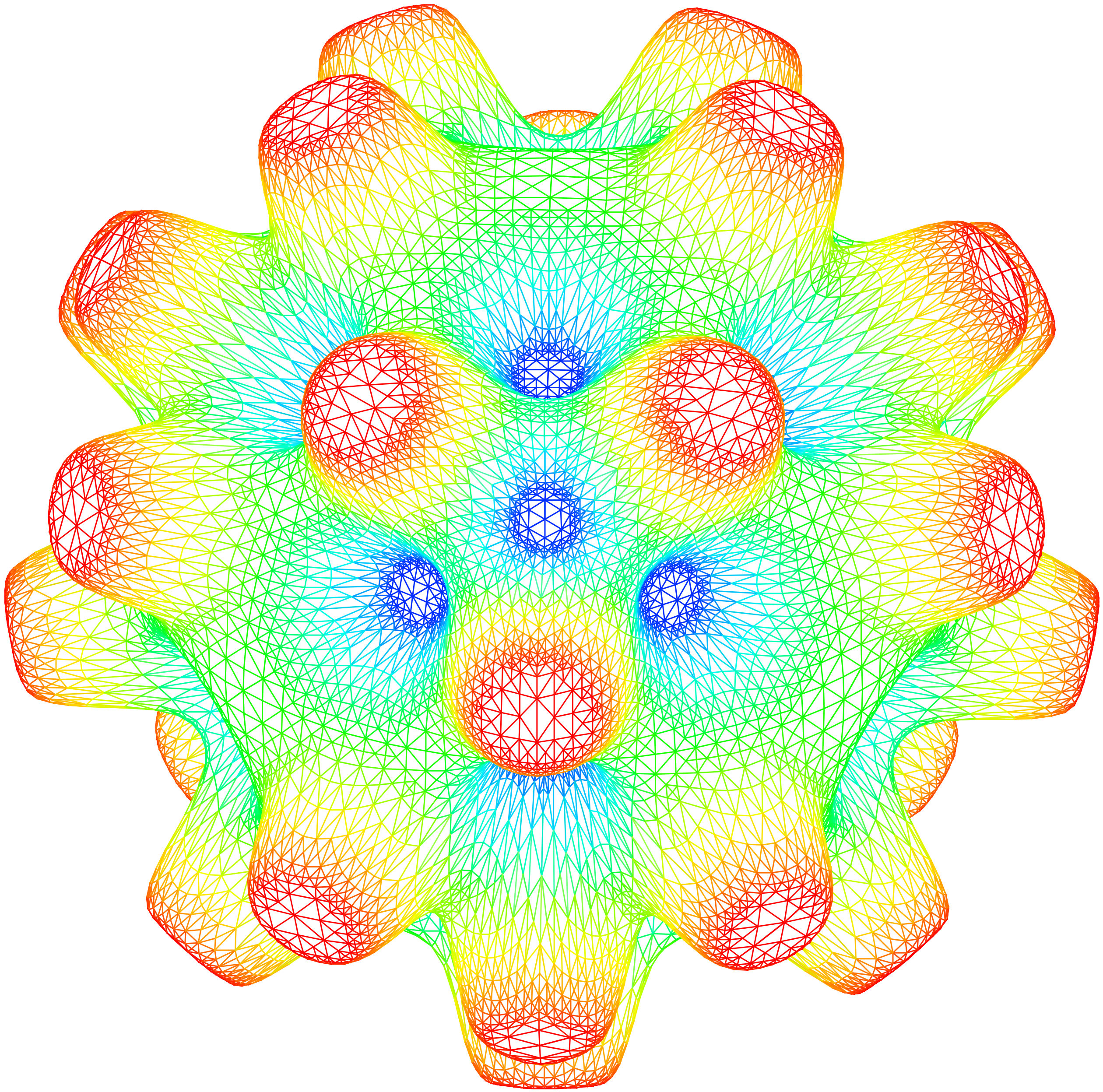


Adjusted velocity field over horizontal planes

Conclusions and Future Research

- **We have introduced an efficient technique for 3-D adaptive mesh generation in environmental problems, with a minimal user intervention and low computational cost.**
- **We have modified a mass consistent model to take into account the observed wind flow and the vertical buoyancy or momentum plume rise. Genetic Algorithms have been applied to estimate parameters involved in the model.**
- **In future works, we will use mixed finite element for a better approximation of the wind field velocity.**
- **We will apply this wind field model to simulate atmospheric pollution.**





ICNPSC3

The Third International Conference on
Neural, Parallel and Scientific Computation
August 9-12, 2006, Atlanta, USA

Applications of 3-D Automatic Triangulations for Wind Field Simulation

R. Montenegro, G. Montero, J.M. Escobar, E. Rodríguez and J.M. González-Yuste*

University Institute for Intelligent Systems and Numerical Applications in Engineering
University of Las Palmas de Gran Canaria, Spain

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Grant contract: CGL2004-06171-C03-02/CLI

http://www.dca.iusiani.ulpgc.es/proyecto0507/html