

B-1272 Proposed by Hideyuki Ohtsuka, Saitama, Japan.

For any integer $n \geq 0$, prove that

$$(i) \sum_{k=0}^n \beta^k \binom{n}{k} \cos \frac{k\pi}{5} = (-\beta)^n \cos \frac{n\pi}{5},$$

$$(ii) \sum_{k=0}^n \beta^k \binom{n}{k} \sin \frac{k\pi}{5} = -(-\beta)^n \sin \frac{n\pi}{5}.$$

B-1273 Proposed by Robert Frontczak, Landesbank Baden-Württemberg, Stuttgart, Germany.

Let $\{u_n\}_{n \geq 0}$ be a generalized Fibonacci sequence defined by $u_n = u_{n-1} + u_{n-2}$ with u_0 and u_1 not both being zero. Let $\{a_n\}_{n \geq 1}$ be an arithmetic progression, that is, $a_n = a_1 + (n-1)d$, where $a_1, d > 0$. Show that

$$\sum_{k=1}^n \frac{u_{k+2}}{a_{k+1}\sqrt{a_k} + a_k\sqrt{a_{k+1}}} = \frac{1}{d} \left(\frac{u_3}{\sqrt{a_1}} - \frac{u_{n+2}}{\sqrt{a_{n+1}}} + \sum_{k=1}^{n-1} \frac{u_{k+1}}{\sqrt{a_{k+1}}} \right).$$

B-1274 Proposed by Ivan V. Fedak, Precarpathian National University, Ivano-Frankivsk, Ukraine.

For all positive integers n , prove that

$$\sum_{k=1}^n \sqrt{F_{2k-1}} \geq \sqrt{F_{2n+3} - 2F_{n+3} + 2}.$$

B-1275 Proposed by Hideyuki Ohtsuka, Saitama, Japan.

Given a real number $c > 0$, for any integer $n \geq 0$, find a closed form expression for the sum

$$\sum_{k=0}^n \prod_{j=k}^n \frac{1}{c(L_{2^{j+1}} + 1) + L_{2^j} - 1}.$$

SOLUTIONS

A Binomial Sum of Generalized Fibonacci Numbers

B-1251 (Corrected) Proposed by Ángel Plaza, University of Las Palmas de Gran Canaria, Spain.

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For any positive integers m and n , prove that

$$\begin{aligned} \sum_{k=0}^n \binom{n}{k} F_{m+k} (x+1)^k &= \sum_{k=0}^n \binom{n}{k} F_{m+2n-k} x^k, \\ \sum_{k=0}^n \binom{n}{k} L_{m+k} (x+1)^k &= \sum_{k=0}^n \binom{n}{k} L_{m+2n-k} x^k. \end{aligned}$$