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Testing the forward volatility unbiasedness hypothesis in exchange rates under long-range dependence

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ABSTRACT

This paper analyses the unbiasedness hypothesis between spot and forward volatility, using both the actual and the continuous path of realised volatility, and focusing on long-memory properties. For this purpose, we use daily realised volatility with jumps for the USD/EUR exchange rate negotiated in the FX market and employ fractional integration and cointegration techniques. Both series have long-range dependence, and so does the error correction term of their long-run relationship. Hence, deviations from equilibrium are highly persistent, and the effects of shocks affecting the long-run relationship dissipate very slowly. While for long-term contracts, there is some empirical evidence that the forward volatility unbiasedness hypothesis does not hold – and, thus, that forward implied volatility is a systematically downward-biased predictor of future spot volatility – for short-term contracts, the evidence is mixed.

1. Introduction

Foreign exchange volatility is an important factor in risk management, reflecting policy uncertainties, and in forming market expectations. Volatility shows how news affects asset prices, which information is important, and how markets process this information. An interesting and well-documented aspect of the empirical literature on exchange rates is the rejection of the uncovered interest parity (UIP) condition which leads to a forward bias in foreign exchange. This violation suggests that the forward exchange rate is a biased predictor of the future spot exchange rate (see, for example, Bilson, 1981; Fama, 1984; Engel, 1996), which in practice implies that high interest rate currencies tend to appreciate rather than to depreciate (Della Corte et al., 2011). As pointed out by Della Corte et al. (2009), forward bias generates high economic value to an investor designing a strategy aimed at exploiting the UIP violation, commonly referred to as "carry trade".

The forward volatility unbiasedness hypothesis (FVUH), which is the volatility analogue of the forward unbiasedness hypothesis (FUH),[1] has received considerably less attention. Exploiting its violation can generate economic value via spot-forward volatility speculation using forward volatility agreements (FVA).[2] Della Corte et al. (2011) study the FVUH for exchange rates. These authors analyse the empirical relation between spot and forward implied volatility in the FVA estimating by ordinary least squares (OLS) a linear model for implied volatility changes rather than for the highly persistent levels. They find strong evidence that forward implied

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[1] The forward unbiasedness hypothesis suggests that the forward foreign exchange rate serves as an unbiased predictor of the future spot rate.

[2] It is noteworthy that in a standard forward volatility contract (FVA), the payoff at expiration is given by the difference between levels of volatility; e.g., realised volatility (historical volatility) or implied volatility (e.g., the parameter to be used in the Black Scholes formula to match a given option price) of a given asset against the volatility delivery price or strike of the forward contract.

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volatility is a systematically upwards-biased predictor of the movements in future spot implied volatility. The returns to volatility speculation will, then, be predictable and a carry trade in volatility strategy (that is, the difference between the spot and forward volatility from issuing a FVA contract at time t with maturity at time $t+k$) can be profitable.

In this paper, we analyse the volatility levels in a fractional cointegration framework rather than the volatility returns in a standard econometric approach for the following two reasons.[3]

First, volatility is well known to be highly persistent with long memory, unlike changes in volatility which are not. As pointed out by Bandi and Perron (2006), the persistence of measures of financial market volatility needs to be taken into account when assessing volatility relationships, in which case, standard cointegration methods are not appropriate. Using fractional methods allows then for a richer analysis of these long-run relationships, in particular, the speed at which they are recovered. In particular, fractional cointegration is shown for the long-run dynamics of realized exchange rate volatility based on the movements in the market's sensitivity to information, jointly with movements in the rate of information arrival by Berger et al (2009); and for the implied-realised volatility relation by Bandi and Perron (2006), Christensen and Nielsen (2006), Nielsen and Frederiksen (2011), and Rossi and Santucci de Magistris (2013), among others.[4]

Second, the analysis of volatility levels in a fractional cointegrating framework can also be useful to risk managers and hedgers for the design of trading strategies in volatility. Fractional cointegration may provide better price forecasts than traditional models (for example, Afzal and Sibbertsen (2019) find that price forecasts based on the FVECM outperform the ones of traditional models based on the MSE and MAE). In this context, the excess volatility (the difference between the future spot volatility and the forward volatility at a certain time horizon) can be directly analysed by studying the cointegrating errors. If they are not less persistent than the series themselves, the FVUH does not hold (even with an estimated unity relationship). The resulting time-varying and predictable excess volatility allows then speculation. In this scenario, an investor could use a carry trade in volatility strategy using FVAs, which is a dynamic strategy implied by the predictions of the forward volatility regression (FVR) in Della Corte et al. (2011). Finally, from the excess volatility forecasts the returns to volatility speculation (conditional excess volatility over spot volatility) can be calculated. The present paper contributes to the empirical literature on exchange rates in the following three ways.

First, our paper is the first to analyse the FVUH between spot and forward volatility in a long memory context. Methodologically, we use the single-equation fractionally cointegrating model by Nielsen and Frederiksen (2011) and the fractionally cointegrated VAR (FCVAR) model by Johansen and Nielsen (2010, 2012), respectively. Fractional cointegration would then imply a long-run relationship between spot and forward volatility whose error correction term (e.g., the excess volatility from issuing an FVA contract) could possess long memory and, thus, highly persistent deviations from equilibrium. The effects of shocks affecting the long-run relationship would then dissipate at a very slow rate. Importantly, a FVUH violation in this fractional cointegration framework would allow implementing trading strategies in volatility. Given that the FVUH requires spot-forward volatility (fractional) cointegration with a unity vector (or a (fractional) stationary forward volatility risk premium), unbiasedness in the forward volatility implies unpredictability in the excess volatility. However, in the case of a FVUH violation (due to the absence of (fractional) cointegration or due to a non-unity relationship), equilibrium errors are non-stationary; i.e. there is no long-run one-to-one co-movement between volatility series. In this case, volatility speculation becomes possible thanks to a systematic bias in the way the market sets forward volatility. This enables trading strategies in volatility based on differences between spot and forward volatility.

Second, we analyse the spot-forward realised volatility relation based on historical data, rather than the spot and forward implied volatility based on options, as Della Corte et al. (2011). The paper of Della Corte et al. (2011) is based on implied rather than realized volatilities and hence takes a different view on these FVA related issues. It is well known that implied and realized volatilities do not have the same behaviour (see the literature review section for an overview). It is, further, noteworthy that Della Corte et al. (2011) use a different statistical approach based on proportional changes and for a different sample period and data.

Third, we investigate the possible influence of jumps in prices (or price discontinuities) on volatility forecasting (Busch et al., 2011). Andersen et al. (2007) show that separating realized volatility into its continuous sample path and jump components can lead to an improved forecasting performance.[5] Thus, to predict future spot prices we distinguish the efficient information contained in the forward realised volatilities and its continuous path (by subtracting estimated jumps). In particular, we assess whether forward volatility (and its continuous path) are unbiased predictors of future realised volatility (and its continuous path), under long-range dependence.

The data used in this study consists of five-minute intraday information on bid-ask quotes for the spot and 3-months and 1-year forward contracts for USD/EUR exchange rate (expressed as the quantity of USD per 1 Euro) for the period 1 January 2000 to 26 February 2015.

The rest of this paper is organised as follows. Section 2 reviews the previous literature in this field. Section 3 describes the theoretical framework and the methodology. Section 4 presents the empirical analysis and the main results. Finally, Section 5 concludes.

[3] It is noteworthy that the FVUH expressed in both levels and returns is equivalent when conditional excess volatility (that is, the difference between expected future spot and forward volatilities at a time horizon) is zero (Della Corte et al., 2011, page 499).

[4] The use of ordinary least squares methods in the relation between implied and realized volatility leads to inconsistent (and downward biased) slope estimates (Bandi and Perron, 2006).

[5] Andersen et al. (2007) show that important gains in volatility accuracy can be obtained by explicitly differentiating the jump and continuous sample path components. In addition, these authors linked macro-announcements to price discontinuities (jumps), due to their possible influence on volatility forecasting.

2. Literature review

The question of whether forward or implied volatilities are unbiased predictors of future volatility has been empirically analysed in two ways: either through the implied-realised volatility relation, or by analysing the spot-forward/future volatility relation.

Regarding the first approach, the literature finds a positive relation between both volatilities using linear regression models, but does not reach a general consensus regarding the role of implied volatility as predictor of future volatility.[6] For example, various studies uphold the hypothesis of implied volatility being an unbiased predictor of future realised volatility, using single linear regression models such as the Mincer-Zarnowitz regression. For example, Christensen and Prabhala (1998), using monthly sampling, report that implied volatility outperforms past volatility in forecasting future volatility and even subsumes the information content of past volatility in some specifications. Similarly, Bandi and Perron (2006), Christensen and Nielsen (2006) and Nielsen and Frederiksen (2011) all report that the implied and future volatility have a stationary fractional co-integration relationship, and that estimates tend to support the long-run unbiasedness hypothesis. Baruník and Hlínková (2016) propose a band regression on the spectrum estimated by wavelet coefficients. Their findings support the unbiasedness of implied volatility as a good proxy for long-run future volatility.

However, other studies find no evidence on the usefulness of implied volatility for predicting future volatility (Canina and Figlewski, 1993), or conclude that the accuracy of implied volatility as a predictor of future volatility may be biased (Day and Lewis, 1992; Lamoureux and Lastrapes, 1993; Fleming, 1998; among others). Busch et al. (2011), use forecasting methods based on long-range dependence methods, such as the heterogeneous autoregressive forecasting model proposed by Corsi (2009), to show that implied volatility contains incremental information about future return volatility regarding both the continuous and the jump components of realised volatility in the equity, bond and currency markets. Finally, the prediction of realised volatilities can also be improved using implied volatilities among other predictive variables in commodity markets (Haugom et al., 2014; and Dutta, 2017), and including jumps (Luo et al., 2016).

Regarding the second approach – the spot-forward/futures volatility relation –, the empirical literature on this topic is scarce, to our knowledge. For example, the forward volatility unbiasedness hypothesis, which may be viewed as the volatility analogue to the extensively researched hypothesis of unbiasedness in forward exchange rates, is studied by Della Corte et al. (2011). They investigate the empirical relation between spot and forward implied volatility in foreign exchange. Using a new dataset of spot implied volatility quoted on over-the-counter currency options, they compute the forward implied volatility that corresponds to the forward volatility agreement. They find evidence that forward implied volatility is a systematically biased predictor which overestimates movements in future spot implied volatility. As an important economic conclusion, this bias generates high economic value to an investor exploiting predictability in the returns on volatility speculation and reflects the presence of predictable volatility term premia in foreign exchange.

Rossi and Santucci de Magistris (2013) study the future spot and future daily-range as a volatility measure in the stock market. Using the S&P500 index, they take an approach comparable to the one used to examine the no-arbitrage relation between futures and spot prices. They base their study on an analysis of the long-memory properties of range-based volatility estimators. As part of this method, fractional cointegration is tested in a semi-parametric framework using the fractional vector error correction model (FVECM) to study the underlying dynamic relationship. Their results highlight the importance of incorporating the long-run equilibrium in the volatility modelling. FVECM lead to better out-of-sample forecasts given the information in the volatility of futures prices rather than alternative models. Further, deviations from equilibrium are corrected by changes only in the spot but not in futures log-range. Therefore, futures volatility leads spot volatility, implicitly confirming Cox (1976)'s hypothesis on the efficiency of the futures market in processing new information.

3. Methodology

3.1. Forward volatility unbiasedness hypothesis

The forward volatility unbiasedness hypothesis (FVUH) is an analogue to the forward unbiasedness hypothesis (FUH) under risk-neutrality and rational expectation (Della Corte et al., 2011). FVUH states that the forward volatility exchange rate should be an unbiased predictor of the future volatility spot exchange rate in terms of their realised volatilities.

Formally, FVUH can be written as: $E_t[RV_{s,t+k}] = RV_{f,t}^k$, where $E_t[\cdot]$ is the conditional expectation at time t, $RV_{s,t+k}$ is the realised volatility of the future spot exchange rate at time $t+k$, and $RV_{f,t}^k$is the realised volatility of the forward exchange rate at time t for a k-period contract. Therefore, if the FVUH holds, then the conditional expectation of the excess volatility is zero, that is, $E_t[RV_{s,t+k}] - RV_{f,t}^k = 0$.

Hence, we test the FVUH within the following linear regression model:

$$RV_{s,t+k} = \alpha + \beta RV_{f,t}^k + u_{t+k}, \tag{1}$$

[6] In general, implied volatility might not be a useful unbiased predictor of future spot volatility for the following reasons: the implied and future volatility relation tends to be affected by measurement errors (Christensen and Prabhala, 1998), by missing variables (Poteshman, 2000), and by overlapping data (Christensen and Prabhala, 1998). Pérez-Rodríguez (2020) shows a positive but asymmetric dependence in the relationship between realized and implied volatilities, using standard copulas, indicating that a linear model is possibly inappropriate.

with which we can test the FVUH under the joint null hypothesis that $\alpha = 0, \beta = 1$ and that the innovations $\{u_{t+k}\}$ are uncorrelated and with zero mean and constant variance. In this sense, the innovation term under the FVUH is $u_{t+k} = RV_{s,t+k} - RV^k_{f,t}$ which is the excess volatility between spot and forward volatilities. If the FVUH holds, $E_t[RV_{s,t+k}] - RV^k_{f,t} = 0$. However, this hypothesis might be rejected on the grounds that agents' expectations regarding exchange rate changes are not rational and/or are influenced by the presence of a premium in the forward volatility.

The estimation of equation (1) depends on the time-series properties of realised volatilities. It is noteworthy that ordinary least squares (OLS) estimation on volatility can lead to spurious results if the variables are persistent (see Chung and Tsay, 2000). Since realised volatilities are found to be fractionally integrated which can be described by long-memory models (Bandi and Perron, 2006; Christensen and Nielsen, 2006; Berger et al., 2009; Nielsen and Frederiksen, 2011; and Baruník and Hlínková, 2016), we use fractional cointegration estimators.

Note that the model estimated by Della Corte et al. (2011) is a predictive regression based on volatility changes rather than the highly persistent volatility levels. Therefore, ordinary least squares (OLS) estimation of the volatility changes does not lead to spurious results.

An interesting aspect of the fractional cointegration framework for trading is the analysis of the innovations in equation (1). They represent the excess volatility and their analysis could be interesting to investors for two reasons: First, they measure the speed with which a non-unity relationship between spot and forward volatility which can be exploitable in speculation is recovered. Second, if their memory is as high as the one of the original series, i.e. if the series are not fractionally cointegrated, the FVUH does not hold, even if the coefficient equals 1.

Finally, with a fractional cointegrated non-unity relationship, the resulting carry trade in volatility strategy corresponds to the one in Della Corte et al. (2011): buy (sell) FVAs when forward implied volatility is lower (higher) than current spot implied volatility. In particular, depending on the estimated coefficient in [1], the expected excess volatility, $E_t[RV_{s,t+k}] - RV^k_{f,t}$, is either negative or positive leading to a negative or positive excess volatility return, $\frac{E_t[RV_{s,t+k}] - RV^k_{f,t}}{RV^k_{f,t}}$, over the riskless rate.

3.2. *Fractional cointegration and FVUH*

In particular, we, first, use the weak fractional cointegrated method by Nielsen and Frederiksen (2011) and, second, the fractionally cointegrated vector autoregressive (FCVAR) method proposed by Johansen (2008) and extended by Johansen and Nielsen (2010, 2012). Next, we describe both methods.

3.2.1. *Single-equation fractionally cointegrating regression*

Following Nielsen and Frederiksen (2011), we formulate the single-equation cointegrating regression between $y_{t+k} = RV_{s,t+k}$ and $x^k_t = RV^k_{f,t}$ defined by equation (1) to accommodate potential non-stationarity as:

$$\Delta^\gamma y_{t+k} = \alpha + \beta\Delta^\gamma x^k_t + \Delta^\gamma u_{t+k}, t = 1, ..., T, \tag{2}$$

where L is a lag operator, $\Delta^\gamma = (1-L)^\gamma$, $\gamma \geqslant 0$ (i.e., γ is any real number which transforms a potentially non-stationary model in equation (1) into one with stationary regressors; that is, $d_x - \gamma < 0.5$, $\Delta^\gamma y_{t+k} \in I(d_y - \gamma)$, $\Delta^\gamma x^k_t \in I(d_x - \gamma)$ and $\Delta^\gamma u_{t+k} \in I(d_u - \gamma)$, with d_y, d_x, and d_u being the fractional integration orders of y, x and the error term, u, respectively. Additionally, the cointegrating error in the transformed model has non-negative memory (so that $d_u - \gamma \geqslant 0$). x_t and u_t could have long memory, and can in fact be non-stationary (d_x, $d_u \geqslant 0.5$), but the error has less memory than the regressor; that is, $d_x > d_u \geqslant 0$.[7] In equation (2), $d_u < d_x \leqslant 1$ implies the existence of a reversion mechanism towards the long-run equilibrium. The model with $d_x - d_u < 0.5$ is termed weak fractional cointegration in Hualde and Robinson (2010). To estimate the parameters of interest (β, d_x and d_u), we use two procedures depending on the values of γ in equation [2].

First, setting $\gamma = 0$, which represents the stationary fractional cointegration case, Robinson (1994) develops a semi-parametric narrow-band least squares estimator (NBLS) in the frequency domain. The NBLS exploits the dominance of the spectral density of $x_t$on u_t at low frequencies (since cointegration implies $d_x > d_u$).

Following Nielsen and Frederiksen (2011), the NBLS estimator of β related to bandwidth m and parameter γ is defined by:

$$\widehat{\beta}_{NBLS}(m, \gamma) = \widehat{F}^{-1}_{xx}(\gamma, 1, m)\widehat{F}_{xy}(\gamma, 1, m), \tag{3}$$

where for two general variables, g and h:

$$\widehat{F}_{gh}(\gamma, k, l) = \frac{2\pi}{T}\sum_{j=k}^{l} Re\left(I_{gh}(\gamma, \lambda_j)\right), 0 \leqslant k \leqslant l \leqslant T-1, \tag{4}$$

is the average co-periodogram with $\lambda_j = 2\pi j/T$, and

[7] According to Granger (1986) and Engle and Granger (1987), cointegration arises when $d_x > d_u \geqslant 0$.

$$\widehat{I}_{gh}(\gamma,\lambda)=\frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}((1-L)^{\gamma}g_t)((1-L)^{\gamma}h_s)'exp(-i(t-s)\lambda) \tag{5}$$

is the co-periodogram matrix.

Second, Nielsen and Frederiksen (2011) choose $\gamma=d_u$ which is always an appropriate choice in the sense that $d_x-\gamma<0.5$ and $d_u-\gamma\geqslant 0$, in a context of weak cointegration. These authors suggest a fully modified narrow-band least squares (FMNBLS) method to take into account the bias in the limiting distribution of the NBLS arising from the presence of a non-zero long-run coherence between cointegrating errors and regressors, in the weak fractional cointegration case ($d_x-d_u<0.5$).[8] This approach, thus, extends the stationary setting of Marinucci and Robinson (2001) and Christensen and Nielsen (2006) to one of weak fractional cointegration.

The FMNBLS estimator of β associated with bandwidth m_3 and γ is expressed as:

$$\widehat{\beta}_{FMNBLS}(m_3,\gamma)=\widehat{\beta}_{NBLS}(m_3,\gamma)-\lambda_{m_3}^{-\widehat{d_u}}\lambda_{m_3}^{\widehat{d_x}}\lambda_{m_2}^{-\widehat{d_u}}\lambda_{m_2}^{\widehat{d_x}}\widehat{\Gamma}_{m_2}(\gamma), \tag{6}$$

whereλ_j is the angular frequency and $\widehat{\Gamma}_{m_2}(\gamma)$ is the estimator of the bias, given by:

$$\widehat{\Gamma}_{m_2}(m_2,\gamma)=\widetilde{F}_{xx}^{-1}(\gamma,m_0+1,m_2)\widetilde{F}_{x\widehat{u}}(\gamma,m_0+1,m_2), \tag{7}$$

where the modified average co-periodogram is defined by:

$$\widehat{F}_{xu}(\gamma,k,l)=\frac{2\pi}{T}\sum_{j=k}^{l}Re\left(e^{i\lambda_j(d_x-d_u)/2}I_{xu}(\gamma,\lambda_j)\right),0\leqslant k\leqslant l\leqslant T-1. \tag{8}$$

Nielsen and Frederiksen (2011) use a variety of bandwidth parameters, such as $m_0=m_3\in\{\lfloor T^{0.4}\rfloor,\lfloor T^{0.5}\rfloor\}$, $m_1\in\{\lfloor T^{0.6}\rfloor,\lfloor T^{0.7}\rfloor,\lfloor T^{0.8}\rfloor\}$, and $m_2\in\{\lfloor T^{0.8}\rfloor\}$, where $\lfloor x\rfloor$ denotes the largest integer less than or equal to x. They also propose a generalised least square (GLS)-version of the FMNBLS, labelled as FMNBGLS.

Once the key parameters β and d_u have been estimated, we can test for fractional cointegration. In the standard cointegration framework, cointegrating errors are assumed to be I(1) or I(0). Unfortunately, the standard unit root tests have low power against the fractional alternative (see Dittman, 2000, for a comparison of some residuals-based tests for fractional cointegration). The null hypothesis of no cointegration against fractional cointegration can then be tested by examining the hypothesis: $H_0: d_u=d_x$ (y_t and x_i are not cointegrated) versus $H_1: d_u<d_x$ (y_t and x_t are fractionally cointegrated, in which case, there would exist a linear combination of y_t and x_t that is of lower memory than the variables themselves).[9] Similar as Nielsen and Frederiksen (2011) in their empirical application, we compare the memory estimates $\widehat{d}_y$ respectively $\widehat{d}_x$ with the one of the cointegration error, $\widehat{d}_u$ respectively $\widetilde{d}_u$, and conclude that the two variables are cointegrated if the latter is smaller than the former.

3.2.2. Fractionally cointegrated vector autoregressive model (FCVAR)

In this section, we evaluate the FVUH in a multi-equational cointegrating framework in which we allow for long memory in the error correction term and for shocks affecting the long-run relationship at a slow rate.

First, we briefly describe the FCVAR model and the testing and estimation method (Johansen and Nielsen, 2010, 2012), following Nielsen and Popiel (2016). The two-dimensional time series of interest is $X_t=\left[RV_{s,t+k}\quad RV_{f,t}^{k}\right]'$. The fractionally cointegrated VAR model for realised volatilities can then be written in its error correction form as:

$$\Delta^d\begin{bmatrix}RV_{s,t+k}\\RV_{f,t}^k\end{bmatrix}=\Delta^{d-b}L_b\alpha\beta'\begin{bmatrix}RV_{s,t+k}\\RV_{f,t}^k\end{bmatrix}+\sum_{i=1}^{k}\Gamma_i\Delta^bL_b^i\begin{bmatrix}RV_{s,t+k}\\RV_{f,t}^k\end{bmatrix}+\begin{bmatrix}\varepsilon_{s,t+k}\\\varepsilon_{f,t}^k\end{bmatrix} \tag{9}$$

where $\varepsilon_t=\left[\varepsilon_{s,t+k}\quad\varepsilon_{f,t}^k\right]'$ is two-dimensional i.i.d.(0,Ω), $\Delta^d=(1-L)^d$ is the fractional difference operator which is defined by its binomial expansion:

$$(1-L)^d=\sum_{j=0}^{\infty}(-1)^j\binom{d}{j}L^j=\sum_{j=0}^{\infty}\frac{\Gamma(j-d)L^j}{\Gamma(-d)\Gamma(j+1)},\Gamma(g)=\int_0^{\infty}x^{g-1}e^{-x}dx, \tag{10}$$

where Γ(.) is the gamma function. This model has long memory, with a real positive parameter, d ($0<d<1$). The process has short memory and a limited spectral density if $d=0$. However, the process is non-stationary when $d\geqslant 0.5$, since it has then infinite variance. Also, $L_b=1-\Delta^b$ is the fractional lag operator. Johansen and Nielsen (2012) impose two restrictions on the parameter space in their

[8] The NBLS estimator is confined to the case $d_x<0.5$ and in practice it is difficult to ensure that we are in the stationary regions of d_x. To overcome this drawback, Nielsen and Frederiksen (2011) suggest the FMNBLS estimator.

[9] Standard errors for $\widehat{d}_u$ in NBLS and $\widetilde{d}_u$ in FMNBLS are based on the same asymptotic distribution as $\widehat{d}$. Asymptotic standard errors are obtained using $\sqrt{m_1}\left(\widehat{d}-d\right)\xrightarrow{d}N(0,1/4)$ by Nielsen and Frederiksen (2011). However, this should be used with caution (see Theorem 2.2 in Nielsen and Frederiksen, 2011).

asymptotic analysis; $d \geqslant b$ and $d - b < 1/2$, although these restrictions are relaxed in Johansen and Nielsen (2018a,b).[10]

The long-run parameters αandβ, which are $2 \times r$ matrices, with $0 \leqslant r \leqslant 2$ being the cointegration, or cofractional, rank, are the most important parameters of equation [9]. The parameters in $\alpha$are the adjustment (or loading) coefficients and represent the speed of adjustment towards equilibrium for each of the variables. The columns of β are the r cointegration (cofractional) vectors such that $\beta' X_t$ are the long-run equilibrium relations. For the case of $r = 1$, $\alpha = [\alpha_y \quad \alpha_x]'$ and $\beta = [1 \quad \beta_x]'$. The short-run dynamics of the variables are governed by the parameters contained in the autoregressive part, $\Gamma_1, \ldots, \Gamma_k$.

Model [9] can be estimated by maximum likelihood, conditional on N initial values (the number of observations used for conditioning). Assuming that the sample is of length $T + N$, the function to be maximised can be written as:

$$logL_T(\lambda) = \frac{2T}{2}(log(2\pi) + 1) - \frac{T}{2}log\,det\left[T^{-1}\sum_{t=N+1}^{T+N}\varepsilon_t(\lambda)\varepsilon_t'(\lambda)\right], \tag{11}$$

where the residuals are defined as:

$$\varepsilon_t(\lambda) = \Delta^d X_t - \alpha\Delta^{d-b}L_b(\beta' X_t + \rho') - \sum_{i=1}^{k}\Gamma_i\Delta^d L_b^i X_t - \xi, \tag{12}$$

and where the parameter ρ, the restricted constant term (so called because the constant term in the model is restricted to the form $\alpha\rho'$), is interpreted as the mean level of the long-run equilibria when these are stationary; and the parameter ξ is the unrestricted constant term, which gives rise to a deterministic trend in the levels of the variables.

To test for fractional cointegration, Johansen and Nielsen (2012) propose a likelihood ratio (LR) test statistic of the hypothesis H_0 : $rank(\alpha\beta') = r$ against $H_A : rank(\alpha\beta') = r + 1$, whose asymptotic distribution under the null depends on the parameter b_r, i.e. b estimated under the null. Similar as in the case of standard cointegration, the test statistic LR_T can be obtained from reduced rank regression. In particular, for $0 < b_r < 1/2$, LR_T follows a χ_1^2 distribution, while for $1/2 < b_r \leqslant d$,it follows a nonstandard distribution (see Johansen and Nielsen, 2012, for details). The approach also allows to test restrictions on the cointegration relationship or the adjustment matrix. In particular, the unbiasedness hypothesis translates into the testable hypothesis,$H_0 : \beta_X = 1$.

4. Data and results

4.1. Exchange rate data

The data used in this study consist of five-minute intraday information on bid-ask quotes for the spot and 3-months and 1-year forward contracts for USD/EUR. The data are provided by Olsen Financial Technologies (OFT). The sample period considered is 1 January 2000 to 26 February 2015. Following Andersen et al. (2003), weekends, public holidays and other inactive trading days are excluded from the sample. This database does not account for the effect of daily overlapping observations.[11] Thus, a total of 3954 trading days are considered.

Regarding the forward contracts for 3-months and 1-year USD/EUR bid-ask quotes, the data supplied by OFT are expressed in basis points. These are then converted into forward rates for delivery on a specific value date using the formula: current spot rate + basis points/10,000. Therefore, forward points are the number of basis points added to or subtracted from the current spot rate of a currency pair. When points are added to the spot rate this is called a forward premium; when points are subtracted from the spot rate, it is a forward discount.

Note that we work with the midpoint quotes of the bid-ask which are constructed as $Q_t = \frac{A_t + B_t}{2}$, with B_t denoting the bid price and A_t denoting the ask price at time t.

4.2. Estimating realised variances and jumps

This section focuses on the theoretical aspects necessary to construct realised volatility (RV), its continuous path (C) and jumps (J) in our empirical exercise.

The daily realised volatility is constructed from five-minute intraday data, supplied by OFT and calculated from high frequency data for USD/EUR bid-quotes. The following measures are calculated using the method by Andersen et al. (2007).

Let $p_{t,i}$ be the i-th intraday log-price on day t, where $t = 1, 2, \ldots, T$ and $i = 1, 2, \ldots, n_t$, where n is the number of observations on day t, and $\forall t$, $r_{t,i} = p_{t,i} - p_{t,i-1}$. is the intraday period return on day t, formed by the differences between the i-th $\left(p_{t,i}\right)$ and (i-1)-th $\left(p_{t,i-1}\right)$intraday log-prices, n_t is the number of observations on day t. If this number is constant, $n_t = n$, $\forall t$.

[10] The cointegrated VAR (CVAR) model of Johansen (1995) is a special case in which $d = b = 1$ in equation [3] (Johansen and Nielsen, 2018b).

[11] It is noteworthy that Della Corte et al. (2011) obtain similar results comparing predictive regression results using the Mincer-Zarnowitz equation, with both overlapping and non-overlapping observations.

4.2.1. Realised volatility

Using the theory of quadratic variation, the realised variation converges uniformly in probability to the increment of the quadratic variation process as the sampling frequency of the underlying returns increases (Andersen and Bollerslev, 1998; Andersen et al., 2001; and Barndorff-Nielsen and Shephard, 2002). The realised volatility with jumps between day t-1 and t is defined by $RV_t = \sum_{i=1}^{n_t} r_{t,i}^2, \; t = 1, 2, ..., T$ and $i = 1, 2, ..., n_t$.

RV_t is a consistent estimator of integrated variance if there is no microstructure noise (which could arise from a bid-ask bounce, asynchronous trading, price discreteness, among others). However, when returns are serially correlated, there is microstructure noise. Then, RV_t is a biased and inconsistent estimator of volatility (see McAleer and Medeiros, 2011, for an overview). To deal with microstructure noise, Barndorff-Nielsen et al. (2008) propose a flat-top kernel-based estimator, defined as:

$$RV_t^* = RV_t + \sum_{h=1}^{H} K\left(\frac{h-1}{H}\right)\left(\widehat{\gamma}_h + \widehat{\gamma}_{-h}\right), \tag{13}$$

where $K(x)$ for $x \in [0, 1]$ is a non-stochastic weight function such that $K(0) = 1, K(1) = 0$ and the h-th autocovariance is defined by $\widehat{\gamma}_h = \frac{n}{n-h} \sum_{j=1}^{n-h} r_{t,j} r_{t,j+h}$.

4.2.2. Jumps

Based on the theoretical results of Barndorff-Nielsen and Shephard (2006) involving so-called bipower variation measures constructed from the summation of approximately scaled cross-products of adjacent high-frequency absolute returns, Andersen et al. (2007) propose a nonparametric realised volatility procedure for separately measuring the continuous sample path variation and the discontinuous jump part of the quadratic variation process.

To estimate and test the presence of jumps, Andersen et al. (2007) show that the contribution to the quadratic variation process due to discontinuities (jumps) in the underlying price process may be consistently estimated via the combined realised volatility and bipower variation.

Bipower variation, which is robust-to-jumps, can be estimated by the expression:

$$BV_t = \mu_1^{-2}(1 - 2\Delta)^{-1} \sum_{i=2}^{n_t} |r_{t,i}||r_{t,i-1}|, \tag{14}$$

where $\mu_1 = \sqrt{2/\pi}$ and $\Delta = \frac{1}{n_t}$.

Andersen et al. (2007) develop the following formal test statistic for daily jumps:

$$Z_t = \frac{\Delta^{-1/2}[RV_t - BV_t](RV_t)^{-1}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5) max\left(1, TQ_t (BV_t)^{-2}\right)}}, \tag{15}$$

where $TQ_t$is the realised Tri-power quarticity, defined by:

$$TQ_t = \Delta^{-1} \mu_{4/3}^{-3} (1 - 4\Delta)^{-1} \sum_{i=5}^{n_t} |r_{t,i}|^{4/3} |r_{t,i-2}|^{4/3} |r_{t,i-4}|^{4/3}, \tag{16}$$

where $\mu_{4/3} = 2^{2/3} \Gamma\left(\frac{7}{6}\right) \Gamma\left(\frac{1}{2}\right)^{-1}$. TQ_t is an estimator of the integrated quarticity.

The Z_t statistic is closely approximated by a standard normal distribution and has reasonable power with respect to several plausible stochastic volatility jump diffusion models. Using this test statistic, Andersen et al. (2007) estimate jumps as:

$$J_{t,\tau} = I(Z_t > \Phi_\tau)[RV_t - BV_t], \tag{17}$$

where $I(Z_t > \Phi_\tau)$ is an indicator function which takes the value 1 when $Z_t > \Phi_\tau$ and 0 otherwise, and Φ_τ is the critical value for the standard normal cumulative distribution function for the probability τ.

4.2.3. Continuous path

To ensure that the estimated continuous sample path component variation and jump variation sum to the total realised variation, Andersen et al. (2007) estimate the residual:

$$C_{t,\tau} = I(Z_t \leqslant \Phi_\tau) RV_t + I(Z_t > \Phi_\tau) BV_t. \tag{18}$$

This expression can then be used to construct the continuous path of the realised volatility in period t at the level τ.

Once realised volatility, RV_t^*, is calculated from equation (13), jumps, $J_{t,\tau}$, and the continuous path, $C_{t,\tau}$, can be constructed using the formulas (17) and (18) respectively. With $\tau = 0.99$, we identify as jumps 25.19% of the observations of the spot series, 25.06% of the 3-months forward contracts series, and 23.87% of the 1-year forward contracts series. In general, their order of magnitude is 10^{-4} and comparable to the one of the realized volatility.

4.3. Empirical results

4.3.1. Statistical properties

Table 1 shows the descriptive statistics and the Jarque-Bera normality test for the daily total realised volatilities for the spot $(RV_{s,t+k})$ and forward $\left(RV_{f,t}^{k}\right)$ time series, and the continuous path for the spot $(C_{s,t+k,\tau})$and forward $\left(C_{f,t,\tau}^{k}\right)$. We consider $k = 66$ days for 3-month data, and 252 days for 1-year data. While for $k = 66$, total realised volatilities and their continuous path have similar distributions, for $k = 252$ they somehow differ. Fig. 1 shows the 100 autocorrelations of these series. These autocorrelations decrease rather slowly towards zero, indicating that they may have long memory.

Table 2 shows the long memory estimates and homogeneity tests of the integration orders: Panel A displays the results of the memory parameter estimates of the realised volatilities of the spot and forward time series obtained from Robinson's (1995) local Whittle estimator for several bandwidth choices ($m_1 = \lfloor T^{0.6} \rfloor, m_1 = \lfloor T^{0.7} \rfloor$, and $m_1 = \lfloor T^{0.8} \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x) and the corresponding asymptotic standard errors, using the expressions derived by Nielsen and Frederiksen (2011).[12] Panel B shows the outcomes of the Robinson (1995) homogeneity test for equality of the orders of integration in the bivariate system (i.e., $H_0 : d_x = d_y$), where d_x and d_y are the orders of integration of the spot and forward series, respectively.

The individual estimates of the orders of integration of the series are virtually the same for the spot and forward contracts and for several bandwidths using Robinson's (1995) local Whittle estimator (Panel A). From the confidence interval at 95% for each memory parameter, the estimated memory parameters significantly differ between different bandwidth choices (in bold), and we reject both hypotheses, $H_0 : d_i = 0,\ i = x, y$, and $H_0 : d_i = 1$ against $H_1 : d_i < 1,\ i = x, y$, and, thus, conclude that the series are fractionally integrated in all cases. However, volatilities are only found to be stationary, i.e. with $d < 0.5$, for $m_1 = \lfloor T^{0.8} \rfloor$, yet, mean reversion ($d < 1$) is observed for both spot and forward volatilities for all bandwidths.

From the homogeneity tests (Panel B), we cannot reject homogeneity at any significance level. Thus, both series have the same order of integration, which is the requisite of the consequent cointegration analysis.

4.3.2. OLS estimation

In this section, we estimate equation [1] in levels by OLS, ignoring the fractional cointegration relationship, and, therefore, potentially obtain spurious results (see Chung and Tsay, 2000). Table 3 shows the OLS estimates, the R^2 and F-statistic, $F_{\alpha=0,\beta=1}$, for the null of the unbiasedness hypothesis; $\alpha = 0$, $\beta = 1$.

From these results, we would reject the null hypothesis of the FVUH in all cases. However, taking into account that the variables are fractionally integrated with the same order of integration, as aforementioned, the following fractional cointegration analysis is more appropriate.

4.3.3. Fractional cointegration analysis

4.3.3.1. Single-equation fractionally cointegrating model. Tables 4 and 5 show the results for the single-equation fractionally cointegrating model using the NBLS and FMNB(G)LS estimators (Nielsen and Frederiksen, 2011).[13] We consider a variety of bandwidth parameters,$m_0 = m_3 \in \left\{ \lfloor T^{0.4} \rfloor, \lfloor T^{0.5} \rfloor \right\}, m_1 \in \left\{ \lfloor T^{0.6} \rfloor, \lfloor T^{0.7} \rfloor, \lfloor T^{0.8} \rfloor \right\}$ and $m_2 \in \left\{ \lfloor T^{0.8} \rfloor \right\}$. A comparison of the estimates of the memory parameters of the two series in Table 2 with the ones of the cointegration error in Table 4 and 5 allows us then to test the hypothesis $H_0 : d_u = d_x$ (no cointegration) against $H_1 : d_u < d_x$ (fractional cointegration).

First, for spot and forward 3-months contracts, from Table 4, the existence of an unobserved risk premium can imply a negative intercept in the regression. Moreover, any volatility risk premium correlated with forward volatility might bias the NBLS estimator. The NBLS estimates of the parameter of interest, β, are found to be in the interval 0.62–0.86, which is below *one*, for all bandwidths. However, in all cases $\widehat{d}_x + \widehat{d}_u \geqslant 0.5$ and, therefore, Theorem 2.1 in Nielsen and Frederiksen (2011) does not apply to the NBLS estimator and the asymptotic standard error is not available (denoted by (–)). In consequence, the FMNBGLS estimates, which correct for the possible correlation between the regressor and the error term, could be a better choice.

Table 4 also shows the FMNBGLS estimates obtained for the parameter of interest, β, in the interval 0.71–1.45, representing mixed evidence for the long-run unbiasedness hypothesis $H_0 : \beta = 1$ for both total and continuous path. The test of the null hypothesis of no cointegration versus fractional cointegration produces the following results: The integration order of the cointegration residual is found to be lower than the integration orders of the individual series in all cases. However, the long-run equilibrium is only stationary for bandwidth $m_1 = \lfloor T^{0.8} \rfloor$ for both the actual and continuous series and for the combination $m_0 = m_3 = \lfloor T^{0.3} \rfloor, m_1 = \lfloor T^{0.7} \rfloor, m_2 = \lfloor T^{0.8} \rfloor$ for the actual series.

In these cases, the cointegrating residuals are stationary, since $\widehat{d}_u$ less than 0.5. In the remaining cases, the memory of the cointegrating error is greater than 0.5, reflecting mean-reverting but non-stationary behaviour of the cointegration residuals. Note that in these cases, in addition to stationary fractional cointegration, also the unbiasedness hypothesis holds.

[12] Ox programs to calculate NBLS, FMNBLS, and other narrow-band least squares estimators can be downloaded from the personal page of Professor Morten Ørregaard Nielsen (Queen's University, Canada) at http://www.econ.queensu.ca/faculty/mon/software/.

[13] These authors describe the NBLS estimation of equation [1] and [2] and derive their asymptotic distribution. They also discuss inference, using the local Whittle estimator of the integration order of the errors when the errors are not observed, using residuals instead.

Table 1
Descriptive statistics for daily total realised volatilities and their continuous path.

	Spot and forward 3-months				**Spot and forward 1-year**			
	Total volatility		**Continuous path**		**Total volatility**		**Continuous path**	
	$RV_{s,t+66}$	$RV_{f,t}^{66}$	$C_{s,t+66,\tau=0.99}$	$C_{f,t,\tau=0.99}^{66}$	$RV_{s,t+252}$	$RV_{f,t}^{252}$	$C_{s,t+252,\tau=0.99}$	$C_{f,t,\tau=0.99}^{252}$
Mean	0.0059	0.0060	0.0057	0.0057	0.0059	0.0062	0.0056	0.0059
Median	0.0056	0.0056	0.0053	0.0054	0.0055	0.0058	0.0053	0.0056
Maximum	0.0323	0.0321	0.0241	0.0240	0.0261	0.0313	0.0235	0.0236
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0026	0.0026	0.0024	0.0024	0.0024	0.0025	0.0023	0.0024
Skewness	1.8492	1.8703	1.5445	1.5458	1.6254	1.8979	1.5180	1.6445
Kurtosis	11.510	11.546	8.3927	8.3755	9.1273	11.533	8.3571	9.0209
Jarque-Bera	13949.2	14099.1	6257.1	6229.61	7421.4	13454.5	5848.5	7260.6
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Observations	3888	3888	3888	3888	3702	3702	3702	3702

Note: The numbers in brackets are the p-values of the Jarque-Bera normality test.

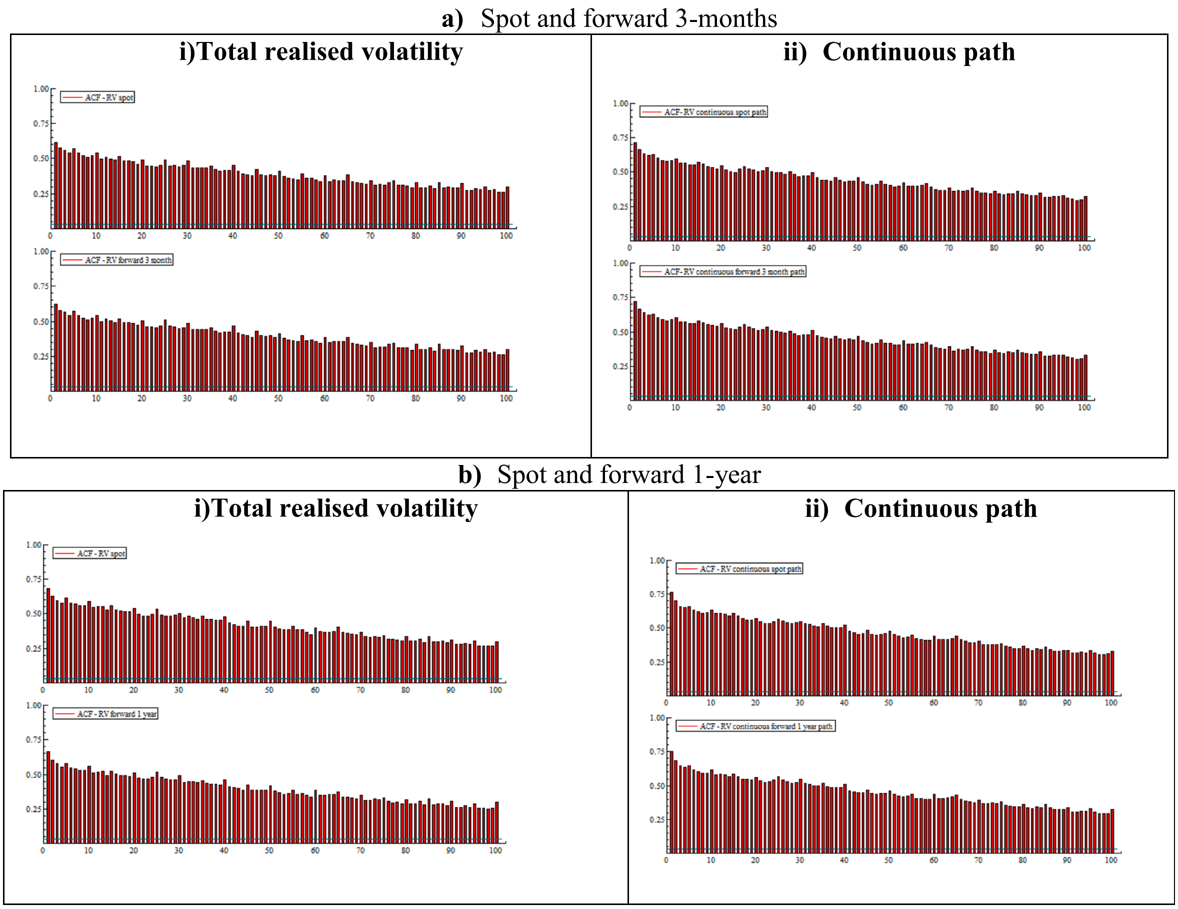

Fig. 1. Autocorrelations of total realised volatilities and continuous path for daily spot and forward time series.

Finally, although jumps make up about a quarter of the sample, only for one bandwidth combination, the results for the actual and for the continuous paths of volatilities actually differ. Therefore, jumps seem to be of rather small importance in the long-run relationship of the spot and forward volatility of USD/EUR exchange rates.

Next, for spot and forward 1-year contracts, Table 5 shows the NBLS and FMNBGLS results. While for FMNBGLS, the point estimates of β ranges from 0.33 to 0.51, but are significantly different from *one*, for all bandwidths, for NBLS, the estimates are even lower. Therefore, the estimates do not support the long-run hypothesis, $\beta = 1$. Thus, in general, the null hypothesis of unbiasedness is rejected. Analysis of the $\widehat{d}_u$ parameter shows that only for the bandwidth $m_1 = \lfloor T^{0.8} \rfloor$ there is evidence supporting stationary fractional cointegration for the relation between spot and forward 1-year values (indicated by a superscript [b]). For the remaining cases, the memory

Table 2
Long memory estimates and homogeneity tests.

	Spot and forward 3-months		Continuous path – spot and forward 3-months		Spot and forward 1-year		Continuous path – spot and forward 1-year	
	Spot	Forward	Spot	Forward	Spot	Forward	Spot	Forward
Panel A: Memory parameter estimates	$\widehat{d}_y$	$\widehat{d}_x$	$\widehat{d}_y$	$\widehat{d}_x$	$\widehat{d}_y$	$\widehat{d}_x$	$\widehat{d}_y$	$\widehat{d}_x$
$m_1 = \lfloor T^{0.6} \rfloor$	0.6154	0.6292	0.6784	0.6378	0.6477	0.6213	0.6482	0.6292
	(0.042)	(0.042)	(0.043)	(0.043)	(0.042)	(0.042)	(0.043)	(0.043)
	[0.53,0.70]	[0.55,0.71]	[0.59,0.76]	[0.55,0.72]	[0.57,0.73]	[0.54,0.70]	[0.56,0.73]	[0.54,0.71]
$m_1 = \lfloor T^{0.7} \rfloor$	0.5139	0.5095	0.5227	0.5220	0.5218	0.5040	0.5169	0.5293
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
	[0.46,0.57]	[0.45,0.56]	[0.47,0.58]	[0.47,0.58]	[0.47,0.58]	[0.45,0.56]	[0.46,0.57]	[0.47,0.58]
$m_1 = \lfloor T^{0.8} \rfloor$	0.4205	0.4221	0.4581	0.4608	0.4355	0.4309	0.4616	0.4633
	(0.018)	(0.018)	(0.019)	(0.019)	(0.018)	(0.018)	(0.019)	(0.019)
	[0.39,0.46]	[0.39,0.46]	[0.42,0.50]	[0.42,0.50]	[0.40,0.47]	[0.40,0.47]	[0.42,0.50]	[0.43,0.50]
Panel B: Homogeneity tests	$H_0 : d_y = d_x$		$H_0 : d_y = d_x$		$H_0 : d_y = d_x$		$H_0 : d_y = d_x$	
$m_1 = \lfloor T^{0.6} \rfloor$	0.01009 [0.92]		0.0092 [0.92]		0.00046 [0.98]		0.0000052 [0.99]	
$m_1 = \lfloor T^{0.7} \rfloor$	0.2230 [0.34]		0.00658 [0.93]		0.01577 [0.90]		0.00511 [0.94]	
$m_1 = \lfloor T^{0.8} \rfloor$	0.04609 [0.83]		0.1670 [0.68]		0.1181 [0.73]		0.00089 [0.99]	

Notes: Long memory estimates are obtained using OxMetrics 5 and codes written by Nielsen and Frederiksen (2011). Between round brackets appear asymptotic standard errors, which are obtained using $\sqrt{m_1}\left(\widehat{d} - d\right) \xrightarrow{d} N(0, 1/4)$ by Nielsen and Frederiksen (2011). Numbers in square brackets show the confidence interval at 95% for each parameter. The p-values of the homogeneity test of equal integration orders, obtained by Robinson's (1995) procedure in STATA 13.1, are shown in square brackets. The statistic is F(1,2(m_1-k)), where k is the number of parameters in the log-periodogram regression and m_1 is the number of ordinates.

parameter of the cointegration residuals is greater than 0.5, reflecting their mean-reverting but non-stationary behaviour.

4.3.3.2. FCVAR model results. In this section, we apply the Johansen and Nielsen (2012) fractionally cointegrated vector autoregressive (FCVAR) model, equation [3]. Table 6 shows the test and estimation results: in particular, the LR test statistic for the cointegration rank, and the FCVAR estimation results, i.e. the estimated values of the parameters (α and β and the memory estimates (d and b) together with their standard errors). It also shows the p-values of a test of the hypothesis $H_0 : \beta_X = 1$ which corresponds to the unbiasedness hypothesis.

The analysis is based on the Johansen and Nielsen (2012) likelihood ratio fractional cointegration tests for the null of rank 0, with unrestricted constant and autoregressive orders chosen as the maximum of the ones chosen by AIC and BIC criteria (for $p \leq 3$). Further, the likelihood is conditional on 20 initial values and d and b are searched within the intervals [0.4, 0.8] and [0, d], respectively, which cover the estimated memory parameters in Table 1 for any combination of bandwidths.[14] First, for spot and forward 3-months contracts, the null of rank *zero* is clearly rejected for both the series with jumps and its continuous path. Given that both test statistics by far exceed the critical value at 5% significance and at 1% significance from a standard χ^2_1 distribution (since the b_r estimated under the null [and not reported] is below 0.5), i.e., 3.84 and 6.64, respectively. The null of rank 1 is clearly not rejected, when comparing the [not reported] test statistics to the corresponding critical value at the 10% significance level of 6.27 and 7.18, respectively, simulated using a FORTRAN routine by MacKinnon and Nielsen (2014). Therefore, the spot and forward 3-months contracts are fractionally cointegrated of rank 1 for both the series with jumps and its continuous path, with $d = 0.69$ and b = 0.52 and $d = 0.67$ and $b = 0.65$,

Table 3
OLS estimates of equation [1]. Variables in levels.

	α	β	R^2	$F_{\alpha=0,\beta=1}$
Spot and forward 3-months	0.0039 [0.00]	0.3445 [0.00]	0.13	171.77 [0.00]
Continuous path – Spot and forward 3-months	0.0034 [0.00]	0.4011 [0.00]	0.17	165.42 [0.00]
Spot and forward 1-year	0.0055 [0.00]	0.0616 [0.01]	0.06	883.52 [0.00]
Continuous path – Spot and forward 1-year	0.0052 [0.00]	0.0729 [0.01]	0.03	695.69 [0.00]

Notes: $F_{\alpha=0,\beta=1}$ is the F-statistic for the unbiasedness null hypothesis, α = 0, β = 1. The p-values appear in brackets.

[14] The reason for this restricted search for d is a certain lack of robustness in both the FCVAR testing and estimation procedure. In fact, for a larger search interval of d, the point estimate can become considerably larger than the estimates in Table 1, and therefore unreasonably large. Similarly, for a restricted rather than unrestricted constant, results are quite volatile. Given that in both cases, constants are very close to zero, we argue that this latter assumption is innocuous.

Table 4
Spot and forward 3-months fractional cointegration results with the Nielsen and Frederiksen (2011) approach for realised volatilities.

	Spot and forward 3-months						Continuous path – spot and forward 3-months					
	NBLS			FMNBGLS			NBLS			FMNBGLS		
Bandwidths	$\alpha(\gamma=0)$	$\beta(\gamma=0)$	$\widehat{d}_u$	$\alpha\left(\gamma=\widehat{d}_u\right)$	$\beta\left(\gamma=\widehat{d}_u\right)$	$\widetilde{d}_u$	$\alpha(\gamma=0)$	$\beta(\gamma=0)$	$\widehat{d}_u$	$\alpha\left(\gamma=\widehat{d}_u\right)$	$\beta\left(\gamma=\widehat{d}_u\right)$	$\widetilde{d}_u$
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00079	0.8614(–)	0.5345	−0.0027	1.4458	0.5497	0.00076	0.8612(–)	0.5379	−0.0024	1.4311	0.5547
			(0.042)		(0.301)	(0.042)			(0.042)		(0.295)	(0.042)
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00079	0.8614(–)	0.4424[b]	−0.0032	1.5269[a]	0.4424[a]	0.00076	0.8612(–)	0.4544	−0.0030	1.5222[c]	0.4626
			(0.028)		(0.361)	(0.028)			(0.028)		(0.363)	(0.028)
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00079	0.8614(–)	0.3618[b]	−0.0033	1.5512[a]	0.3698[a]	0.00076	0.8612(–)	0.4004[b]	−0.0032	1.5468[a]	0.4082[a]
			(0.018)		(0.376)	(0.018)			(0.018)		(0.379)	(0.018)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00184	0.6862(–)	0.5411	0.0013	0.7812[c]	0.5365	0.00184	0.6862(–)	0.5436	0.0013	0.7751[c]	0.5398
			(0.042)		(0.140)	(0.042)			(0.042)		(0.138)	(0.042)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00184	0.6862(–)	0.4489	0.0004	0.9261[c]	0.4414	0.00184	0.6862(–)	0.4600	0.0005	0.9141[c]	0.4536
			(0.028)		(0.172)	(0.028)			(0.028)		(0.172)	(0.028)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00184	0.6862(–)	0.3664[b]	−0.0002	1.0221[a]	0.3612[a]	0.00184	0.6862(–)	0.4045[b]	0.00006	0.9841[a]	0.3997[a]
			(0.018)		(0.180)	(0.018)			(0.018)		(0.183)	(0.018)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00225	0.6182(–)	0.5459	0.0025	0.5682	0.5502	0.00225	0.6182(–)	0.5476	0.0026	0.5978	0.5502
			(0.042)		(0.087)	(0.042)			(0.042)		(0.088)	(0.042)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00225	0.6182(–)	0.4532	0.0017	0.7061	0.4479	0.00225	0.6182(–)	0.4635	0.0016	0.7218	0.4587
			(0.028)		(0.100)	(0.028)			(0.028)		(0.105)	(0.028)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00225	0.6182(–)	0.3695[b]	0.0011	0.8170[a]	0.3625[a]	0.00225	0.6182(–)	0.4070[b]	0.0011	0.7960[a]	0.4015[a]
			(0.018)		(0.112)	(0.018)			(0.018)		(0.111)	(0.018)

Notes: The fractional cointegration estimates are obtained using OxMetrics 5 and codes written by Nielsen and Frederiksen (2011). Standard errors are in round brackets. FMNBGLS is the GLS version of FMNBLS. Following Nielsen and Frederiksen (2011), the FMNBLS estimator has no asymptotic bias and has the same asymptotic variance as the NBLS estimator. The asymptotic standard errors for the NBLS and FMNBGLS estimates are based on equations (2.12) and (3.8), respectively, in Nielsen and Frederiksen (2011). Standard errors for $\widehat{d}_u$ and $\widetilde{d}_u$ in NBLS and FMNGBLS are based on the same asymptotic distribution as d, and should be used with caution; see Theorem 2.2 in Nielsen and Frederiksen (2011). The asymptotic standard errors are obtained using $\sqrt{m_1}\left(\widehat{d}-d\right)\xrightarrow{d}N(0,1/4)$ by Nielsen and Frederiksen (2011). Superscripts should be interpreted as follows: [a] indicates that the FVUH holds (β is statistically not different from 1 and there is stationary fractional cointegration). [b] indicates that there is stationary fractional cointegration because $\widehat{d}_u$ and $\widetilde{d}_u$ are lower than 0.5. [c] indicates that β is statistically not different from 1 analysing $\beta\left(\gamma=\widehat{d}_u\right)$ for FMNBGLS.

Table 5

Spot and forward 1-year fractional cointegration results with the Nielsen and Frederiksen (2011) approach for realised volatilities.

	Spot and forward 1-year						Continuous path – spot and forward 1-year					
	NBLS			FMNBGLS			NBLS			FMNBGLS		
Bandwidths	$\alpha(\gamma=0)$	$\beta(\gamma=0)$	$\widehat{d}_u$	$\alpha\left(\gamma=\widehat{d}_u\right)$	$\beta\left(\gamma=\widehat{d}_u\right)$	$\widetilde{d}_u$	$\alpha(\gamma=0)$	$\beta(\gamma=0)$	$\widehat{d}_u$	$\alpha\left(\gamma=\widehat{d}_u\right)$	$\beta\left(\gamma=\widehat{d}_u\right)$	$\widetilde{d}_u$
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00519	0.1092(–)	0.5667	0.0035	0.3837	0.6735	0.00492	0.1178(–)	0.6550	0.00369	0.3251	0.6641
			(0.042)		(0.269)	(0.042)			(0.042)		(0.256)	(0.042)
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00519	0.1092(–)	0.5210	0.0035	0.4450	0.5165	0.00492	0.1178(–)	0.5151	0.00334	0.3847	0.5112
			(0.028)		(0.255)	(0.028)			(0.028)		(0.239)	(0.028)
$m_0=m_3=\lfloor T^{0.3}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00519	0.1092 (–)	0.4333[b]	0.0035	0.4581	0.4274[b]	0.00492	0.1178(–)	0.4591[b]	0.00328	0.3946	0.4544[b]
			(0.018)		(0.239)	(0.018)			(0.018)		(0.229)	(0.018)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00521	0.1069 (–)	0.6565	0.0035	0.3763	0.6732	0.00494	0.1148(–)	0.6548	0.00363	0.3365	0.6645
			(0.042)		(0.169)	(0.042)			(0.042)		(0.164)	(0.042)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00521	0.1069 (–)	0.5210	0.0033	0.4194	0.5169	0.00494	0.1148(–)	0.5152	0.00339	0.3767	0.5113
			(0.028)		(0.161)	(0.028)			(0.028)		(0.154)	(0.028)
$m_0=m_3=\lfloor T^{0.4}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00521	0.1069(–)	0.4334[b]	0.0032	0.4817	0.4277[b]	0.00494	0.1148(–)	0.4591[b]	0.00334	0.3844	0.4545[b]
			(0.018)		(0.116)	(0.018)			(0.018)		(0.149)	(0.018)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.6}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00516	0.1160(–)	0.6572	0.0029	0.5682	0.6767	0.00490	0.1258(–)	0.6554	0.00256	0.5119	0.6685
			(0.042)		(0.087)	(0.042)			(0.042)		(0.118)	(0.042)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.7}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00516	0.1160(–)	0.5209	0.0029	0.4837	0.5159	0.00490	0.1258(–)	0.5150	0.00269	0.4933	0.5098
			(0.028)		(0.109)	(0.028)			(0.028)		(0.109)	(0.028)
$m_0=m_3=\lfloor T^{0.5}\rfloor, m_1=\lfloor T^{0.8}\rfloor, m_2=\lfloor T^{0.8}\rfloor$	0.00516	0.1160(–)	0.4332[b]	0.0029	0.4808	0.4271[b]	0.00490	0.1258(–)	0.4589[b]	0.00275	0.4846	0.4534[b]
			(0.018)		(0.104)	(0.018)			(0.018)		(0.105)	(0.018)

Notes: The fractional cointegration estimates are obtained using OxMetrics 5 and codes written by Nielsen and Frederiksen (2011). Standard errors are in round brackets. FMNBGLS is the GLS version of FMNBLS. Following Nielsen and Frederiksen (2011), the FMNBLS estimator has no asymptotic bias and has the same asymptotic variance as the NBLS estimator. The asymptotic standard errors for the NBLS and FMNBGLS estimates are based on equations (2.12) and (3.8), respectively, in Nielsen and Frederiksen (2011). Standard errors for $\widehat{d}_u$ and $\widetilde{d}_u$ in NBLS and FMNBLS are based on the same asymptotic distribution as d, and should be used with caution; see Theorem 2.2 in Nielsen and Frederiksen (2011). The asymptotic standard errors are obtained using $\sqrt{m_1}\left(\widehat{d}-d\right)\xrightarrow{d}N(0,1/4)$ by Nielsen and Frederiksen (2011). Superscript b indicates that there is stationary fractional cointegration analysing $\widehat{d}_u$ and $\widetilde{d}_u$.

Table 6
Fractional cointegration analysis with the Johansen and Nielsen (2012) approach.

	LR test for cointegration rank	Fractionally cointegrated VAR: Estimation results					
	Rank	LR test statistic	α_y	α_x	β_x	***d***	***b***
Spot and forward 3-months	0	27.97 [0.00]	0.064	0.295	0.51 [0.012]	0.69 (0.041)	0.52 (0.071)
Continuous path –Spot and forward 3-months	0	31.05 [0.00]	0.010	0.123	0.72 [0.063]	0.67 (0.071)	0.65 (0.071)
Spot and forward 1-year	0	6.50 [0.01]	0.204	0.230	0.49 [0.561]	0.80 (0.072)	0.39 (0.085)
Continuous path –Spot and forward 1-year	0	2.83 [0.09]	0.060	0.100	0.54 [0.751]	0.80 (0.078)	0.44 (0.101)

Notes: The MATLAB software package FCVAR model (downloaded from http://www.econ.queensu.ca/faculty/mon/software/) is used for this analysis. We allow for an unrestricted constant and the autoregressive orders are chosen by the AIC and BIC criteria. Standard errors are in round brackets. The p-value of a test of $H_0 : \beta_X = 1$ is displayed in square brackets below the estimates of β_x.

respectively. The point estimates of β are 0.51 and 0.72 which are below *one*, thus providing evidence against the unbiasedness hypothesis. Indeed, testing the hypothesis $H_0 : \beta_X = 1$, is rejected at the 5% level for the actual series and at the 10% level for its continuous path.

Next, for spot and forward 1-year contracts, the hypothesis of rank *zero* is still rejected at the 5% significance level for the actual series, for the continuous path it is rejected only at the 10% level. In both cases, the estimated strengths of the cointegrating relationship, *b*, are 0.39 and 0.44. These estimates are below the ones for the 3-months' case and represent weak fractional cointegration. The point estimates of β of 0.49 and 0.54 are below the ones for the 3-months contracts and, thus, seem to provide even stronger evidence against the unbiasedness hypothesis. However, interestingly, the hypothesis $H_0 : \beta_X = 1$ is not rejected for both the actual series and its continuous path. In any case, estimates that far from the null might be taken as a sign of imprecise estimation and therefore low power rather than as evidence in favour of the null hypothesis of unbiasedness.

In comparison to the results of NBLS and FMNBGLS, the FCVAR analysis coincides with the finding of a stronger fractional cointegration relationship for the spot and forward 3-months contracts than for the 1-year case. Further, the point estimates of the cointegration relationship for the 1-year case are quite below *one*.

In summary, results for the total and continuous path are quite similar indicating that there are no substantial differences between them. Therefore, jumps do not fundamentally affect the relationship between realised spot and forward volatilities. Further, depending on whether forward volatilities are used for short or long-term contracts, our study provides evidence of fractional cointegration in some cases for both types of contracts. However, while there is stronger evidence against the unbiasedness hypothesis for long-term forward contracts, for short-term forward contracts, the evidence is rather mixed.

5. Conclusion

This paper tests the forward volatility unbiasedness hypothesis using long-range dependence techniques. More specifically, we analyse fractional cointegration for realised volatilities associated with spot and forward contracts for the USD/EUR exchange rates (spot and forward 3-months and 1-year contracts) using both the Nielsen and Frederiksen (2011) and the Johansen and Nielsen (2012) fractional cointegration procedures. Realised volatilities are constructed using the five-minute intraday USD/EUR exchange rates and sampling procedures consistent with the methods by Andersen et al. (2003) and Andersen et al. (2007). Therefore, to construct realised volatilities we distinguish between the actual series of the daily realised volatility and its continuous path (subtracting the jump effects), and, by doing so, analyse the effects of jumps in prices on the analysis of the unbiasedness and cointegration hypotheses. The empirical analysis shows several interesting results for the spot-forward relationship for USD/EUR exchange rate.

First, we find evidence for the presence of long range dependence in realised volatilities for spot and forward contracts, both in the actual series and its continuous path. The fractional integration parameters of all series are in the stationary region, excluding the possibility of strong fractional cointegration. Moreover, given that the two realised volatility series exhibit the same order of integration, we examine the possibility of a long-run equilibrium relationship between them.

Second, we empirically examine the volatility relation between the spot and forward exchange rate market for USD/EUR and find evidence for fractional cointegration. More specifically, the cointegration analysis indicates the presence of stationary fractional cointegration over the full sample between both variables for large bandwidths with NBLS and FMNBGLS in the single equation approach and also with the multiequational FCVAR approach. Therefore, the error correction term has long memory, implying that deviations from the equilibrium are highly persistent, and so the effects of shocks affecting the long-run relationship dissipate rather slowly. Further, it is worth noting that results for the actual series and their continuous paths are similar in all analysed cases and, therefore, jumps, despite occurring rather frequently, do not affect the link between realised spot and forward volatilities.

Third, we find no statistical evidence that forward volatility is a systematically biased predictor that overestimates movements in future spot volatility. More concretely, our evidence for the forward volatility unbiasedness hypothesis is mixed for both long-term and for short-term contracts. For example, for short-term contracts, the FMNBGLS estimates in the single equation, depending on the bandwidths, vary around unity, representing mixed evidence for the unbiasedness hypothesis. However, for long-term forward contracts, the unbiasedness hypothesis does not hold. With the FCVAR approach, the unbiasedness hypothesis is rejected for short-term but not for long-term contracts, which as aforementioned might be due to its imprecise estimation.

In general, these mixed results indicate that there might be a bias in both short-term and long-term contracts. Specially, for one-year future contracts, there is a bias in the excess volatility issuing the FVA. A violation of the FVUH can then be profitable to speculate and, therefore, a carry trade in volatility strategy could be used to generate economic value. However, for short-term future contracts (one month), the FVUH might hold, at least for some bandwidths. In these cases, deviations from the equilibrium are highly persistent, and so the effects of shocks affecting the long-run relationship dissipate rather slowly. Knowledge on the persistence of these deviations might be profitable given that an investor could exploit slow reversion of the equilibrium errors. Examining the possibility of using a mean reversion strategy with volatility futures/forwards within pairs trading in volatility is left for further research.

Finally, the results hint at the presence of predictable volatility term premia in foreign exchange. It could be interesting to analyse the FVUH for a set of different exchange rates. By doing so it could be possible to discover systematic differences and factors leading to violations of the FVUH. This could provide investors with useful information in their volatility speculation and, therefore, generate economic value.

Author Contribution Statement

All Authors: Conceived and designed the analysis, Analysis tools, Performed the analysis, Wrote parts of the paper.

6. Data availability Statement

The data that support the findings of this study are available from Olsen Financial Technologies GmbH (Zurich, Switzerland). Restrictions apply to the availability of these data, which are used under license for this study. The data supplied continues to be owned by Olsen Financial Technologies and non-transferable license for use of supplied data is granted to the licensee institution (University of Las Palmas de Gran Canaria).

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