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Formulation and calibration of a Pasternak model for seismic analysis of pile foundations

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1. Introduction – Boundary and Finite Elements Methods (BEM and FEM) based models are accurate, but often require heavy computational costs. A simplified, analytical model is presented to obtain a faster yet reliable response of single pile foundations subjected to vertically-incident shear waves. This will enable us to analyse and incorporate in an intuitive way the most relevant dynamic properties of a determined system.
More specifically, the soil is assumed to be a homogenous, linear, viscoelastic half space, while the pile foundation is considered as a Timoshenko beam with circular cross-section. Soil-Structure Interaction (SSI) is represented through a Pasternak model [1], instead of a usual Winkler. The reason lies in Winkler weaknesses. Although it is conveniently simple, considering independent linear springs would create discontinuities in a charged surface boundary, which means that Winkler model does not represent properly SSI in many cases. By approaching the problem through a Pasternak model, it is expected to achieve a better response, introducing soil cohesion and a more efficient force transmission through near field without losing simplicity. The aim of this study is the calibration of this Pasternak model for seismic analysis, using a multidomain BEM model as the reference.

2. Formulation – From the equilibrium of a differential length dx of pile, the following equations are obtained:

$$\frac{\partial V}{\partial x} - \rho A \ddot{u}_y + q_y = 0 \qquad \frac{\partial M}{\partial x} + V - \rho I \ddot{\theta} + q_\theta = 0$$

where u is horizontal pile displacement, θ is its rotation, V is the shear force, M is the bending moment, ρ is the pile's density, A is the pile cross-section area, I is the pile's inertia, q_{H} is the horizontal soil reaction and q_θ is the reaction associated with rotation. From classical strength of materials, we obtain the shear force (V) and bending moment (M) as:

$$V = -\iint_A \tau_{xy}\, dA \;\rightarrow\; V = \kappa A G \left(\frac{\partial u_y}{\partial x} - \theta\right) \qquad M = \iint_A y \cdot \sigma_{xx}\, dA \;\rightarrow\; M = EI\,\frac{\partial \theta}{\partial x}$$

Where σ_{xx} and τ_{xy} are respectively the normal and tangential stresses of the pile's cross-section, κ is the shear correction factor, G is the pile's shear modulus, E is the pile's elastic modulus:

$$\frac{\partial^4 u_y}{\partial x^4} - \frac{\rho}{G\kappa} \cdot \frac{\partial^2 \ddot{u}_y}{\partial x^2} + \frac{\rho A}{EI} \cdot \ddot{u}_y + \frac{1}{GA\kappa} \cdot \frac{\partial^2 q_y}{\partial x^2} - \frac{1}{EI} \cdot q_y - \\ - \frac{\rho}{E} \cdot \frac{\partial^2 \ddot{u}_y}{\partial x^2} + \frac{\rho^2}{EG\kappa} \cdot \ddddot{u}_y - \frac{\rho^2}{EG\kappa} \cdot \ddot{q}_y + \frac{1}{EI} \cdot \frac{\partial q_\theta}{\partial x} = 0$$

Changing to the frequency domain and defining ξ ($\xi = x/L$), the following governing equation is obtained:

$$\frac{\partial^4 \bar{u}_y}{\partial \xi^4} + \kappa_a^2 \alpha \cdot \frac{\partial^2 \bar{u}_y}{\partial \xi^2} - \kappa_l^4 \beta \cdot \bar{u}_y + \frac{L^2}{GA\kappa} \cdot \frac{\partial^2 \bar{q}_y}{\partial \xi^2} + \frac{L^4}{EI}\beta \cdot \bar{q}_y + \frac{L^3}{EI} \cdot \frac{\partial \bar{q}_\theta}{\partial \xi} = 0$$

$$\kappa_a^2 = \left(\frac{\omega \cdot L}{c_a}\right)^2 \quad \alpha = \left(1 + \frac{E}{G \cdot \kappa}\right) \quad \kappa_l^4 = \left(\frac{\omega \cdot L^2}{r_g \cdot c_a}\right)^2 \quad \beta = \left(1 - \omega^2 \frac{\rho I}{GA\kappa}\right)$$

where C_a is the pile axial phase velocity, ω is the frequency and r_g is the radius of gyration. At this point, we can define the reactions as:

$$q_H = -K_W \cdot (u_y - u_y^I) + K_P \cdot \frac{\partial^2 u_y}{\partial x^2} \qquad \bar{u}_y^I = \frac{1}{2} \cdot \left(e^{j\, a_0 \frac{L}{D} \xi} + e^{-j\, a_0 \frac{L}{D} \xi} \right)$$

$$q_\theta = -K_R \cdot \theta \qquad q_\theta^I = -K_D \cdot \frac{\partial u_y^I}{\partial x} \quad / \quad K_D = G_s D^2 \frac{\pi}{4}$$

where u^{I} is the displacement produced by the incident field wave, $a_0$is the dimensionless frequency, K_{W} is a horizontal impedance (or K_{H}, conceptually equal to Winkler's), K_θ is a rocking impedance, K_{D} is a constant that synthetize the effect of shear forces on pile surface and K_{P} is Pasternak impedance. In order to obtain the rotation, we have to go back to equilibrium, and after some operations:

$$\bar{\theta} = g_3 \cdot \frac{\partial^3 \bar{u}_y}{\partial \xi^3} + g_1 \cdot \frac{\partial \bar{u}_y}{\partial \xi} + g_I \cdot \frac{\partial \bar{u}_y^I}{\partial \xi}$$

where,

$$g_3 = \frac{1 + \frac{K_P}{GA\kappa}}{\left(\frac{GA\kappa L^2}{EI} + \frac{L^2 K_R}{EI} \right)} \qquad g_1 = \frac{\frac{GA\kappa L^2}{EI} - \frac{L^2 K_H}{GA\kappa}}{\left(\frac{GA\kappa L^2}{EI} + \frac{L^2 K_R}{EI} \right)} \qquad g_I = \frac{\frac{L^2 K_H}{GA\kappa} - \frac{L^2 K_D}{EI}}{\left(\frac{GA\kappa L^2}{EI} + \frac{L^2 K_R}{EI} \right)}$$

Introducing the reactions and the rotation expressions into the governing equation and reorganizing it, we obtain the following governing equation:

$$\frac{\partial^4 \bar{u}_y}{\partial \xi^4} + C_2 \cdot \frac{\partial^2 \bar{u}_y}{\partial \xi^2} + C_0 \cdot \bar{u}_y = C_{I2} \cdot \frac{\partial^2 \bar{u}_y^I}{\partial \xi^2} + C_{I0} \cdot \bar{u}_y^I$$

$$C_2 = \frac{1}{1 + \frac{L^2 K_R}{EI} \cdot g_3 - \frac{K_P}{GA\kappa}} \cdot \left[\kappa_a^2 \alpha - \frac{L^2 K_H}{GA\kappa} - \frac{L^2 K_R}{EI} \cdot g_1 - \frac{\beta L^2 K_P}{EI} \right]$$

$$C_0 = \frac{1}{1 + \frac{L^2 K_R}{EI} \cdot g_3 - \frac{K_P}{GA\kappa}} \cdot \left[-\kappa_l^4 \beta + \frac{L^4 K_H}{EI} \beta \right]$$

$$C_{I2} = \frac{1}{1 + \frac{L^2 K_R}{EI} \cdot g_3 - \frac{K_P}{GA\kappa}} \cdot \left[\frac{L^2 K_R}{EI} \cdot g_I - \frac{L^2 K_H}{GA\kappa} - \frac{L^2 K_D}{EI} \right]$$

$$C_{I0} = \frac{1}{1 + \frac{L^2 K_R}{EI} \cdot g_3 - \frac{K_P}{GA\kappa}} \cdot \left[\frac{L^4 K_H}{EI} \beta \right]$$

Once the incident field is introduced in the governing equation, the solution of this fourth-order ordinary differential equation can be obtained. The problem is now almost defined. At this point, the impedances K_{W}, K_θ and the only parameters of the model that we have not defined so far. K_{W} and K_θ are calculated with Novak procedure [2], K_{D}as defined above and K_{P} is defined by:

$$K_P = G_s \cdot D^2 \cdot S_P$$

where G_S is the soil shear modulus, D is the pile diameter and S_{P} is a dimensionless Pasternak impedance. The dimensionless Pasternak impedance S_{P} is the free parameter of the model, which will be calibrated in the next section.

3. Verification and optimization – A multidomain BEM model [3] is taken as reference to calculate a relative error between BEM and Pasternak model results, defined as:

$$Error(\omega) = \frac{1}{N_\xi} \sum_\xi \frac{|m^{AN}(\omega,\xi) - m^{BEM}(\omega,\xi)|}{|\, m_{max}^{BEM}(\omega,\xi) \, - \, m_{min}^{BEM}(\omega,\xi) \,|}$$

where $Error$ is the error; m is a generic response variable (displacement, rotation, shear force or bending moment), AN refers to analytical model, BEM to BEM model. From the error formula, it can be seen that $Error$ depends on the frequency and the response variable, while pile depth has been synthesized by averaging

along the pile in order to simplify the problem. Moreover, by studying S_P behaviour regarding the error value, a smooth dependence is observed. This way, the optimal value of S_P is calculated as that one that minimizes the error defined above. The possibility of an optimal $S_P = 0$ (a Winkler model) is also considered, so the situation in which the Pasternak assumption does not improve the results can be noticed.
In order to carry out the calibration process, the parameters whereof S_P (and, consequently, the Pasternak model) depends on should be identified. By doing so, we can adjust the Pasternak model response curves according to the ones obtained with the BEM model. Considering that the error depends on the problem configuration, the excitation frequency and which response variable is measured, so will do S_P. It is important to notice that an adequate definition of the error measure is required in order to draw good conclusions. Such optimization is made using a specific optimisation *MatLab* function named *fmincon*. This function is selected for its versatility when choosing input and output parameters and defining them. In our specific case, the sensitivity of the response variables related to S_P is calculated with central finite differences. S_P is assumed to be a real number.
Apart from that, and equally important, BEM model takes nearly 2h in calculations for each case, while the analytic one, error calculation and Pasternak impedance value identification process take less than a few seconds.

Table 1. Constants

Pile diameter:	$D = 0.6\ (m)$	**Soil damping coefficient:**	$\xi_S = 0.05$
Pile Young's modulus:	$E = 30\ (GPa)$	**Shear factor:**	$\kappa = 0.9$
Pile-soil density ratio:	$\rho_S/\rho_P = 0.7$	**Number of frequencies:**	$N_\omega = 15$
Pile Poisson 's coefficient:	$\nu_P = 0.25$	**Number of points:**	$N_\xi = 42$

Table 2. Values of dimensionless parameters considered

L/D	E_P/E_S	ν_S
10	50	0.30
15	100	0.40
20	200	0.49

4. Results – Different physical configurations have been tested. All of them have in common the data shown in Table 1, while pile-length ratio (L/D), pile-soil Young's modulus ratio (E_P/E_S) and soil Poisson's coefficient (ν_S) have the values shown in Table 2. That makes a total of 27 different configurations tested.
To summarise the results, some images are shown. First, we can see a comparison of the bending moment (real part) obtained with BEM, Winkler and Pasternak models with similar problem properties (Image 1). As it can be seen, the improvement of the Pasternak with respect to the Winkler model is remarkable.

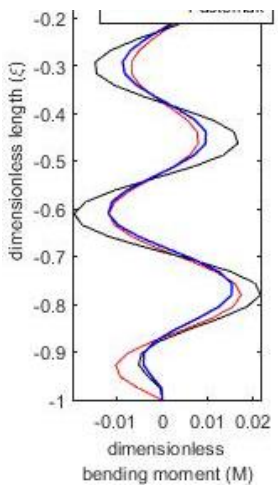

Image 1. Model's response comparison. $L/D = 20$, $E_P/E_S = 100$, $\nu_S = 0.40$,

Image 2 shows how the error $Error$ and S_P behave within the frequency range considered. It is interesting to highlight two things: first, error in lower frequencies is still far too high. It may be caused by using Novak impedances. It could be that the cohesion effect achieved through a Pasternak model depends directly on the frequency. Second, intimately related to what has just been said, the calibrated Pasternak model is able to reproduce BEM results very well.

Finally, some graphs regarding the evolution of S_P are presented. In Image 3, we have a comparison of how optimal S_P grows depending on which response variable S_P is been optimized for. Ignoring those peeks in low frequencies in V and M curves, which would not lead to a bigger error according to what is seen in Image 2, a linear trend can be observed in every response variable.

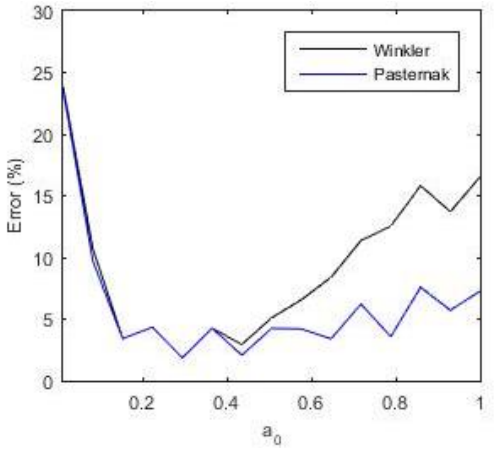

Image 2. Comparison of errors with respect to BEM model obtained with Winkler and Pasternak model. $L/D = 20$, $E_{\mathrm{P}}/E_{\mathrm{S}} = 100$, $\nu_{\mathrm{S}} = 0.40$, $a_0 = 1$, optimizing M

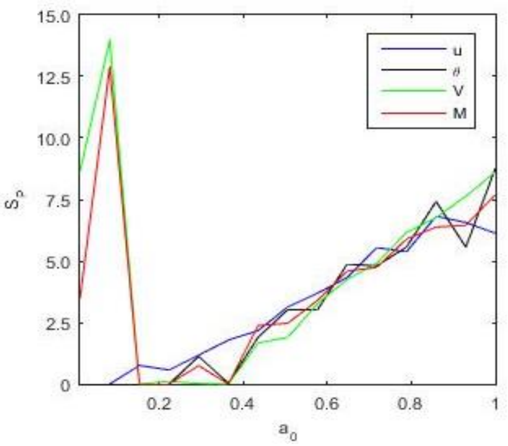

Image 3. Sp depending on the response variable optimized. $L/D = 20$, $E_{\mathrm{P}}/E_{\mathrm{S}} = 100$, $\nu_{\mathrm{S}} = 0.40$

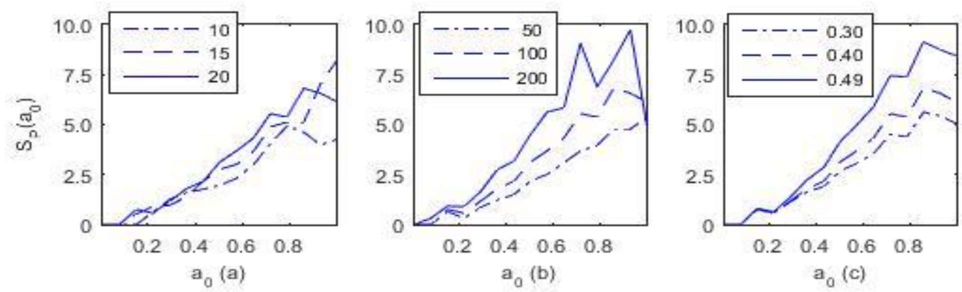

Image 4. $\boldsymbol{S_P}$ comparison changing $\boldsymbol{L/D}$, $\boldsymbol{E_P/E_S}$ and $\boldsymbol{\nu_S}$ parameters, taken as reference

to compare each other $\boldsymbol{L/D = 20}$, $\boldsymbol{E_P/E_S = 100}$ and $\boldsymbol{\nu_S = 0.40}$.

In Image 4 we have a comparison of how optimal S_{P} depends on L/D, $E_{\mathrm{P}}/E_{\mathrm{S}}$ and ν_{S} parameters (Image 4 (a), (b) and (c), respectively). In contrast with Image 3, the linear trend does change its angle with x-axis depending on which value of L/D, $E_{\mathrm{P}}/E_{\mathrm{S}}$ or ν_{S} we introduce in the problem.

5. Conclusions – It has been found that introducing the Pasternak impedance enhance the Winkler model response in all studied cases, which let us get closer to that simplified yet reliable model we pursue. This way, it has been observed that optimal S_P depends linearly on frequency and directly on pile aspect ratio (L/D), pile-soil Young's modulus ratio (E_P/E_S) and soil Poisson's coefficient (ν_S), while stays constant no matter what response variable is optimized. In a view of these circumstances, further investigation following this path is encouraged, as it may help to deeply understand how this model with such potential would work when well treated.

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