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SIMPLE DAMAGE ZONE NUMERICAL MODEL FOR CAPTURING THE CHANGE IN MODAL PARAMETERS OF CRACKED CONCRETE ELEMENTS

Luis A. Padrón*[1], Juan J. Aznárez[1], Héctor Cifuentes[2], Fernando Medina[2], Orlando Maeso[1]

[1]University Institute SIANI, Universidad de Las Palmas de Gran Canaria, Las Palmas de G.C., Spain
{lpadron,jjaznarez,omaeso}@siani.es

[2]Universidad de Sevilla, Seville, Spain
{bulte,medinaencina}@us.es

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Abstract. *Cracks are a common defect in plain concrete and reinforced concrete structures. Sometimes, they can lead to structural failure, but in most cases they develop and modify the structural behavior without reaching catastrophic propagation. When analyzing the dynamic response of these structures, for example, under a seismic event, it would be necessary to take into account the influence of cracks on the dynamic behavior of the whole. To do so, there exist mainly two types of approaches: discrete (i.e. fictitious crack model) or smeared cracking models (i.e. continuum damage models). The first group of models, being more appealing for being closer to the real situation in plain concrete structures, implies the modeling of individual cracks, which makes it not viable for large real structures with multiple cracks. On the contrary, and specially in cases where the structure is analyzed under dynamic loads that do not induce crack growth, and when the points of interest are not close to the cracks themselves, linear continuum damage models are a viable alternative if they are capable of reproducing the general influence of the crack on the response of the overall structure.*

In this paper, the modal parameters estimated from a set of experimental impact hammer tests on intact and cracked plain concrete beams is presented, and the way in which cracks affect natural frequencies and modes of the beam is discussed. Then, a simple damage zone model for capturing those changes in the modal parameters is presented.

1 INTRODUCTION

There exist mainly two types of approaches to study the behavior of cracks in concrete structural elements: discrete (i.e. fictitious crack model) or smeared cracking models (i.e. continuum damage models). The first group of models is generally used to predict crack growth and study the behavior in the immediate vicinity of the damage. However, and being more appealing for being closer to the real situation in plain concrete structures, discrete crack models imply the modeling of individual cracks, which makes it not viable for large real structures with multiple cracks. On the contrary, and specially in cases where the structure is analyzed under dynamic loads that do not induce crack growth, and when the points of interest are not close to the cracks themselves, linear continuum damage models are a viable alternative if they are capable of reproducing the general influence of the crack on the response of the overall structure.

This work focuses in the situation in which a concrete structural element is already cracked and is being submitted to dynamic loads (seismic, for instance) which are not large enough to cause crack growth. The presence of those cracks can modify the dynamic response of the structure but, as said above, is not viable in real structures to model precisely each individual crack.

Therefore, and considering that the dynamic behavior of the structure (under loads not large enough to produce crack propagation) can be defined by its modal parameters (natural frequencies, modal shapes and modal damping ratios), the aim of this work is looking into the possibility of capturing the change in those modal parameters by using a simple damage zone numerical model for cracked concrete elements. To do so, plain concrete specimens are firstly dynamically identified with and without cracks in order to find out how cracks affect the first three flexural modes of the element. Then, modal analysis is performed by finite elements in order to estimate the material properties of the specimens and validate the results. Finally, a boundary element – finite element coupled code is used to find a model of the cracked beam able to reproduce the experimental results in terms of frequency shifts and modal damping ratios. One of the parameters of such a simplified model is the length of an equivalent notch. Its value will be compared to the effective crack length given by the two-parameter fracture model for concrete of Jenq and Shah [1], and a very good coincidence is found.

2 METHODOLOGY

2.1 Overview

This work contains two distinct parts: *a)* an experimental part in which the modal parameters of plain-concrete intact and cracked single-edge notched bending specimens are estimated; and *b)* a numerical part involving different numerical models to address the problem at hand. The details are given in the following sections.

2.2 Specimen description

The test specimens are prismatic single-edge notched bending plain concrete elements of $120 \times 60 \times 570\,\mathrm{mm}$ (height, width, length), with a center notch $3\,\mathrm{mm}$ width and $60\,\mathrm{mm}$ deep (i.e. 50% of the specimen's height). One specimen is shown in Figures 1 and 2.

Three specimens were produced and tested with a 1/1.7/2.9/0.5 (CEM-I-52.5 cement, sand, crushed rock, water) concrete. The resulting concrete had a density (after 35 days) of $\rho = 2418.8$ kg/m^3. They were dynamically identified (see section 2.3) after approximately 35 days. Immediately after identifying the intact specimens, they were subjected to a three-point bend-

Figure 1: Specimen during a three-point flexural test (left) and detail of cracks after test (right).

ing single-edge notched fracture test in order to produce visible cracks with lengths around $30-35\,\mathrm{mm}$ but without reaching total fracture of the element (see Figure 1). The state of the damaged specimen could be understood to be close to that of a beam with a plastic hinge.

2.3 Experimental modal analysis description

The characteristics of the specimen recommend the use of the impact hammer technique [2, 3] to perform the experimental modal analysis. Free-free boundary conditions were used, implemented by hanging the specimen by rubber bands. Since 4 accelerometers were available for the tests, and in order to be able to record in 7 positions on the top edge of the specimen, two different setups (see Figure 2) were used in the tests (one accelerometer stays in the same place to work as reference sensor). To be sure that all significant modes were properly excited, the impacts were applied on two different points (A and B in Figure 2). 5 repetitions were recorded for each setup and impact point and for each one of the three intact or damaged specimens, what makes a total of 60 impact hammer tests.

The technical characteristics of all equipment is detailed in the following: a) Impact hammer: Brüel & Kjaer 8206-001. Reference sensitivity: 11.42 mV/N. Full Scale Force Range Compression: 445 N. Effective seismic mass with enhancement: 140 gr. Aluminum tip; b) Accelerometers: Brüel & Kjaer 4508-002 and 4507-002. Reference sensitivity: 1 V/g. Frequency: 0.3-8000 Hz. Residual noise level (rms): $\pm 0.35\,\mathrm{mg}$. Mass: 4.8 gr. Beewax is used for mounting; c) DAQ system: NI PXIe-4496, 24-Bit, Sigma-Delta ADCs, on NI PXIe-1073; and d) Acquisition parameters: sampling frequency: 25000 Hz. Pre-trigger at 0.05 N with 100 buffer points. The response is recorded during 0.4 s (10000 samples), time in which the response damps out completely. No windowing is necessary.

2.4 Numerical analyses

The calibration of the Young's modulus of the concrete was made by matching the natural frequencies of a numerical model with the experimental natural frequencies. To do so, modal analysis was performed by finite elements using ANSYS® software [4].

Numerical frequency response functions (FRFs), on the other hand, were obtained numerically by a 2D BEM-FEM code in the frequency domain. The concrete specimen has been modeled as an isotropic viscoelastic domain with histeretic damping model of the type $E =$

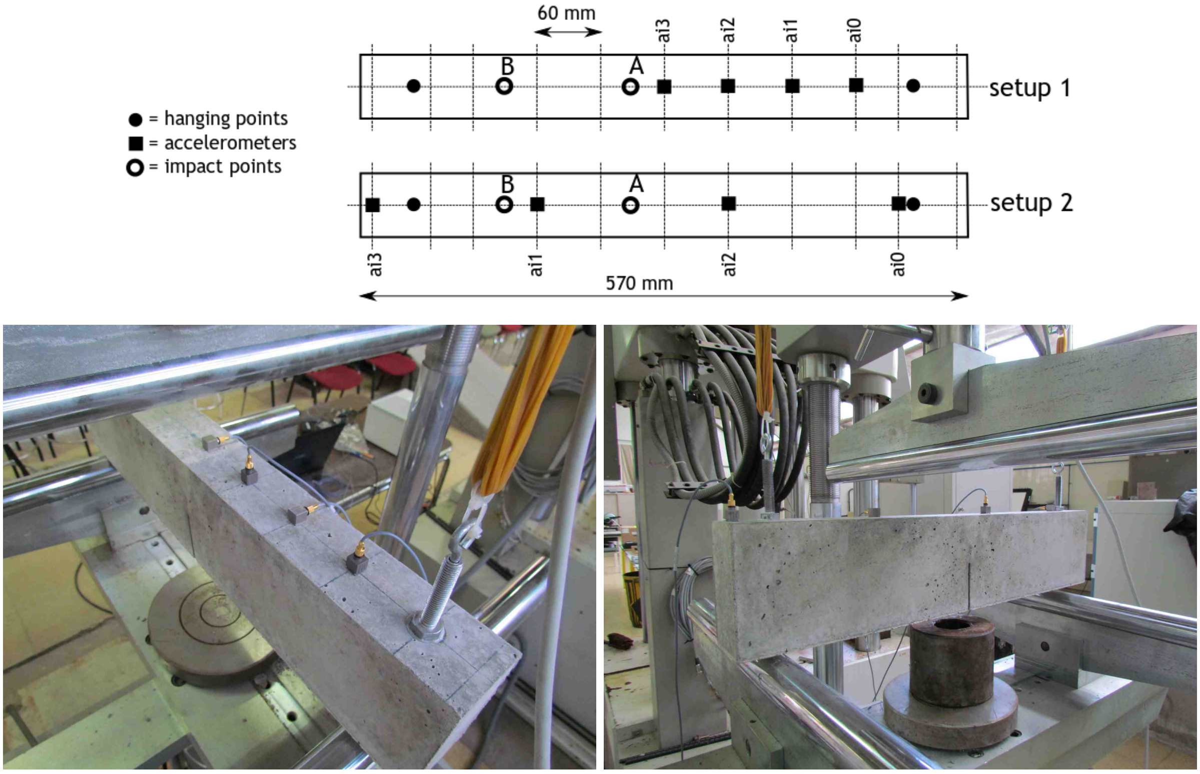

Figure 2: Experimental setups used in the experimental modal analysis by impact hammer. Details of setup 1 (bottom left) and 2 (bottom right).

$\mathrm{Re}[E](1+2\mathrm{i}\xi)$, being ξ the histeretic damping ratio, and analyzed with the boundary element method [5] using quadratic elements. The rubber bands are modeled as Euler-Bernoulli beams using 2-node 6-degrees of freedom beam finite elements [6]. The coupling between boundary elements and finite elements is made through plane perfectly rigid surfaces defined by any number of boundary elements, allowing the establishment of equilibrium and compatibility conditions between those boundary elements and the finite element structure attached to them.

2.5 Effective crack length by LEFM assumptions.

Several adaptations of the linear elastic fracture mechanics (LEFM) assumptions have been proposed which take into account the nonlinear fracture behavior in an approximate manner [7]. These adaptations are usually called effective crack models and they take into account the pre-peak nonlinear behavior of concrete and are able to give a good estimate of the peak load. One of the most universal effective crack model is the two parameter model from Jenq and Shah [1]. On the basis of this method, an equivalent elastic structure containing an effective crack whose length is larger than the length of the crack in the real structure should be determined [8]. Then LEFM can be applied to study the fracture behavior of the concrete structure considering the effective crack length. The model of Jenq and Shah will be compared here with the results by the BEM-FEM model in terms of modal parameters.

	Mode 1	**Mode 2**	**Mode 3**
Intact specimen	760	2960	4817
Damaged specimen	∼300	2702	4718

Table 1: Experimentally identified natural frequencies for specimen one (Hz).

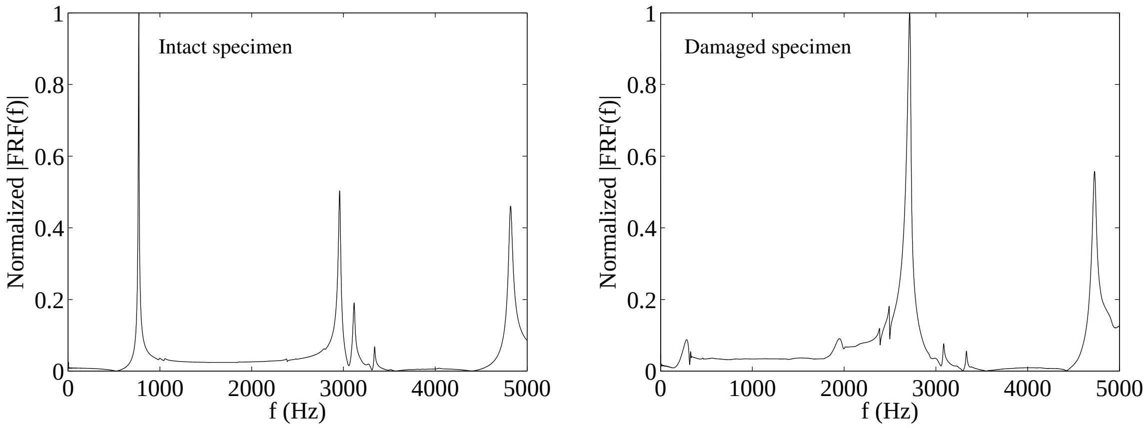

Figure 3: Frequency response functions obtained from experimental modal analysis. Setup 1. Measured at 60 mm left from the center (ai2). Impact on point A. Intact (left) and cracked (right) specimens.

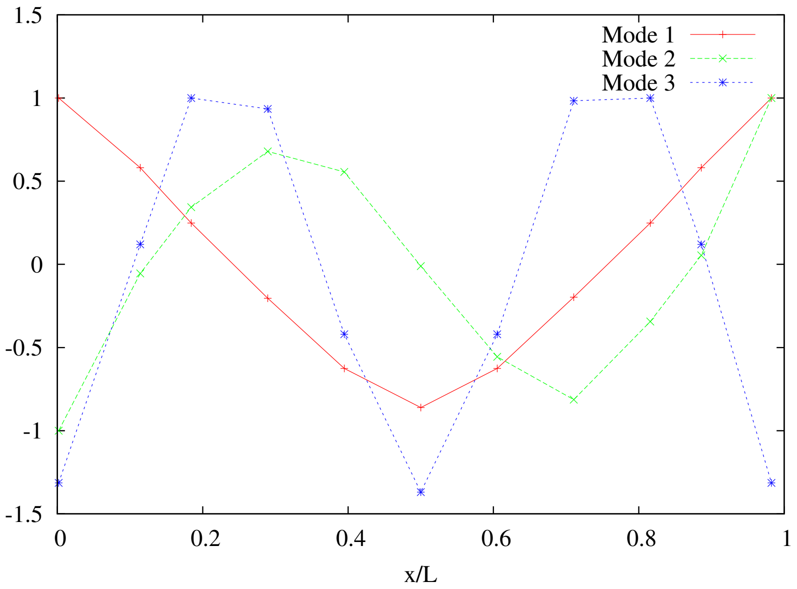

Figure 4: Modal shapes obtained from experimental modal analysis.

3 RESULTS

3.1 Results from experimental modal analysis.

This subsection summarizes some of the results obtained from the experimental modal analysis performed on the specimens. The identified parameters are consistent across repetitions and even across specimens, which suggests that the experimental results are robust. Therefore, in the following, results corresponding only to specimen one will be shown.

Table 1 presents the experimentally identified natural frequencies for the intact and damaged states. In this last case, the length of the visible cracks is of around 30 mm. Figure 3 shows the corresponding normalized frequency response functions corresponding to an impact on point A (i.e. 30 mm to the left of the center) and measuring in channel ai2 from setup 1 (i.e. at 60 mm to the right of the center, see Figure 2). As expected, frequencies are lower when the element is damaged due to the smaller stiffness of the damaged zone. It can also be seen that, while the fundamental flexural frequency gets reduced by more than 50%, the second and third frequencies are reduced by only 8 and 2% respectively, showing that, in this specific configuration of the element, the damage affects mainly to the fundamental mode. It is also apparent how the first mode is extremely damped in the damaged specimens, probably due to the fact that most of the deformation corresponding to that fundamental mode occurs in the central damaged part.

This can be seen in Figure 4, where the three modal shapes experimentally identified are presented for an intact specimen. It is worth mentioning here that the modes do not change significantly between intact or damaged specimens in terms of shapes, although they obviously change in terms of amplitude and frequency.

Modal damping ratios were also estimated from the experimental frequency response functions, finding a value of $\xi = 0.005$ (0.5%) for all the three modes of the intact specimen, which supports the use of a histeretic damping model for the numerical analysis. In the damaged specimen, damping ratios for the second and third modes are slightly larger while the fundamental mode is completely overdamped.

3.2 Results from finite elements modal analysis.

This section presents results from numerical modal analyses performed in order to identify the material Young's modulus by finite–element updating. The density of the concrete was measured to be $\rho = 2418.8$ kg/m^3. Poisson's ratio was assumed to be $\nu = 0.2$. A Young's modulus $E = 3.5 \times 10^{10}$ Pa, which is an expected value for the concrete used for the specimens, was found to match the experimental flexural fundamental frequency. Table 3.2 shows the very good agreement found also for the higher modes. Figure 5 shows the corresponding modal shapes, that agree very well with the experimental ones presented in Figure 4, giving more information about the deformation of the element, not only on the top surface, but as a whole.

Mode	Numerical (Hz)	Experimental (Hz)	Difference (%)
1	760	760	0
2	2917	2960	1.4
3	4651	4817	3.4

Table 2: Comparison between numerical (FEM) and experimental natural frequencies for the intact state.

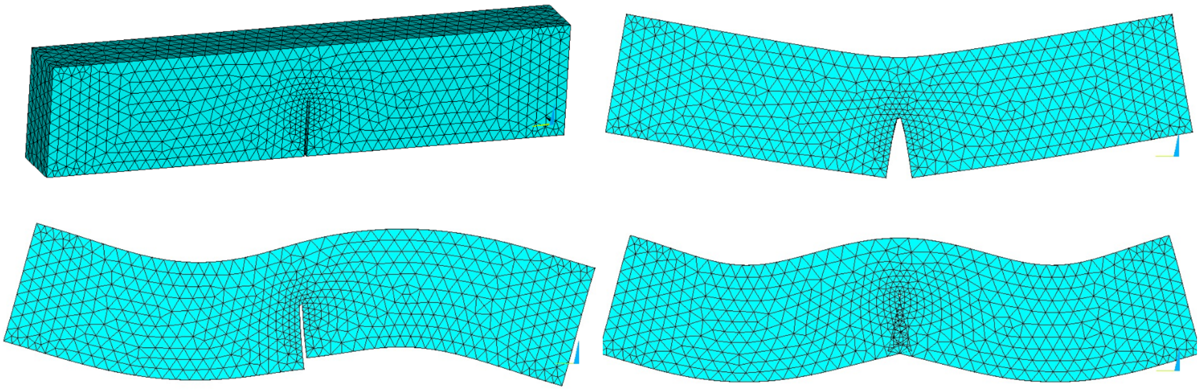

Figure 5: Mesh and first three flexural modal shapes obtained from finite element modal analysis for the intact state.

3.3 Results in the frequency domain from BEM-FEM analysis.

Using the material parameters found in the previous section ($\rho = 2418.8$ kg/m^3, $\nu = 0.2$, $E = 3.5 \times 10^{10}$ Pa, $\xi = 0.005$), the two-dimensional BEM-FEM formulation described above has been used to find a simple damage model for capturing the change in the modal parameters of the specimen due to the cracks. As shown above, such a model should be able of capturing not only the frequency shifts but also the extremely high damping suffered by the first mode in relationship with the the other modes.

Experimental results for the intact specimen are very closely reproduced using the original geometry and properties. On the other hand, in the case of the damaged configuration, the frequency shift (much larger for the fundamental mode than for the higher ones) was found to be reproduced by using an equivalent notch length of approximately 94 mm, i.e., slightly more than the addition of original notch and crack lengths. This model, however, is not able to capture the overdamping of the first mode. A model in which the link zone (see shadowed area Ω_2 in Figure 6) is given the same density and stiffness than the rest of the element but a much higher damping ratio ($\xi_2 = 2.00$) has been found to reproduce very closely the experimental results. Obviously, the addition of such a high damping reduces the damped natural frequencies, reason why a new equivalent notch length of 90 mm gives a better approximation for both modal frequencies and dampings.

	Mode 1	**Mode 2**	**Mode 3**
Intact specimen (exp.)	760	2960	4817
Intact specimen (num.)	760	2910	4650
Damaged specimen (exp.)	∼300	2702	4718
94 mm notch (num.)	∼300	2578	4646
94 mm notch & $\xi_2 = 2.0$	∼300	2473	4606
90 mm notch & $\xi_2 = 2.0$	∼300	2535	4610

Table 3: Numerical BEM-FEM natural frequencies vs experimental natural frequencies (Hz).

Table 3 summarizes these results in terms of frequencies, while Figure 7 shows the normalized frequency response functions for all the described cases. The comparison between the top

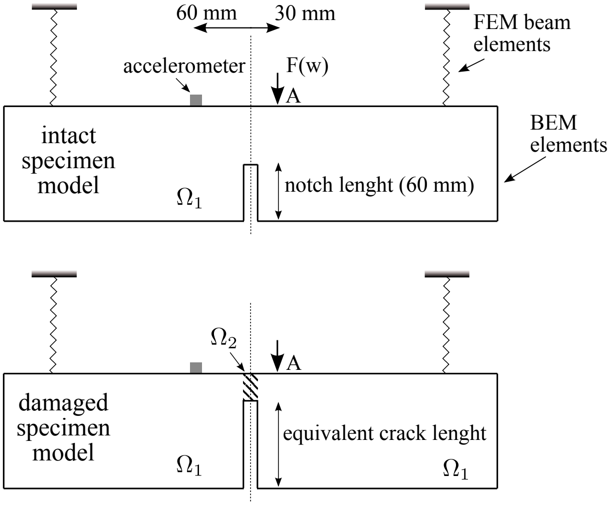

Figure 6: BEM-FEM models for intact (top) and damaged (bottom) specimens.

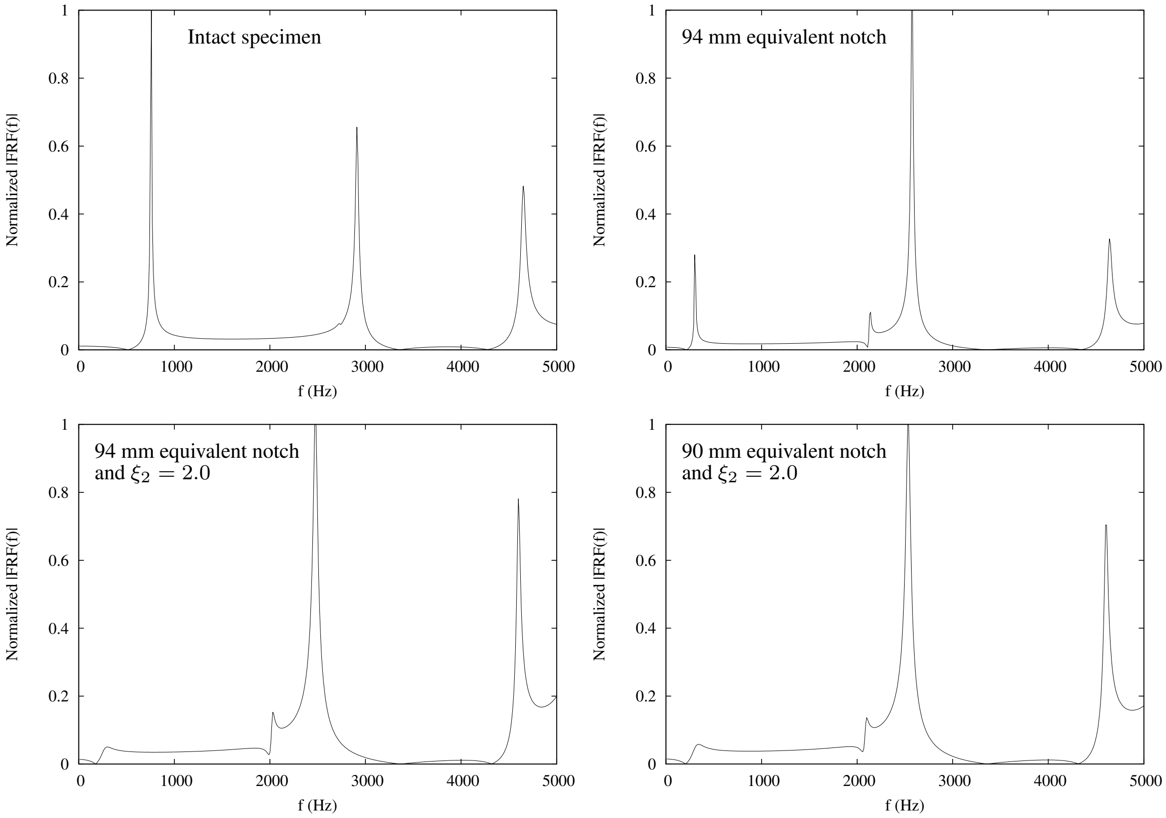

Figure 7: Numerical normalized frequency response functions for different models by BEM-FEM formulation. Measured at 60 mm left from the center (ai2). Impact on point A. To be compared with Figure 3.

left plot of Figure 7 with the left plot of Figure 3, and between the bottom right plot of Figure 7 with the right plot of Figure 3, shows the good agreement between this simple model and the experiments.

3.4 Equivalent elastic crack length according to Jenq and Shah

Critical effective crack length a_e according to LEFM was obtained with P-CMOD curves in order to compare it with the results of the previous sections and see if these equivalent linear approximations could be used in the future for more complex cases. This was calculated using the compliance method of Jenq and Shah [1]. According to these indications, after obtaining the initial compliance C_i of the P-CMOD curves, the elastic modulus of concrete is obtained with as $E_c = 6Sa_oV_1(\alpha')/(C_iBD^2)$, where B is the specimen width, $V_1(\alpha')$ is a geometric function given by

$$V_1(\alpha') = 0.8 - 1.7\alpha' + 2.4\alpha'^2 + \frac{0.66}{(1-\alpha')^2} + \frac{4D}{S}(-0.04 - 0.58\alpha' + 1.47\alpha'^2 - 2.04\alpha'^3), \quad (1)$$

$\alpha' = (a_o + HO)/(D + HO)$ with HO the clip gauge holder thickness, and the rest of symbols are defined in Figure 8. In this case, HO was 1 mm and there was no accuracy loss if $\alpha' \approx \alpha$ was considered (being $\alpha = a_o/D$ the relative notch depth). The value obtained for $V_1(\alpha)$ was 2.972 and the elastic modulus of concrete was obtained.

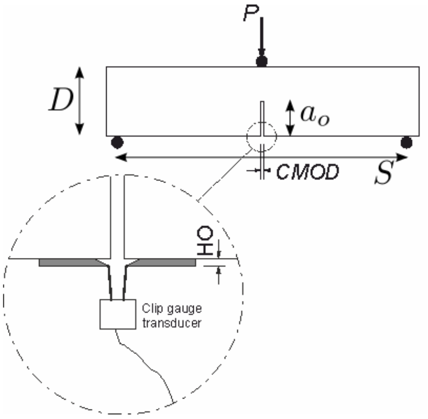

Figure 8: Details of the CMOD instrumentation for the three-point bending tests

Now, a_e is calculated from E_c and the unloading compliance C_u measured at 95% of the maximum load and assuming that the unloading path return to the origin. The critical effective crack length is found when the relationship $E_c = 6Sa_eV_1(\alpha'_e)/(C_uBD^2)$ is satisfied. This way, the obtained unloading compliance from three-point bending tests is 6.477×10^{-5} mm/N $\pm 15\%$ and the obtained value of the critical crack is $a_e = 90\,\text{mm}$, which coincides exactly with the result of the previous sections once damping has been taken into account.

4 CONCLUSIONS

This work presents a simple damage zone numerical model for cracked concrete elements to capture the change in modal parameters due to damage. A very specific case has been investigated: a single-edge notched plain concrete bending specimen, which leads to an specific configuration close to plastic hinge when damaged.

Several specimens were experimentally identified with and without damage. Several numerical analyses are performed and a model comprised by an equivalent crack length and an overdamped zone is proposed. It is shown that such a model is able to capture the main features of the problem, in terms of both modal frequency shifts and modal dampings, and that the equivalent crack length coincides with that proposed by Jenq and Shah [1]. However, in order to be able to draw general conclusions, future developments should involve different and more general configurations, as well as different concrete mixes.

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