Applications of 3-D Automatic Triangulations for Wind Field Simulation

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Abstract

The objective of this work is to present our recent results in the numerical solution of a mass consistent wind field model, which could be also introduced in an air pollution model. We summarize the evaluation of the wind field based on the contribution of the observed wind flow. Besides, this mass consistent model includes effects of chimney emissions with vertical buoyancy or momentum plume rise defined by a Gaussian plume model. All these techniques are applied to realistic and test problems.

1. INTRODUCTION

In finite element simulation of wind fields it is essential to adapt automatically the three-dimensional discretization to geometry and solution. We construct a tetrahedral mesh that approximates the orography of the terrain with a given precision (Montenegro et al., 2002a; Montenegro et al., 2002b). To do so, we only have digital terrain information. Our domain is limited on its lower part by the terrain and on its upper part by a horizontal plane placed at a height at which the magnitudes under study may be considered steady. The lateral walls are formed by four vertical planes. The generated mesh could be used for numerical simulation of natural processes, such as wind field adjustment (Montero et al., 1998; Montero et al., 2005), fire propagation or atmospheric pollution (Montero et al., 2004).

The following procedures are mainly used for the automatic mesh generation: a Delaunay triangulation method (George et al., 1991; Escobar & Montenegro, 1996), a refinement/derefinement algorithm (Ferragut et al., 1994) and a simultaneous untangling and smoothing algorithm (Escobar et al., 2003; Knupp, 2001; Djidjev, 2000). Besides, we have recently developed a new method for quality improvement of surface triangulations, by using optimal local projections (Montenegro et al., 2005), which can be introduced in the mesh generator. In order to improve the numerical solution of the model, a local refinement algorithm (González-Yuste et al., 2004) for tetrahedral meshes, based on the 8-subtetrahedron subdivision (Löhner & Baum, 1992; Liu & Joe, 1996; Bornemann et al., 1993), can be also applied.

2. WIND FIELD MODELLING

We consider a mass consistent model for wind field adjustment which are based on the continuity equation and the impermeability conditions on the terrain Γ_b,

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \text{in } \Omega \tag{1}$$

$$\vec{n} \cdot \vec{u} = 0 \qquad \text{on } \Gamma_b \tag{2}$$

assuming that the air density is constant in the whole domain. We formulate a least-square problem in the domain Ω in which the wind $\vec{u}(\widetilde{u}, \widetilde{v}, \widetilde{w})$ must be adjusted to the observed wind $\vec{v}_0(u_0, v_0, w_0)$. Lagrange multiplier technique is used to solve this problem, whose minimum comes to form the Euler-Lagrange equations and yields an elliptic equation and boundary conditions defined on the Lagrange multiplier ϕ

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{T_v}{T_h}\frac{\partial^2 \phi}{\partial z^2} = -\frac{1}{T_h}\left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} + \frac{\partial w_0}{\partial z}\right) \quad \text{in } \Omega \tag{3}$$

$$\phi = 0 \quad \text{on} \quad \Gamma_a \tag{4}$$

$$\vec{n} \cdot T\, \vec{\nabla}\phi = -\vec{n} \cdot \vec{v}_0 \quad \text{on} \quad \Gamma_b \tag{5}$$

To obtain the observed wind, horizontal interpolation of the station measures is carried out. Then, a log-linear wind profile is built up to the surface layer taking into account the horizontal interpolation, the effect of roughness on the wind velocity and air stability. Above the surface layer, a linear interpolation is carried out using the geostrophic wind. We propose a discretization adapted to geometry, roughness and solution for solving the above problem. For more details see (Montero et al., 1998; Montero et al., 2005). In order to introduce a vertical velocity along the trajectory of a pollutant plume arising from a chimney, a Gaussian plume model is considered to modify the observed wind (Montero et al., 2004). For computing the effective height of the plume, we use Briggs' equations; see e.g. (Boubel et al., 1994).

3. REALISTIC NUMERICAL SIMULATION

As a practical application of our mesh generator and the optimisation procedure we have taken under consideration a rectangular area in North-West of *Isla de Gran Canaria* (Canary Islands) of 16.5×9.5 km. A representation of the orography of this region is shown in figure 1. The upper boundary of the domain has been placed at $h = 7$ km. To define the topography we used a digitalisation of the area where heights were defined over a grid, with a spacing step of 25 m in directions x and y, with a precision of 5 m. Starting from a uniform 2-D mesh τ_1 of the rectangular area with a size of elements about 3×3 km, eight global refinements were carried out using Rivara 4-T algorithm (Rivara, 1987). Once the data were interpolated on this refined mesh, the derefinement algorithm developed in (Ferragut et al., 1994) with a derefinement parameter of $\varepsilon = 10$ m was used. Thus, the adapted mesh nears the terrain surface with an error less than that value. The node distribution of τ_1 is the one considered on the upper boundary of the domain.

The mesh has 215707 tetrahedra and 44832 nodes, see figure 2. This initial mesh has not inverted tetrahedra, its average quality measure $\overline{q}_\kappa = 0.471$ and its minimum quality

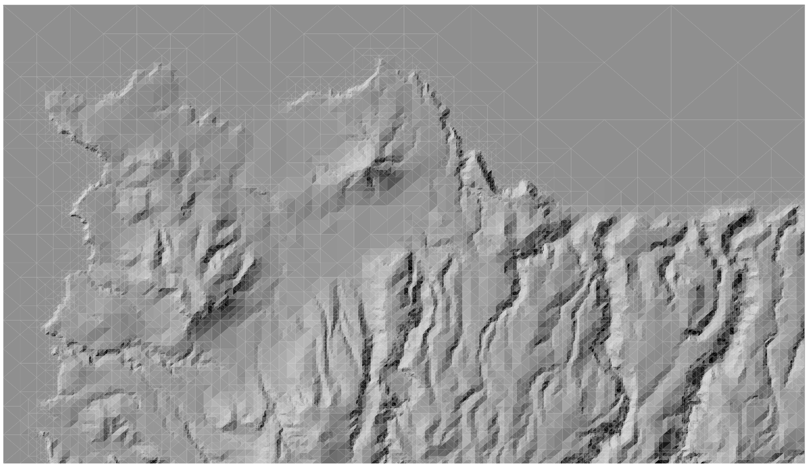

Figure 1: Orography of North-West of *Isla de Gran Canaria* (Canary Islands)

is 0.091, see reference (Escobar et al., 2003) and figure 3. The node distribution is hardly modified after ten steps of the optimisation process by using our modified objective function.

The evolution of the mesh quality during the optimisation process is represented in figure 3. This measure tends to stagnate quickly. The quality curves corresponding to the 5-th and 10-th optimisation steps are very close. The average quality measure increases to $\overline{q}_\kappa = 0.752$. After this optimisation process, the worst quality measure of the optimised mesh tetrahedra is 0.204. Finally, we remark that the number of parameters necessary to define the resulting mesh is quite low, as well as the computational cost. The initial mesh was generated in less than 1 minute and optimised in about 10 minutes on a computer with two Intel Xeon processors, 2.1 GHz and 4 Gb RAM memory.

As example of air pollution modelling of a test power plant located in a region of *Isla de La Palma* (Canary Islands), we have to add the chimney geometry to the topographical data and apply the 3-D mesh generator. We consider a chimney with a height of 200 m over the terrain and diameter of 20 m at its top and 40 m at its bottom. In this case, we have applied six local refinement steps with the 8-subtetrahedron subdivision in the plume trajectory to the initial mesh. Figure 4 represents a detail of the adjusted velocity field $\vec{u}$ where the effect of chimney emission has been introduced.

CONCLUSIONS

We have presented an efficient technique for automatic and adaptive 3-D mesh generation in environmental problems. So, we can discretize domains defined over complex terrains which may include several chimneys, with a minimal user intervention and low computational cost. The local mesh refinement in the pollutant plume allows to define a velocity field that takes into account the observed wind and the emission of gases from chimneys. This field may be used for air pollution simulation.

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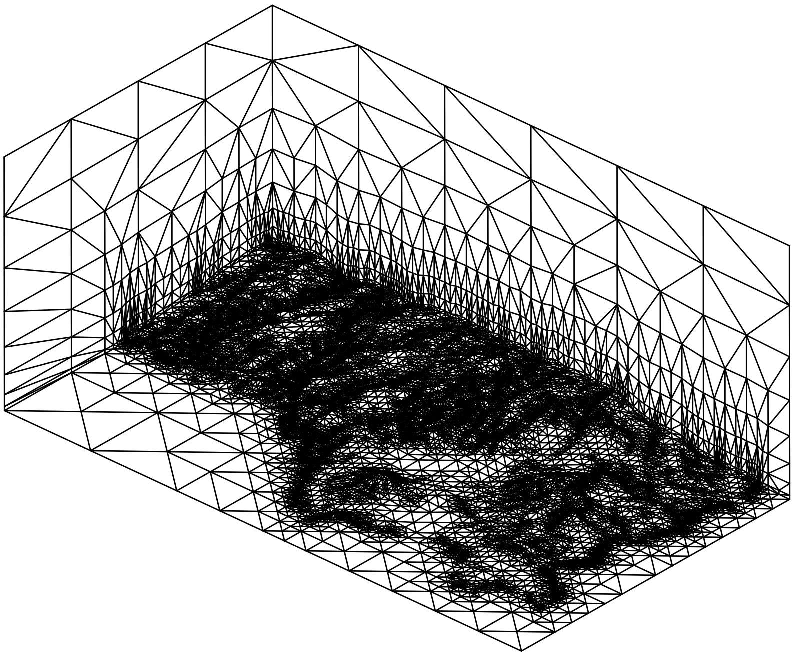
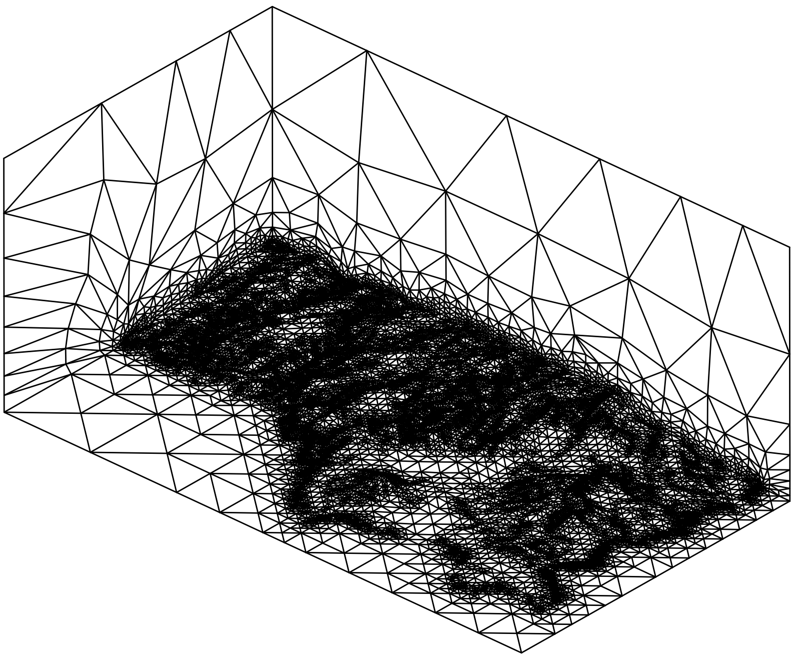


(a)



(b)

Figure 2: Rectangular area of *Isla de Gran Canaria* (Canary Islands): (a) initial mesh and (b) resulting mesh after ten steps of the optimisation process

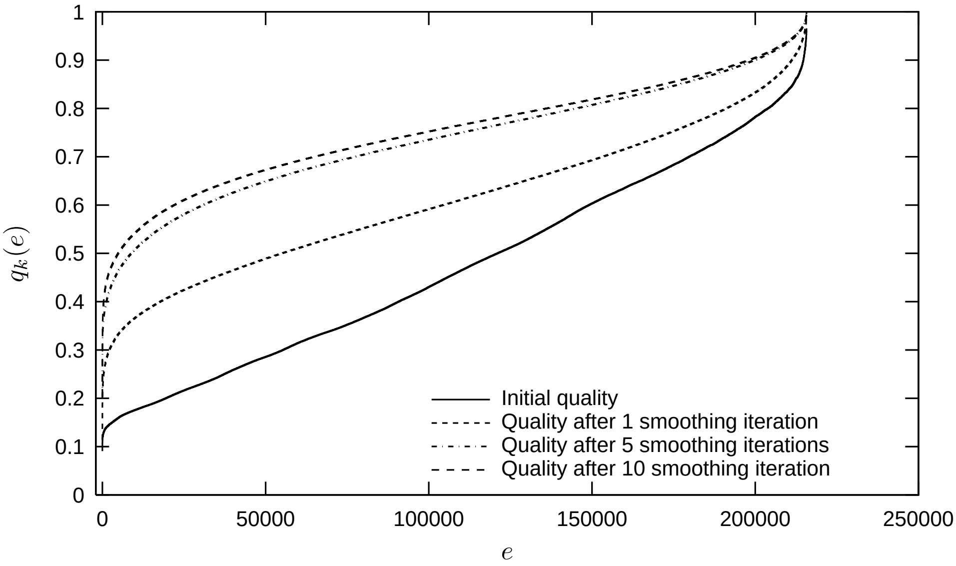


Figure 3: Evolution of quality curves during the optimisation process. Function $q_\kappa(e)$ is a quality measure for tetrahedron e

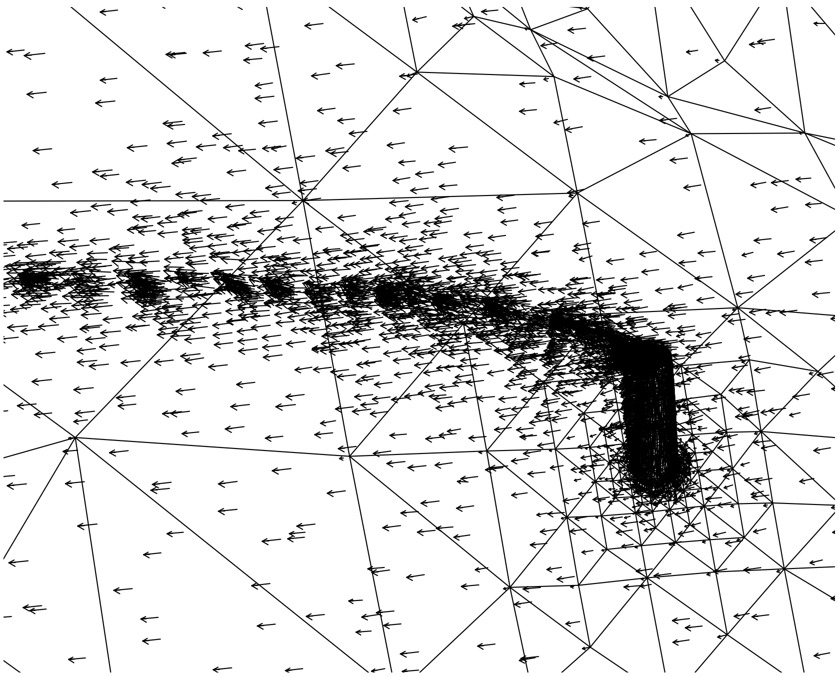


Figure 4: Mesh and velocity field in the surrounding of a chimney

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