Improving on-line signature skillfulness

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***Abstract*—One of the biggest challenges in on-line signature verification is the detection of counterfeited signatures. Recently, novel schemes based on the kinematic theory of rapid human movements and its associated Sigma-Lognormal model has been proposed to improve the detection of on-line skilled forgeries. But for a more realistic and reliable estimation of the forgery detection rate, we would need more challenging on-line forgeries than those included in current databases. To get better on-line skilled forgeries, this paper aimed at leveraging the Sigma-Lognormal model to improve the skill of any online forged signature. Specifically, we propose to replace the original velocity profile of any on-line signature by a synthetic Sigma-Lognormal profile. The new profile emulates a genuine-like velocity profiles without modifying the original ballistic trajectory. Experimental results were performed with the 132 on-line users of publicly BiosecureID database. It is shown that the detection rate of forgeries is significantly worsened when the velocity profile is replaced by the synthetic one. A countermeasure to detect this kind of improved fake signatures is also proposed.**

Keywords—Automatic Signature Verification, Sigma-Lognormal model, forged signatures.

I. Introduction

Biometrics have emerged as a reliable, fast and automatic identification technology. Among the different biometric traits (i.e., fingerprint, face, voice, iris, etc.), one of the most widely accepted is the signature. Even though the verification performance rates of Automatic Signature Verifiers (ASV) have reached significant ratios, skilled forgeries still remains a major challenge for those systems.

According to forensic handwriting examiner nomenclature, the spectrum of signature forgeries spans from random to simple or zero effort up to skilled or freehand specimens. Although efforts have been put in recent years to incorporate better forgeries in test database, most of these cannot be considered as fully skilled forgeries, at least in a forensic document perspective, where it refers to the action of a forger who tries to imitate, after time and practice, as closely as possible the static and dynamic information hidden in a specimen.

Recently, several papers have considered the use of the kinematic theory of rapid human movements and its associated Sigma-Lognormal model [4] to improve the forgery detection. Briefly, this theory models the velocity profile of a rapid movement, like a signature, as a weighted sum of delayed lognormals. Each of these lognormals represent a stroke, the complete movement being a composition of overlapped strokes. One of the advantage of this model is that it considers physical body features such as the neuromuscular system response for the production of a signature difficult to imitate.

In the literature we can find some articles which use the kinematic theory of rapid human movements to detect skilled forgery. For example, the lognormal parameters are combined with classical parameters to improve an ASV in [5]. In the same context, in [6] a dissimilarity measure between lognormal features are proposed. These parameters have been also used to train ASV with only one signature [7].

However, it could be said that the results reported in such papers are biased as most of the skilled forgeries available in the current databases usually reproduce the trajectory of a genuine signature accurately but the dynamic of the signature is poorly imitated [2]. It arguably explains the poor performance of off-line ASV against skilled forgeries and the better accuracy of on-line ASV as they analyze not only the trajectory on the paper (pen-down), but also consider the trajectory in the air (pen-up) and its dynamic properties.

Obviously, impersonating pen-ups and dynamic properties is really challenging for a forger. On the one hand, a genuine signer signs quickly and swiftly which corresponds to a well-learnt movement. On the other hand, the forgeries sign carefully producing a larger and slower velocity profile than the genuine counterpart. This fact is easily detectable by on-line ASV such as those based on Dynamic Time Warping (DTW) [3].

To get a more realistic estimate of an ASV performance, it would be required to build a new database with more realistic on-line skilled forgeries. But this is an almost unrealistic goal to reach from a time, monetary and legal perspective.

In this paper we address this problem by providing, on the one hand, a methodology to generate better synthetic skilled forgeries at the testing phase of an ASV design and, on the other hand, a countermeasure technique to avoid this methodology to be used against such an ASV system in real life application.

For this purposes, to improve the skillfulness of any on-line

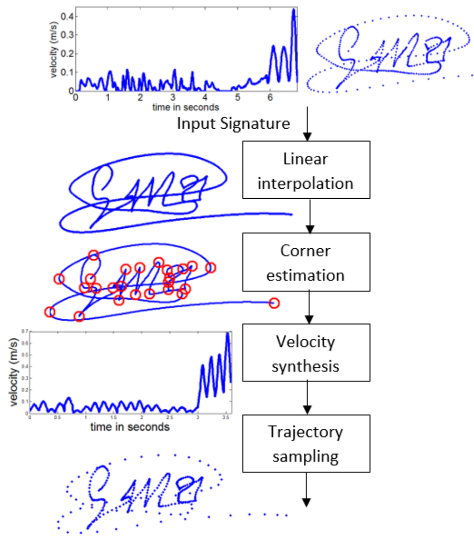


Figure 1. Proposed procedure to improve the skill of a forgery with the Sigma-Lognormal model.

forged signatures, we propose the following algorithm, outlined in Figure 1, which is mainly based on replacing the real velocity profile by a synthetic one as follows: 1) The sampling points of an on-line signature are interpolated and the perceptual important points of the 8-connected trajectory are estimated. 2) A time is given to each stroke between perceptual important points. 3) A lognormal is assigned to each stroke between perceptual important points. Each lognormal is overlapped with the adjacent lognormals. 4) The 8-connected trajectory is resampled with the new velocity profile.

In this way, it is expected that the performance of skilled forgeries in a database fully resampled will be worse than the original one. At the same time, it is also expected that the performance in random forgeries will be similar to the original. In other words, the resampling will move the probability density function (pdf) of skilled forgeries scores toward the pdf of genuine signature scores. Besides, the pdf of genuine signature scores will not change.

The outline of the paper is as follows: Section 2 describes the procedure to estimate the perceptual important points of the trajectory. The velocity synthesis based on lognormal and resampling is detailed in the third section. The results are given in Section 4 while in Section 5 the conclusions are discussed which are drawn upon the found results in our study.

II. PERCEPTUAL IMPORTANT POINTS ESTIMATION

Even though many proposals have been issued, the accurate estimation of the perceptual important points in handwriting is still a challenging problem. According to [8], some approaches worked out the curvature at point p as the tangent of the angle between the lines $< p, p+d >$ and $< p, p-d >$, d being a predefined distance. Different values of d were used in multiscale estimators. Other approaches estimated the curvature as the radius of the osculating circle [9] or spline reconstruction [8].

Focusing on handwriting segmentation, two relevant approaches have been found in the literature. The first ones [10] proposed a method on the basis that handwriting is composed of curvilinear and angular strokes. The second one suggested a multiresolution algorithm [11].

These methods calculated the curvature of the trajectory in order to estimate the perceptual important points by thresholding. The thresholding works reasonably well when it is applied to handwriting text. But some problems arise with Western signatures. This results mainly from the wide variety of curvatures because they combine text with flourishes. As a consequence, it produces corners of different sharpness in the same specimen.

To address these effects, a Two-Steps Perceptual Important Points Estimator (TS-PIPE) for handwriting signatures has been proposed in [12]. In the first step, the more salient corners are worked out by means of a multiscale estimation of the curvature [11]. In the second step, a novel approach is applied to work out the missed salient curvature points based on the fact that each single stroke can be approximated by a circumference arc[4]. Consequently, it could be thought that the trajectory between two salient curvature points is fairly circular and will change from one circle to the next one around the corners. In other words, if the trajectory between two detected corners in the first step is circular enough, no more corner will be added in the middle. Otherwise, new corner points will be added in the middle. For further details, please go through [12].

III. VELOCITY PROFILE SYNTHESIS AND RESAMPLING

This section is devoted to generate genuine-like synthetic lognormal velocity profile. For this purposes, we use the location of the perceptual important points to set up the minima in the velocity profile. The new velocity is obtained from the kinematic theory of rapid movements. It claims that the human being performs their movements with a velocity profile $\bar{v}(t)$ which can be modeled as a linear combination of lognormals [4] as follows:

$$\bar{v}(t) = \sum_{j=1}^{M} \bar{v}_j(t; D_j, \tau_j, \mu_j, \sigma_j^2) \qquad (1)$$

being the velocity profile of each stroke $v_j(t)$ defined as:

$$v_j(t; \tau_j, \mu_j, \sigma_j^2) = \frac{D_j}{\sigma_j\sqrt{2\pi}(t-\tau_j)} exp\left\{-\frac{\left[ln(t-\tau_j)-\mu_j\right]^2}{2\sigma_j^2}\right\} \qquad (2)$$

where t is the time basis, τ_j the time of stroke occurrence, D_j the amplitude of each stroke, τ_j the stroke time delay and σ_j the stroke response time, both on a logarithmic time scale. The distance $s(t)$ traveled at time t is obtained as:

$$s(t) = \int_{-\infty}^{\infty} v_j(t)dt = \frac{D_j}{2}\left(1 + \text{erf}\left(\frac{\ln(t-\tau_j)-\mu_j}{\sqrt{2}\sigma_j}\right)\right) \qquad (3)$$

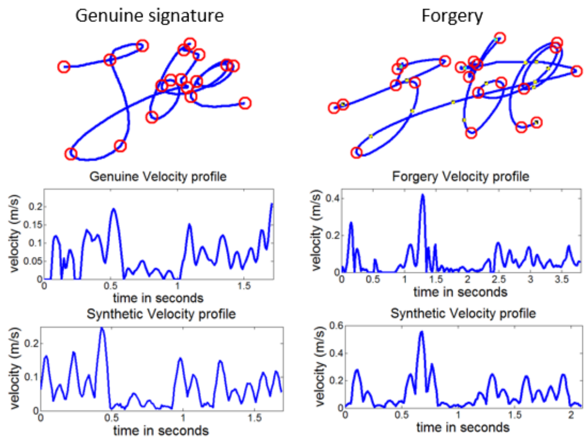


Figure 2. Examples of synthetic velocity profiles for both genuine and forged signatures

which is the lognormal cumulative function. Solving for t this equation, we get the time in terms of the distance as:

$$t(s) = \exp\left\{\sqrt{2}\sigma_j \operatorname{erf}^{-1}\left(\frac{2s(t)}{D_j} - 1\right) + \mu_j\right\} + \tau_j \tag{4}$$

Consequently, to generate synthetic specimens, two tasks have to be performed, the first one to synthesize the velocity profile (working out the parameters of Eq. (1)) and the second one to sample the signature trajectory with Eq. (4).

A. *Velocity profile synthesis*

In this section, we adapt the procedure proposed in [13] to the present problem. Basically, to synthetize the velocity profile, the signature is segmented in strokes. Then a velocity profile is generated for each stroke which are accumulated to obtain the global synthetic velocity profile of the signature. Each stroke is located between two consecutive important points obtained by the TS-PIPE algorithm [12]. Let be Ns the number of strokes and $ls_j, \forall j = 1, \dots, Ns$ the length of the j^{th} stroke.

To each perceptual important point, it is assigned a time$ts_j, \forall j = 0, \dots, Ns$, where $ts_0 = 0$ is the beginning of the signature. Therefore, the j^{th} stroke spans from ts_{j-1} up to ts_j. As has been said in section II, each perceptual important point corresponds to a minimum in the synthetic velocity profile.

The time of each perceptual important point is obtained as follows. The time between perceptual important points or velocity minima $ts_j - ts_{j-1}$is fixed to a fairly constant time following the hypothetical existence of the so-called Central Pattern Generators (CPG). The CPG produces rhythmic patterned outputs, without sensory feedback, to activate different motor pools [14]. This can be observed in the clearly periodic pattern of the handwriting velocity. Therefore, if the velocity generation is assimilated to the CPG step cycle, the duration of each stroke should be similar. Specifically, in the BiosecureID-132 database [16], the time between velocity minima has been calculated and modeled by a Normal distribution of average 0.1 and variance 0.005. Consequently, the time $\Delta ts_j = ts_j - ts_{j-1}$, $\forall j = 1, \dots, Ns$ is worked out following such distribution. This time corresponds with the assigned duration for each stroke.

Once defined the time scale of the signature, we generate a synthetic velocity profile for each stroke. Let $v_j(t)$ be the velocity profile of the j^{th} stroke. Then it must be overlapped with the previous and next stroke. For this reason, the starting time of the stroke is set to $\tau_j = ts_{j-1} - \Delta ts_j, \forall j = 1, \dots, Ns$.

The values of D_j, μ_j and σ_j^2 are set from the following two hypotheses: firstly, the margins for natural human handwriting given in [4] and secondly, it was heuristically observed that most of the lognormals were centered, i.e. the lognormal peak approaches the center of the stroke. Therefore, our skewness is close to zero, but positive and the kurtosis is around three.

The calculation of these values is suggested as follows. From Eq. (3) we deduce:

$$ls_j = \frac{D_j}{2}\left(1 + \operatorname{erf}\left(\frac{\ln(ts_j - \tau_j) - \mu_j}{\sqrt{2}\sigma_j}\right)\right) \tag{5}$$

Note that $ts_j - \tau_j = 2\Delta ts_j$. As $\operatorname{erf}(3) = 1$, a possible solution for Eq. (5) is:

$$D_j = ls_j \tag{6}$$

$$\mu_j = \ln(2\Delta ts_j) - 3\sqrt{2}\sigma_j \tag{7}$$

Furthermore, if the lognormals were centered in the middle of the stroke with a low positive skew, their maximum or mode, defined by $e^{\mu_j - \sigma_j^2}$, is around $ts_{j-1} + \Delta ts_j/2$ with a slight left skew. Therefore, it holds that:

$$ts_{j-1} + k_j \cdot \Delta ts_j - \tau_j = \Delta ts \cdot (1 + k_j) = e^{\mu_j - \sigma_j^2} \tag{8}$$

where the value k_j, which provide a slight left skew, is worked out randomly for each stroke, following a uniform distribution in the margin $[0.4\ 0.5]$ .This procedure is useful only for isolated strokes.

Finally, combining Eq. (7) and Eq. (8) we obtain:

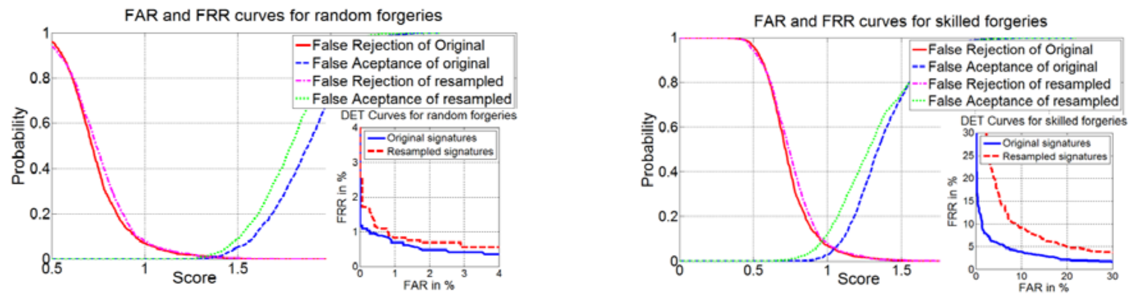
$$\sigma^2 + 3\sqrt{2}\sigma - \ln\left(1 + {k_j}/_2\right) = 0 \tag{9}$$

Thus, this approach leads to assign to the parameters D_j the value of ls_j (see Eq. (6)), σ_j as the positive solution of a simple second order equation (see Eq. (9)), and μ_j by substituting σ_j in Eq. (7).

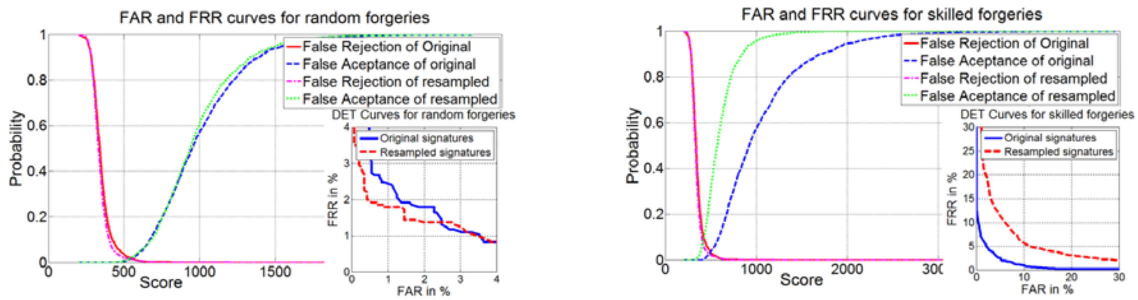
Once the velocity profile of individual strokes are obtained, the velocity profile of the signature is worked out following Eq. (1). As a check of the obtained velocity profile of the signature, we must be sure that the integral of the signature velocity profile is equal to the total length of the signature.

B. *Lognormal sampling of the trajectory*

The time at every pixel in the signature trajectory is calculated



a) Curves for DTW-based ASV



b) Curves for Manhattan distance-based ASV

Figure 3. FAR and FRR curves for original and resampled databases in the random and skilled forgeries experiments. DET curves have been added to complement the illustrations.

as the integral of the velocity from zero up to each multiple of $1/f_m$, f_m being the sample frequency. Figure 2 shows a synthetic example of a reconstructed velocity profile for a genuine and a forgery specimen, highlighting that the synthetic profile in the forged signature contains less lognormals than the original one.

IV. Experiments

The experiments aimed to verify whether the resampled on-line profile is genuine-like enough. As such, the genuine signature should be similarly detected by an ASV whereas skilled forgeries should be more difficult to detect than the original ones. Therefore, the experimental methodology is designed as follows: 1. the performance of the original database is worked out for both random and skilled forgeries by using two on-line ASV, 2. the database is completely resampled with the proposed method, 3. the performance of the resampled database is obtained and compared to the original one.

The experiments have been run with the BiopsecureID-132 database [16] which contains 132 users, 16 genuine and 12 forgeries per user.

A. *Comparing False Acceptance Rate and False Rejection Rate curves*

The experiments in this subsection aimed at assessing whether the proposed method makes on-line skilled forgeries more skillful. For this purposes, we compare the False Acceptance Rate (FAR) and False Rejection Rate (FRR) curves of both original and resampled signatures. Such comparison is performed for both random and skilled forgeries (RF and SF, respectively) experiments. The random forgery experiment use as forgeries the genuine signatures of other signers. The so-called skilled forgery experiment uses as forgeries the 12 ones provided by the database. For the evaluation, two state-of-the-art on-line ASV have been taken into account: a Dynamic Time Warping (DTW) [3] and a Manhattan-based distance ASV [17].

Table I. Equal Error Rates (EER) obtained with DTW

Experiment	With penups		Without penups	
	RF	SF	RF	SF
Original	0.71%	5.46%	1.19%	13.53%
Resampled	0.88%	9.51%	1.41%	16.67%

RF: Experiment with Random Forgeries.
SF: Experiment with Skilled Forgeries.

Table II. Equal Error Rates (EER) with Manhattan Distance

Experiment	With penups		Without penups	
	RF	SF	RF	SF
Original	1.91%	3.72%	2.93%	6.94%
Resampled	2.32%	8.07%	3.07%	13.17%

Both ASV have been trained with the first 5 genuine signatures of the BiosecureID database [16]. For FRR, we have used the remainder 11 genuine signatures of the same signer. In the random forgery experiment we have used the remainder 11 signatures of all the other signers for calculating the FAR. In the skilled forgery experiment, we have used the 12 available skilled forgeries of each signer to calculate the corresponding FAR. These experiments were performed for the original and resampled database under the same conditions.

The results in terms of EER can be seen in Table I and II for DTW and Manhattan distance ASV respectively. As expected, the EER in the random forgery (RF) experiment, which only involves genuine signatures, are similar between original and resampled signatures. However, the EER increases significantly in the skilled forgery (SF) experiment.

This experiment was repeated removing the pen-ups from the trajectory, that is to say discarding the samples that corresponds

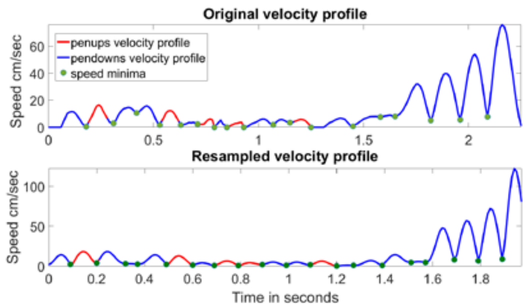


Figure 4. Original and resample velocity profile of a genuine signature

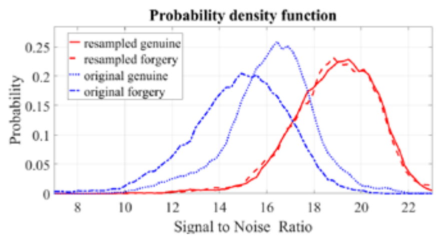


Figure 5. Probability Density Function for SNR score which distinguish between original and resampled signatures for genuine and forgery cases.

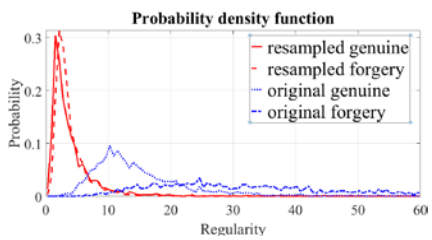


Figure 6. Probability Density Function for regularity score which distinguish between original and resampled signatures for genuine and forgery cases.

to pen lifts and training and testing both ASV under the same conditions than above mentioned. As can be seen in Table I and II, the results without pen ups keep the same tendency than with pen ups.

These results are reinforced by the FAR and FRR curves shown in Figure 3. In the case of skilled forgeries, the FRR curves are similar whereas the FAR curves of the resampled databases are displaced toward left in all the cases. It means that resampled skilled forgeries are nearer to the genuine than the original ones. This effect is clearer in the case of the Manhattan distance based ASV.

Figure 3 also shows that the FAR and FRR curves roughly stay the same in the random experiment, which suggests that the genuine signatures are barely affected by the proposed resampling method.

V. COUNTERMEASURES

Previous section showed that using Sigma-Lognormal model is feasible to improve the skill of a forgery. In this section, we analyse if it is possible to detect the synthetic velocity profile of the resampled signatures. As an example, Figure 4 shows the velocity profile of an original and resampled genuine signature.

Two ways of detecting the resampling of a signature are devised in this paper based on the speed profile:

Lognormality-wise: Following the Sigma-Lognormal model, a handwritten signature can be decomposed as a sum of weighted and overlapped lognormals. However, there are many factors that modify the free performance of the motor system, which leads to deviations in the lognormal speed profile. For instance, some joint pain, uncomfortable wear or posture, emotional state of the signer, and so on. As consequence, the Sigma-Lognormal model is only able to approach the velocity profile of an original handwritten signature up to a reasonable Signal-to-Noise ratio (SNR), which is defined as:

$$SNR = 10\log\left(\frac{\int_{t=0}^{T} v_o(t)^2\,dt}{\int_{t=0}^{T}(v_0(t) - v_r(t))^2 dt}\right)$$

Where $v_o(t)$ is the original velocity profile and $v_r(t)$ is the resampled velocity profile.

On the contrary, the velocity profile of the resampled signature is purely a sum of lognormals. Therefore, it is expected that its Sigma-Lognormal decomposition reached higher Signal-to Noise ratios. As example of the performance of the SNR score as countermeasure, Figure 5 shows the SNR distribution of original and resampled signatures for the genuine and forgery cases along the on-line real dataset in BiosecureID database.

Regularity-wise: Following the above rationale, the velocity profile of resampled signatures is expected to be more regular than original ones. This regularity can be measured as the variance of the time between minima of the velocity profile. It is expected that the regularity is more stable for resampled than for original profiles. As example, Figure 6 shows the distribution of the regularity for original and resampled signatures for the genuine and forgery cases through the on-line real dataset in BiosecureID database.

In both cases, to detect a counterfeited signature, the countermeasure algorithm compares the above mentioned SNR or regularity score with a threshold. In this case, we use the Bayes threshold. If the SNR score of a given signature is greater than its threshold or the Regularity score is lower than its threshold, then the signature is supposed to be a resampled signature and discarded as original.

To evaluate the performance of both methods, the next measures have been worked out for genuine, forgeries and all together: precision, recall or sensitivity, specificity and accuracy which are defined as:

$$precision = \frac{tp}{tp + fp} \qquad recall = \frac{tp}{tp + fn}$$

$$specificity = \frac{tn}{tn + tp} \qquad Accuracy = \frac{tp + tn}{tp + tn + tf + fn}$$

Being tp: true positive, tn: true negative, fp: false positive and fn: false negative. The positive hypothesis is that the signature has been resampled.

The precision is the ratio of signatures classified as resampled that are truly resampled. Recall refers to the ratio of resampled signatures detected. Specificity appertains to the ratio of signatures classified as original that are really original. Finally, accuracy is the ratio of signatures rightly classified.

The results are displayed in Table III for SNR and Table IV for Regularity. The measure based on regularity is more effective detecting resampled signatures than SNR one as was expected as original signatures are also lognormals. Therefore, we recommend the countermeasure based on regularity score.

VI. CONCLUSION

This paper proposes a procedure based on the Sigma-Lognormal model to make on-line forgeries more skilful. While a well-trained forger can imitate accurately the genuine signature trajectory, they usually fail to emulate feasibly the velocity profile. Therefore, this paper proposes to improve the skill of a forged signature by modifying the speed profile of the signature by resampling. It is expected that the new synthetic velocity profile will be more genuine-like.

On the other hand, it could be said that the minima in the velocity profile of a genuine signature coincide with the perceptual important points in the trajectory. So that, the skillfulness of the skilled forgeries is improved by resampling the on-line signature producing minima in the estimated perceptual important points. The perceptual important points are estimated with a two steps algorithm (TS-PIPE) [12]. Then, the synthetic velocity profile is build up based on the kinematic theory of rapid movements: A set of lognormals are fitted to the trajectory regarding the location of the estimated perceptual important points. This strategy is applied to original signatures, genuine or forgeries, to produce a synthetic (resampled) version of the signature with a more genuine-like velocity profile.

The conducted experiments show that the resampled skilled forgeries contain a similar number of speed minima than their respective genuine. Additionally, the EER is significantly increased for skilled forgeries while it is barely modified in random forgeries. These experiments have been conducted in two different on-line ASV. The robustness of these observations have been confirmed in on-line signatures with and without pen-up trajectories.

Some further work is still required to reduce the differences between the False Rejection Curves of the original and resampled databases. Additionally, the False Acceptance Curves should be more similar in the random forgery experiments and move more toward left in the skilled forgery experiments. The paper ends by proposing a countermeasure to detect this kind of fake signatures in terms of measuring the regularity of the speed profile of the given signature.

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TABLE III. PRECISION, RECALL, SPECIFICITY AND ACCURACY FOR SNR

	Genuine	**Forgeries**	**All**
Precision	0.79	0.87	0.83
Recall	0.80	0.87	0.83
Specificity	0.50	0.50	0.50
Accuracy	0.79	0.87	0.83

TABLE IV. PRECISION, RECALL, SPECIFICITY AND ACCURACY FOR REGULARITY

	Genuine	**Forgeries**	**All**
Precision	0.89	0.98	0.92
Recall	0.89	0.97	0.91
Specificity	0.50	0.50	0.50
Accuracy	0.89	0.97	0.91

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