HM-LM-AM Inequalities

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Let \( a, b \in \mathbb{R} \), with \( a \neq b \). The harmonic, logarithmic, and arithmetic means of \( a \) and \( b \) are respectively defined by \( H(a, b) = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b} \), \( L(a, b) = \frac{b-a}{\ln b - \ln a} \), and \( A(a, b) = \frac{a+b}{2} \).

**Theorem.** For \( 0 < a < b \), \( \frac{2}{a+b} < \frac{\ln b - \ln a}{b-a} < \frac{a+b}{2ab} \), which may be written as \((AM)^{-1} < (LM)^{-1} < (HM)^{-1}\).

**Proof.** Let us consider functions \( \frac{2}{a+b} \), \( \frac{1}{x} \), and the linear interpolation between points \((a, \frac{1}{a})\) and \((b, \frac{1}{b})\).

![Graph showing the functions and the linear interpolation.]

For \( x \in (0, (b-a)/2) \), \( \frac{1}{a+x} + \frac{1}{b-x} \geq \frac{4}{a+b} \) by the AM-HM inequality.

Then, by the mean value theorem for definite integrals there exists \( c \in (a, (a+b)/2) \) such that \( \frac{1}{c} = \frac{\ln b - \ln a}{b-a} \).

**Summary.** We demonstrate visually the inequalities among the harmonic mean, the logarithmic mean and the arithmetic mean of two positive numbers.

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