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PROOFS WITHOUT WORDS

HM-LM-AM Inequalities

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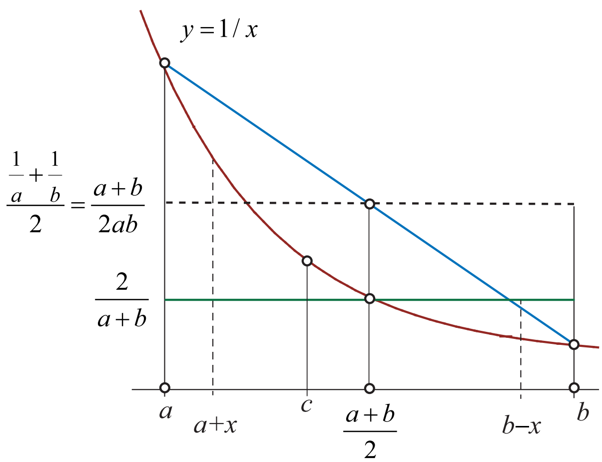
Let $a, b \in \mathbb{R}$, with $a \neq b$. The harmonic, logarithmic, and arithmetic means of a and b are respectively defined by $H(a,b) = \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b}$, $L(a,b) = \frac{b-a}{\ln b - \ln a}$, and $A(a,b) = \frac{a+b}{2}$.

Theorem. For $0 < a < b$, $\dfrac{2}{a+b} < \dfrac{\ln b - \ln a}{b-a} < \dfrac{a+b}{2ab}$, which may be written as $(AM)^{-1} < (LM)^{-1} < (HM)^{-1}$.

Proof. Let us consider functions $2/(a+b)$, $1/x$, and the linear interpolation between points $(a, 1/a)$ and $(b, 1/b)$.



$$\frac{2}{a+b} < \frac{\ln b - \ln a}{b-a} < \frac{a+b}{2ab}$$

For $x \in (0, (b-a)/2)$, $\dfrac{1}{a+x} + \dfrac{1}{b-x} \geq \dfrac{4}{a+b}$ by the AM-HM inequality.

Then, by the mean value theorem for definite integrals there exists $c \in (a, (a+b)/2)$ such that $\dfrac{1}{c} = \dfrac{\ln b - \ln a}{b-a}$. $\square$

Summary. We demonstrate visually the inequalities among the harmonic mean, the logarithmic mean and the arithmetic mean of two positive numbers.

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