and, equating the coefficients of ε^2 and ε^3 to 0, we obtain respectively $a = 0$ and

$$b + \frac{1}{24} + b - \frac{1}{48} = 0 \qquad \text{or} \qquad b = -\frac{1}{96}.$$

(We get the same values of a and b using the second arrangement of the equation.)

But the two respective approximations yield $x = \frac{1}{2}\varepsilon\left(\ + 0\varepsilon^3\right)$ and

$$x = \frac{\sqrt{4 + 2\varepsilon^2} - 2}{\varepsilon} = \frac{\varepsilon}{2} - \frac{\varepsilon^3}{16} + \cdots,$$

so they only agree as far as terms in ε^2. However, the terms in ε^3 do align with the inequality established at the end of Example 2.

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$\sum_{n=2}^{\infty} \frac{1}{nH_{n-1}}$ diverges while $\sum_{n=2}^{\infty} \frac{1}{nH_n^{1+\varepsilon}}$ converges

The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is an example of a divergent series with positive terms where the general term tends to zero as n tends to infinty. For any integer number $n \geqslant 1$, the nth harmonic number H_n, is defined by $H_n = \sum_{k=1}^{n} \frac{1}{k}$. Then the terms of $\sum_{n=2}^{\infty} \frac{1}{nH_{n-1}}$ grow much more slowly than those of $\sum_{n=1}^{\infty} \frac{1}{n}$ since $\lim_{n \to \infty} \frac{\frac{1}{nH_{n-1}}}{\frac{1}{n}} = 0$. However,

Theorem 1: $\sum_{n=2}^{\infty} \frac{1}{nH_{n-1}}$ is divergent.

Proof: Here we present in Figure 1 a visual proof of this fact, following [1].

Raising the second factor of each summand to the $1 + \varepsilon$ power 'only slightly' shrinks the size of each summand, especially if ε is very small. But this modification is enough to transform divergence into convergence.

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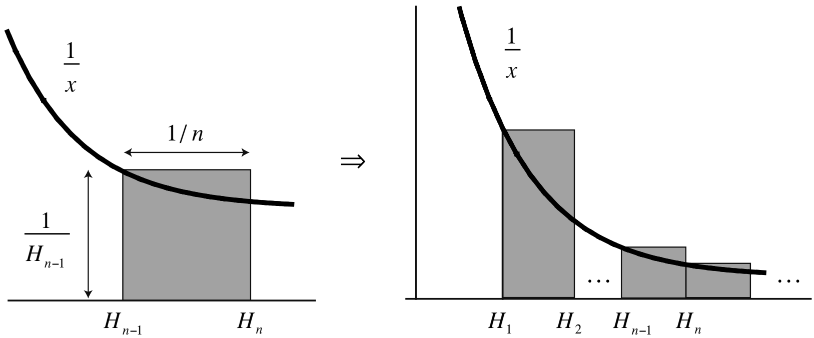

FIGURE 1

Theorem 2: Let $\varepsilon > 0$. Then $\sum_{n=2}^{\infty} \frac{1}{nH_n^{1+\varepsilon}}$ converges.

Proof: The proof follows from considering Figure 2 below.

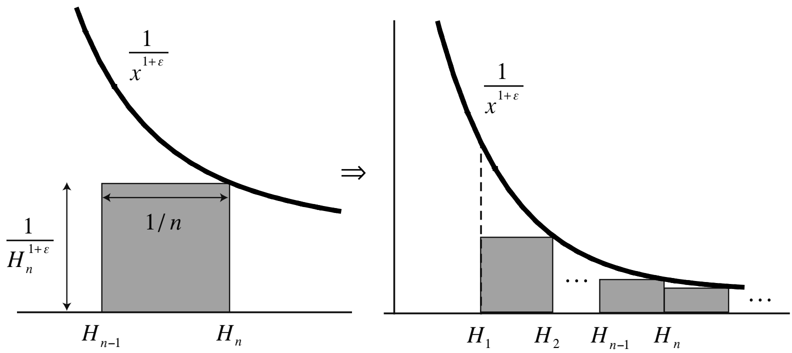

FIGURE 2

Remark: Each term of the series of Theorem 1 is a rational number. If ε is chosen to be a positive whole number, then this is also true for the series of Theorem 2.

Reference

1. J. Marshall Ash, Neither a Worst Convergent Series nor a Best Divergent Series Exists, *The College Mathematics J*. **28** (4) (1997) pp. 296-297.

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